Interaction Notes
Note 157
February 1974

ON THE RESPONSE OF AN INFINITELY LONG, PERFECTLY CONDUCTING, CYLINDRICAL ANTENNA TO AN ELECTROMAGNETIC PLANE WAVE PULSE*

by

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ABSTRACT

In this note, the transient response of an infinitely long, perfectly conducting, cylindrical antenna to an electromagnetic plane wave pulse is considered. The infinite cylindrical antenna is located in free space and the two antenna elements are separated by a finite gap. Early time asymptotic expansions are derived for the antenna short-circuit current, open-circuit voltage, load current, and load voltage responses. A parametric study of the antenna response was performed in graphical form. Also, the responses of a selected infinite cylindrical antenna to a double exponential electromagnetic pulse are calculated and graphed for several types of loads as an example problem.

*Research sponsored by the Air Force Weapons Laboratory under Union Carbide Corporation's contract with the U.S. Atomic Energy Commission.

ACKNOWLEDGMENT

I wish to thank Dr. Donald Arnurius of ORNL for his interest and computer programming assistance. I also wish to thank Ruby N. Thurmer of ORNL for typing the manuscript.

I. INTRODUCTION

In communications systems, the antenna is an important means of coupling transient electromagnetic energy to some of the most vulnerable electronic components in the system. In order to specify surge protection requirements for communications equipment, knowledge of the early time response of the antenna to a transient electromagnetic source is desirable.

The singularity expansion method as formalized by Baum[1] is a new method for determining the solution of electromagnetic interaction problems. It involves expanding the solution in terms of its singularities in the Laplace transform plane. The time domain solution is given by the inverse Laplace transform of each term in the singularity expansion. For late and moderate times, a solution can often be obtained by considering only a few singularities. This method is more efficient and accurate than the conventional brute-force numerical integration methods. However, for early times nearly all of the singularities contribute to the solution and the singularity expansion method also becomes inaccurate and inefficient.

In this note, the exact solution of an infinitely long, perfectly conducting, cylindrical antenna response to an electromagnetic plane wave pulse is discussed. This solution can be used to provide early time information about the finite cylindrical dipole antenna response. The infinite cylindrical antenna and incident wave are shown in Fig. 1. Each antenna element of radius a is separated by a finite gap with a half-width b. The incident electromagnetic plane wave source is polarized with the magnetic field perpendicular to the axis of the cylinder.

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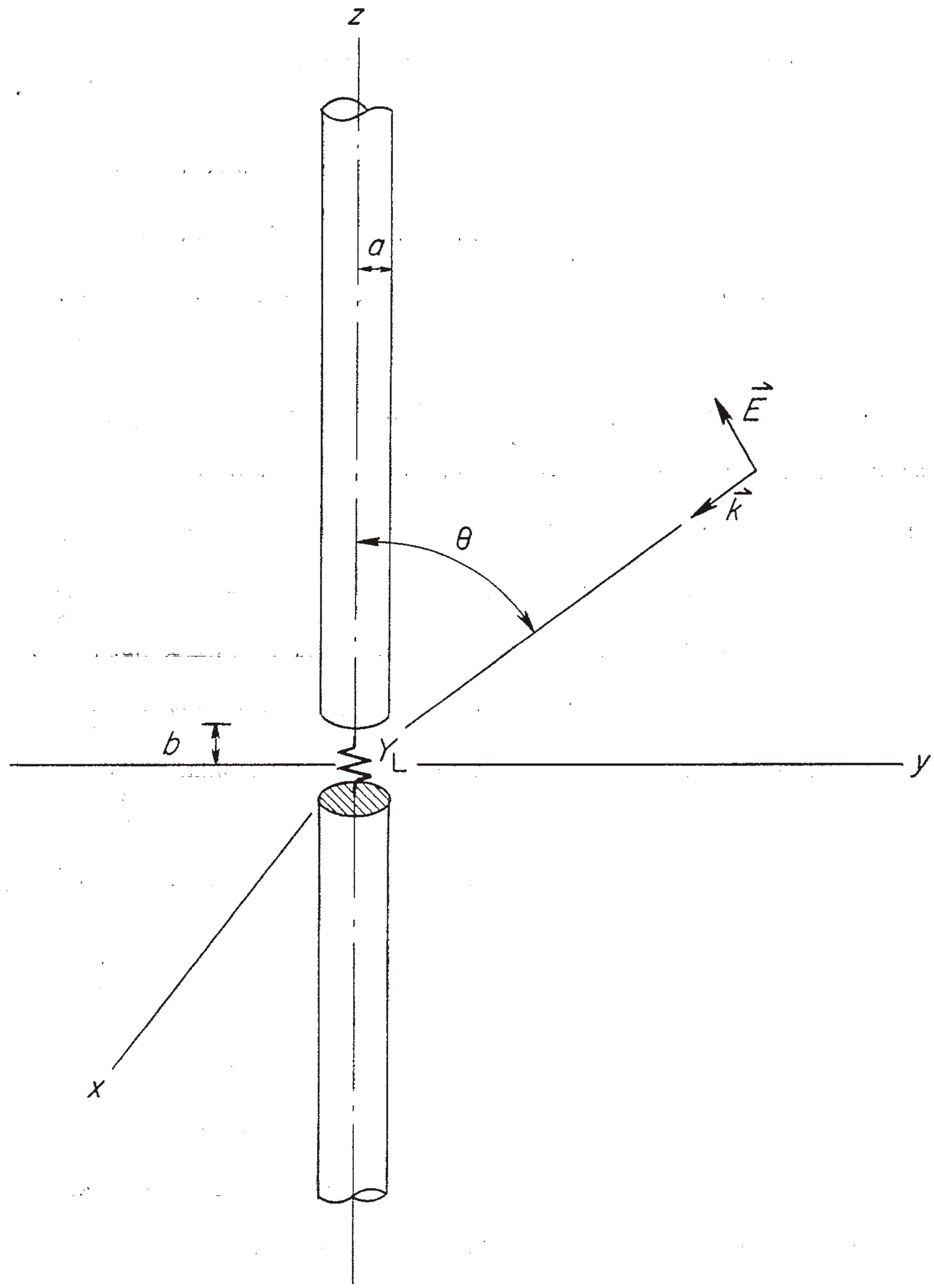

Fig. 1. Infinitely Long, Circular, Cylindrical Antenna and Incident Vector Plane Wave.

The response of the finite antenna is the same as that of the infinite antenna for times before the ends of the finite structure have effect. The applicable time interval is[2]

$$0 \le t \le \frac{\ell}{c}(1-\cos\theta) \qquad (1.1)$$

where zero time is reference to the time that the leading edge of the incident wave reaches the cylindrical surface where the response is observed. The constant c is the speed of light in free space, ℓ is the half-length of the finite dipole, and θ is the incident angle defined in Fig. 1. For long antennas which are several hundred meters long such as low-frequency communications antennas, the applicable time interval is several hundred nanoseconds. The solution for the infinite cylindrical antenna response is an early time solution for the finite cylindrical dipole response, and thus complements the singularity expansion method.

II. ANALYSIS

The Norton equivalent circuit for an antenna may be employed to analyze the response of the infinite cylindrical antenna as a receiving device. The Norton equivalent circuit is shown in Fig. 2. The Norton equivalent circuit parameters are I_a, the Norton equivalent current source of the antenna and the electromagnetic environment and Y_a, the admittance of the antenna. Once I_a and Y_a have been determined, the voltage across the load admittance Y_L is given by

$$V = \frac{I_a}{Y_a + Y_L} \quad , \qquad (2.1)$$

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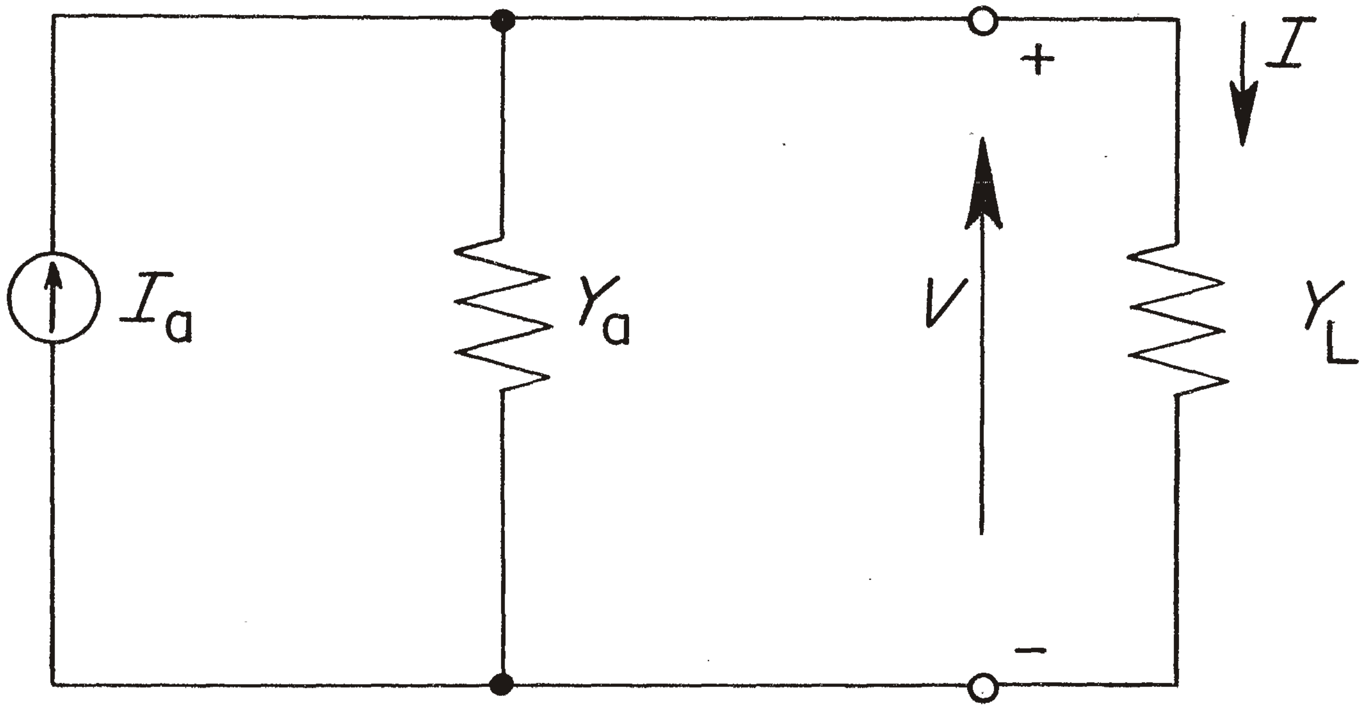

Fig. 2. Norton Equivalent Circuit for Determining the Responses of an Antenna.

and the current through the load is

$$I = \frac{I_a Y_L}{Y_a + Y_L} \quad . \tag{2.2}$$

And the open circuit antenna voltage, the Thevenin equivalent voltage source, is given by

$$V_a = \frac{I_a}{Y_a} \quad . \tag{2.3}$$

The Norton equivalent current source is equal to the short circuit current response of the antenna. The current induced on an infinitely long, perfectly conducting cylinder is given by[2]

$$\tilde{I}_a = \frac{2\pi c \; E(s)}{Z \sin\theta \; s \; K_o \; (as \sin\theta/c)} \quad , \tag{2.4}$$

where s is the Laplace transform variable, E(s) is the Laplace transform of the incident wave time history, and Z is the wave impedance of free space approximately equal to 120π ohms. The tilde, ~, over a quantity indicates the frequency domain expression of the quantity. Equation (2.4) can be written as a function of time as[2]

$$I_a = \frac{2\pi \; c}{Z \sin\theta} \int_o^t E(t-\tau) \; F(c\tau/a \sin\theta) \; d\tau \tag{2.5}$$

where $F(c\tau/a \sin\theta)$ is the normalized impulse response given by

$$F(c\tau/a \sin\theta) = \int_o^\infty \frac{e^{-\xi\left(\frac{c\tau}{a \sin\theta} - 1\right)} \; I_o(\xi)}{\xi[K_o^2(\xi) + \pi^2 \; I_o^2(\xi)]} \; d\xi \quad . \tag{2.6}$$

The admittance of an infinitely long, perfectly conducting, cylindrical antenna excited by a uniform finite distributed source has been derived in a previous report.[3] The distributed source is a much more realistic source than the infinitesimal-gap voltage source so often employed for the purpose of deriving the antenna driving point admittance since the finite gap of the distributed source does not introduce a singularity in the admittance function. However, the admittance of an antenna excited by a distributed source is dependent on the field distribution of the source. For frequencies with wavelengths greater than the source gap, this dependence is slight. Even for frequencies with wavelengths equal to or less than the source gap, the uniform source is as good as any realistic source that one might select for the purpose of deriving the admittance of the antenna.

The infinite cylindrical antenna admittance is given in terms of the Laplace transform variable s as[3]

$$Y_a = \frac{as}{c\psi Z} \int_{-\infty}^{\infty} \frac{\sin \psi\xi}{\xi} \frac{K_1(u)}{u\,K_0(u)} \, d\xi \qquad (2.7)$$

where

$$u = \left(\xi^2 + \left(\frac{as}{c}\right)^2\right)^{1/2} , \qquad (2.8)$$

and ψ is an antenna parameter given by

$$\psi = b/a \quad . \qquad (2.9)$$

The Laplace transform of the load response can be obtained by the substitutions of Eqs. (2.7) and (2.5) into Eqs. (2.1) or (2.2). The time history of the load response can be obtained by a numerical Laplace inversion.

III. ASYMPTOTIC EXPANSIONS

B. Early Time Behavior

The early time asymptotic expansions of the open circuit voltage, load current, and load voltage responses are of interest since they are applicable to the finite cylindrical dipole antenna, as well as the infinite cylindrical antenna. To derive the early time asymptotic expansions, the Theorem in Ref. 4 which relates the large s asymptotic expansion of the Laplace transform of a function to its small time asymptotic expansion in the time domain will be employed.

The asymptotic expansion for $\tilde{I}_a$ as $s \to \infty$ is given by[2]

$$\tilde{I}_a = \frac{2\,E(s)}{Z}\sqrt{\frac{2\pi ca}{\sin\theta}}\;\frac{e^{as\,\sin\theta/c}}{\sqrt{s}}\left[1 + \frac{c}{8\,as\,\sin\theta} - \frac{7\,c^2}{128\,a^2\,s^2\,\sin^2\theta} + O(|s|^{-3})\right] \quad . \tag{3.1}$$

And the asymptotic expansion for Y_a as $s \to \infty$ is[3]

$$Y_a = \frac{\pi}{\psi Z}\left[1 + \frac{c}{2as} - \frac{c^2}{4a^2 s^2} + O(|s|^{-3})\right] \quad . \tag{3.2}$$

The large s asymptotic expansion of the open circuit voltage response is found as

$$\tilde{V}_a = \frac{\tilde{I}_a}{Y_a}$$

$$= 2\,\psi E(s)\sqrt{\frac{2\,ca}{\pi\,\sin\theta}}\;\frac{e^{as\,\sin\theta/c}}{\sqrt{s}}\left[1 + \frac{c - 4c\,\sin\theta}{8\,as\,\sin\theta} + \frac{64\,c^2\,\sin^2\theta - 8\,c^2\,\sin\theta - 7\,c^2}{128\,a^2\,s^2\,\sin^2\theta} + O(|s|^{-3})\right] \quad . \tag{3.3}$$

For the incident wave with a decaying exponential time history given by

$$E(t) = E_o \, e^{-\alpha t} \quad , \tag{3.4}$$

the large s asymptotic expansion for V_a is

$$\tilde{V}_a = 2 \, \psi \, a \, E_o \sqrt{2/\pi} \, \frac{A \, e^{as}}{As \sqrt{As}} \left[1 - \frac{c_1}{As} + \frac{c_2}{(As)^2} + O(|As|^{-3}) \right] \tag{3.5}$$

where

$$A = a \sin\theta / c \quad ,$$

$$c_1 = \frac{8 \, A\alpha + 4 \sin\theta - 1}{8} \quad ,$$

and

$$c_2 = (128 \, A^2 \alpha^2 + 64 \sin^2\theta + 64 \, A\alpha \sin\theta - 16 \, A\alpha - 8 \sin\theta - 7)/128 \quad .$$

The inverse Laplace transform of Eq. (3.5) is

$$V_a = \frac{4 \, \psi \, a \, E_o}{\pi} \sqrt{\frac{2 \, ct^*}{a \sin\theta}} \left[1 - \frac{2}{3} \frac{c_1 \, ct^*}{a \sin\theta} + \frac{4 \, c_2}{15} \left(\frac{ct^*}{a \sin\theta} \right)^2 + O\left[\left(\frac{ct}{a \sin\theta} \right)^3 \right] \right] \quad , \tag{3.6}$$

where t^* is a shifted time given by

$$t^* = t + \frac{a \sin\theta}{c} \quad . \tag{3.7}$$

For the double exponential incident wave time history given by

$$E(t) = E_o(e^{-\alpha t} - e^{-\beta t}) \quad , \tag{3.8}$$

the asymptotic expansion of V_a becomes

$$V_a = \frac{8\,\psi\,a\,E_o\,t^*\,(\beta-\alpha)}{3\pi}\sqrt{\frac{2\,ct^*}{a\,\sin\theta}}\Bigg[1 - \frac{2t^*}{S}\,[\beta + \alpha$$

$$+ \frac{c}{2a}\,(1 - \frac{1}{2\,\sin\theta})] + 0\,[(t^*)^2]\Bigg] \quad . \tag{3.9}$$

The asymptotic expansion of the load voltage response can be derived from the open circuit voltage by the relation

$$\tilde{V} = \frac{\tilde{V}_a\,Y_a}{Y_a + Y_L} \quad . \tag{3.10}$$

The substitution of Eq. (3.2) into (3.10) gives

$$\tilde{V} = \frac{\pi\,\tilde{V}_a}{\psi\,Z\,Y_L + \pi}\Bigg[1 + \frac{c}{2as}\,(1 - \frac{\pi}{\psi\,Z\,Y_L + \pi})$$

$$- \frac{c^2}{4\,a^2 s^2}\,(1 - \frac{\pi^2}{(\psi\,Z\,Y_L + \pi)^2}) + 0\,(s^{-3})\Bigg] \quad . \tag{3.11}$$

For a resistive load $Y_L = 1/R$, Eq. (3.11) becomes

$$\tilde{V} = \frac{\pi R\,\tilde{V}_a}{\psi Z + \pi R}\Bigg[1 + \frac{c}{2as}\,(1 - \frac{R\pi}{\psi Z + R\pi})$$

$$- \frac{c^2}{4\,a^2 s^2}\,(1 - \frac{\pi^2 R^2}{\psi Z + R\pi)^2}) + 0(s^{-3})\Bigg] \quad , \tag{3.12}$$

and the inverse Laplace transform of Eq. (3.12) is

$$V = \frac{\pi R V_a}{\psi Z + \pi R} + O\left[\int_0^{t^*} V_a(\tau)\, d\tau\right] . \tag{3.13}$$

The voltage response across a resistive load due to a double exponential incident wave is obtained by the substitution of Eq. (3.9) into Eq. (3.13). The asymptotic form is

$$V = \frac{8 R\psi\ a\ E_o\ t^* (\beta-\alpha)}{3\ (\psi Z + \pi R)} \sqrt{\frac{2\ ct^*}{a\ \sin\theta}} \left[1 + O(t^*)\right] . \tag{3.14}$$

Of course, the load current can be obtained by dividing Eq. (3.14) by R.

For an inductive load $Y_L = 1/sL$, the asymptotic expansion of the load voltage response for $s \to \infty$ is

$$\tilde{V} = \tilde{V}_a \left(1 - \frac{\psi Z}{\pi s L} + O(s^{-2})\right) . \tag{3.15}$$

And the inverse Laplace transform of Eq. (3.15) is

$$V = V_a + O\left[\int_0^{t^*} V_a(\tau)\, d\tau\right] . \tag{3.16}$$

For the double exponential incident waveform, the early time asymptotic expansion of the load voltage is

$$V = \frac{8\ \psi\ a\ E_o\ t^* (\beta-\alpha)}{3\pi} \sqrt{\frac{2\ ct^*}{a\ \sin\theta}} \left[1 + O(t^*)\right] . \tag{3.17}$$

The load current can be derived from Eq. (3.17) by the relation

$$I = \frac{1}{L} \int_0^{t^*} V(\tau)\, d\tau \quad . \tag{3.18}$$

The asymptotic form of the load current response to a double exponential incident wave is obtained by the sbustitution of Eq. (3.17) into (3.18) and is given by

$$I = \frac{16\, \psi\, a\, E_o\, (\beta-\alpha)\, (t^*)^2}{15\, \pi\, L} \sqrt{\frac{2\, ct^*}{a\, \sin\theta}} \left[1 + O(t^*)\right] \quad . \tag{3.19}$$

B. Late Time Behavior

The late time asymptotic behavior of the infinite cylindrical antenna response can be deduced from the behavior of its Laplace transform near the singularity $s = 0$ [Ref. 5]. The small s asymptotic form of the antenna admittance is[3]

$$Y_a \sim \frac{-\pi}{Z\, \ell n\, (as/c)} \quad . \tag{3.20}$$

The substitution of Eq. (3.20) into (2.3) gives the small s asymptotic behavior of the open circuit voltage as

$$\tilde{V}_a \sim \frac{-\tilde{I}_a\, Z\, \ell n\, (as/c)}{\pi} \quad . \tag{3.21}$$

The small s asymptotic form of $\tilde{I}_a$ is[2]

$$\tilde{I}_a \sim \frac{-2\pi c\, E(s)}{Z\, \sin\theta\, s\, \ell n\, (\frac{as\, \sin\theta}{c})} \quad . \tag{3.22}$$

Equation (3.21) can be rewritten as

$$\tilde{V}_a \sim \frac{2c}{\sin\theta}\frac{E(s)}{s} + \frac{\tilde{I}_a Z}{\pi} \ln(\sin\theta) \quad . \tag{3.23}$$

The late time behavior of V_a can be written from the inverse Laplace transform of Eq. (3.23), the result is

$$V_a \sim \frac{2c}{\sin\theta}\int_0^t E(\tau)\, d\tau + \frac{Z \ln(\sin\theta)}{\pi} I_a(t) \quad . \tag{3.24}$$

For the step function incident wave given by Eq. (3.4) with $\alpha = 0$, the late time behavior of the open circuit voltage is[2]

$$V_a \sim \frac{2\, c\, E_o\, t}{\sin\theta}\left[1 + \frac{\ln(\sin\theta)}{\ln\left(\frac{ct}{a \sin\theta}\right)}\right] \quad . \tag{3.25}$$

The open circuit voltage response to the decaying exponential incident wave given by Eq. (3.4) with $\alpha \neq 0$ is

$$V_a \sim \frac{2\, c\, E_o}{\alpha \sin\theta}\left[1 - e^{-\alpha t} + \frac{\ln(\sin\theta)}{\ln\left(\frac{ct}{a \sin\theta}\right)}\right] \quad . \tag{3.26}$$

Now consider the late time behavior of the load voltage response for a resistive load. The substitution of Eq. (3.20) into (2.1) gives the small s asymptotic behavior of the load voltage as

$$\tilde{V} \sim \frac{R\, \tilde{I}_a}{1 - \frac{\pi R}{Z \ln(as/c)}} \sim R\tilde{I}_a \quad . \tag{3.27}$$

The late time asymptotic behavior of the load voltage response is

$$V_a \sim RI_a(t) \quad . \tag{3.28}$$

For a step function incident wave, the late time response is[2]

$$V_a \sim \frac{2\pi\, c\, E_o\, R\, t}{\sin\theta\ Z\, \ln\left(\frac{ct}{a \sin\theta}\right)} \quad . \tag{3.29}$$

And the late time load voltage response to a decaying exponential incident wave is[2]

$$V_a \sim \frac{2\pi\, c\, E_o\, R}{Z\, \alpha\, \sin\theta\, \ln\left(\frac{ct}{a \sin\theta}\right)} \quad . \tag{3.30}$$

IV. TIME DOMAIN RESULTS

One method of transforming from the Laplace domain to the time domain is to employ the Gaussian integration technique. This method is described in Ref. 6 and is convenient to use for analytic functions of s which transform to nonoscillatory functions of time. In contrast to the usual method of computing the inverse Laplace transform which employs a set of arbitrarily selected and equally spaced frequency points, the Gaussian technique prescribes the frequencies at which the response function is to be calculated. It does this in such a way so as to minimize the errors for nonoscillatory functions of time. It can also give good results for some oscillatory functions of time. This Gaussian inverse Laplace transform technique has been used to obtain the time domain responses presented in this section.

The normalized open-circuit voltage responses to an incident plane wave with a step-function time history are shown in Figs. 3 and 4 as a function of normalized time T where

$$T = \frac{ct^*}{a} \quad . \tag{4.1}$$

In Fig. 3, the curves are plotted with $\psi = 1$ and θ as a parameter. In Fig. 4, the curves are plotted with $\psi = 10$ and θ as a parameter. The early time asymptotic forms have been computed from the first three terms of Eq. (3.6) with $\alpha = 0.0$.

In Figs. 5 and 6, the normalized load voltage response to a step-function incident wave for a 75-ohm load are presented for the same range of ψ and θ values used for the open-circuit voltage plots. The early time asymptotic forms were computed from the first two terms as derived from Eq. (3.12). Curves for the load current can be obtained from Figs. 5 and 6. Curves from the short-circuit current responses are available in Ref. 2.

In Figs. 3 and 4, the late time asymptotic forms have been calculated by Eq. (3.25). And in Figs. 5 and 6, the late time asumptotic forms have been calculated by Eq. (3.29).

As an example problem, consider the response of an infinite cylindrical antenna with a = 0.75 m and $\psi = 1$ to a double exponential electromagnetic pulse. The time history of the incident wate is given by Eq. (3.8) with E_o = 100 kV/m, $\alpha = 0.01$ nsec^{-1}, and $\beta = 0.5$ nsec^{-1} and is shown in Fig. 7.

In Figs. 8 through 10, the response curves are presented with θ as a parameter. Figure 8 shows the short-circuit current and open-circuit

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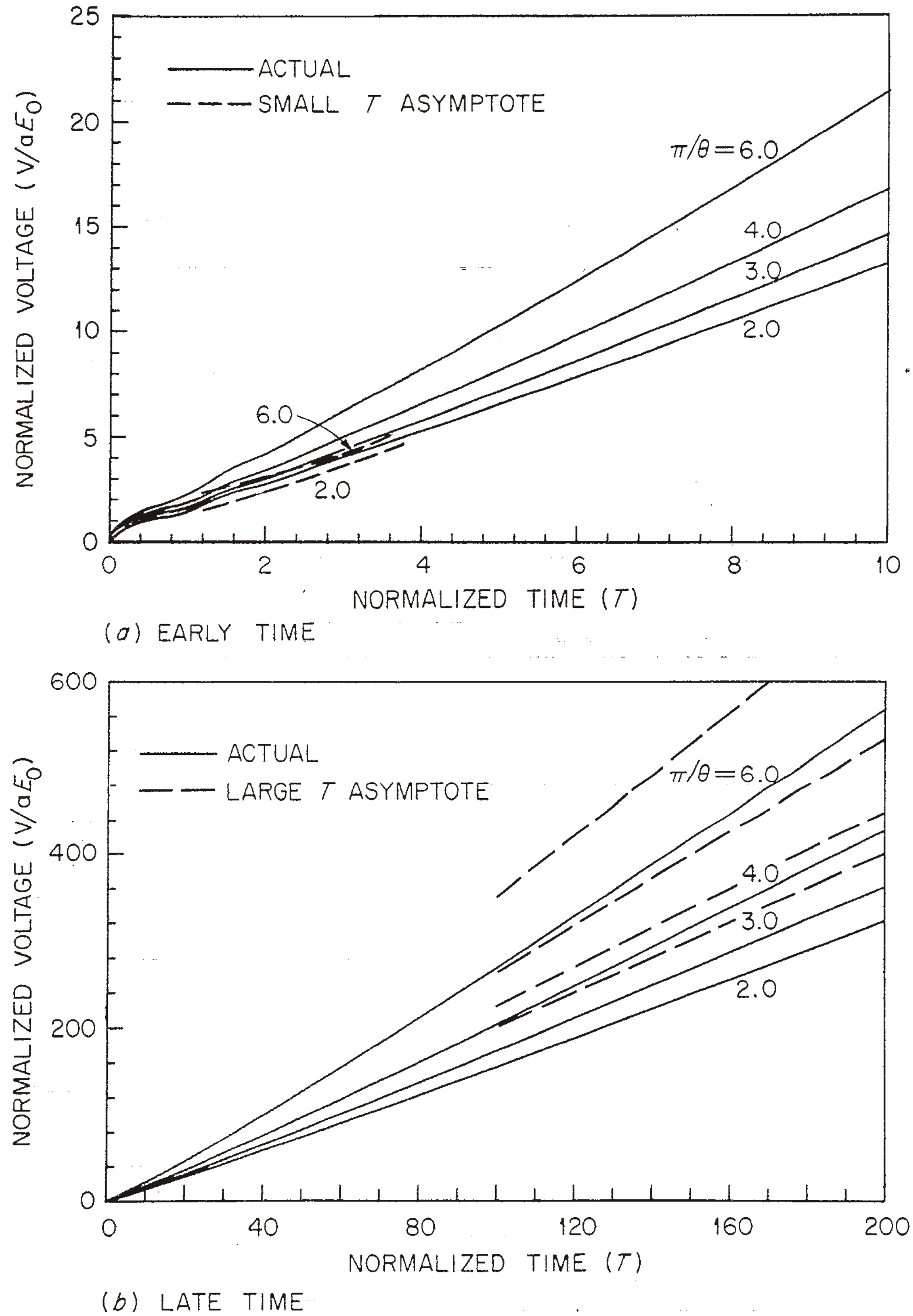

Fig. 3. Normalized Open Circuit Voltage Response of an Infinite Cylindrical Antenna to a Step Function Incident Electromagnetic Plane Wave with ψ = 1.0 and θ as a Parameter.

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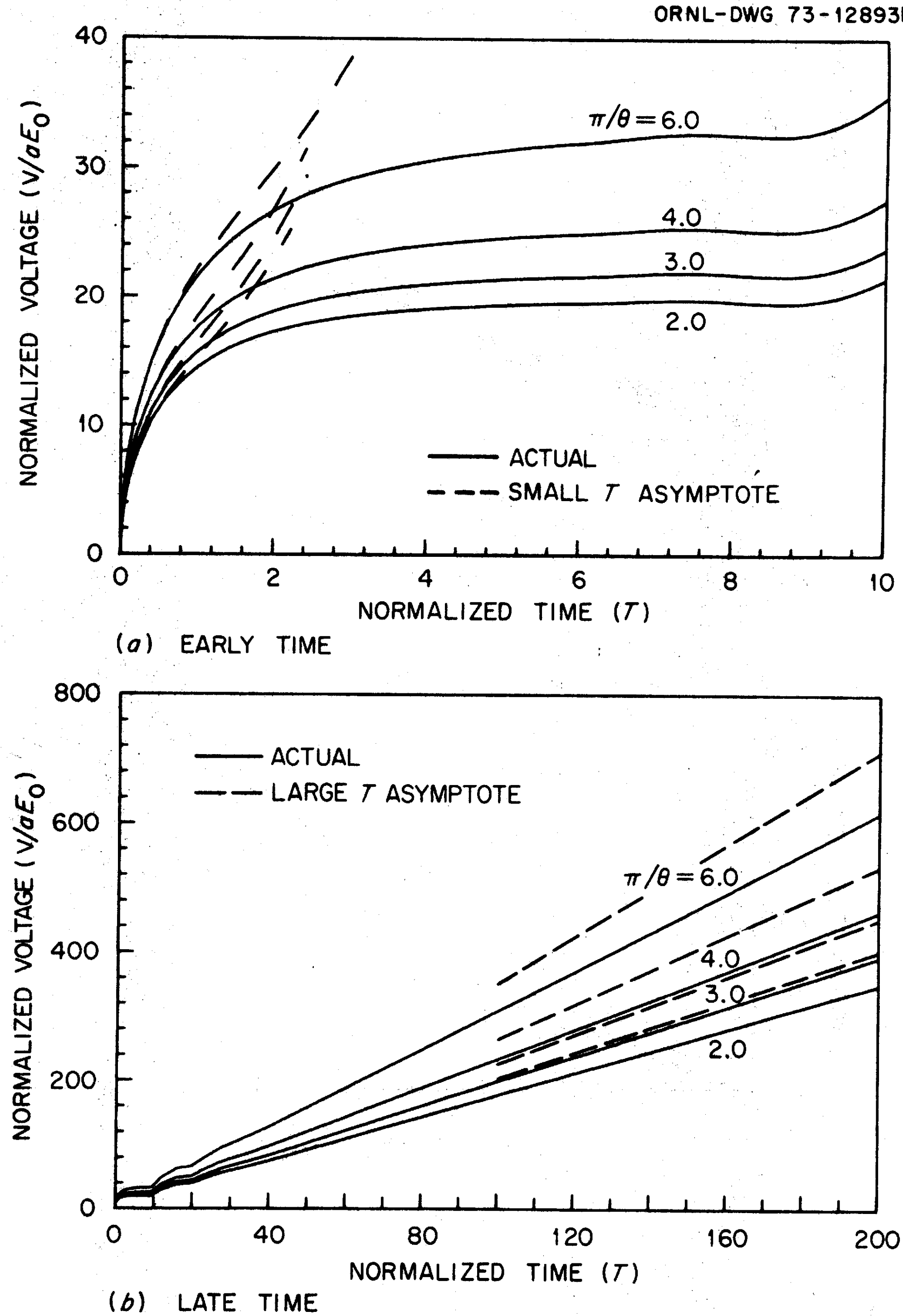

Fig. 4. Normalized Open Circuit Voltage Response of an Infinite Cylindrical Antenna to a Step Function Incident Electromagnetic Plane Wave with ψ = 10.0 and θ as a Parameter.

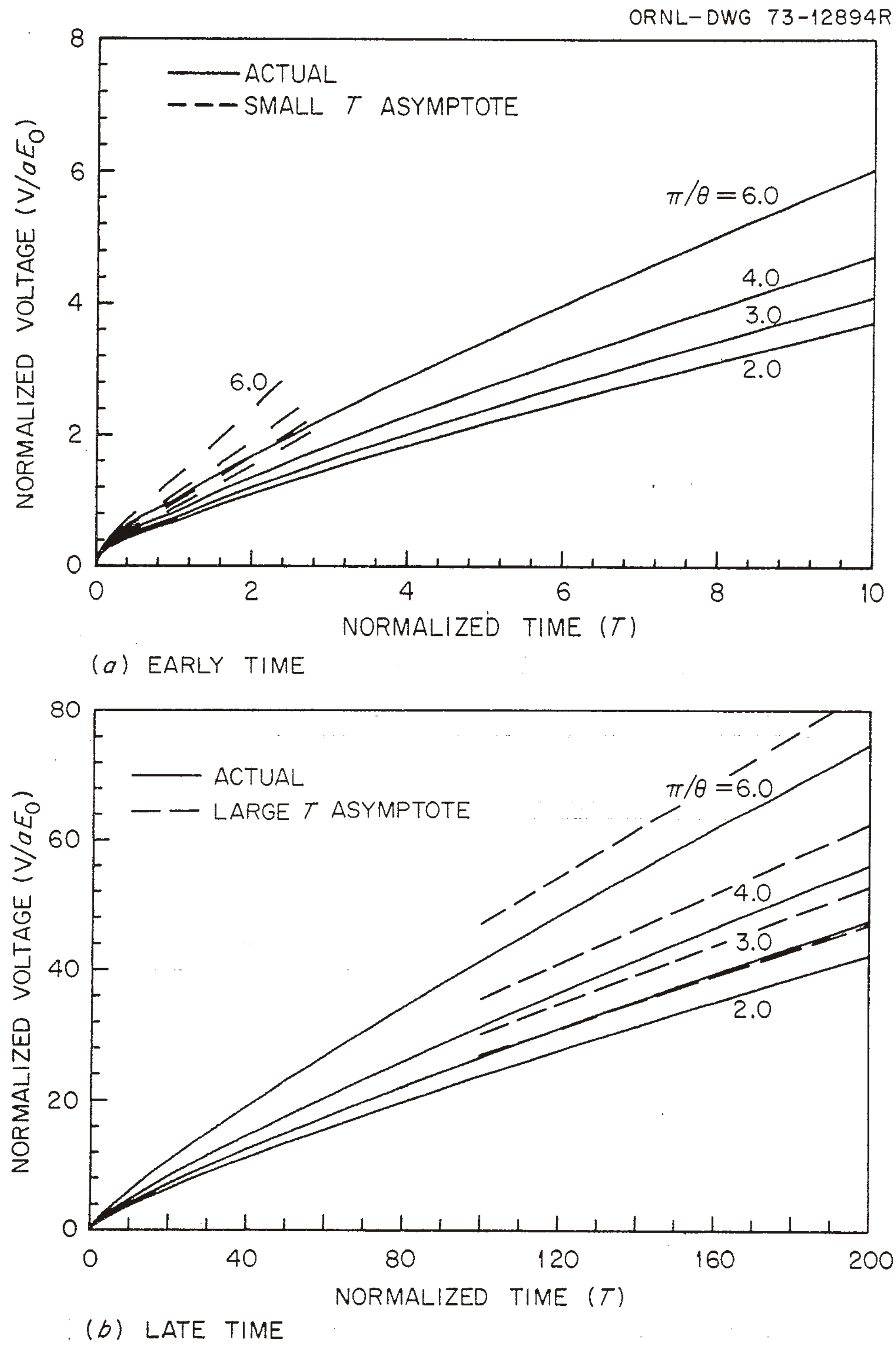

Fig. 5. Normalized Voltage Response Across a 75-ohm Load by an Infinite Cylindrical Antenna to a Step Function Incident Electromagnetic Plane Wave with ψ = 1.0 and θ as a Parameter.

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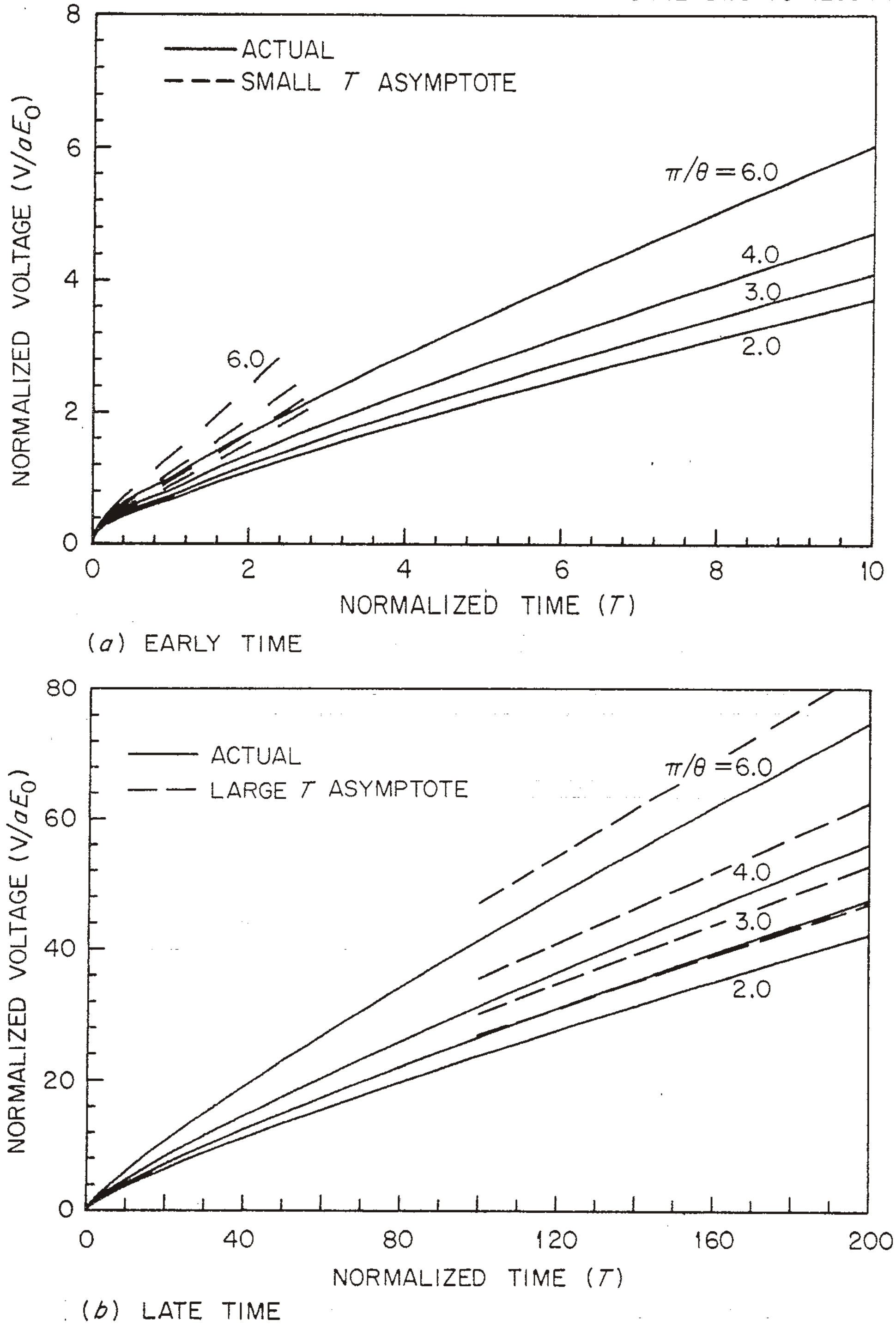

Fig. 5. Normalized Voltage Response Across a 75-ohm Load by an Infinite Cylindrical Antenna to a Step Function Incident Electromagnetic Plane Wave with ψ = 1.0 and θ as a Parameter.

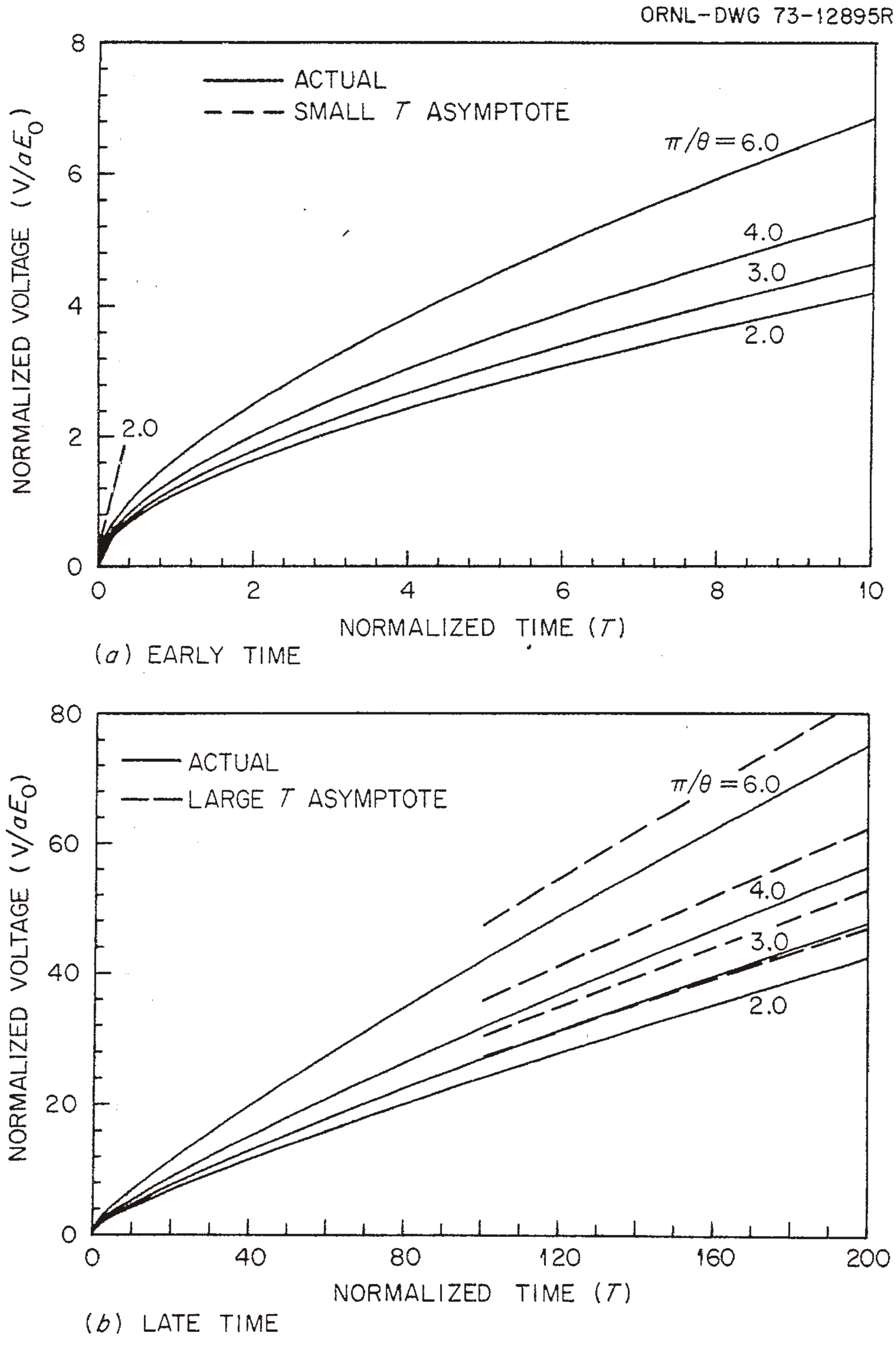

Fig. 6. Normalized Voltage Response Across a 75-ohm Load of an Infinite Cylindrical Antenna to a Step Function Incident Electromagnetic Plane Wave with ψ = 10.0 and θ as a Parameter.

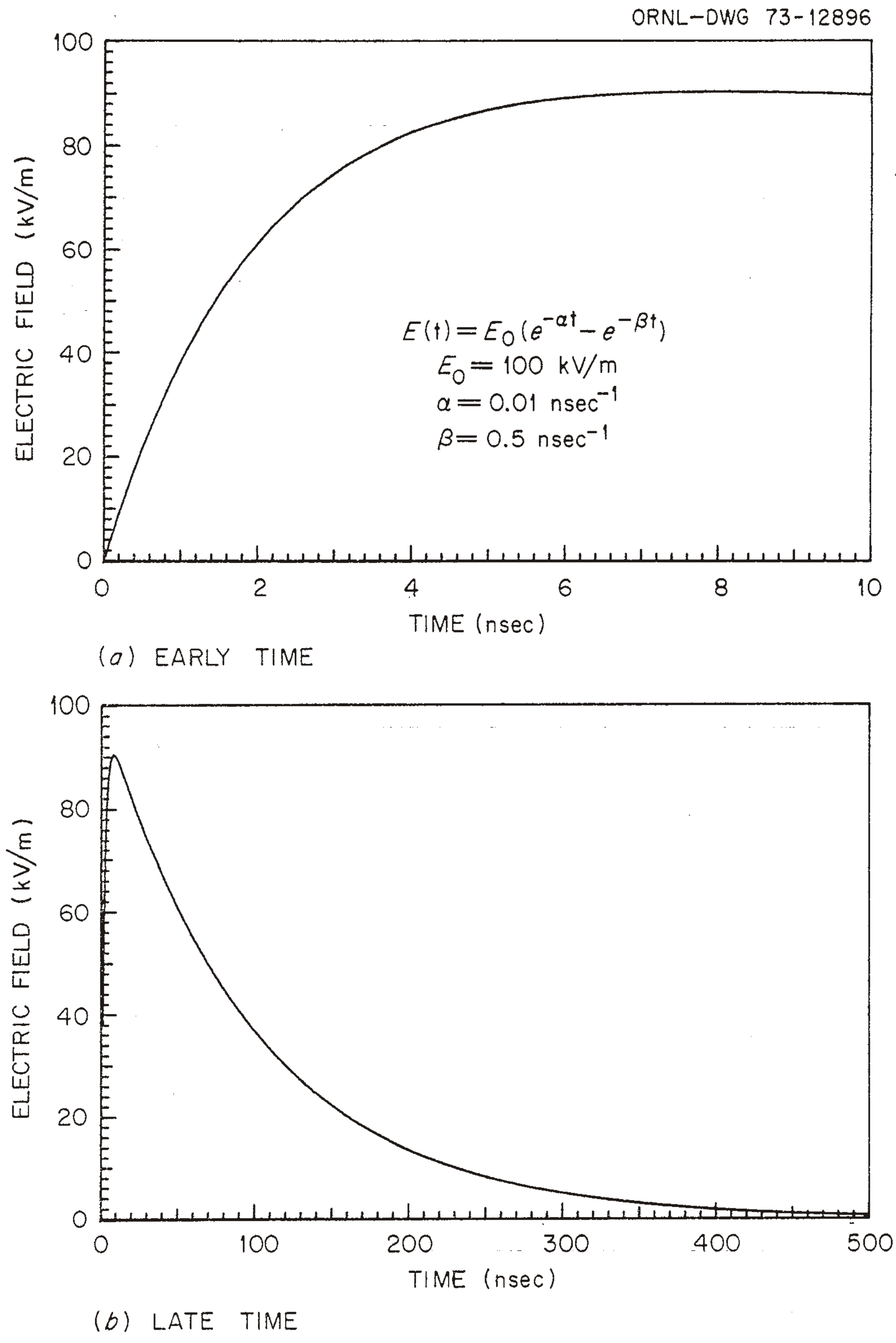

Fig. 7. Time History of the Electromagnetic Plane Wave Pulse Used for the Example Calculations.

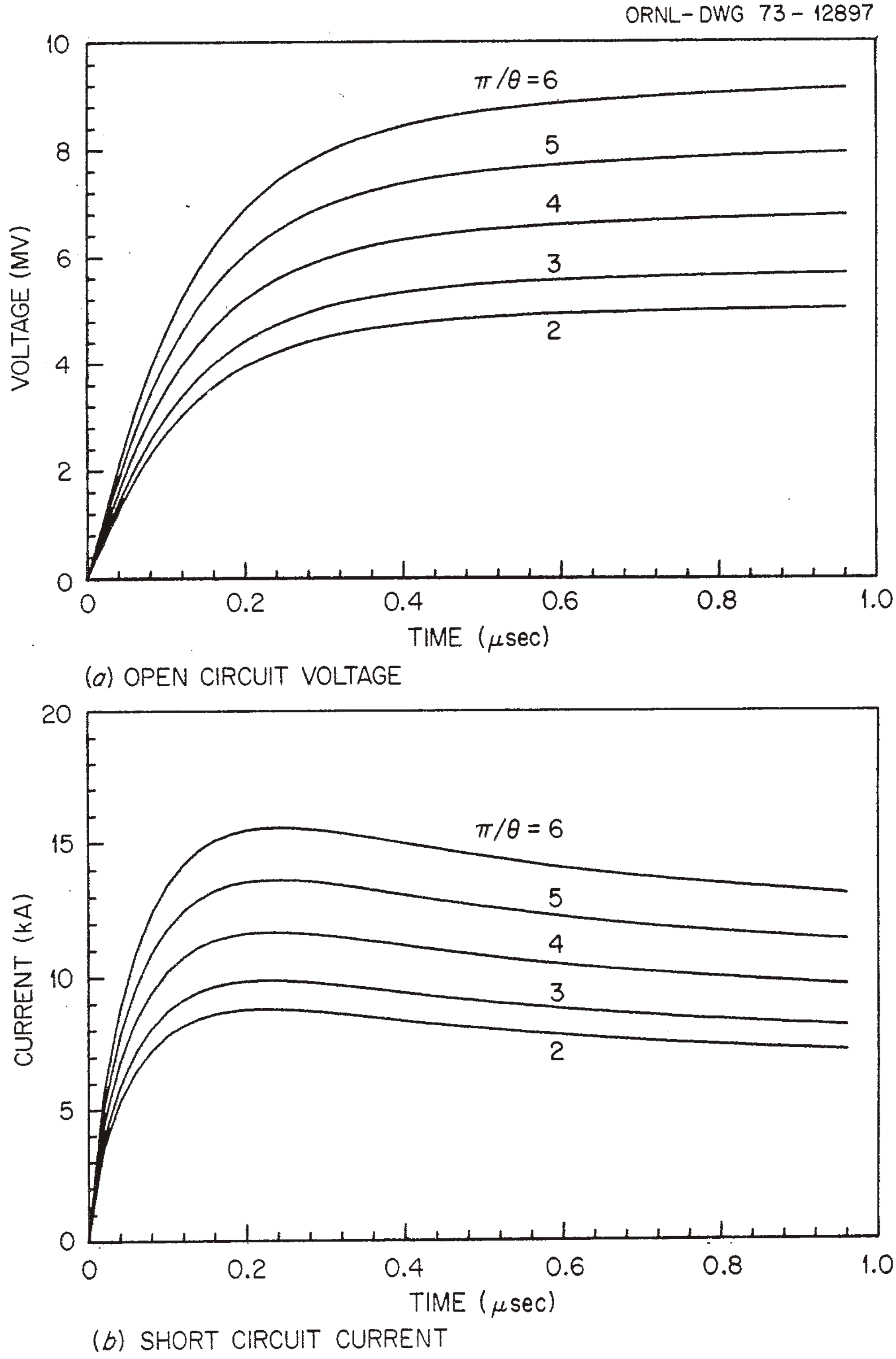

Fig. 8. Response of an Infinite Cylindrical Antenna to the Example Electromagnetic Pulse with ψ = 1.0, a = 0.75 m, and θ as a Parameter.

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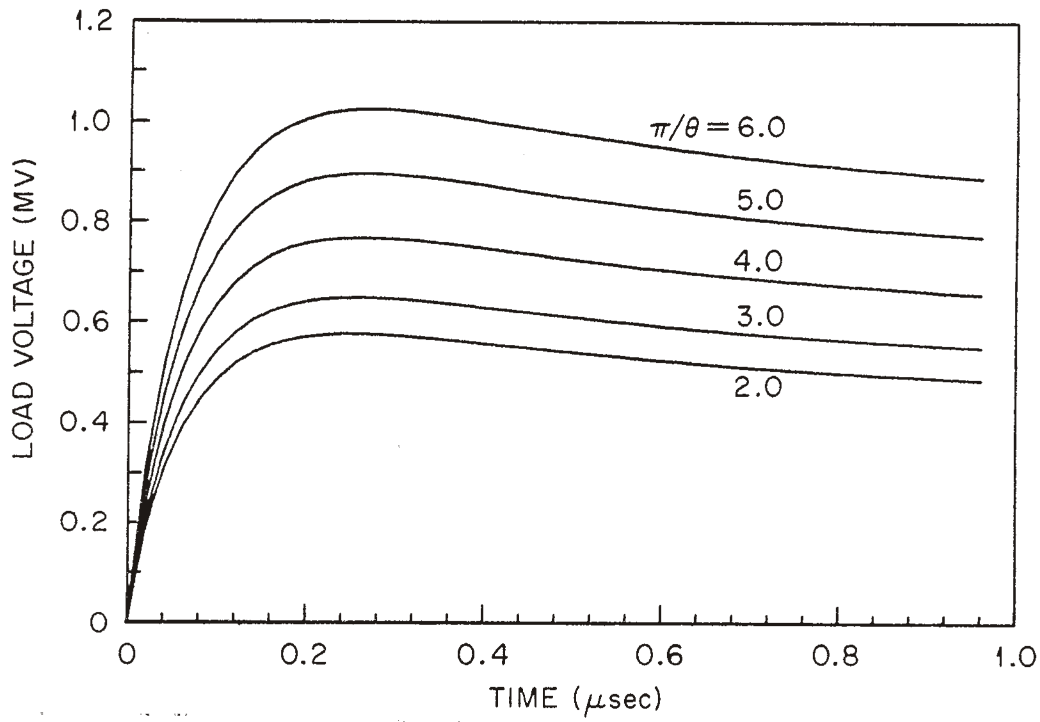
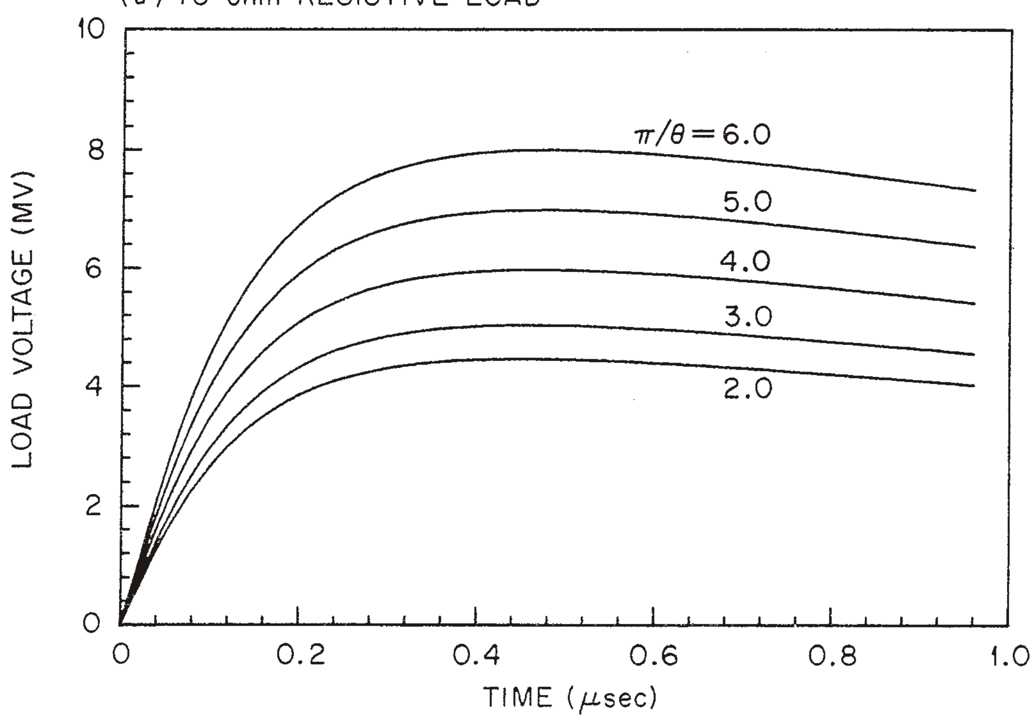


(*a*) 75-ohm RESISTIVE LOAD



(*b*) 2-mh INDUCTIVE LOAD

Fig. 9. Voltage Response Across the Load of an Infinite Cylindrical Antenna to the Example Electromagnetic Pulse with $\psi = 1.0$, $a = 0.75$ m, and θ as a Parameter.

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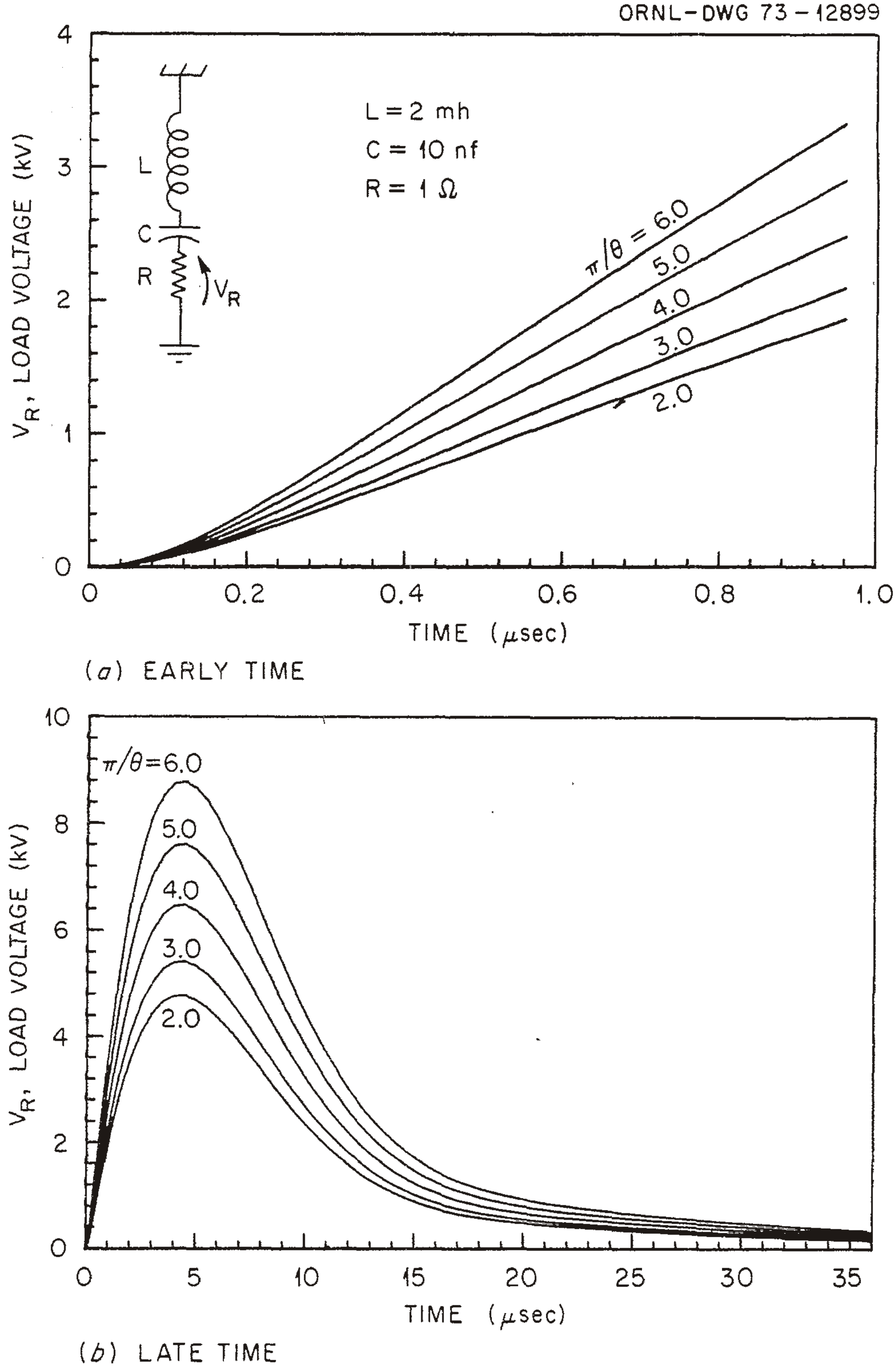

Fig. 10. Voltage Response Across the Resistor of an RLC Load Connected to an Infinite Cylindrical Antenna Subjected to the Example Electromagnetic Pulse with ψ = 1.0, a = 0.75 m, and θ as a Parameter.

voltage responses. Figure 9 shows the load voltage responses for a 75-ohm resistive load and a 2-mh inductive load. In Fig. 10, the voltage response across the resistor of a series RLC load with L = 2 mh, C = 10 nf, and R = 1 ohm is shown. The series RLC circuit is at about 35.6 kHz.

V. SUMMARY

In this note, the responses of an infinitely long, perfectly conducting, cylindrical antenna to a transient electromagnetic pulse have been considered. The exact solutions of the Norton equivalent circuit parameters have been used to calculate the open-circuit voltage, short-circuit current, load current, and load voltage responses. The antenna input admittance used in the calculations was derived from the axial current obtained at the driving point when the antenna is driven by a finite uniform distributed source. It was found that the early time behavior of the open-circuit voltage, short-circuit current, and a resistive load voltage and current responses to a double exponential wave are proportional to the three halves power of time. This result is due, in part, to the discontinuous time derivative of the incident field at time equal to zero. It was also found that the late time behavior of the voltage response across a resistive load to a decaying exponential wave is inversely proportional to the logarithm of time. The results of a parametric study and an example problem have been presented as graphical data.

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