SCATTERING OF ELECTROMAGNETIC RADIATION BY APERTURES VI. FIRST DISCUSSIONS ON GENERALIZING BABINÉT'S PRINCIPLE IN TWO DIMENSIONS

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ABSTRACT: We present in this report our first efforts to thoroughly understand Babinét's principle for electromagnetic diffraction and the results of our initial efforts to extend the principle beyond its usual range of application. Proof is given that the principle in its present form is invalid for general two dimensional, infinitely thin, perfectly conducting surfaces containing apertures. The only case in which it is valid is the conventional one of an infinite plane.

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I. INTRODUCTION

This paper contains the results of our first efforts to investigate the scope of validity of a principle that has proven to be of great value in electromagnetic diffraction theory. All published proofs of Babinét's principle have been limited to the case when the diffracting surface is a perfectly conducting, infinitely thin plane screen. Booker[1] presented the first published account of the principle for electromagnetic fields. Other early references may be found in the review article on diffraction theory by Bouukamp[2]. Born and Wolf[3] and Jackson[4] present more lucid and somewhat more recent discussions of Babinet's principle.

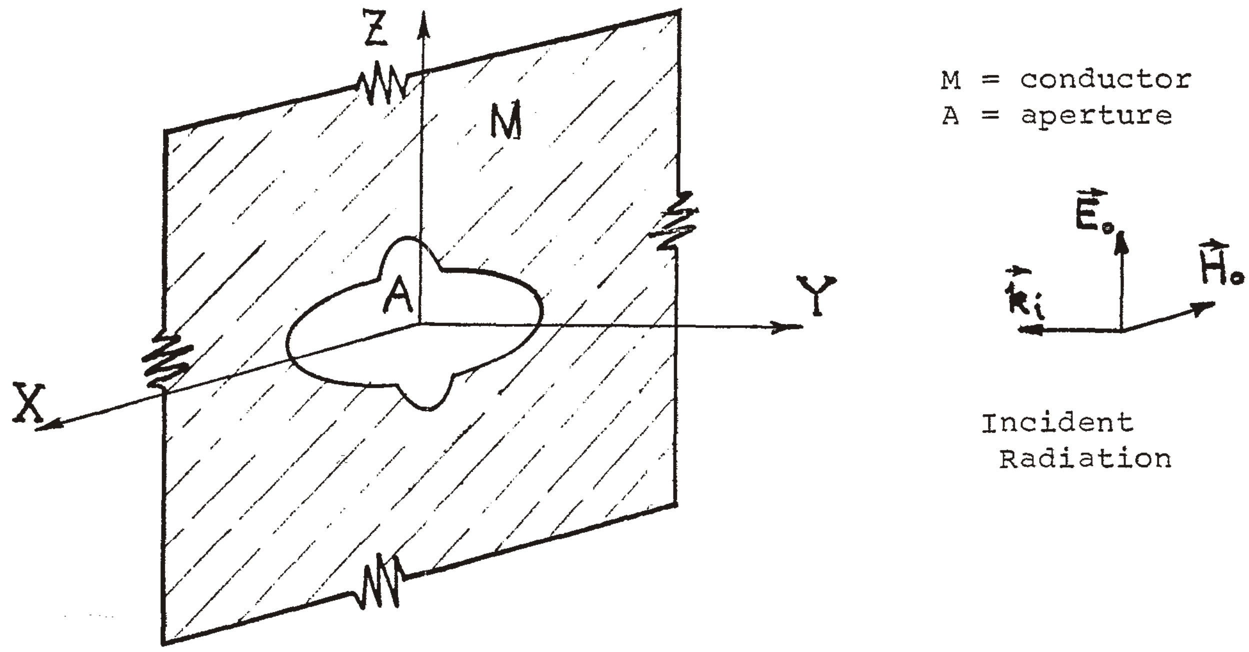
The principle states that if the solution of the diffraction problem posed by a perfectly conducting, infinitely thin plane screen with apertures of arbitrary size and shape is known, then the solution of the diffraction problem posed by the complementary screen, with the incident field suitably transformer, is also known. Specifically, the following theorem has been proved:

Let the electromagnetic field $\underset{\sim}{E}_o$, $\underset{\sim}{H}_o$ be incident in the half-space $y > 0$ on a perfectly conducting, infinitely thin screen M with apertures A in the plane $y = 0$ as illustrated in Figure 1. Let the total field in $y < 0$ be $\underset{\sim}{E}$, $\underset{\sim}{H}$.

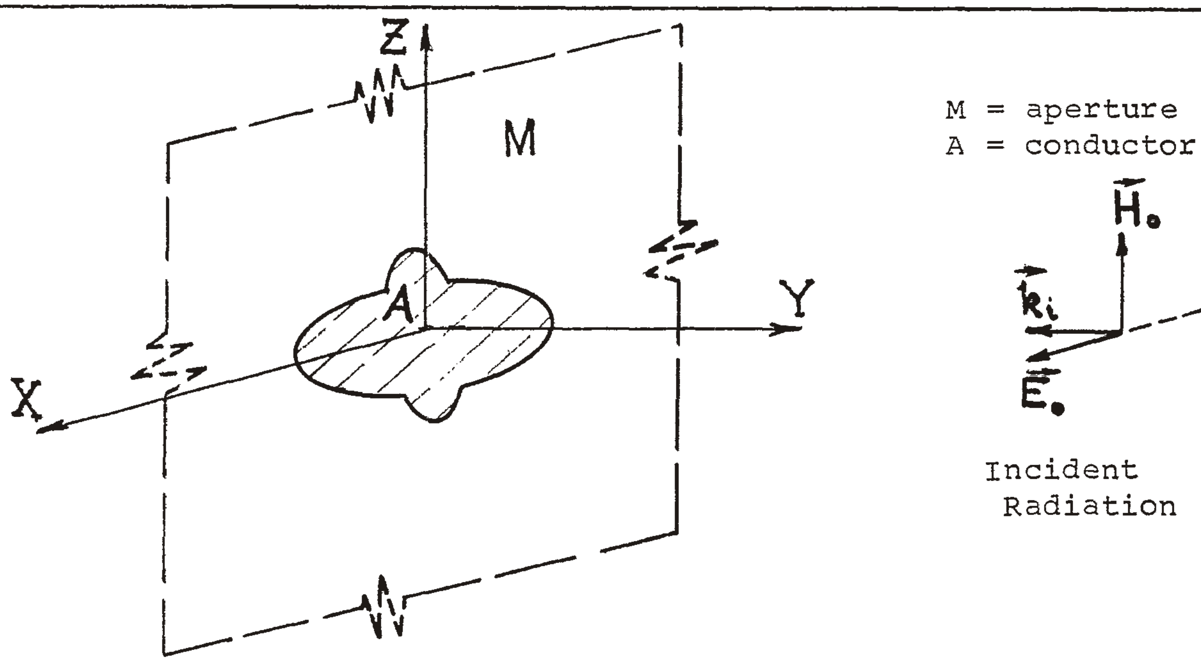
Replace the screen with its complement which we shall denote symbolically as $(M \leftrightarrow A)$. Let the incident electric and magnetic fields be $-\underset{\sim}{H}_o$ and $\underset{\sim}{E}_o$, respectively, and let the total field in $y < 0$ be $\underset{\sim}{E}'$ and $\underset{\sim}{H}'$. If the incident field is linearly polarized, the transformation $E_o \rightarrow -H_o$, $H_o \rightarrow E_o$ is equivalent to rotating the plane of polarization through a right angle counter-clockwise looking in the direction of propagation. Then

$$\underset{\sim}{E} = \underset{\sim}{H}' = \underset{\sim}{E}_o, \qquad \underset{\sim}{H} - \underset{\sim}{E}' = \underset{\sim}{H}_o. \tag{1}$$

This is Babinét's principle in its rigorous form. It is natural to inquire if this principle can be extended to include perforated diffracting surfaces of more general shape. The potential value of such a generalization should be quite clear. Our conclusion is that for two-dimensional diffraction problems (those independent of one Cartesian coordinate) direct application of the principle does not appear to be valid beyond the case of the plane. Whether



a) The Direct Diffraction Problem



b) The Complementary Problem

Figure 1. Babinét's Principle - The Plane Screen In z=0 Plane

Babinet's principle as it is known can be extended to special surfaces in three dimensions (besides a plane screen with arbitrary apertures) remains a completely open question. The null result for the two-dimensional case does, however, make the success of this program somewhat doubtful.

In Section II, we discuss the essential ingredients of Babinet's principle. In Section III, we show that the necessary conditions for an extension of Babinet's principle in its presently known form are not satisfied for two-dimensional diffraction problems posed by surfaces deviating from a plane.

II. ANALYSIS OF BABINET'S PRINCIPLE

The bases of Babinet's principle applied to a plane screen are:

(A) The special symmetry of an infinitesimally thin plane screen by which the total tangential magnetic field in its apertures is exactly equal to the value of an incident field, and

(B) The uniqueness theorem.

In essence, one shows that the boundary value problems for diffraction from the initial screen and its complement, with the incident electromagnetic and $\underset{\sim}{E}_o$, $\underset{\sim}{H}_o$ replaced with $-\underset{\sim}{H}_o$, $\underset{\sim}{E}_o$, are the same. This correspondence is a result of (A). Hence, if one problem has a unique solution (condition B), the two mutually complementary problems are simply identical, and Eq. (1) follows immediately.

To see this, consider the diffraction problems defined in the introduction. Let A denote the screen apertures of the initial problem and M its metal surface. The surface current induced on M by the incident field $\underset{\sim}{E}_o$, $\underset{\sim}{H}_o$ radiates a field $\underset{\sim}{E}_s$, $\underset{\sim}{H}_s$ in $y \leq 0$ that satisfied the conditions

(i) Maxwell's equations in $y < 0$,
(ii) the boundary conditions

$$\underset{\sim}{n} \times \underset{\sim}{E}_s = -\underset{\sim}{n} \times \underset{\sim}{E}_o \qquad \text{on } M,$$

$$\underset{\sim}{n} \times \underset{\sim}{H}_s = 0 \qquad \text{on } A,$$

where n is a unit vector normal to the screen;
(iii) the radiation condition in $y < 0$.
The boundary condition $\underset{\sim}{n} \times \underset{\sim}{H}_s = o$ on A is simply a restatement of

Condition A. Now consider the complementary problem with the incident field $-\underset{\sim}{H}_o$, $\underset{\sim}{E}_o$. Here the roles of A and M are interchanged. The total field $\underset{\sim}{E}'$, $\underset{\sim}{H}'$ in $y \leq 0$ is the superposition of the new incident field and the field radiated by the induced surface current on A. This field satisfies the conditions:

(iv) Maxwell's equations in $y < 0$

(v) the boundary conditions

$$\underset{\sim}{n} \times (-\underset{\sim}{H}') = -\underset{\sim}{n} \times \underset{\sim}{E}_o \qquad \text{on M,}$$

$$\underset{\sim}{n} \times \underset{\sim}{E} = 0 \qquad \text{on A,}$$

(vi) the radiation condition in $y < 0$.

The boundary condition $\underset{\sim}{n} \times \underset{\sim}{H}' = \underset{\sim}{n} \times \underset{\sim}{E}_o$ on M is a restatement of condition A. The last condition is an assumption. Physically, this requirement insures that the diffracted field $\underset{\sim}{E}'$, $\underset{\sim}{H}'$ is $y < 0$ can be calculated from knowledge of the field at $y = 0$ alone. If we make this assumption, then we note that the two fields $\underset{\sim}{E}_s$, $\underset{\sim}{H}_s$ and $-\underset{\sim}{H}'$, $\underset{\sim}{E}'$ are both solutions of Maxwell's equations in $y < 0$ and satisfy the same boundary conditions. If, in addition, we assume that there is only one field $\underset{\sim}{E}_s$, $\underset{\sim}{H}_s$ satisfying conditions (i) - (iii) in the source-free space $y < 0$ (condition B), then we can write the identities

$$\underset{\sim}{E}_s = -\underset{\sim}{H}' \quad , \quad \underset{\sim}{H}_s = \underset{\sim}{E}' \qquad (2)$$

These are equivalent to (1) since $\underset{\sim}{E} = \underset{\sim}{E}_o + \underset{\sim}{E}_s$ and $\underset{\sim}{H} = \underset{\sim}{H}_o + \underset{\sim}{H}_s$. Note that no mention has been made about the type of incident wave (e.g. plane, spherical, cylindrical, etc.) or its state of polarization. Secondly, no explicit mention has been made of possible singularities in the fields at the screens infinitely sharp edges. Nevertheless, restrictions on edge singularities are implicit here, since they are required in the proof of the uniqueness theorem. The reader is referred to the literature for details, (e.g. see the discussion of Wilcox[5] or that of Jones[6].)

We will now show by a rather simple argument based on the preceeding analysis that Babinet's principle for the plane does not generalize as is to the case of two-dimensional diffraction from slotted surfaces deviating from a plane.

III. DIFFRACTION FROM NONPLANAR SURFACES

Let the incident electromagnetic field E_o, H_o be diffracted by an infinitely thin, perfectly conducting slotted screen as illustrated by Figure 2. Let the screen be independent of one Cartesian coordinate, say Z. Suppose the orthogonal curvilinear coordinates ξ_1, ξ_2, and Z, with scale factors h_1, h_2, and 1, respectively, are appropriate for the diffracting screen under consideration. The scale factors are defined by the square of the distance ds between two infinitesimally separated points:

$$ds^2 = dx^2 + dy^2 + dz^2 = h_1{}^2 d\xi_1^2 + h_z^2 \, d\xi_2^2 + dz^2.$$

The diffracting screen is assumed to be, for example, in the surface $\xi_1 = C$. Let A denote the apertures and M the conducting surface. Then, we define the complementary diffraction problem by interchanging the roles of M and A and assume that M + A complete the entire surface $\xi_1 = C$. The incident field is assumed to be in $\xi_1 > C$. Our goal is to relate the fields in $\xi_1 < C$ when the roles of M and A are interchanged and the incident field is suitable transformed (not necessarily $\underset{\sim}{E}_o \rightarrow -\underset{\sim}{H}_o$, $\underset{\sim}{H}_o \rightarrow E_o$. See Eq. 4 below). The surface currents induced on M by the incident field radiate a field E_s, H_s in $\xi_1 \leq C$ that satisfies the conditions:

(vii) Maxwell's equations in $\xi_1 < C$,

(viii) The boundary condition

$$\underset{\sim}{n} \times \underset{\sim}{E}_s = -\underset{\sim}{n} \times \underset{\sim}{E}_o \qquad \text{on M,}$$

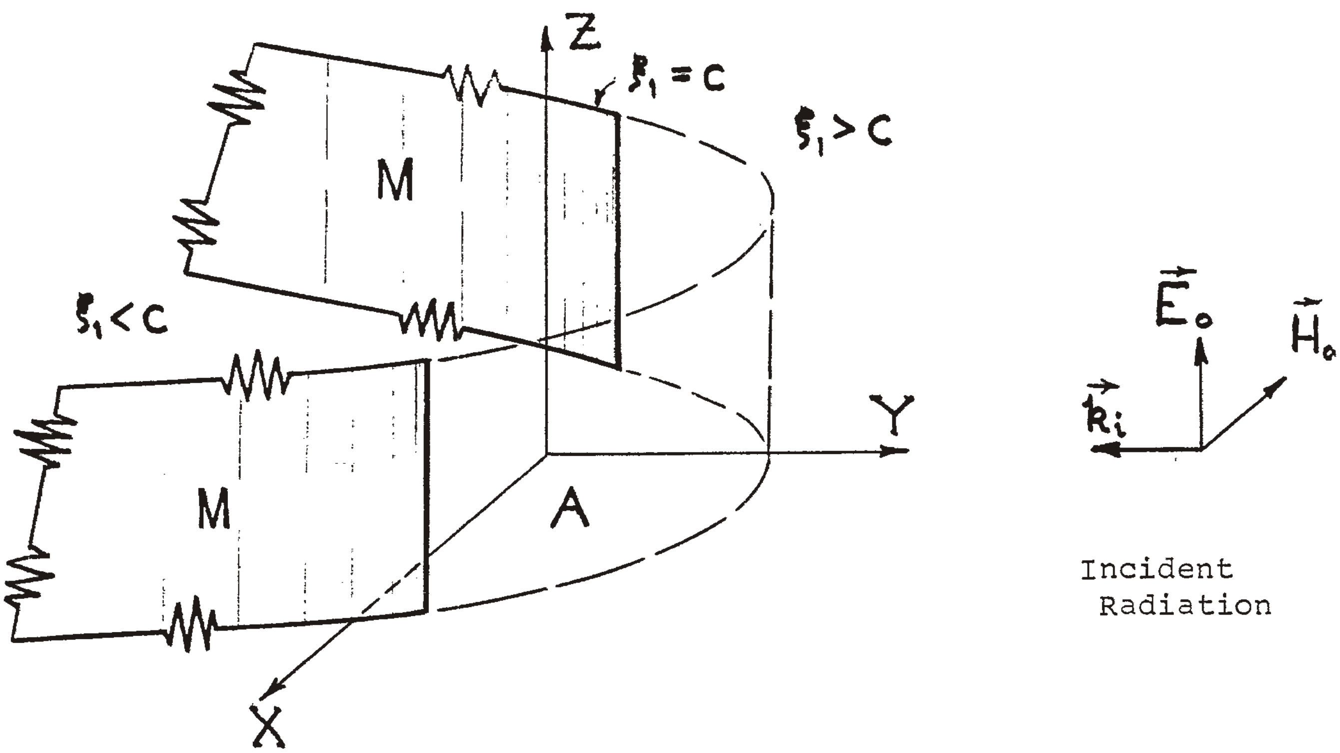
where n is a unit vector normal to M; and also

(ix) the radiation condition at infinity in $\xi_1 < C$.

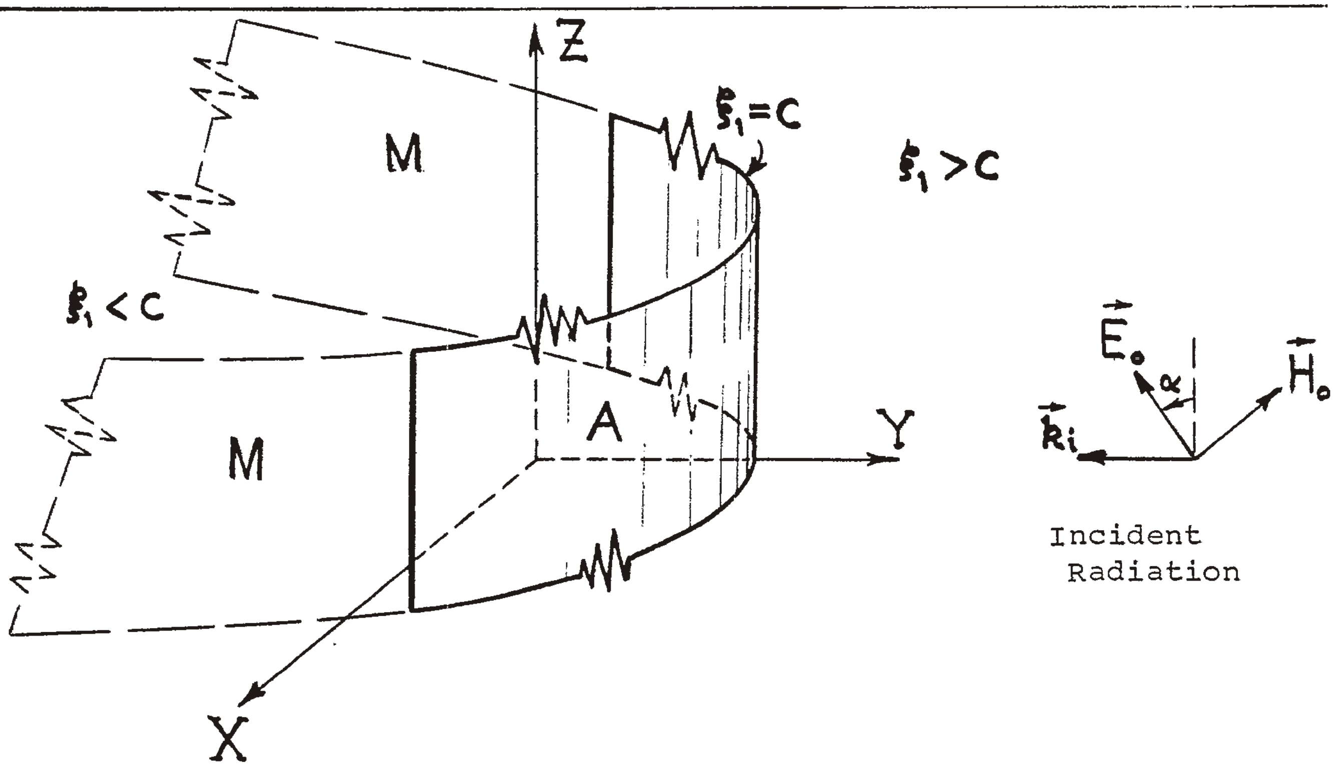
Throughout this section, unprimed (primed) fields will refer to the initial (complementary) diffraction screen. Following the analysis of Sec. II, we seek a field $\underset{\sim}{F}'$ in $\xi_1 \leq C$ of the complementary problem that satisfies condition (ix) and which, in conjunction with another field $\underset{\sim}{G}'$, satisfies condition (vii). In addition, $\underset{\sim}{F}'$ must satisfy the boundary condition

$$\underset{\sim}{n} \times (\underset{\sim}{F}' + \underset{\sim}{E}_o) = 0 \qquad \text{on M.} \qquad (3)$$

The existence of such a field is certainly a necessary condition for establishing a correspondence between the boundary value problems posed by the initial and complementary screens. This correspondence is required in order to invoke the uniqueness theorem



a) The Direct Problem



b) The Complementary Problem

Figure 2. Babinét's Principle - General Two-Dimensional Screen Problem

by which the field $\underset{\sim}{E}_s$ in $\xi_1 < C$ for the initial screen is related to $\underset{\sim}{F}'$ for the complementary screen. For the case of a plane screen, we found in Sec. II that $\underset{\sim}{F}' = -\underset{\sim}{H}'$. We will now show that $\underset{\sim}{F}'$ does not exist when the diffracting screen deviates from a plane surface.

Consider the complementary problem. The incident fields $\underset{\sim}{E}_o'$, $\underset{\sim}{H}_o'$ are assumed to be obtained from the fields $\underset{\sim}{E}_o$, $\underset{\sim}{H}_o$ according to the transformation

$$\begin{pmatrix} \underset{\sim}{E}_o' \\ \underset{\sim}{H}_o' \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \underset{\sim}{E}_o \\ \underset{\sim}{H}_o \end{pmatrix} \qquad (4)$$

The fields $\underset{\sim}{E}_o'$, $\underset{\sim}{H}_o'$ satisfy Maxwell's equations in free space if $\underset{\sim}{E}_o$, $\underset{\sim}{H}_o$ do. When $\alpha = \pi/2$, we obtain the incident field appropriate to the complementary diffraction problem posed by a plane screen.

The vector potential $\underset{\sim}{A}_s$ of the current $\underset{\sim}{J}$ induced on A by $\underset{\sim}{E}_o'$, $\underset{\sim}{H}_o'$ is

$$\underset{\sim}{A}_s(x) = \frac{1}{C} \int_A d^3x' \, \frac{J(x')}{|x-x'|} \, e^{ik|x-x'|} ,$$

where we have assumed a monochromatic incident wave with wave number k. The integral is taken over the conducting surface of the complementary screen, which now lies on A. The field $\underset{\sim}{E}_s'$, $\underset{\sim}{H}_s'$ radiated into $\xi_1 \leq C$ by $\underset{\sim}{J}$ is

$$\underset{\sim}{H}_s' = \nabla \times \underset{\sim}{A}_s \qquad (5a)$$

$$-ik\underset{\sim}{E}_s = k^2 \underset{\sim}{A}_s + \nabla(\nabla \cdot A_s). \qquad (5b)$$

These fields obviously satisfy conditions (vii) and (ix). However, since the tangential components of $\underset{\sim}{E}_s'$ and $\underset{\sim}{H}_s'$ do not generally vanish on M, they are not suitable candidates for the vector $\underset{\sim}{F}' + \underset{\sim}{E}_o$. To show this, we first put $\underset{\sim}{A}_s$ in a form more suitable for the two-dimensional diffraction problem posed by an infinitely thin screen. Since the complementary screen A is assumed to lie on the surface $\xi_1 = C$ and the problem is Z-independent, $\underset{\sim}{J}(\underset{\sim}{x}) \longrightarrow \underset{\sim}{J}(\xi_2)$. The element of length ds_1 normal to the surface $\xi_1 = C$ is $h_1 d\xi_1$. Thus, we define a surface current $\underset{\sim}{K}(\xi_2)$ by

$$\underset{\sim}{K}(\xi_2) = \underset{\sim}{J}(\xi_2) h_1 d\xi_1,$$

and obtain ($d^3x = h_1 h_2 d\xi_1 d\xi_2 dZ$)

$$\underset{\sim}{A}_s(\underset{\sim}{x}) = \frac{1}{c}\int_A h_2 d\xi_2' \int_{-\infty}^{\infty} dz' \frac{K(\xi_2)}{|\underset{\sim}{x}-\underset{\sim}{x}'|} e^{ik|\underset{\sim}{x}-\underset{\sim}{x}'|}$$

Performing the integral over Z', we get

$$\underset{\sim}{A}_s(\xi_1,\xi_2) = \frac{i\pi}{c}\int_A h_2 d\xi_2' \; H_o(k\rho)\, \underset{\sim}{K}(\xi_2'), \qquad (6)$$

where $\rho^2 = (x-x')^2 + (y-y')^2$ and $x = x(\xi_1,\xi_2)$, $y = y(\xi_1,\xi_2)$.

The Hankel function in (6) is of the first kind. Let us now calculate $\underset{\sim}{H}_s'$. Since $\underset{\sim}{K}$ has no component normal to the surface $\xi_1 = C$, this component of $\underset{\sim}{A}_s$ likewise vanishes. Hence,

$$\underset{\sim}{H}_s' = \frac{1}{h_2}\frac{\partial(\underset{\sim}{i}_3\cdot\underset{\sim}{A}_s)}{\partial\xi_2}\underset{\sim}{i}_1 - \frac{1}{h_1}\frac{\partial(\underset{\sim}{i}_3\cdot\underset{\sim}{A}_s)}{\partial\xi_1}\underset{\sim}{i}_2 + \frac{1}{h_1h_2}\frac{\partial(h_2\underset{\sim}{i}_2\cdot\underset{\sim}{A}_s)}{\partial\xi_1}\underset{\sim}{i}_3,$$

where $\underset{\sim}{i}_n$ are unit vectors normal to the surfaces ξ_n = constant and $\xi_3 = Z$. Accordingly,

$$\underset{\sim}{n}\times\underset{\sim}{H}_s' = -\frac{1}{h_1}\frac{\partial(\underset{\sim}{i}_3\cdot\underset{\sim}{A}_s)}{\partial\xi_1}\underset{\sim}{i}_3 - \frac{1}{h_1h_2}\frac{\partial(h_2\underset{\sim}{i}_2\cdot\underset{\sim}{A}_s)}{\partial\xi_1}\underset{\sim}{i}_2 \qquad (7)$$

For $\underset{\sim}{H}_s'$ to be a suitable candidate for $\underset{\sim}{E}' + \underset{\sim}{E}_o$, the coefficients of $\underset{\sim}{i}_2$ and $\underset{\sim}{i}_3$ in (7) must separately vanish on M. Consider the coefficients of $\underset{\sim}{i}_3$:

$$\frac{\partial(\underset{\sim}{i}_3\cdot\underset{\sim}{A}_s)}{\partial\xi_1} = \frac{i\pi k}{c}\int_A h_2 d\xi_2' \; H_o'(k\rho)\, K_3(\xi_2')\frac{\partial\rho}{\partial\xi_1} \qquad (8)$$

The value of the integrand in (8) on $M(\xi_1 = C)$ is, in general, nonvanishing, since $\partial\rho/\partial\xi_1 > 0$ for a convex surface as can be seen in Fig. 3. Thus, the Z-component of $\underset{\sim}{n}\times\underset{\sim}{H}_s'$ is not identically zero on M. For the coefficient of $\underset{\sim}{i}_2$ in (7), we get

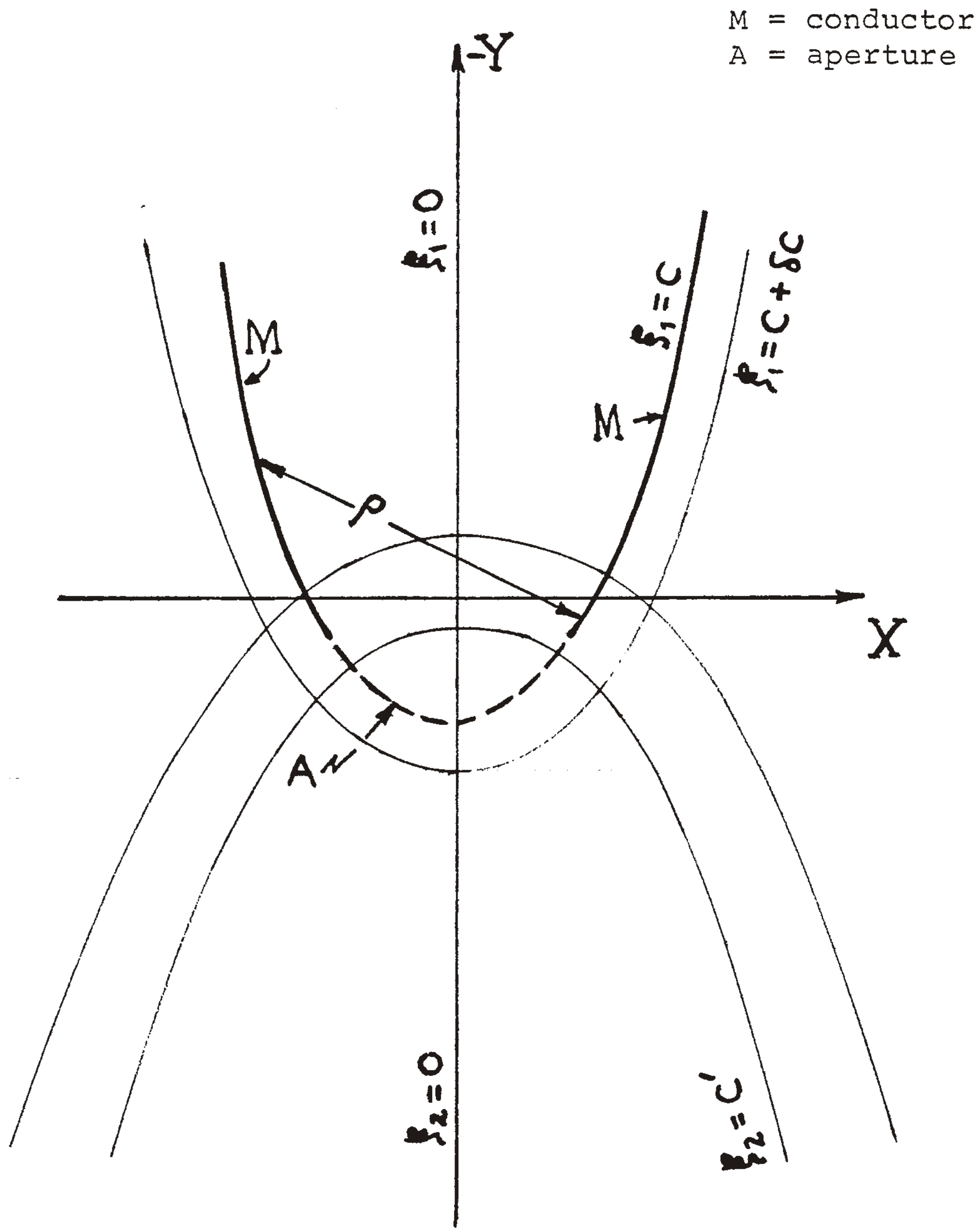


Figure 3. Cross-section of the Slotted Cylinder $\xi_1=c$ in the z=0 Plane Showing the Chord ρ satisfies $\frac{\partial\rho}{\partial\xi_1}\neq 0$

$$\frac{\partial (h_2 \underset{\sim}{i}_2 \cdot \underset{\sim}{A}_s)}{\partial \xi_1} = i\pi \int_A h_2 d\xi_2' \, K_2(\xi_2') \left[H_o(k\rho) \frac{\partial h_2(\xi_1, \xi_2)}{\partial \xi_1} + kh_2(\xi_1, \xi_2) \cdot H_o'(k\rho) \frac{\partial \rho}{\partial \xi_1} \right] \quad (9)$$

Both terms within the square brackets of Eq. (9) are, in general, nonvanishing on M. The cancellation of the two terms everywhere on M would involve an intricate relation between the screen geometry and the frequency of the incident wave. We reject this possibility as unphysical. Since the integral (9) is not identically zero on M, $\underset{\sim}{H}_s'$ cannot be a candidate for $\underset{\sim}{F} + \underset{\sim}{E}_o$. Nor is $\underset{\sim}{E}_s'$ a suitable candidate. For we have from (5b)

$$-ik \, \underset{\sim}{n} \times \underset{\sim}{E}_s' = -\left\{ k^2 (\underset{\sim}{i}_3 \cdot \underset{\sim}{A}_s) + \frac{1}{h_2} \frac{\partial}{\partial \xi_2} \left[\frac{1}{h_1 h_2} \frac{\partial}{\partial \xi_2} (h_1 \underset{\sim}{i}_2 \cdot \underset{\sim}{A}_s) \right] \right\} \underset{\sim}{i}_2 + k^2 (\underset{\sim}{i}_2 \cdot \underset{\sim}{A}_s) \, \underset{\sim}{i}_3 \quad (10)$$

Again, the coefficients of $\underset{\sim}{i}_2$ and $\underset{\sim}{i}_3$ in (10) must separately vanish for $\underset{\sim}{E}_s'$ to be acceptable. The coefficient of $\underset{\sim}{i}_3$, the second component of $\underset{\sim}{A}_s$, is obviously not identically zero on M for arbitrary polarization of the incident wave. The vanishing of the coefficient of $\underset{\sim}{i}_2$ over M would again require the existence of a relation between screen geometry and incident wave frequency, which we reject.

The next possibility for $\underset{\sim}{F}' + \underset{\sim}{E}_o$ is a linear combination of $\underset{\sim}{E}_s'$ and $\underset{\sim}{H}_s'$. The most general combination satisfying conditions (vii) and (ix) above is

$$\underset{\sim}{E}_s' \longrightarrow \underset{\sim}{\mathcal{E}}_s' = \cos\varphi \, \underset{\sim}{E}_s' + \sin\varphi \, \underset{\sim}{H}_s' ,$$

$$\underset{\sim}{H}_s' \longrightarrow \underset{\sim}{\mathcal{H}}_s' = \cos\varphi \, \underset{\sim}{H}_s' - \sin\varphi \, \underset{\sim}{E}_s' ,$$

where φ is an arbitrary angle. Take, for example, the Z-component of $\underset{\sim}{n} \times \underset{\sim}{\mathcal{E}}_s'$:

$$\underset{\sim}{i}_3 \cdot (\underset{\sim}{n} \times \underset{\sim}{\xi}'_s) = ik\,(\underset{\sim}{i}_2 \cdot \underset{\sim}{A}_s)\cos\varphi = \frac{1}{h_1}\frac{\partial(\underset{\sim}{i}_3 \cdot \underset{\sim}{A}_s)}{\partial\xi_1}\sin\varphi = \frac{ik\pi}{c}\int_A h_2 d\xi'_2 \cdot$$

$$\left[iH_o(k\rho)\,K_2(\xi'_2)\cos\varphi - \frac{1}{h_1(\xi_1,\xi_2)}\,H'_o(k\rho)\,\frac{\partial\rho}{\partial\xi_1}\,K_3(\xi'_2)\sin\varphi\right]$$

A necessary condition for this integral to vanish identically on M is for

$$\frac{1}{h_1(\xi_1,\xi_2)}\,\frac{\partial}{\partial\xi_2}\,\ln H_o\,(k\rho)\Big|_{\xi_1 = C} = \xi\text{-independent.}$$

Such a constraint, for surfaces other than a plane, mixes screen geometry with the incident wave frequency and is, therefore, physically unacceptable. Examination of the remaining tangential field components indicates that they cannot vanish everywhere on M without equally unacceptable constraints on frequency and geometry.

The final nontrivial candidates for the vector $\underset{\sim}{F}' + \underset{\sim}{E}_o$ are the total diffracted electric ($\underset{\sim}{E}'$) and magnetic ($\underset{\sim}{H}'$) fields in $\xi_1 < C$, where

$$\underset{\sim}{E}' = \underset{\sim}{E}'_o + \underset{\sim}{E}'_s \quad , \quad \underset{\sim}{H}' = \underset{\sim}{H}'_o + \underset{\sim}{H}'_s \; ,$$

and where the fields $\underset{\sim}{E}'_o$, $\underset{\sim}{H}'_o$, and $\underset{\sim}{E}'_s$, $\underset{\sim}{H}'_s$ are given by Eqs. (4) and (5), respectively. We assume that $\underset{\sim}{E}'$ and $\underset{\sim}{H}'$ satisfy the radiation condition at infinity, $\xi_1 < C$, in order that they may be calculated everywhere in $\xi_1 < C$ from knowledge of their value on the boundary $\xi_1 = C$ alone. We must now check the consistency of condition (3). Take $\underset{\sim}{E}'$ for example:

$$\underset{\sim}{n} \times \underset{\sim}{E}' = \underset{\sim}{n} \times \underset{\sim}{E}'_s + \cos\alpha\; \underset{\sim}{n} \times \underset{\sim}{E}_o - \sin\alpha\; \underset{\sim}{n} \times \underset{\sim}{H}_o \quad . \qquad (11)$$

Now is the limit when the screen A of the complementary problem vanishes, $\underset{\sim}{E}'_s = 0$. Therefore, the incident field on the aperture M of the complementary problem, which is now the whole surface $\xi_1 = C$, must satisfy the constraint

$$\underset{\sim}{n} \times \underset{\sim}{E}_o = \tan\alpha\; \underset{\sim}{n} \times \underset{\sim}{H}_o \quad \text{on M} \qquad (12)$$

if $\underset{\sim}{E}'$ is to be a candidate for $\underset{\sim}{F}' + \underset{\sim}{E}_o$. The constraint (12) is obviously too severe. Likewise, the condition that $\underset{\sim}{n} \times \underset{\sim}{H}' = 0$ on M becomes, in the limit A = 0,

$$\underset{\sim}{n} \times \underset{\sim}{E}_o = -\cot\alpha\ \underset{\sim}{n} \times \underset{\sim}{H}_o \quad \text{on M.}$$

Since the requirement that the tangential components of $\underset{\sim}{E}'$ and $\underset{\sim}{H}'$ vanish on M leads to physically unacceptable constraints on the incident field in the trivial case of no diffracting screen, we reject $\underset{\sim}{E}'$ and $\underset{\sim}{H}'$ as possible candidates for $\underset{\sim}{F}' + \underset{\sim}{E}_o$. Possible linear combinations of $\underset{\sim}{E}'$ and $\underset{\sim}{H}'$ that are consistent with (vii) and (ix) are ruled out by arguments similar to that which lead us to reject linear combinations of $\underset{\sim}{E}_s'$ and $\underset{\sim}{H}_s'$.

In summary, we are led to conclude that the vector $\underset{\sim}{F}'$ does not exist when the initial and complementary surfaces deviate from a plane.

IV. CONCLUSION

We have studied the bases of the conventional form of Babinet's principle applied to a plane. In essence, one finds two sets of fields from the initial and complementary problems, respectively, that satisfy the same boundary conditions on a common closed surface S, and which satisfy Maxwell's equations inside S. The interior of S is assumed to be source-free. Having found the two sets of fields that pose identical boundary value problems, the uniqueness theorem is invoked to justify a strict equality between them. The result is a well-defined set of rules for obtaining the solution of the complementary diffraction problem from the (supposedly) known solution of the initial diffraction problem.

We have attempted to generalize Babinet's principle to nonplanar surfaces that are independent of one Cartesian coordinate. The screens of the initial and complementary problems, together with the points at infinity, are assumed to complete a closed surface S whose interior is source-free. From our analysis of the plane, we concluded that a necessary condition for a generalization of Babinet's principle was the existence of a field $\underset{\sim}{\mathfrak{F}}$ ($\underset{\sim}{F}' + \underset{\sim}{E}_o$ in the analysis of Sec. III) whose tangential components vanished identically on the apertures of the complementary screen. This field corresponds to the vanishing of the total tangential diffracted electric field on the screen of the initial problem. To complete the correspondence, $\mathfrak{F}$, when combined with another field $\underset{\sim}{\mathcal{G}}$, must satisfy Maxwell's equation in the interior of S and the radiation condition at infinity on S. Our proof that Babinet's principle, as we know it for the plane, cannot be generalized to nonplanar surfaces consisted in showing that $\mathfrak{F}$ does not exist. Our proof is independent of the type of incident wave (plane, cylindrical, parabolic, etc) and its state of polarization. In addition, the incident fields on the complementary screen may be a rather general linear combination of the incident fields on the initial screen (see Eq. 4). Finally, we expect our proof to hold when the initial and complementary screens cover a surface S that is infinite in extent, as would be the case for a slotted circular or elliptic cylinder. In this case, the requirement that $\mathfrak{F}$ satisfy the radiation condition at infinity on S is replaced by the condition that $\mathfrak{F}$ remains finite inside S.

We cannot, of course, rule out completely the existence of a more generalized set of rules that enable one to calculate the diffracted fields of the complementary screen from the diffracted fields of the initial screen when these screens are nonplanar. If such rules do exist, we suspect that what one defines as the "complementary diffraction problem" will have to be somewhat modified from the way it is defined here.

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First Discussions on Generalizing Babinet's Principle in Two Dimensions

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13. ABSTRACT

We present in this report our first efforts to thoroughly understand Babinet's principle for electromagnetic diffraction and the results of our initial efforts to extend the principle beyond its usual range of application. Proof is given that the principle in its present form is invalid for general two dimensional, infinitely thin, perfectly conducting surfaces containing apertures. The only case in which it is valid is the conventional one of an infinite plane.

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