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# COMPUTER ANALYSIS OF THE FAN DOUBLET ANTENNA*

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## ABSTRACT

The EMP response of the fan-doublet, an HF-VHF communications antenna which resembles a bow tie viewed from broadside, has been measured using a biconic simulator which produces a peak field strength at the antenna of 5,600 V/m. Computations for comparison with the experimental data were performed, modeling the antenna with an integral equation approach in the frequency domain and Fourier transforming the data to the time domain to determine the transient response of the antenna. Results were obtained for the antenna located in free space and over a finitely conducting ground plane, the latter employing a modified image approach based upon the Fresenel plane wave reflection coefficients to model the antenna-ground interaction. In both cases the exciting field was assumed to be a plane wave at broadside incidence, having an estimated electric field based upon near-ground measurements of the tangential magnetic field. Results of the calculations are presented for comparison with the measurements. The effect of the ground-reflected field is also demonstrated. Possible refinements in the computer model that will more realistically analyze the antenna-biconic system are discussed. Finally, effects of various load conditions are investigated using a time-domain integral equation approach.



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# I. INTRODUCTION

During the past few years numerical analysis has become a subdiscipline within electromagnetics as the number and diversity of problems for which successful numerical methods have been developed and applied has rapidly expanded. Problems which formerly could be solved in only relatively approximate fashion after a great deal of analytic manipulation can now be routinely evaluated with high accuracy using standard computer methods. The thin linear antenna is an example of such a problem, together with its many variants such as the veedipole, log-periodic arrays, loop, conical spiral antennas, etc. More complicated problems such as the interaction of an antenna with a ground plane of an irregular conducting body like an aircraft are also amenable to computer modeling and evaluation. Scattering from complex shaped bodies, both perfectly conducting and dielectrics, and in both the time and frequency domains, is another problem area where computer solution methods have proven useful.

This paper describes the use of computer modeling for calculating the transient response of structures that may collect EMP energy. The efficient application of a given numerical technique to the EMP protection problem depends not only on the ease of performing the calculations but identification of the uncertainties associated with any approximations involved. In this paper, which presents some preliminary calculations for the fan doublet antenna (shown in Fig. 1), the effects of several such approximations are investigated. Our primary goal is to demonstrate the applicability of numerical analysis to real-world EMP protection problems. Section II of this report deals with the theoretical approach and Section III provides an example (that of the numerical and experimental analysis of the fan doublet antenna). Experimental data were obtained by Harry Diamond Laboratories (HDL) and numerical calculations were performed at Lawrence Livermore Laboratory (LLL).

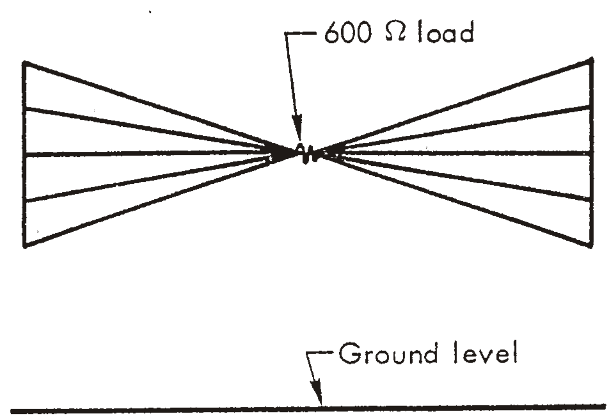


Fig. 1. The fan doublet antenna.

## II. THEORETICAL APPROACH

An electromagnetic problem may be formulated in terms of the differential or integral forms of Maxwell's equations together with the appropriate boundary, continuity, or radiation conditions on the field. The reduction of these types of equations to forms suitable for numerical solution may be found in Refs. 1 through 4. While both types of equations can be solved by a combination of analytical and numerical anlaysis, modern computational techniques have made the integral equation approach the more generally useful. In addition, a great deal of analytic simplification may result if the structure is wire-like (i.e., the maximum cross-sectional dimension $<<\lambda$). The thin-wire approximation to the electric field integral equation can then be used to develop a numerical model of the structure by representing it as a collection of straight-wire segments which is electromagnetically indistinguishable, within some specified accuracy criteria, from the original structure. Thus, not only structures consisting entirely of straight wires, but curved wires and solid surface objects as well (using wire grids) are amenable to modeling in this fashion.

Thin-wire integral equations can be formulated in either the time or frequency domain. The process of generating a linear system to replace either of these integral equations via the moment method is essentially one of evaluating the tangential electric field at a point on the structure due to the current on all segments of the structure. The linear system which thus represents the integral equation may be written in matrix form as:

$$[Z]\,[I] = [E]$$

with the admittance matrix [Y] found from the impedance matrix [Z] as $[Y] = [Z]^{-1}$. It is then a simple matter to evaluate the current distribution on the structure for any primary field through the matrix operation:

$$[I] = [Y]\,[E].$$

All other electromagnetic field quantities (such as near and far fields, polarization, antenna driving point, impedance, etc.) can then be found through other straightforward matrix operations. These processes are entirely consistent with Maxwell's equations and, except for the approximation involved in deriving the integral equation and obtaining its subsequent numerical solution, do not involve any <u>a priori</u> (and possibly restrictive) assumptions. Detailed descriptions of the theoretical foundations of the computer programs used in this study are given in Refs. 5 through 7. Samples of calculations are presented in Refs. 8 through 10. The computer programs have been validated wherever possible by comparing the numerical results with other analytic and experimental results. Even through these comparisons inspire a high level of confidence in the programs, a continuing effort is required to check their accuracy and applicability for new problem requirements.

For completeness, a brief summary of the theoretical basis for the numerical method and computer program follows.

Frequency Domain

The thin-wire electric field integral equation can be written

$$p \int_{C(\overline{r})} I(s')\ G(s,s')\ ds' = E^{p}(s);\ S \varepsilon C(\overline{r}) \tag{1a}$$

where $p = i\omega\mu_0/4\pi$, $I(s')$ is the wire current, $E^p(s)$ is the primary field, $C(\overline{r})$ is the structure geometry and $G(s,s')$ is a suitable kernel function with s and s' observation and source coordinates along $C(\overline{r})$, respectively. For a wire structure in free space, we have

$$G(s,s') = \left[\hat{s} \cdot \hat{s}' + \frac{1}{k^2} \frac{\partial^2}{\partial s \partial s'}\right] g(\overline{r}, \overline{r}') \tag{1b}$$

with $g(\overline{r}, \overline{r}')$ the free-space Green's function given by $\exp(-ik|\overline{r} - \overline{r}'|)/|\overline{r} - \overline{r}'|$, k the wave number, and $\overline{r}$ - $\overline{r}'$ the coordinate vectors to s and s', respectively, where the tangent vectors are correspondingly $\hat{s}$ and $\hat{s}'$. The thin-wire approximation requires that the s be located on the wire surface and s' on the wire axis so that $|\overline{r} - \overline{r}'|$ has a minimum value of a(s'), the wire radius at s'.

Equation (1), while simple in appearance, is extremely versatile and capable of providing good numerical accuracy for analyzing wire structures in free space.

This particular thin-wire integral equation has been chosen for the analysis of wire structures because: 1) it is easily applicable to general geometries [or $C(\overline{r})$], 2) it maintains accuracy for small wire radius/wavelength values, and 3) the required current integration is analytically possible for certain types of current variation. Note that $E^p$ is arbitrary and may be due to an incident plane wave (in which case the scattering characteristics of the structure are obtained) or due to localized excitation (from which the radiation properties of the structure as an antenna are derived).

An approximate solution of I(s') in Eq. (1) can be obtained by reducing the integral equation to an Nth order system of linear equations in which the N unknowns are sampled values of the structure currents. The N equations are generated by enforcing the integral equation at N points (wire segments) on the structure (N values of $\overline{r}$). The coefficients in these equations are interpretable as mutual impedances and are dependent on structure geometry. While there are many methods for accomplishing this reduction of the integral equation to linear system (representable in matrix form), they differ only in detail and the computational effort required to obtain the matrix elements. Common to all such methods are the representation of the current in terms of its samples values; a matching of the integral equation over the structure in some prescribed fashion; the numerical calculation of the $N^2$ "mutual impedance" coefficients; and subsequent solution of the linear system via inversion, factorization, or iteration.

The preceding discussion has dealt explicitly with only the free-space problem. Equation (1) can, however, be used for more involved interaction problems through the use either of a modified kernel which introduces the effect of a more complex environment, or as it stands but with a more complex $C(\overline{r})$ than that of the structure alone. An example of the former is provided by the antenna half-space problem where the interface reflected field is accounted for by adding to Eq. (1b) the Green's function for the half-space geometry. The modified integral equation then permits the treatment of such problems as finding the impedance of an antenna located near the earth's or determining the currents excited by a plane wave incident on a system of buried conductors. In the latter area can be put the perturbing effect of a conducting body (such as an aircraft) on the behavior of an antenna located in its vicinity.

The transient response is developed from the frequency domain calculations using standard procedures: (1) the frequency response of a structure is calculated by repeating the solution of Eq. (1a) for a number of frequencies; (2) the Fourier transform of the total response is calculated by multiplying this structure frequency response by the spectrum of the excitation; and (3) an inverse transformation is performed, yielding the transient response.

Time-Domain Methods

There has recently developed more interest in obtaining the short-pulse electromagnetic response of various objects as opposed to their single frequency or cw behavior. Fortunately, the numerical techniques for evaluating such problems have become feasible with increasing computational power. One method for obtaining the transient characteristics of a given structure (excited as either an antenna or scatterer by a wide-band pulse) is to perform a Fourier transform to the time domain of its frequency-dependent behavior as found from a frequency domain analysis. This method can be quite inefficient, however, and furthermore it does not easily permit the analysis of nonlinear effects. Therefore, a formulation directly in the time domain is desirable.

The time-domain analog of Eq. (1) can be written

$$\frac{\mu_0}{4\pi}\int_{C(\overline{r})}\left[\frac{\hat{s}\cdot\hat{s}'}{R}\frac{\partial}{\partial t'}I(s',t') + c\,\frac{\hat{s}\cdot\overline{R}}{R^2}\frac{\partial}{\partial s'}I(s',t') - c^2\,\frac{\hat{s}\cdot\overline{R}}{R^3}q(s',t')\right]ds' = E^p(s,t);\ S\epsilon C(\overline{r})$$

where $\overline{R} = \overline{r} - \overline{r}'$ and $R = |\overline{R}|$, $q$ is the charge density, and $t' = t - R/c$ is the retarded time with $c$ the free-space velocity of light. This equation is amenable to a treatment similar to that used for Eq. (1) in that a space segmentation of the structure and point fitting of the boundary conditions are involved. The added dimension of the time dependence in Eq. (2) is the essential feature which distinguishes it from Eq. (1)

The straightforward computer solution of Eq. (2) is possible because the quantities under the integral are functions of the retarded time, whereas those outside it are not. Consequently, the integral equation can be solved as an initial value problem, i.e., given the starting values of the source and unknown charge and current, the subsequent values in time of the unknowns can be found in terms of these initial values by a time-stepping procedure. Thus, in addition to the space segmentation there is introduced a time segmentation as well.

The time-domain computer program operational at LLL does not presently have the capability of evaluating performance of structures near ground. Consequently, in the analysis of the fan doublet antenna, the frequency-domain program was primarily used even though it was found to be approximately 20 times less efficient than the time-domain program (for those cases to which the time-domain code could be applied). Extension of the time-domain code to handle grounds is underway.

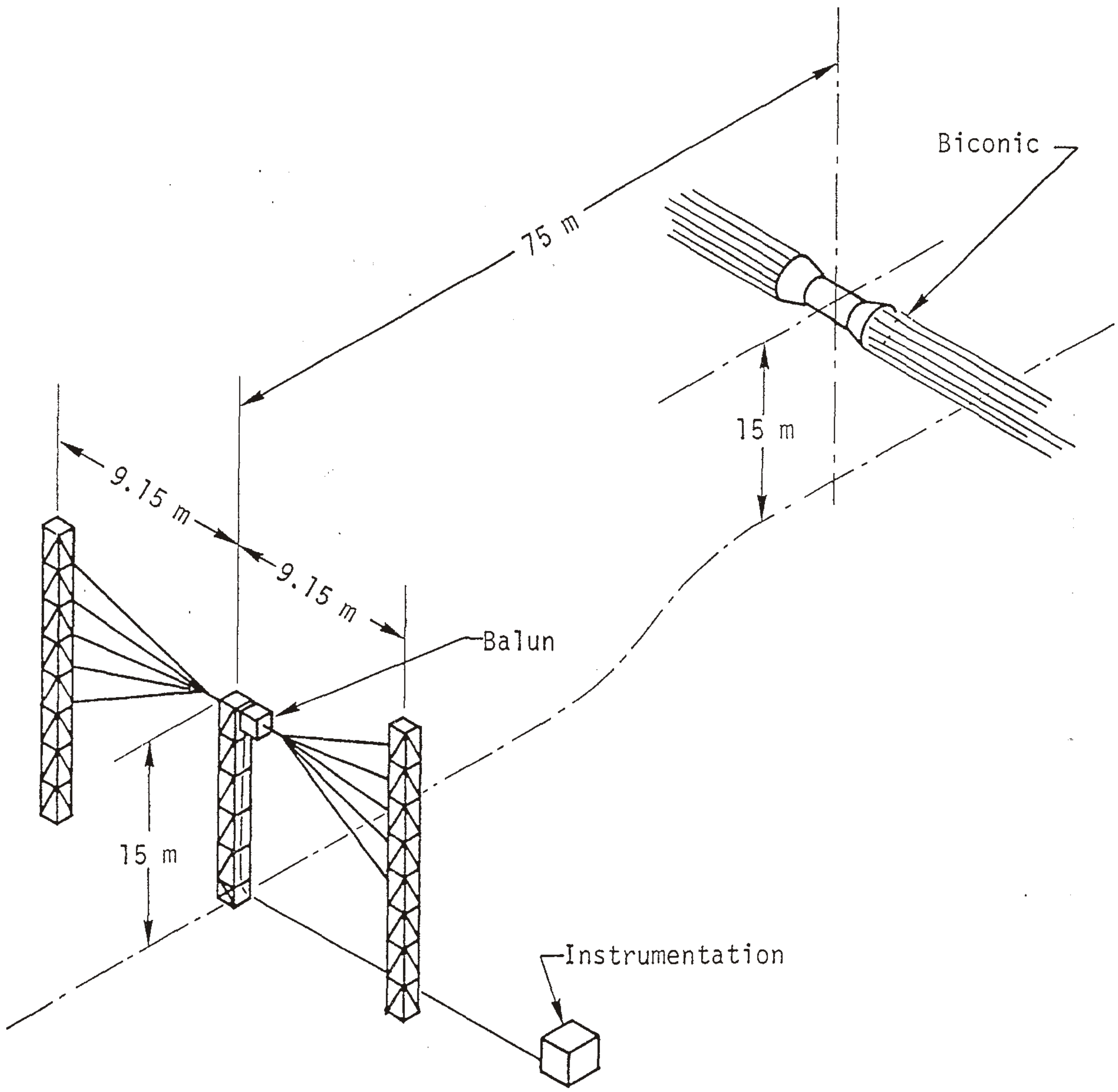


Fig. 2. Experimental geometry.

## III. ANALYSIS OF THE FAN DOUBLET ANTENNA

### The Experiment

The fan doublet antenna is shown in Fig. 1. The antenna is constructed of wire and is supported by steel towers. The experimental data was obtained by illuminating the antenna by a biconic simulator as shown in Fig. 2. HDL obtained measurements of the current through various loads connecting the two sides of the antenna. Experimental results obtained using a 600-ohm resistive load are shown in Fig. 3.

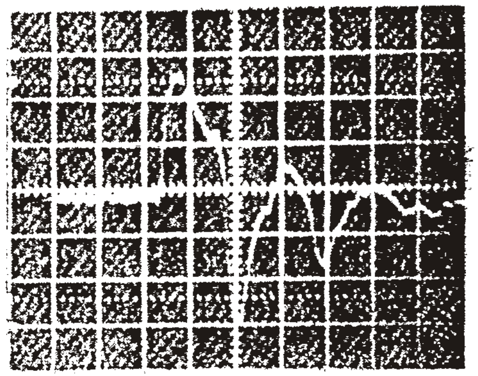


Fig. 3. Measured load current.

Also measured was the horizontal component of the magnetic field produced near ground level by the biconic in the proximity of the fan doublet. This data was used, together with a reflection coefficient approximation, to obtain an estimate of the tangential electric field at the apex of the antenna. The time wave form of this estimated field and its corresponding frequency spectrum are shown in Figs. 4a and 4b.

### The Numerical Calculation

A necessary prerequisite to using an integral equation treatment is the development of a suitable numerical model to represent the structure of interest. In some cases (e.g., a sphere) the selection of the model is obvious, with only the positions and sizes of the sample areas (for a surface integral equation) or wire segments (for a wire grid model) to be chosen. Other structures, such as the fan doublet, may involve considerably more latitude in the numerical model development, depending upon the degree of approximation which may be acceptable in terms of the problem requirements.

It is consequently possible to identify a sequence of computer models for the fan doublet-biconic simulator in order of the approximations involved. One such sequence, in order of anticipated decreasing accuracy, follows:

(1) Model all wires, insulators, and support towers for both the biconic and fan doublet including the ground effect via the rigorous Sommerfeld theory.

(2) Remove the biconic support towers from the biconic model.

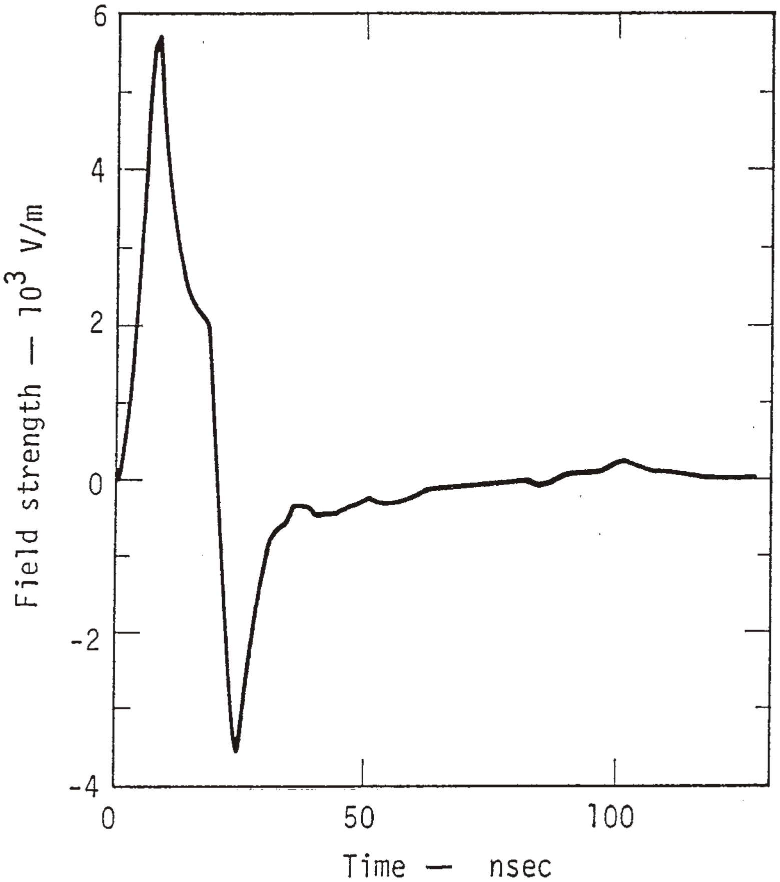


Fig. 4a. Electric field estimated to exist at the apex of the antenna.

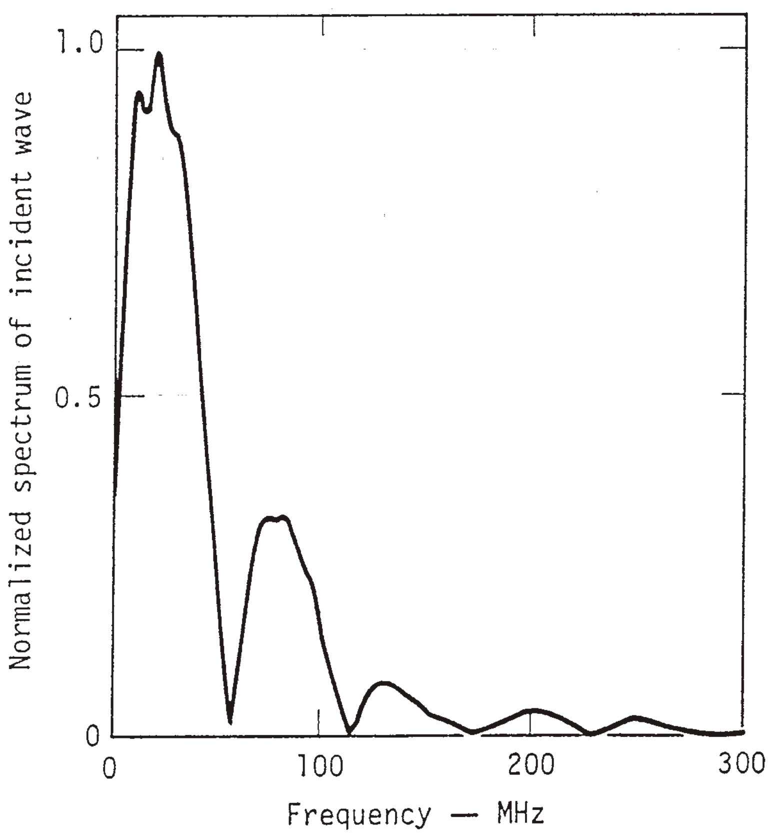


Fig. 4b. Spectrum of the electric field estimated to exist at the apex of the antenna.

(3) Neglect mutual interaction between the biconic and fan doublet by modeling each separately and using the computed simulator fields as the fan doublet excitation.

(4) Reduce the biconic model to a single resistively loaded wire.

(5) Eliminate the support towers and insulators from the fan doublet model.

(6) Replace the Sommerfeld theory with the reflection coefficient approximation, or image approach, for the ground interaction.

(7) Assume plane-wave broadside incidence of the simulator field, using the amplitude and polarization obtained at the apex of the fan doublet antenna over the entire antenna.

(8) Assume a perfect ground plane for both the fan doublet and biconic model.

(9) Neglect interaction of the fan doublet with the ground.

(10) Neglect the ground in computing the simulator field.

We have attempted to order the above levels in terms of the increasing estimated error contribution between successive steps, i.e., the relative error between steps N and N + 1 is less than that between steps N + 1 and N + 2. It is impossible to be sure that this is the actual case without performing the calculations.

In obtaining the results presented below, we used models equivalent to levels (7) and (9) above, except that the experimentally determined simulator field (Fig. 4) has been used as the primary or incident field in place of a computed one. The latter model is thus a very simple representation of the fan doublet itself. In addition, the ground reflected fields are ignored for this case except for the exciting field description, which is assumed to be planar and normally incident. The more accurate model allows for the antenna-ground interaction while using the same exciting field description. If the accuracy of the experimental data warrants it, additional calculations using various of the other numerical models suggested above might be useful to obtain more complete correlation between experimental-computational discrepancies and the modeling approximations employed.

## Numerical Results

The frequency domain code may be used in a straightforward manner to calculate the required transient response. Briefly, this approach consists of calculating the desired transfer function, multiplying by the spectrum of the transient excitation, and finally performing an inverse fast-Fourier transform.

The actual form of the transfer function depends on the particular numerical model employed. For the numerical model corresponding to the 9th level of simplification, the desired transfer function is $Y_T = I_L/E_{inc}$ where $I_L$ is the load current resulting from the field of magnitude $E_{inc}$, for the fan doublet antenna located in free space. This transfer function is shown in Fig. 5 for the fan doublet antenna with a resistive load of 600 ohms. Figure 6 compares the measured current to the current calculated from the incident wave spectrum (Fig. 4b) and the transfer function of Fig. 5. The period of ringing, peak currents, and general character of the measured response is well predicted by the numerical calculations considering the level of simplification of the numerical model.

The calculations were repeated for a model improved by including the interaction of the fan doublet antenna with ground via a reflection coefficient technique (level 7 above).

Nominal ground values of $\sigma$ = 0.01 mho/meter and $\varepsilon_r$ = 25 were used. The resulting transfer function is compared with that obtained in the previous case in Fig. 7, and the corresponding transient response is shown in Fig. 8. Since the apex of the antenna is 50 ft from the ground, the current of Fig. 8 should not differ from that of Fig. 6 until after 100 nsec. This is evident in the comparisons of the two calculated currents shown in Fig. 9. Also note that including the ground interaction improved the agreement with the measured current at the late times.

The transfer functions shown in Figs. 5 and 7 can also be used to calculate the response for loads other than 600 ohm resistive load. To illustrate how this might be done, consider the equivalent circuit of Fig. 10. Terminals A-B are the antenna terminals, $Y_L$ is the load attached to these terminals, $Y_A$ is the antenna driving point admittance, and $I_{SC}$ is the short-circuit current which depends on the arrival direction and polarization of the wave incident on the antenna. In the terminology of circuit analysis, the portion to the left of terminals AB is the Norton's equivalent of the antenna. The current through the load, $I_L$, is thus

$$I_L = \frac{Y_L}{Y_L + Y_A} I_{SC} = \underbrace{\left[\frac{Y_L}{Y_L + Y_A}\right]\left[Y_{SC}\right]}_{Y_T} E_{inc} = Y_T E_{inc}$$

where $Y_{SC} = I_{SC}/E_{inc}$. $Y_T$ is the "transfer admittance" and has been plotted in Figs. 5 and 7 for two numerical models for $Y_L$ = 1/600 mho. Consequently, $Y_{SC}$ and $Y_T$ for loads other than the 600 $\Omega$ load used in Figs. 5 and 7 may be found given $Y_A$, the antenna driving point admittance which is shown in Fig. 11. However, in the study reported here, we investigated the effect of different load conditions using the time-domain code.

Frequency stepping with the frequency-domain code is inefficient. It took approximately 20 minutes on a CDC-7600 to generate the transfer function of Fig. 5. While this time could be reduced by using larger (and consequently fewer) frequency increments, use of the time-domain code reduces computation time to about 30 seconds, or about 40 times less than that required by the frequency-domain code. Presently, the time-domain code is limited to analysis of antennas in free space and can be applied only to certain types of wire junctions. Even though these limitations require modeling the antenna by five straight wires as shown in Fig. 12, the

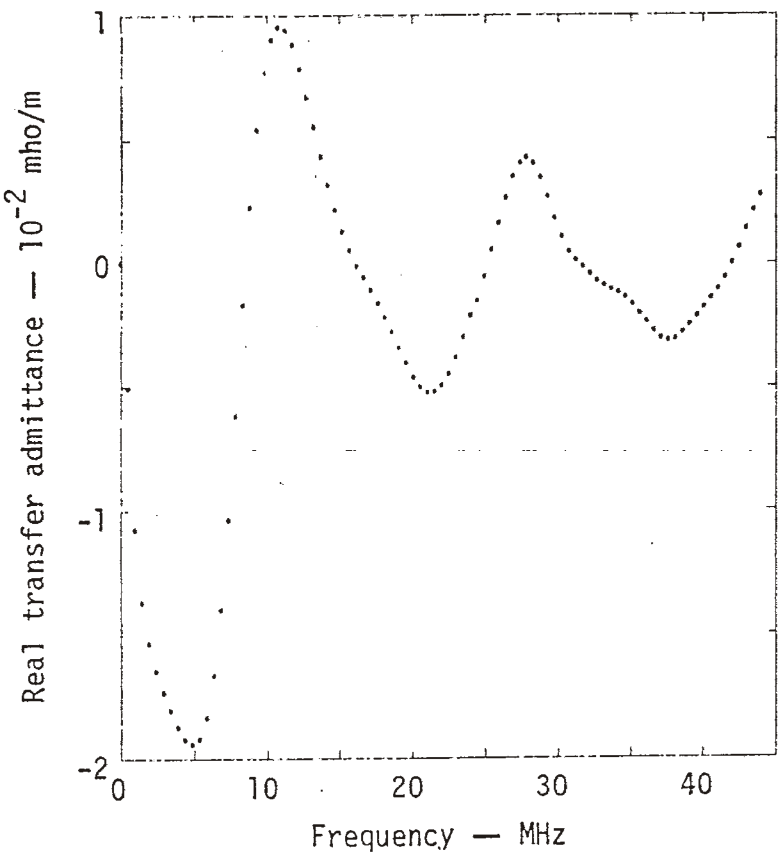


Fig. 5a. Real part of the transfer admittance for the fan doublet antenna in free space.

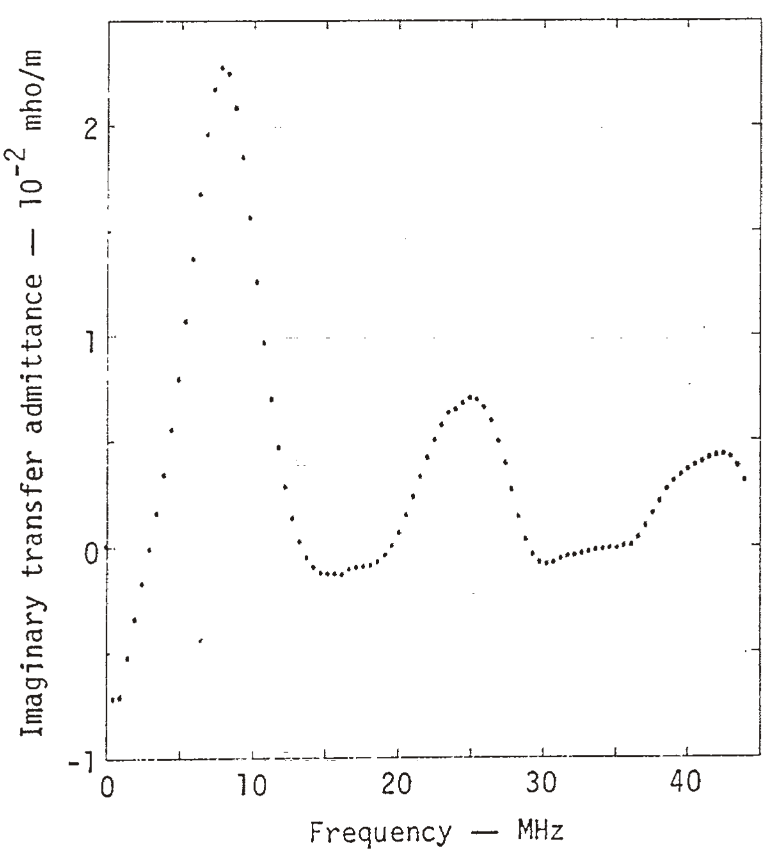


Fig. 5b. Imaginary part of the transfer admittance for the fan doublet antenna in free space.

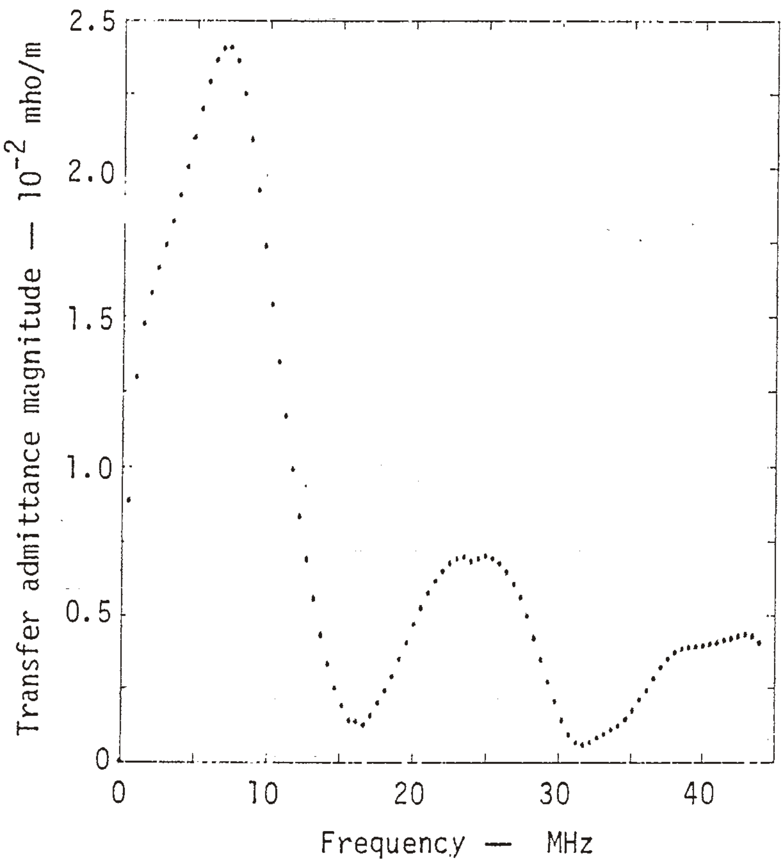


Fig. 5c. Magnitude of the transfer admittance for the fan doublet antenna in free space.

vast savings in computational effort justifies this approximation in view of the general agreement between the measured current and that calculated using a 600-ohm load and the time-domain code as shown in Fig. 13. Again, note that the period of ringing is well predicted, but the peak currents are slightly higher than those predicted by the more complete model of the frequency-domain analysis.

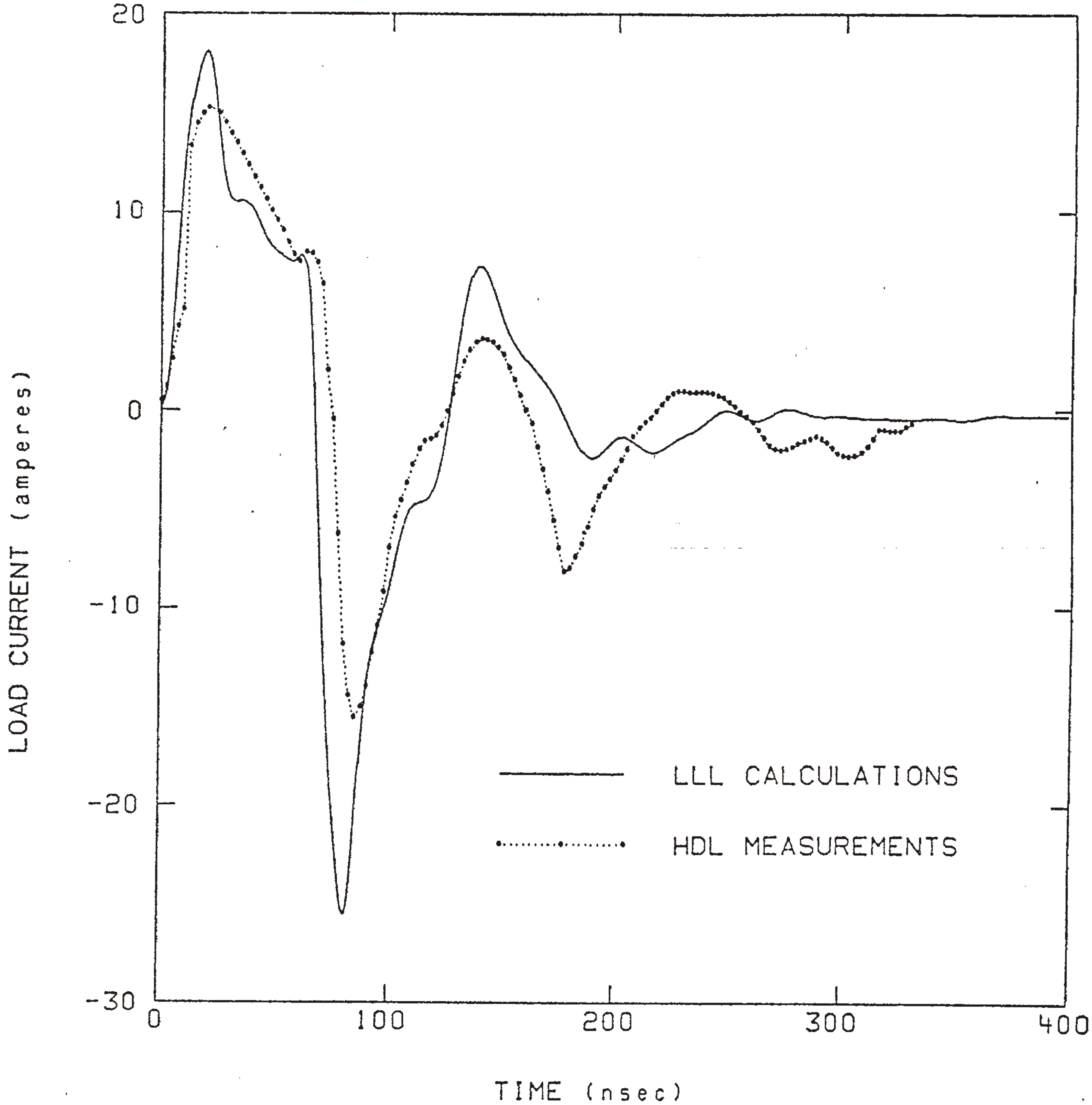


Fig. 6. Comparison of calculated and measured current with calculations made as if antenna were in free space.

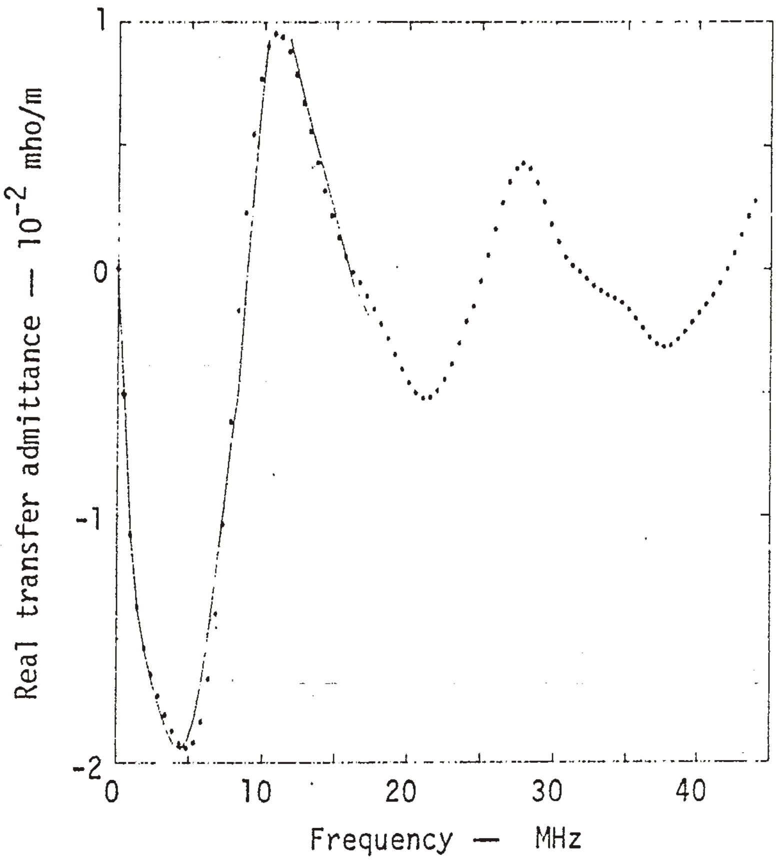


Fig. 7a. Comparison of free-space and over-ground imaginary transfer admittance. Over-ground curve is solid line.

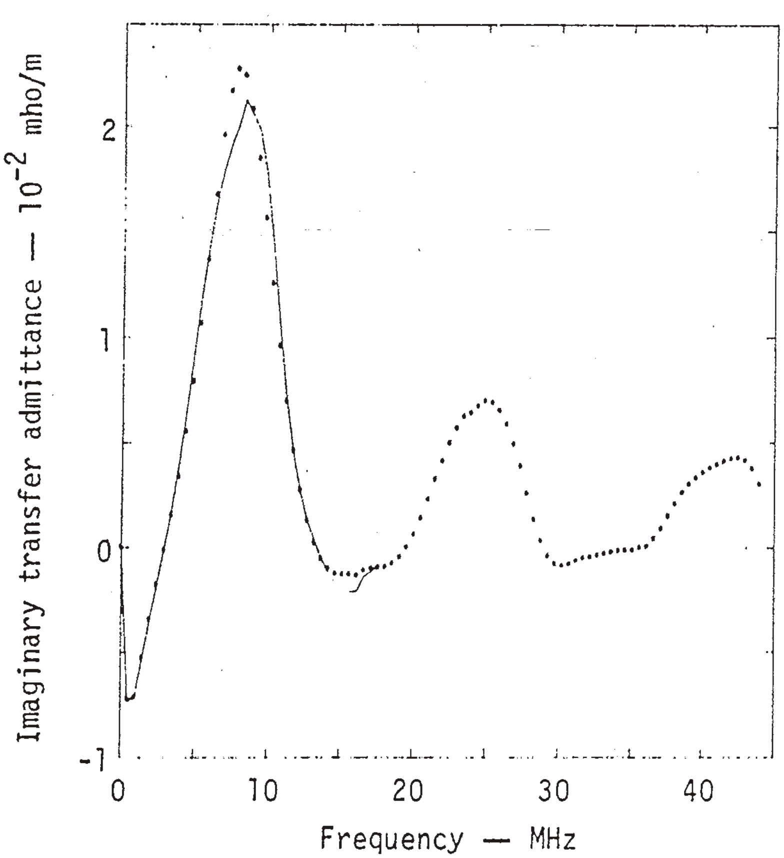


Fig. 7b. Comparison of free-space and over-ground imaginary transfer admittance. Over-ground curve is solid line.

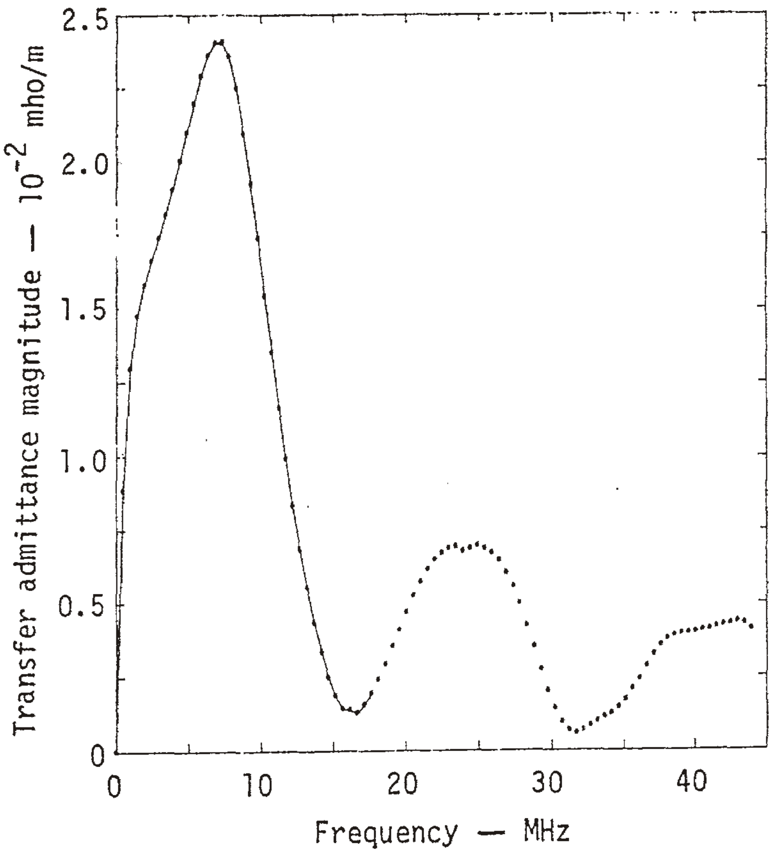


Fig. 7c. Comparison of free-space and over-ground magnitude of transfer admittance. Over-ground curve is solid line.

The efficiency of the time-domain code permits economical analysis of different load conditions and the changes due to different arrival directions and polarizations of the incident wave. For example, Fig. 14 shows the current under short-circuit conditions. In this case, the period of ringing halves, the peak current increases by a factor of about three, and the currents decay much slower than in the case where the load was 600 ohms.

## IV. SUMMARY

The current through a resistive load attached to a fan doublet antenna in the vicinity of a biconic simulator has been calculated by a combination of numerical analysis of the antenna and measurements of the fields produced by the simulator. These calculations agree favorably with measurements of the actual currents in view of the engineering approximations used to make the numerical analysis of the antenna tractable. We expect that refinements in the numerical model will increase the accuracy of the numerical results, but most of these refinements will also increase the cost of the calculations.

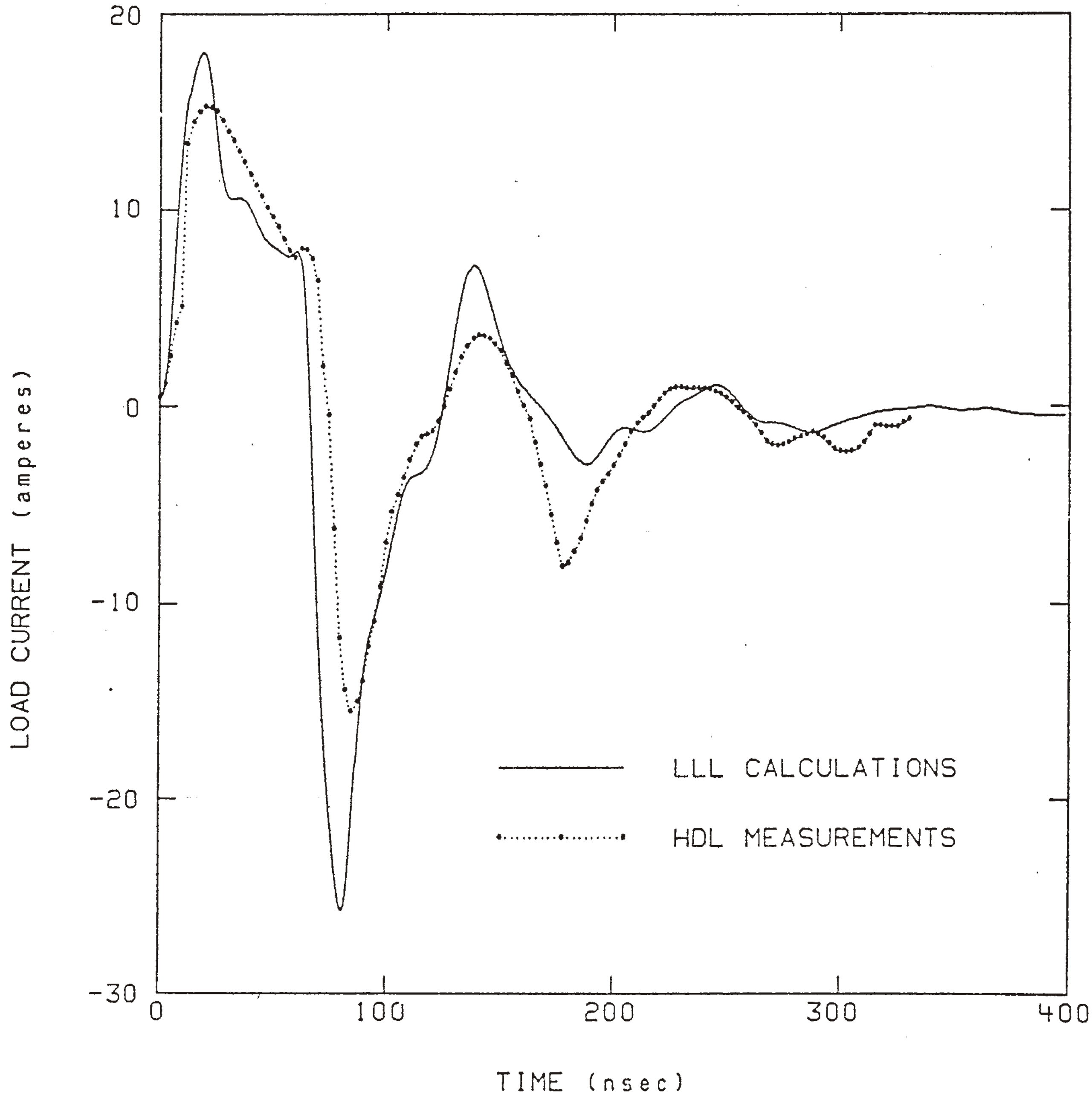


Fig. 8. Comparison of calculated and measured current. Antenna calculations performed for the antenna over ground.

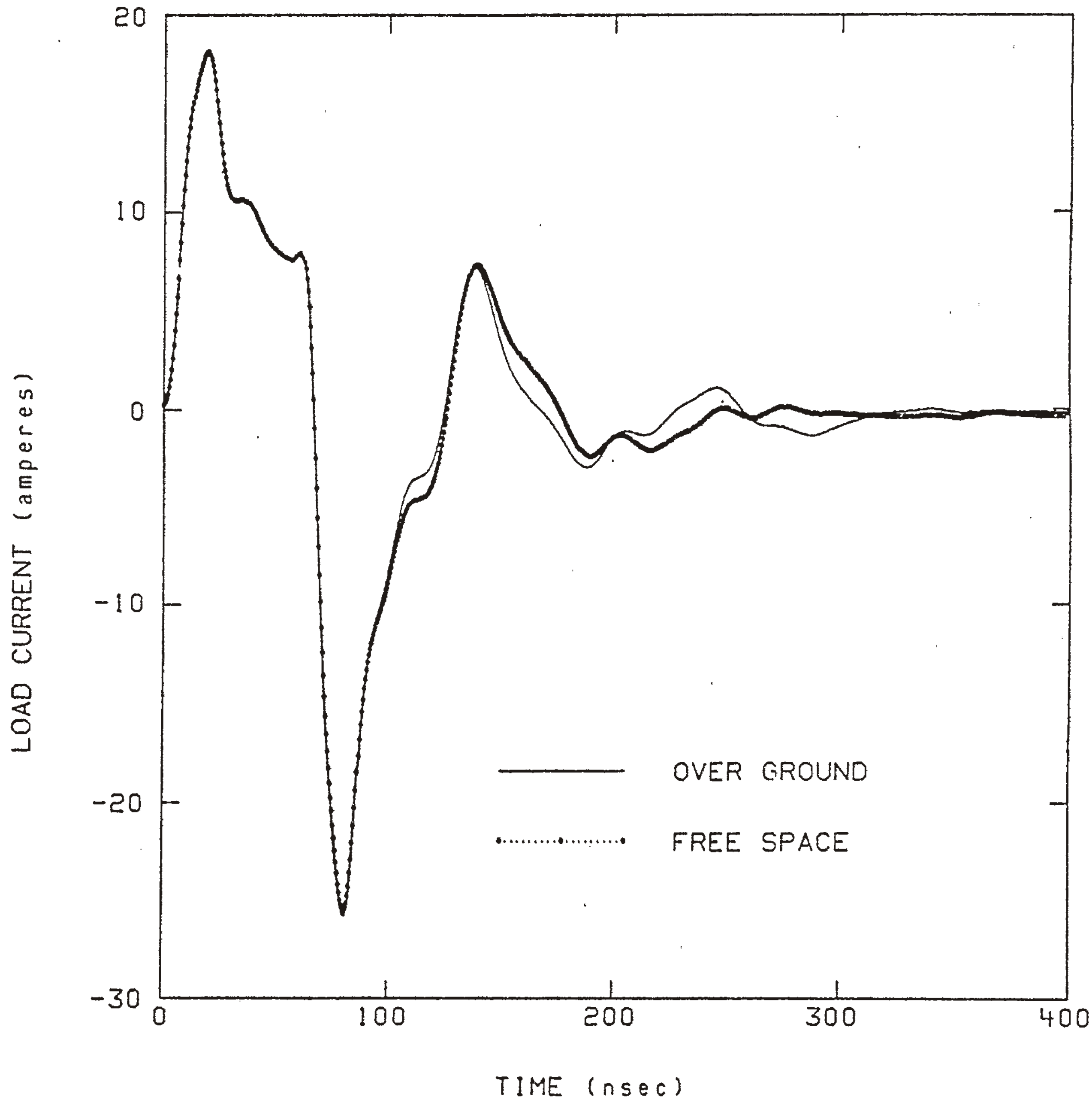


Fig. 9. Comparison of calculated currents.

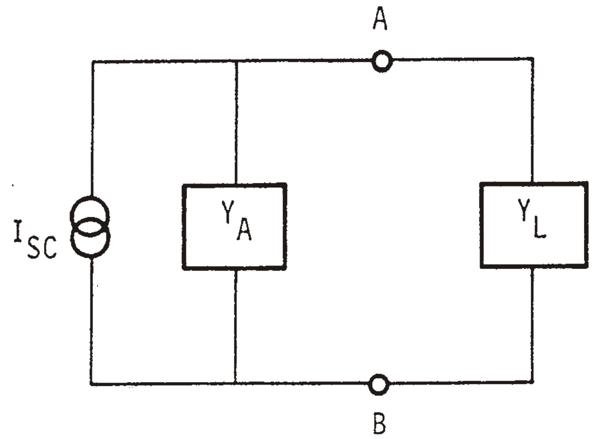


Fig. 10. Equivalent circuit for antenna.

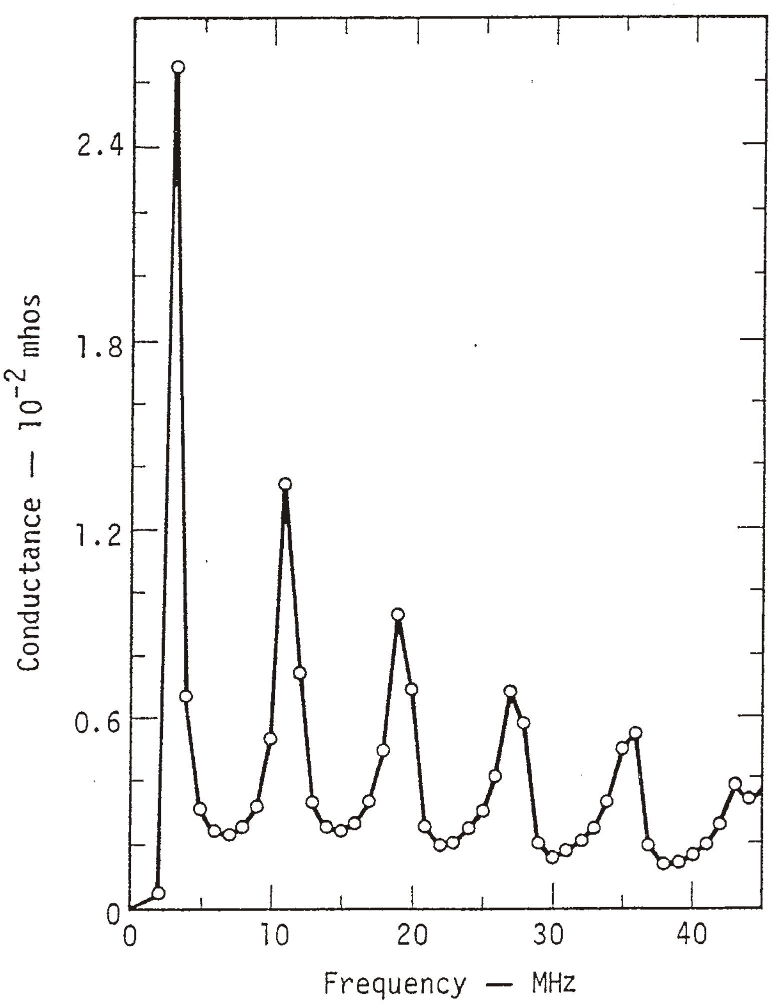


Fig. 11a. Real part of input admittance to the fan doublet antenna in free space.

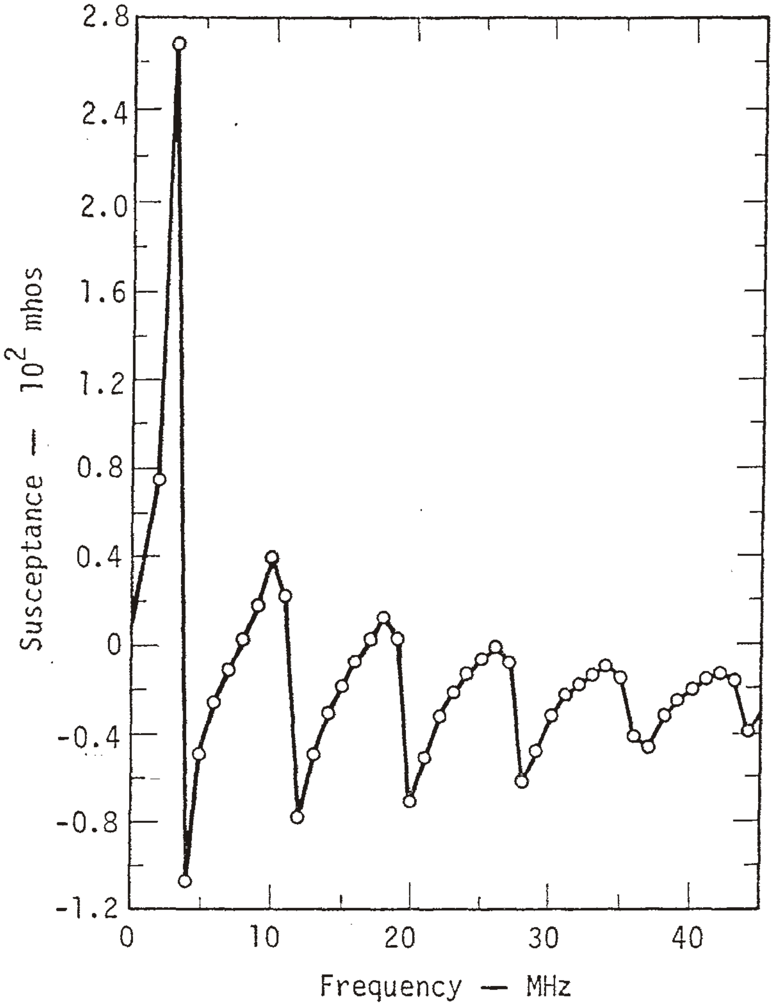


Fig. 11b. Imaginary part of input admittance to the fan doublet antenna in free space.

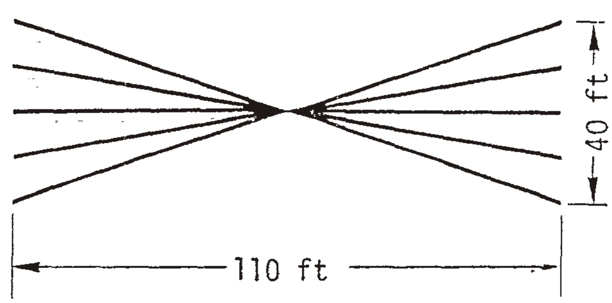


Fig. 12. Wire model of fan doublet antenna used in time-domain analysis.

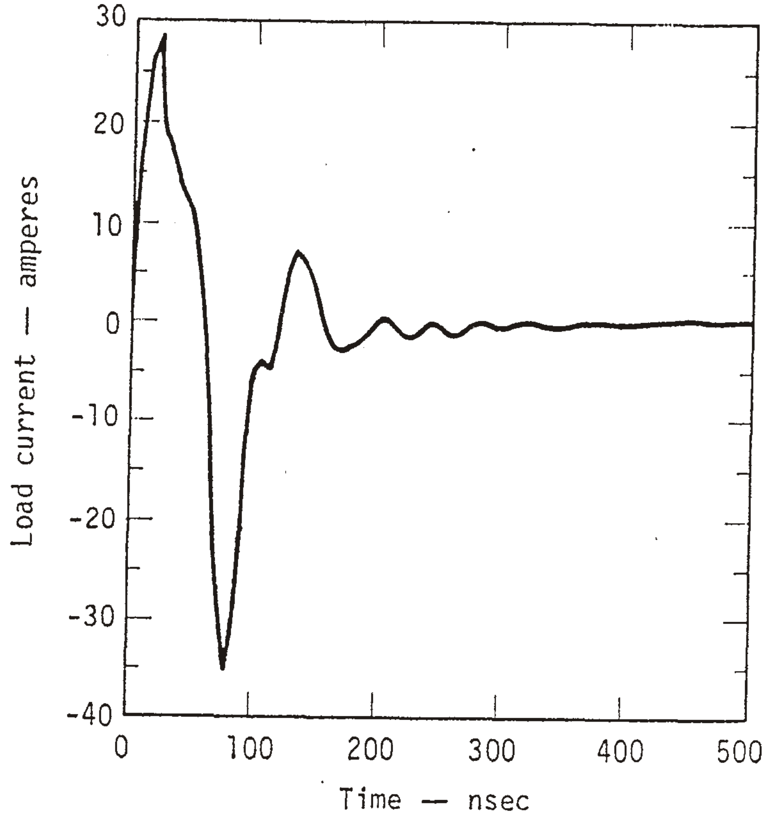


Fig. 13. Current calculated for 600-ohm load with time-domain code.

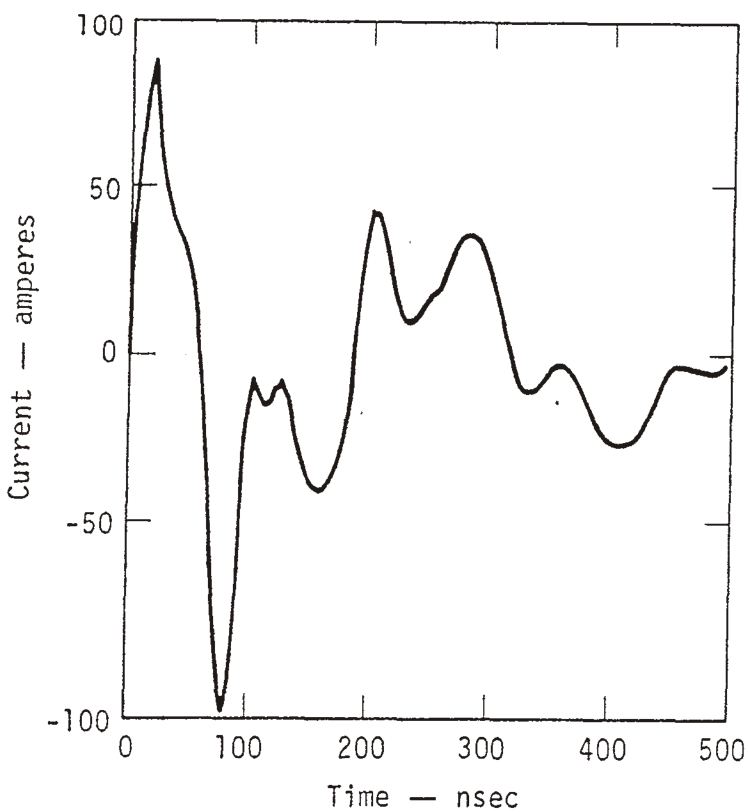


Fig. 14. Short-circuit current calculated with time-domain code.

## REFERENCES


1. R. F. Harrington, Field Computation by Moment Methods (The MacMillian Company, New York, 1968).

2. E. Gee, E. K. Miller, A. J. Poggio, E. S. Selden, and G. J. Burke, "Computer Techniques for Electromagnetic Scattering and Radiation," in Proc. 1971 IEEE Inter. Symp. Electromagnetic Compatibility, Philadelphia, Pa., July 13-15, 1971.

3. C. L. Bennett and W. L. Weeks, "A Technique for Computing Approximate Electromagnetic Impulse Response of Conducting Bodies," Purdue Univ. Rept. TR-EE-68-11 (1968).

4. E. K. Miller, Some Computational Aspects of Transient Electromagnetics, Lawrence Livermore Laboratory Rept. UCRL-51276 (1972), Interaction Note 143.

5. A. J. Poggio and E. K. Miller, "Integral Equation Solutions of Three-Dimensional Scattering Problems," in course notes for Numerical Methods in Electromagnetics and Antennas, Univ. of Illinois, Sept. 1970, R. Mittra, Ed.

6. E. K. Miller, F. J. Deadrick, and W. O. Henry, "Computer Evaluation of Large, Low-Frequency Antennas," IEEE Trans. Ant. and Prop. AP-21, 386-389 (1973).

7. E. K. Miller, A. J. Poggio, and G. J. Burke, "An Integro-Differential Equation Technique for the Time-Domain Analysis of Thin Wire Structures, Part I - The Numerical Method," J. Compt. Phys. 12, 24-28 (1973).

8. A. J. Poggio, E. K. Miller, and G. J. Burke, "An Integro-Differential Equation Technique for the Time-Domain Analysis of Thin Wire Structures, Part II - Numerical Results," J. Compt. Phys. 12, 210-233 (1973).

9. E. K. Miller and M. L. VanBlaricum, "The Short-Pulse Response of A Straight Wire," IEEE Trans. Ant. Propagat. AP-21, 396-398 (1973).

10. J. A. Landt and E. K. Miller, "Short Pulse Response of A Circular Loop," to be published in IEEE Trans. Ant. and Prop. Jan. 1974.