Interaction Notes

Note 178

May 1974

Effects of a Dielectric Jacket of a Braided-Shield Cable on EMP Coupling Calculations

Lennart Marin
Dikewood Corporation, Westwood Research Branch
Los Angeles, California 90024

Abstract

A detailed analysis is presented of how the electric polarizability of an aperture in a cable shield varies with the thickness and the permittivity of the dielectric jacket as well as with the permittivity of the insulating medium within the shield. From a knowledge of this variation the influence of the dielectrics is then determined on the current source term of the transmission-line equations describing the braided-shield cable.

This project has been fully supported by the
Defense Nuclear Agency (DNA) under

DNA Subtask EB088
EMP Interaction and Coupling
DNA Work Unit 32
Coaxial Cables

I. Introduction

The shield of a cable is used, in addition to being part of the cable itself in some cases, to shield against external electromagnetic disturbances. A commonly used type of shields is the braided-wire shield because of its mechanical flexibility. A schematic picture of a braided-shield cable is shown in Figure 1. Exterior to the shield is a dielectric jacket and inside the shield are conductors immersed in an insulating medium. Since there are always small holes between the wires in the braid this type of shield is not perfect in that a small fraction of the electric field outside the cable can penetrate through these holes into the cable.

In the past, a considerable effort has been directed towards the calculation of leakage of electromagnetic fields through the cable shield into the cable. One usual approach is to derive a set of transmission-line equations from which calculations can be made of the currents and voltages induced in the load by the fields outside the cable shield. In deriving the transmission-line equations for a braided-shield cable one common approach is to incorporate the concepts used in statics into the conventional transmission-line theory[1-5]. Another approach[6] starts out from the Maxwell equations and the transmission-line equations are then derived by using a modal analysis and by keeping only the dominant mode (which is justified in the frequency regime where transmission-line theory is applicable).

So far the effects of the dielectric jacket and the dielectric insulation within the shield have been neglected in all shielding calculations except in some limiting cases[3,7]. The purpose of this note is to incorporate these effects into the transmission-line equations previously derived for braided-shield cables. It will be assumed that the apertures are small compared with the radius of the cable, so that their electromagnetic effect can be described by an electric dipole moment $\underline{p}$ and a magnetic dipole moment $\underline{m}$. The dielectric properties of the jacket and the insulating medium in the cable will, of course, only influence the electric polarizability of the aperture.

By applying a first order perturbation theory to the Maxwell equations the following transmission-line equations have been derived for the voltage and current in the braided shield[6]

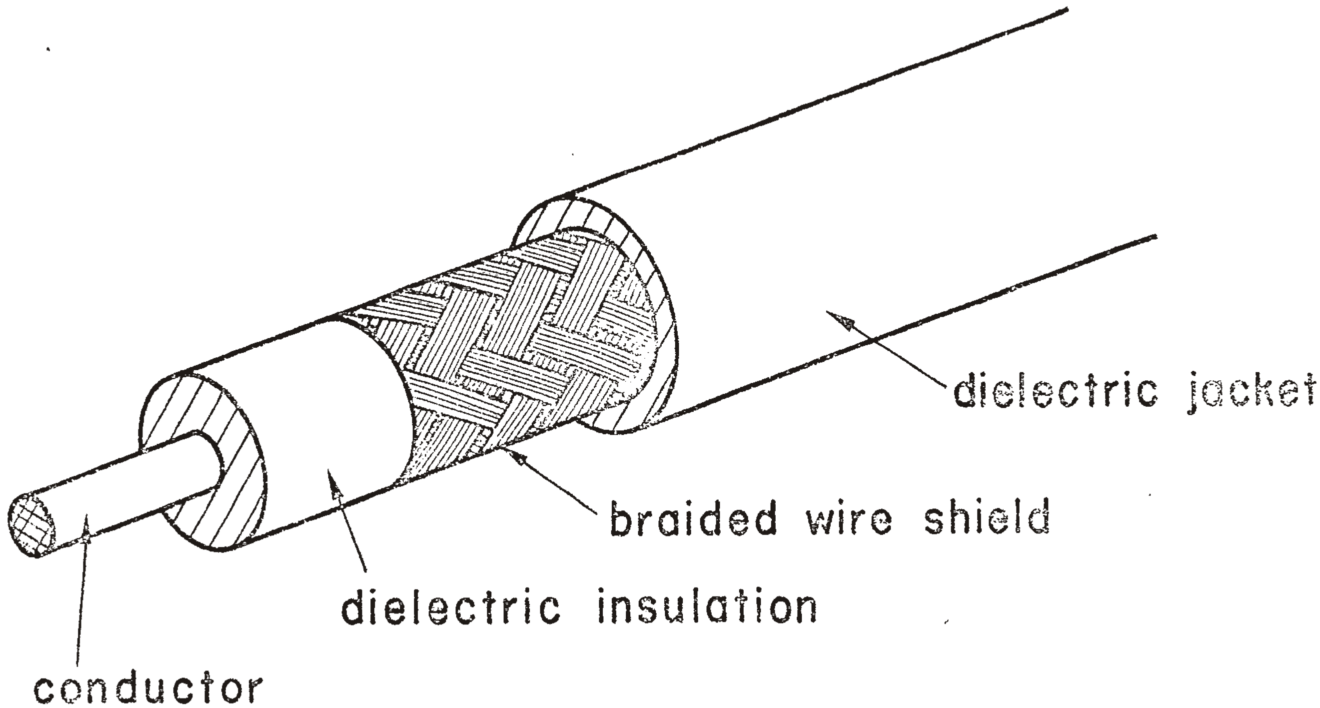


Figure 1. A braided-shield cable with a dielectric jacket.

$$\frac{dV}{dz} = i\omega L(1 + \delta_m)I + i\omega L\delta_m I^{ext}$$

$$\frac{dI}{dz} = i\omega C(1 - \delta_e)V - i\omega\delta_e Q^{ext} .$$

This set of equations is valid when no dielectrics are present or when the entire space outside the cable is filled with one homogeneous dielectric and/or the insulating medium inside the cable consists of another homogeneous medium. Usually $\delta_e << 1$ so that the factor $\delta_e C$ modifying the characteristic capacitance of the cable is small compared to C. In this note we will therefore concentrate our efforts on the calculation of the dielectric effects on the electric-source driving term in the transmission-line equations for a more realistic model of the cable.

In order to develop a feeling for the dielectric effects on the electric polarizability of the aperture we will "isolate" this effect by treating a problem where, for example, the effects due to the curvature of the shield, the interactions between different apertures, the presence of other conductors, and the hole shape are neglected. An estimate of all these effects have been made previously[3].

In Section II, a derivation is given of the dipole moment of a circular aperture in a plane screen sandwiched between two layers of dielectrics. One of these layers represents the dielectric jacket and is of finite thickness, while the other represents the cable insulation and can be taken as a half-space. The dipole moment can be expressed in terms of the solution of a set of dual integral equations and the solution of these equations is presented in Section III. Finially, in Section IV, certain dielectric effects on the dipole moment of a longitudial slot in a circular cylinder is studied.

II. Formulation of a Boundary-Value Problem

In this section we will formulate a boundary-value problem, the solution of which shows the effects of the dielectrics on the polarizability of the aperture. In the model that we use it is assumed that the aperture is small compared to the radius of curvature of the cable, so that we can consider the aperture to be located in an infinite ground plane. Moreover, since our main effort is concentrated on the dielectric effects we choose the aperture's shape to be circular thereby simplifying the analysis somewhat. Finally, the distance between the shield and any conductor inside the braided cable is assumed to be large compared to the size of the aperture and so we can neglect the influence of these inner conductors when calculating the polarizability of the aperture.

Keeping these assumptions in mind we can formulate the boundary-value problem depicted in Fig. 2. The relative dielectric constant of the insulation inside the cable is denoted by ε_1 and the relative dielectric constant of the surrounding jacket is ε_2. The medium outside the jacket is assumed to be air having the relative dielectric constant $\varepsilon_{air} = 1$. The incident electric field, which is perpendicular to the ground plane, is denoted by $\underline{E}_o$, the radius of aperture by a, and the thickness of the jacket by h.

The electric field in region 1 (see Fig. 2) far away from the aperture can be expressed in terms of the dipole moment $\underline{p}$ of the aperture[6],

$$\underline{p} = -\varepsilon_1 \varepsilon_o \hat{z} \int_A \Phi dS \tag{1}$$

where Φ is the electrostatic potential in the aperture. To find this dipole moment we will solve the Laplace equation for the electrostatic potential in regions 1, 2, 3 together with the proper boundary conditions at the junctions of the media and the correct behavior of the field at infinity. Let $\Phi_n(\rho, z)$ denote the electrostatic potential in region n $(n = 1,2,3)$ and we can derive the following expressions for the electrostatic potential by performing a Hankel transform on the Laplace equation, in cylindrical coordinates,

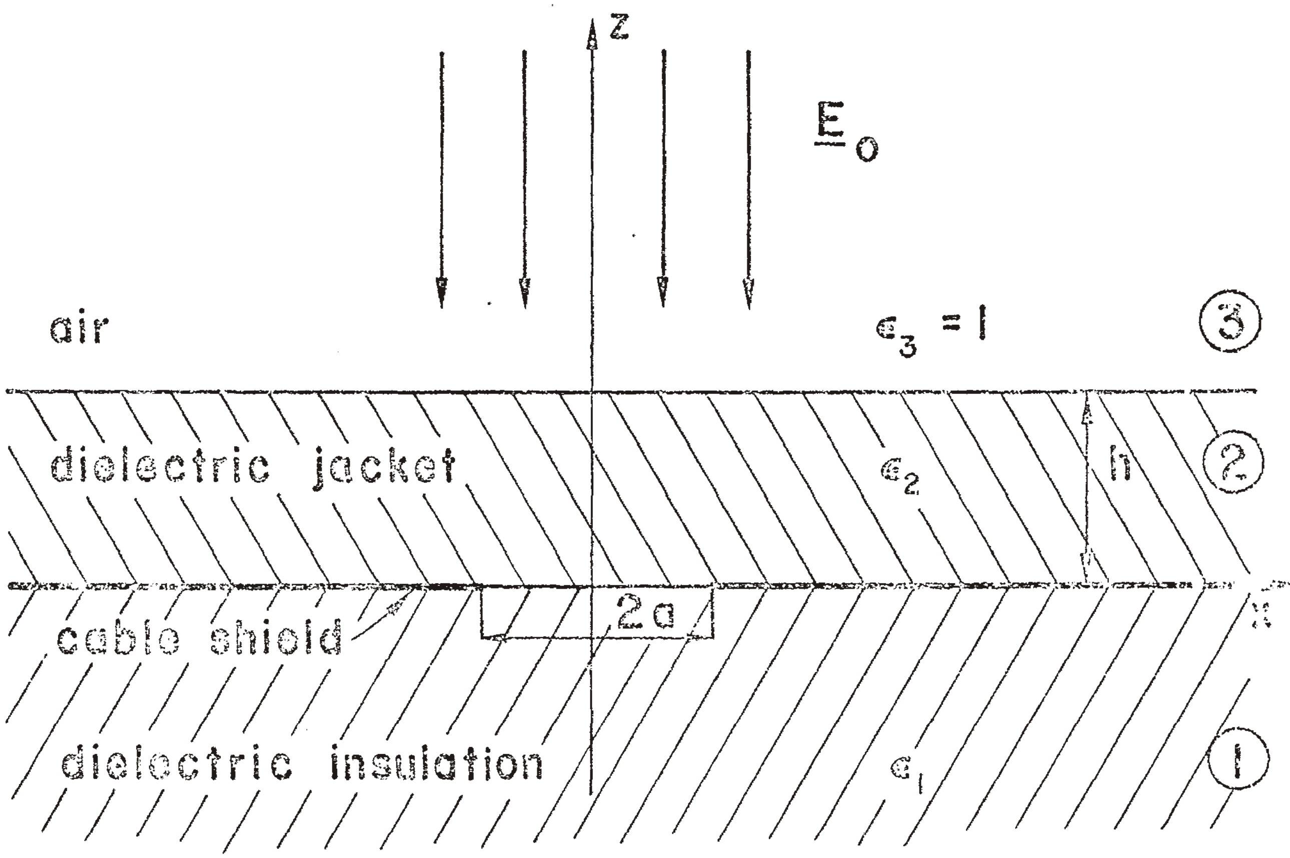


Figure 2. A circular aperture in a plane screen sandwiched between two layers of dielectrics.

$$\Phi_1(\rho,z) = \int_0^\infty A(\lambda)\exp(\lambda z)J_o(\lambda\rho)d\lambda, \qquad z < 0$$

$$\Phi_2(\rho,z) = \int_0^\infty [B(\lambda)\exp(\lambda z) + C(\lambda)\exp(-\lambda z)]J_o(\lambda\rho)d\lambda + E_o z/\varepsilon_2, \qquad 0 < z < h \qquad (2)$$

$$\Phi_3(\rho,z) = \int_0^\infty D(\lambda)\exp(-\lambda z)J_o(\lambda\rho)d\lambda + E_o z - E_o h(\varepsilon_2 - 1)/\varepsilon_2, \qquad z > h$$

where $J_o(x)$ is the Bessel function of the first kind and the functions $A(\lambda)$, $B(\lambda)$, $C(\lambda)$, and $D(\lambda)$ are to be determined from the boundary conditions at $z = 0$ and $z = h$. The integrals in (2) can be viewed as the "scattered" potential, i.e., the influence on the field due to the aperture, and this part of the potential approaches zero as the radius of the aperture tends to zero.

The boundary conditions at $z = h$ (i.e., the electrostatic potential and the normal component of the displacement vector being continuous) imply that

$$\Phi_2(\rho,h) = \Phi_3(\rho,h), \qquad \rho \geq 0$$

$$\varepsilon_2 \frac{\partial\Phi_2}{\partial z}(\rho,h) = \frac{\partial\Phi_3}{\partial z}(\rho,h), \qquad \rho \geq 0. \qquad (3)$$

Similarly, the boundary conditions at $z = 0$ can be expressed mathematically in the following way:

$$\Phi_1(\rho,0) = \Phi_2(\rho,0), \qquad \rho \geq 0$$

$$\Phi_1(\rho,0) = \Phi_2(\rho,0) = 0, \qquad \rho > a \qquad (4)$$

$$\varepsilon_1 \frac{\partial\Phi_1}{\partial z}(\rho,0) = \varepsilon_2 \frac{\partial\Phi_2}{\partial z}(\rho,0), \qquad 0 \leq \rho < a.$$

Of these equations the first one expresses the continuity of the potential at $z = 0$, the second one the fact that the potential is zero on the plate, and the third one the continuity of the normal component of the displacement vector across the aperture.

From (2), (3) and (4) we can derive the following relationships:

$$B(\lambda) = \frac{(\varepsilon_2-1)A(\lambda)}{\varepsilon_2-1+(\varepsilon_2+1)\exp(2\lambda h)}$$

$$C(\lambda) = \frac{(\varepsilon_2+1)A(\lambda)}{\varepsilon_2+1+(\varepsilon_2-1)\exp(-2\lambda h)} \tag{5}$$

$$D(\lambda) = \frac{2\varepsilon_2 A(\lambda)}{\varepsilon_2+1+(\varepsilon_2-1)\exp(-2\lambda h)} \, .$$

Substituting the expressions (2) and (5) into the boundary condition (4) we arrive at the following set of dual integral equations for $A(\lambda)$,

$$\int_0^\infty A(\lambda)J_o(\lambda\rho)d\lambda = 0, \qquad \rho > a$$

$$\int_0^\infty F(\lambda)A(\lambda)J_o(\lambda\rho)d\lambda = E_o, \qquad \rho < a \tag{6}$$

where

$$F(\lambda) = (\varepsilon_1 + \varepsilon_2)\lambda[1 + k_1(\lambda)] \tag{7}$$

and

$$k_1(\lambda) = \frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2}\left[\frac{\cosh(\lambda h) + \varepsilon_2 \sinh(\lambda h)}{\sinh(\lambda h) + \varepsilon_2 \cosh(\lambda h)} - 1\right]. \tag{8}$$

It should be pointed out that $k_1(\lambda)$ tends to zero when $\varepsilon_2 \to 1$ (no dielectric jacket) or when $h \to \infty$ (a very thick dielectric jacket). In these special cases an explicit solution of the set of dual integral equations (6) can be found[8].

To find a solution of (6) in the general case we introduce the normalized quantities:

$$u = \rho/a, \qquad \xi = a\lambda,$$

$$k(\xi) = k_1(\lambda), \qquad X(\xi) = \frac{\varepsilon_1+\varepsilon_2}{E_o a^2} A(\lambda) \tag{9}$$

and we arrive at the following set of equations for $X(\xi)$,

$$\int_0^\infty \xi[1 + k(\xi)]X(\xi)J_o(u\xi)d\xi = 1, \qquad 0 \leq u < 1$$

$$\int_0^\infty X(\xi)J_o(u\xi)d\xi = 0, \qquad u > 1. \tag{10}$$

Before we go on to solve (10) let us see how we can express the quantity of interest, namely the dipole moment, in terms of the solution of (10). After some simple manipulations on (1), (2) and (10) we can derive the following representation of $\underline{p}$,

$$\underline{p} = -2\pi\varepsilon_1\varepsilon_o E_o a^3(\varepsilon_1 + \varepsilon_2)^{-1}\hat{z} \int_0^\infty \xi^{-1}X(\xi)J_1(\xi)d\xi. \tag{11}$$

In conclusion, we have in this section reduced the problem of finding the electrostatic potential in the aperture with a dielectric coating to the solution of the set of dual integral equations (10). Once the solution of these equations has been found, the equivalent dipole moment of the aperture can be obtained by performing the simple integration (11), and the dielectric effects on the polarizability of the aperture can be determined. In the next section we will study the polarizability of the aperture by solving (10), analytically in some limiting cases and numerically in the general case.

III. Solution of the Integral Equations

In this section we will solve the set of dual integral equations (10). In the most general case it is not possible to find a closed form solution of (10). Numerical methods must therefore be used to "solve" (10). Later in this section we will formulate a Fredholm integral equation of the second kind with compact kernel and from the solution of this integral equation we can calculate the dipole moment of the aperture. However, when $h \gg a$ we can use perturbation techniques to obtain an approximate solution of (10). Also, we can derive some limiting forms of the solution of (10) when $h \ll a$.

A. The case $h \gg a$

The set of dual integral equations (10) can be reduced to the following Fredholm integral equation of the second kind[8]

$$X(\xi) + \frac{1}{\pi}\int_0^\infty \left[\frac{\sin(\xi-\eta)}{\xi-\eta} - \frac{\sin(\xi+\eta)}{\xi+\eta}\right] k(\eta)X(\eta)d\eta = G(\xi), \qquad \xi \geq 0 \tag{12}$$

where

$$G(\xi) = \frac{2(\sin \xi - \xi \cos \xi)}{\pi\xi} . \tag{13}$$

The function $k(\xi)$ (see (8)) can be expanded in the series

$$k(\xi) = \frac{2\varepsilon_2}{\varepsilon_1+\varepsilon_2} \sum_{n=1}^{\infty} \gamma^n \exp(-n\beta\xi) \tag{14}$$

where

$$\beta = 2h/a, \qquad \gamma = (1 - \varepsilon_2)/(1 + \varepsilon_2). \tag{15}$$

For $\beta \gg 1$ we can, after some simple but tedious algebraic manipulations, derive the following approximate solution of (10),

$$X(\xi) = G(\xi) - rQ_3(\gamma)G(\xi)\beta^{-3} - rQ_5(\gamma)[2G''(\xi) - 1.2G(\xi)]\beta^{-5}$$
$$+ r^2Q_3^2(\gamma)G(\xi)\beta^{-6} + O(\beta^{-7}) \tag{16}$$

where

$$r = \frac{8\varepsilon_2}{3\pi(\varepsilon_1+\varepsilon_2)} ,$$

$$Q_m(\gamma) = \sum_{n=1}^{\infty} \gamma^n n^{-m}, \qquad m = 3,5. \tag{17}$$

The dipole moment of the aperture as defined by (1) can be expressed in the following way,

$$\underline{p} = \underline{p}_o F(\varepsilon_1,\varepsilon_2,\beta) \tag{18}$$

with

$$\underline{p}_o = -\frac{4\varepsilon_1\varepsilon_o E_o a^3}{3(\varepsilon_1+\varepsilon_2)} \hat{z}. \tag{19}$$

Equations (11) and (16) enable us to derive the following expression for $F(\varepsilon_1,\varepsilon_2,\beta)$

$$F(\varepsilon_1,\varepsilon_2,\beta) = 1 - rQ_3(\gamma)\beta^{-3} + 2.4rQ_5(\gamma)\beta^{-5} + r^2Q_3^2(\gamma)\beta^{-6} + O(\beta^{-7}). \tag{20}$$

It is observed from (20) that the normalized factor $F(\varepsilon_1,\varepsilon_2,\beta)$ approaches unity rather fast for large values of β. From the numerical results that we will present in the next section it will be seen that the asymptotic expansion (20) is valid for $\beta \geq 2$, and that in this regime the dipole moment differs less than 2% from its value for $\beta \to \infty$.

B. The case $h \ll a$

To obtain an integral equation, which can be solved with perturbation techniques when $\beta \ll 1$, we first make the substitution

$$Y(\xi) = \frac{1+\varepsilon_1}{\varepsilon_1+\varepsilon_2} X(\xi) \tag{21}$$

and $Y(\xi)$ satisfies the set of dual integral equations

$$\int_0^\infty \xi[1 + \ell(\xi)]Y(\xi)J_o(u\xi)d\xi = 1, \qquad 0 \leq u < 1$$

$$\int_0^\infty Y(\xi)J_o(u\xi)d\xi = 0, \qquad u > 1, \tag{22}$$

where

$$\ell(\xi) = (\varepsilon_2 - 1)/(\varepsilon_2 + 1) + (\varepsilon_1 + \varepsilon_2)(\varepsilon_1 + 1)^{-1}k(\xi) \tag{23}$$

and

$$\ell(\xi) = (\varepsilon_2^2 - 1)[2\varepsilon_2(\varepsilon_1 + 1)]^{-1}\xi\beta^{-1} + O(\beta^{-3}), \qquad \beta << 1. \tag{24}$$

An asymptotic solution of (22), valid as $\beta \to 0$, can be obtained by first reducing (22) to a Fredholm integral equation of the second kind similar to (12) and then solving the resulting integral equation with perturbation techniques. With this method we derive the following asymptotic representation of $Y(\xi)$

$$Y(\xi) = \frac{2(\sin\xi - \xi\cos\xi)}{\pi\xi} + \frac{(\varepsilon_2^2-1)\sin\xi}{\pi^2\varepsilon_2(\varepsilon_1+1)}\beta\ln\beta + O(\beta). \tag{25}$$

For small values of β the normalized factor $F(\varepsilon_1,\varepsilon_2,\beta)$ is asymptotically given by

$$F(\varepsilon_1,\varepsilon_2,\beta) = \frac{\varepsilon_1+\varepsilon_2}{\varepsilon_1+1} + \frac{3(\varepsilon_2^2-1)(\varepsilon_1+\varepsilon_2)}{2\pi\varepsilon_2(\varepsilon_1+1)^2}\beta\ln\beta + O(\beta). \tag{26}$$

Comparing the two expressions (20) and (26) we observe that, as expected, the dipole moment of the aperture without the dielectric jacket $(\beta = 0)$ is larger than its value when the thickness of the dielectric is large compared to the aperture size $(\beta = \infty)$. We also expect $F(\varepsilon_1,\varepsilon_2,\beta)$ to vary between the two limiting values given by (26) and (20) as β varies between zero and infinity. The numerical calculations show that this is indeed the case.

C. The general case

To find an equation suitable for a numerical evaluation of the dipole moment in the general case we introduce the normalized potential $f(u)$ in the aperture, defined by

$$f(u) = (\varepsilon_1 + \varepsilon_2)\phi(ua,0)/(aE_o) \tag{27}$$

and $f(u)$ is related to the solution of the set of dual integral equations (10) by

$$f(u) = \int_0^\infty J_o(\xi u)X(\xi)d\xi. \tag{28}$$

After some algebraic manipulations on (10) and (28) we arrive at the following Fredholm integral equation for $f(u)$[8]

$$f(u) + \int_0^1 L(u,v)f(v)dv = \frac{2}{\pi}\sqrt{1 - u^2}, \qquad 0 \leq u \leq 1 \tag{29}$$

where

$$L(u,v) = \frac{2v}{\pi}\int_0^\infty\int_u^1 \frac{\sin s\eta}{\sqrt{s^2-u^2}}\eta k(\eta)J_o(v\eta)d\eta ds. \tag{30}$$

To find the solution of (29) we first extend the domain of definition of $L(u,v)$ to $(-1,1)\times(-1,1)$, so that $L(u,v)$ is an even function in both u and v. Similarly, we extend the domain of definition of $f(u)$ to $(-1,1)$, so that $f(u)$ is an even function of u. Expanding $f(u)$ in terms of the Chebyshev polynomials of the second kind, $U_{2n}(u)$[9],

$$f(u) = \sqrt{1 - u^2}\sum_{n=0}^\infty f_n U_{2n}(u) \tag{31}$$

we can transform the integral equation (29) to the following set of algebraic equations by applying the Galerkin method,

$$f_n + \sum_{m=0}^\infty L_{nm}f_m = \frac{2}{\pi}\delta_{no} \tag{32}$$

where δ_{nm} is the Kronecker symbol and

$$L_{nm} = \frac{2}{\pi}\int_0^\infty \eta k(\eta) G_n(\eta) H_m(\eta) d\eta. \tag{33}$$

The function $G_n(\eta)$ can be expressed in terms of the spherical Bessel functions,

$$G_n(\eta) = (-1)^n \eta [j_{n-1}(\eta/2) y_{n-1}(\eta/2) - j_{n+1}(\eta/2) y_{n+1}(\eta/2)] \tag{34}$$

while $H_m(\eta)$ is given by

$$H_m(\eta) = \sum_{\ell=0}^{m} \frac{(-1)^m (m+\ell)!(2\ell)!}{(\ell!)^2 (m-\ell)! \eta^{2\ell+1}} \left[1 - \cos\eta \sum_{k=0}^{\ell} \frac{(-1)^k \eta^{2k}}{(2k)!} \right.$$

$$\left. + \sin\eta \sum_{k=1}^{\ell} \frac{(-1)^k \eta^{2k-1}}{(2k-1)!} \right]. \tag{35}$$

The normalized factor $F(\varepsilon_1, \varepsilon_2, \beta)$ can be obtained from $f(u)$ and the solution of (33) as follows:

$$F(\varepsilon_1, \varepsilon_2, \beta) = \frac{3\pi}{2} \int_0^1 u f(u) du$$

$$= \frac{3\pi}{2} \sum_{n=0}^{\infty} \frac{(-1)^{n+1} f_n}{(2n-1)(2n+3)} . \tag{36}$$

In the next section we will present the results of the numerical calculations performed based on the formulas presented in Sections II and III.

IV. Numerical Results

In this section we will present the results of the numerical calculations that have been performed. Especially, detailed results will be presented of the variation of the electric polarizability of the aperture with the thickness and permittivity of the dielectric jacket.

Equation (32) constitutes a form that is suitable for numerical solution. The integrand in (33) decays exponentially as $\exp(-4ha^{-1}\eta)$ (c.f. (8)), but for small values of h/a this decaying is very slow thus making it difficult to accurately evaluate (33) with numerical means. We therefore had to limit the numerical computations to $h/a \geq 0.1$. In the limiting case as h/a tends to infinity it can immediately be seen from (33) that the solution of (32) is given by $f_n = 2/\pi\delta_{no}$ and that the normalized factor $F(\varepsilon_1, \varepsilon_2, \infty) = 1$. In the numerical solution it was found that satisfactory accuracy (<1%) was obtained by truncating the infinite set of equations (32) to a set of 4 equations and 4 unknowns. This fast convergence of (31) can partly be attributed to the fact that each term in the expansion (31) satisfies the edge condition at $\rho = a$ $(u = 1)$.

The parameters in the transmission-line model of a braided-shield cable are the electric and magnetic polarizabilities[6]. Only the electric polarizability α_e is affected by the presence of the dielectric jacket and is related to the dipole moment p through

$$p = \alpha_e q \tag{37}$$

where q is the charge density on the exterior surface of the conducting wall when the aperture is short circuited, i.e., (note that normal component of $\underline{D}$ is continuous)

$$q = \varepsilon_o E_o . \tag{38}$$

We therefore get

$$\alpha_e = p/\varepsilon_o E_o$$

$$= \frac{2\varepsilon_o a^3}{3} \frac{2\varepsilon_1}{\varepsilon_1+\varepsilon_2} F(\varepsilon_1,\varepsilon_2,\beta)$$

$$\equiv \alpha_e^f \overline{\alpha}_e . \quad (39)$$

where α_e^f is the polarizability of a circular aperture in a conducting plane in vacuo,

$$\alpha_e^f = 2\varepsilon_o a^3/3 \quad (40)$$

and $\overline{\alpha}_e$ is given by

$$\overline{\alpha}_e = \frac{2\varepsilon_1}{\varepsilon_1+\varepsilon_2} F(\varepsilon_1,\varepsilon_2,\beta). \quad (41)$$

In Figures 3a-3g we have graphed $\overline{\alpha}_e(\varepsilon_1,\varepsilon_2,h/a)$ versus h/a for different values of ε_1 and ε_2. It is seen from these figures that $\overline{\alpha}_e$ is an increasing function of ε_1, whereas it is a decreasing function of ε_2 and h/a, as expected.

The asymptotic form (20) for $F(\varepsilon_1,\varepsilon_2,\beta)$ can be used to obtain an asymptotic form of $\overline{\alpha}_e$ for large values of h/a. This asymptotic form for $\overline{\alpha}_e$ deviates less than 10% from the exact form when $h/a \geq 0.8$. However, the difference between the asymptotic form and the exact form increases rapidly when $h/a < 0.8$. The limiting form (26) can be used to obtain the following expression for $\overline{\alpha}_e$ in the special case of vanishing thickness of the dielectric jacket

$$\overline{\alpha}_e(\varepsilon_1,\varepsilon_2,0) = 2\varepsilon_1/(1 + \varepsilon_1). \quad (42)$$

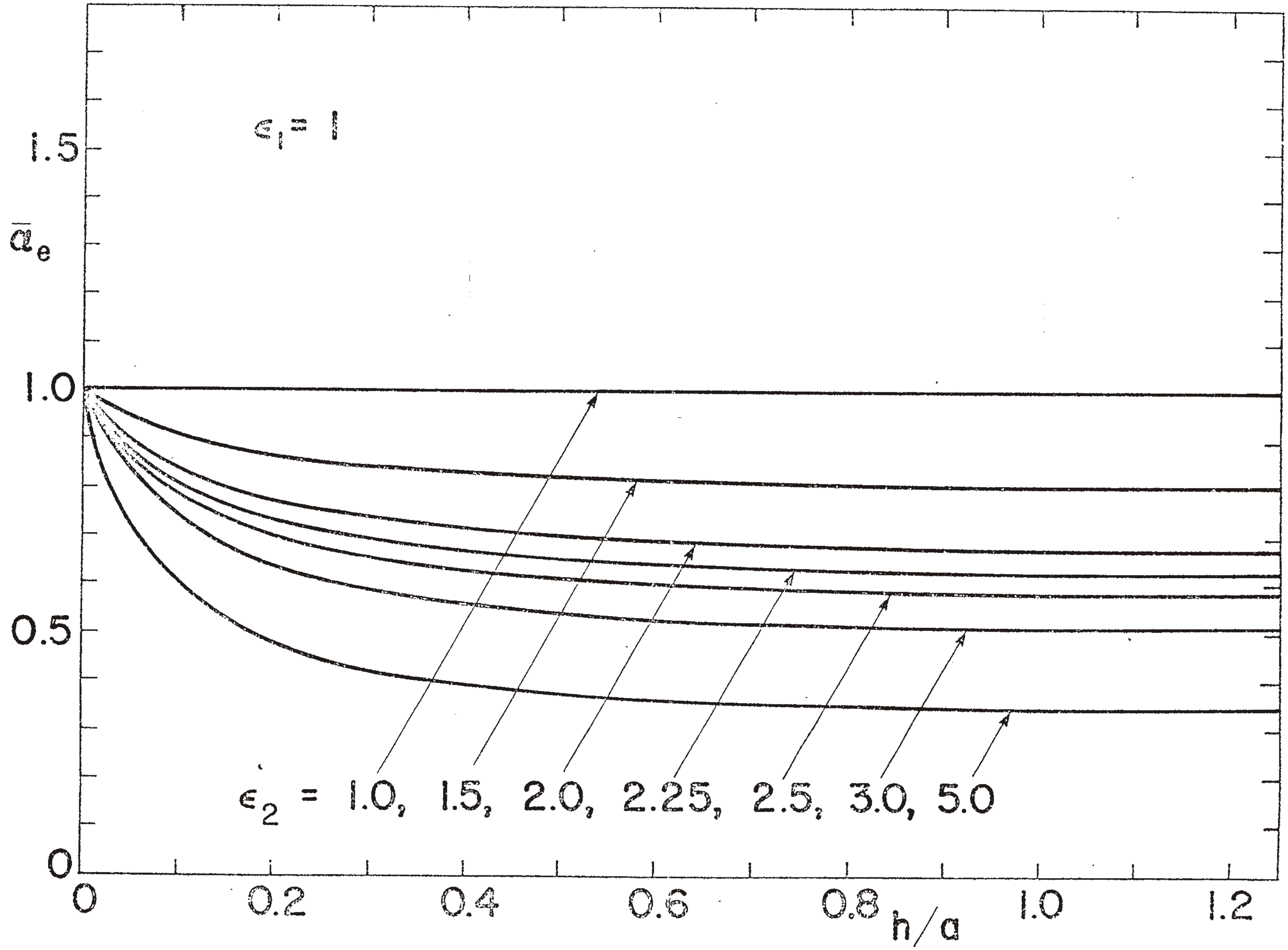


Figure 3a. The variation of the electric polarizability with the dielectric constants and the thickness of the dielectric layers.

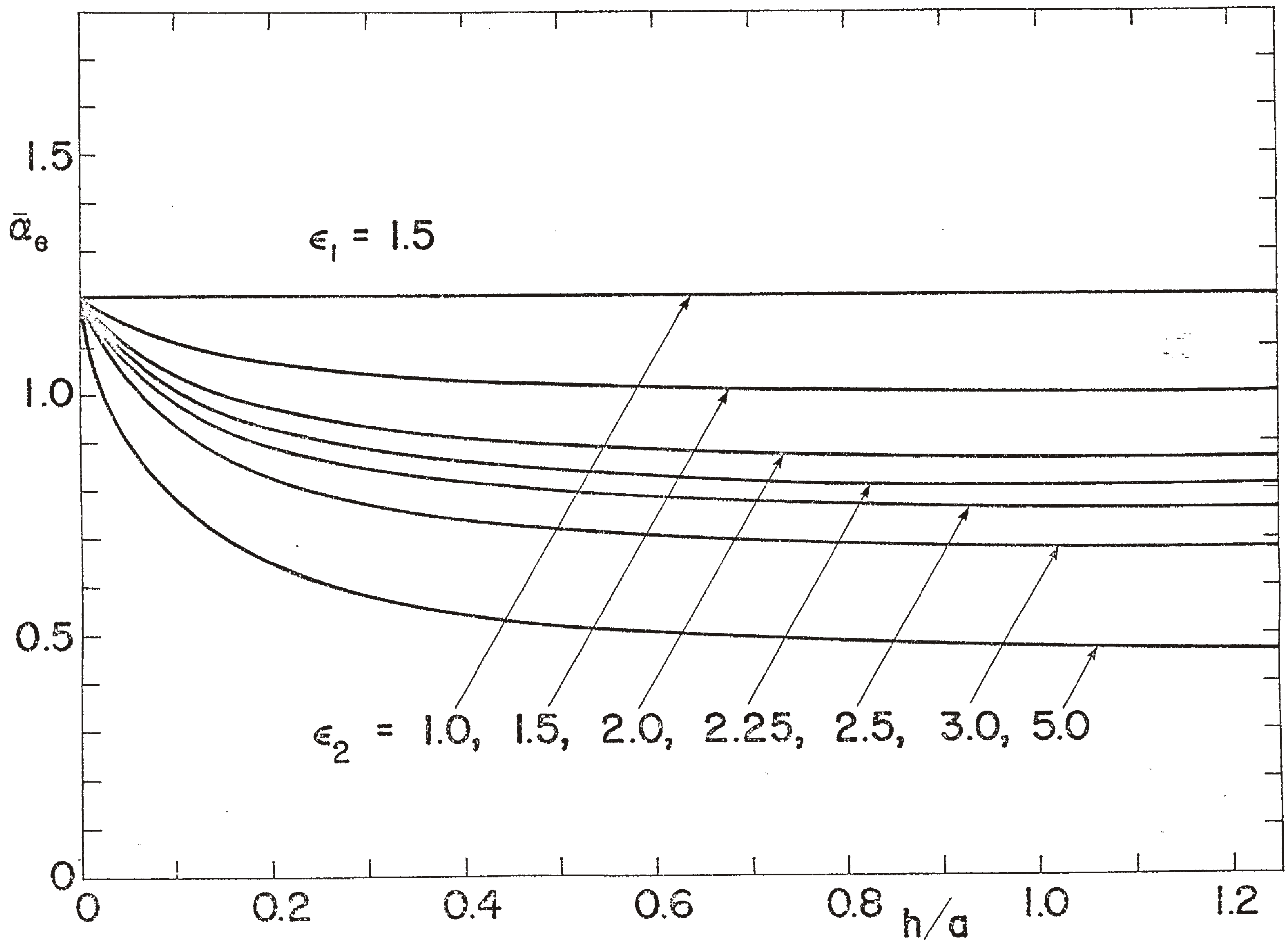


Figure 3b. The variation of the electric polarizability with the dielectric constants and the thickness of the dielectric layers.

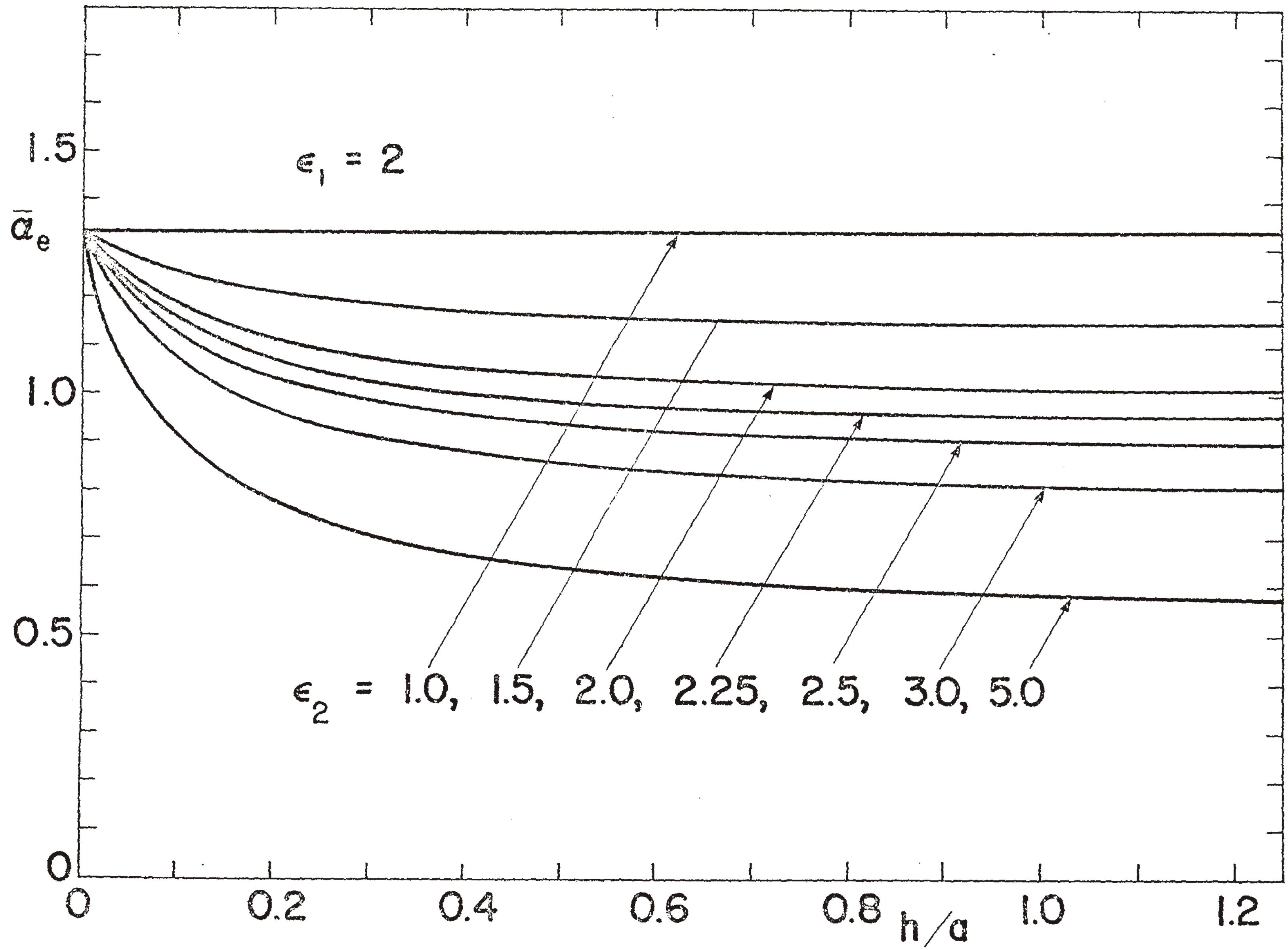


Figure 3c. The variation of the electric polarizability with the dielectric constants and the thickness of the dielectric layers.

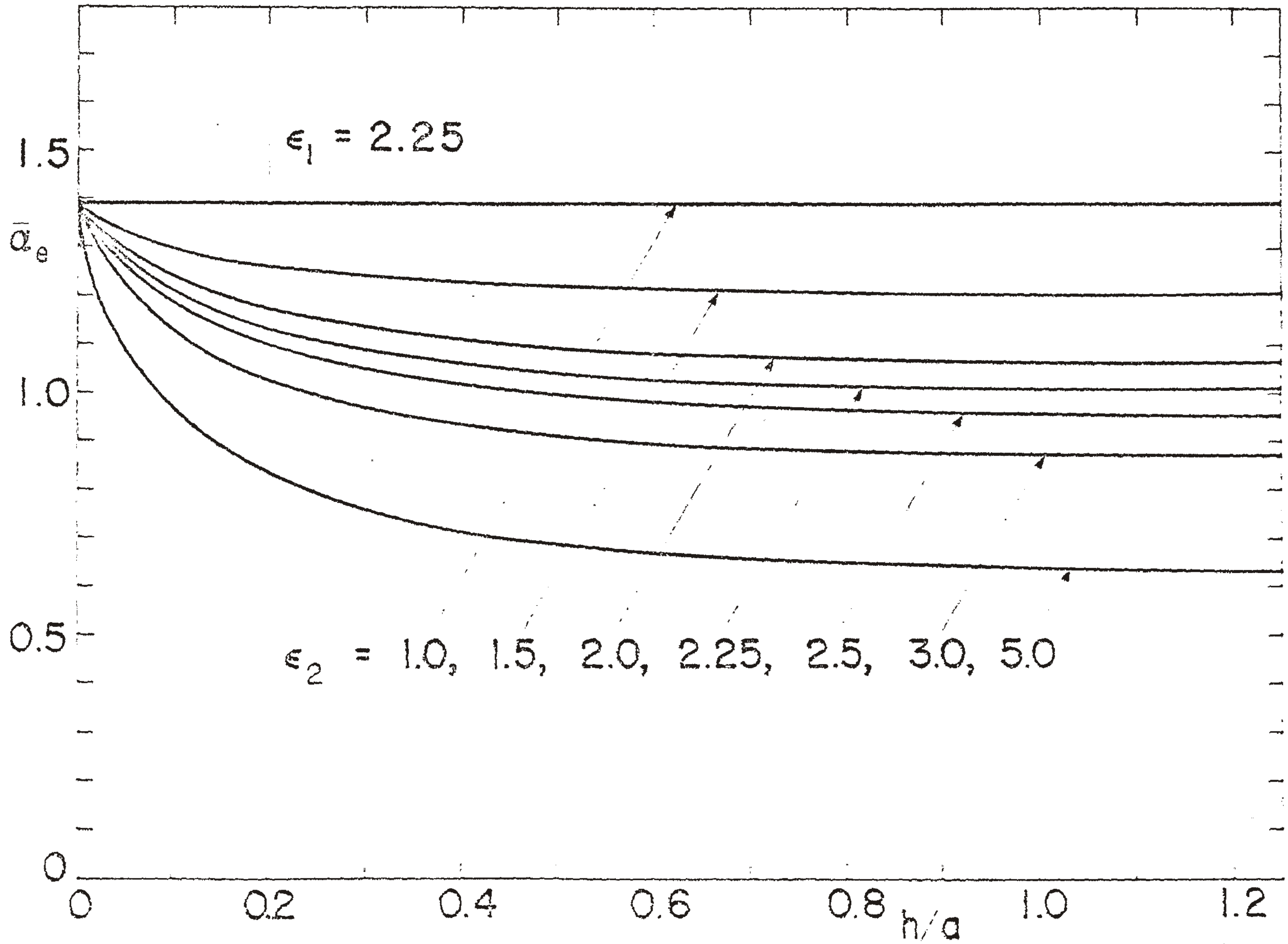


Figure 3d. The variation of the electric polarizability with the dielectric constants and the thickness of the dielectric layers.

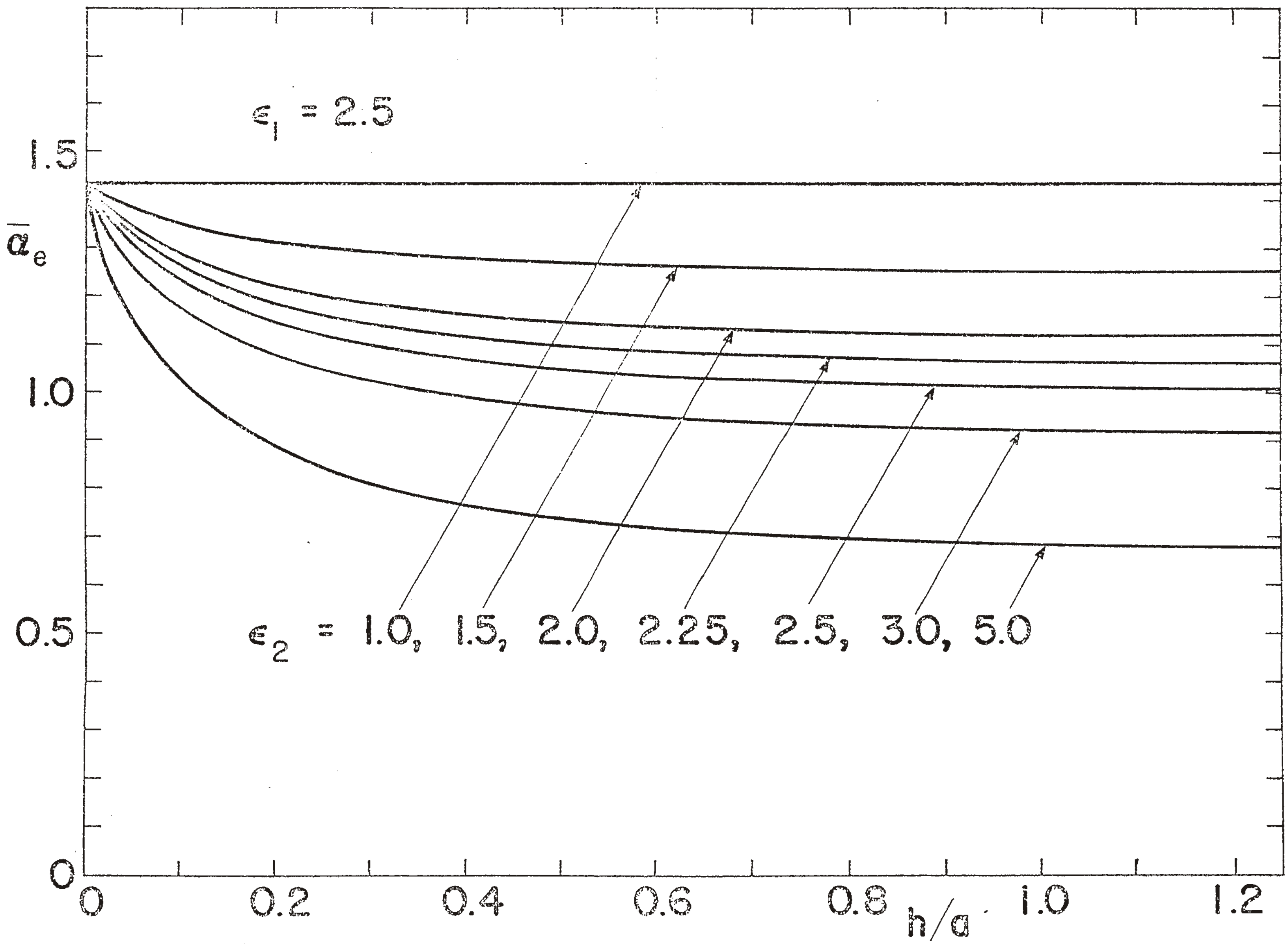


Figure 3e. The variation of the electric polarizability with the dielectric constants and the thickness of the dielectric layers.

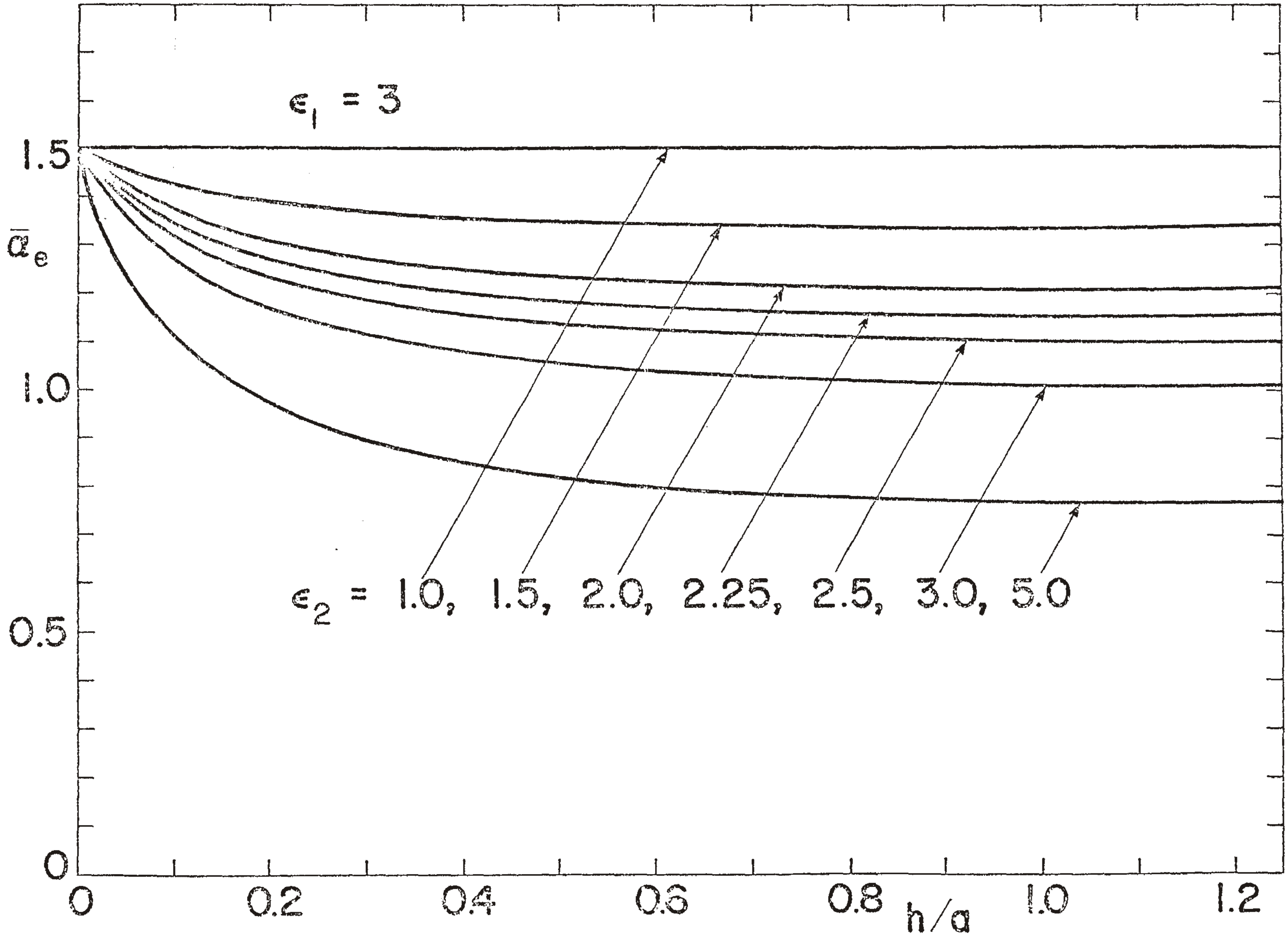


Figure 3f. The variation of the electric polarizability with the dielectric constants and the thickness of the dielectric layers.

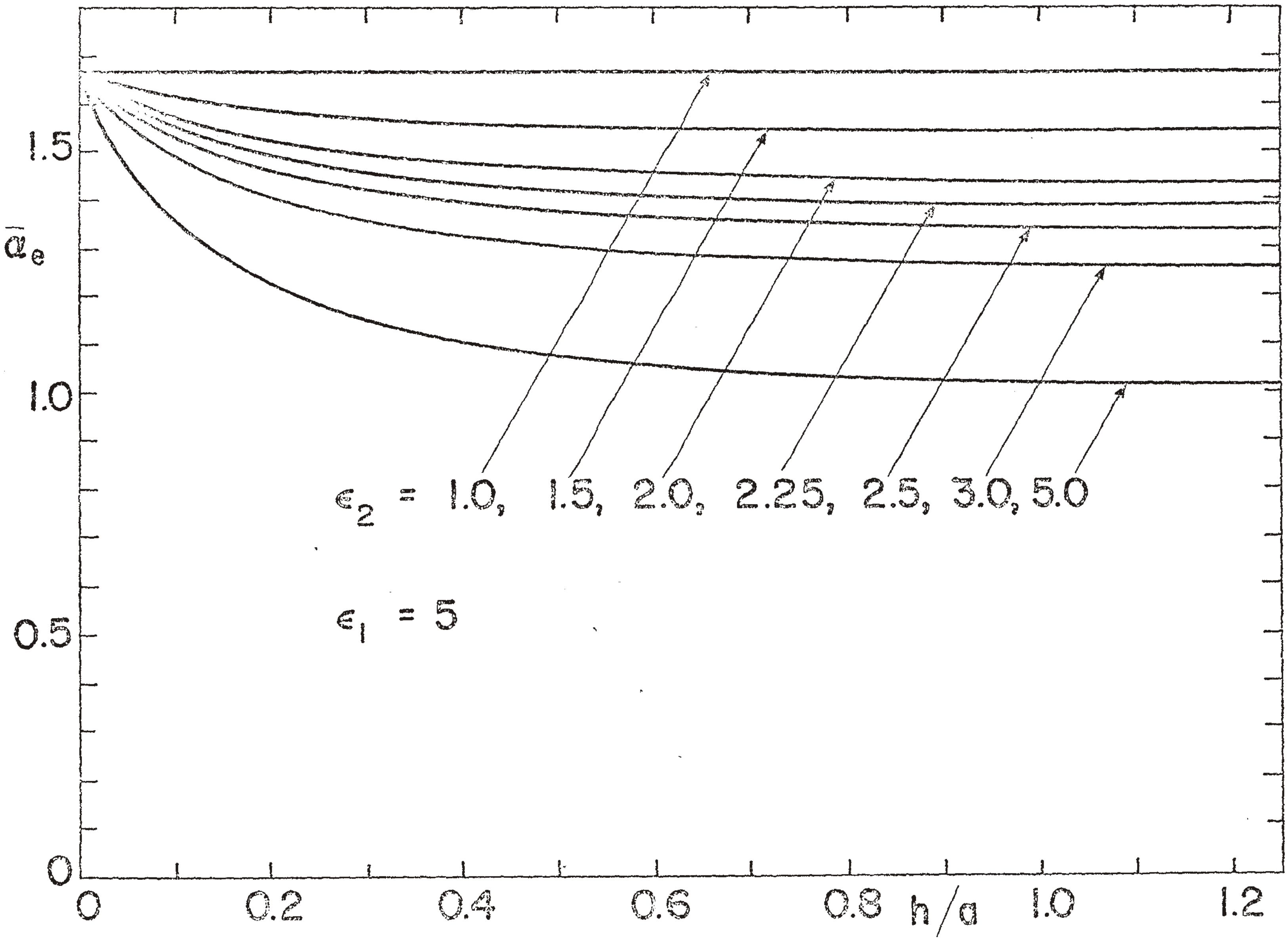


Figure 3g. The variation of the electric polarizability with the dielectric constants and the thickness of the dielectric layers.

V. A Longitudinal Slot in the Shield

In this section we will investigate the dielectric effects and the effects of the radius of curvature on the dipole moment of a longitudinal slot in a cable shield. It will be assumed that the transverse dimensions of the slot is small in terms of wave lengths of the incident field but that the length of the slot is large compared to the radius of the cable. The model we use is that of a cylinder with an infinite longitudinal slot, and a schematic representation of the structure is shown in Fig. 4. The radius of the cable is denoted by a, the exterior radius of the dielectric jacket by b, and the slot occupies the angle $2\phi_1$. The angle between the direction of the homogeneous incident electric field $\underline{E}_o$ and the outward normal at the center of the slot is denoted by ϕ_o. The permittivity of the insulating medium inside the cable $(\rho < a)$ is ε_1 and the dielectric constant of the jacket $(a < \rho < b)$ is ε_2.

Before we carry out the analysis of the problem just posed it is perhaps appropriate to point out the differences between our problem and the associated problem where the slotted cylinder has a net charge q per unit length. The latter problem can be solved trivially by conformal mapping[3] whereas our cannot be solved by conformal mapping because of the form of the excitation field. As we will see later, the solution to the problem formulated in the beginning of this section can be obtained from the theory of dual series equations. Also, for very narrow slits it is customary to think that the related problem is more appropriate (and, of course, more managable) for cable calculations. However, for slots of considerable width one cannot really separate the exterior and interior regions of the cable. In this sense, the solution of our problem complements the results already reported in [3] from both practical and theoretical view points.

The electric field can be determined from the electrostatic potential $\Phi(\rho,\phi)$ which satisfies the Laplace equation and certain regularity conditions at $\rho = 0$, $\rho \to \infty$ and therefore has the following representation in the respective regions:

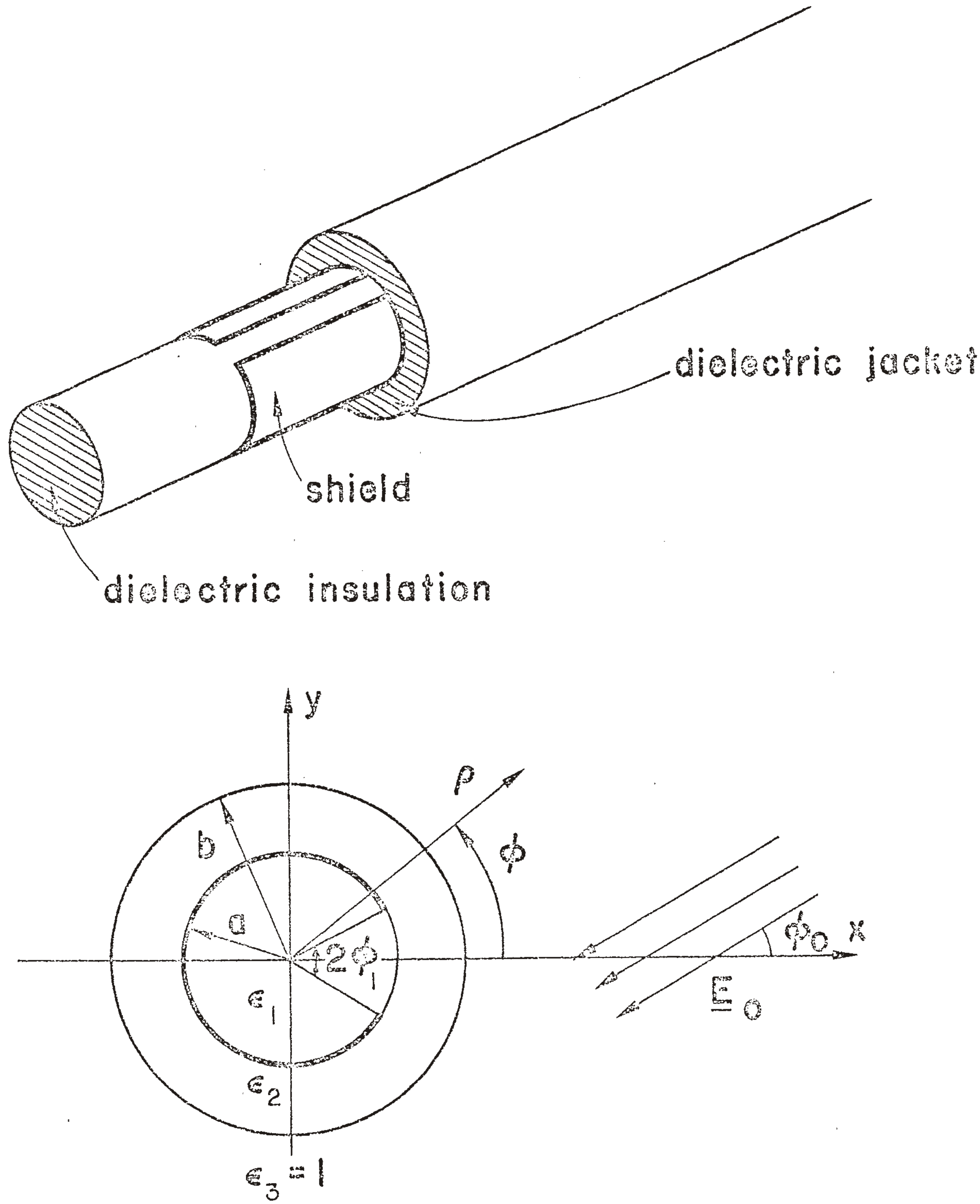


Figure 4. A slotted cylinder with a dielectric jacket.

$$\Phi_1(\rho,\phi) = \sum_{n=-\infty}^{\infty} A_n \rho^{|n|} e^{in\phi}, \qquad \rho \le a$$

$$\Phi_2(\rho,\phi) = \sum_{n=-\infty}^{\infty} (B_n \rho^{-n} + C_n \rho^{n}) e^{in\phi}, \qquad a \le \rho \le b \tag{42}$$

$$\Phi_3(\rho,\phi) = \sum_{n=-\infty}^{\infty} D_n \rho^{-|n|} e^{in\phi} + \phi^{inc}(\rho,\phi), \qquad \rho \ge b$$

where

$$\phi^{inc}(\rho,\phi) = E_o \rho \cos(\phi - \phi_o) \tag{43}$$

and E_o is the strength of the incident field. The conditions on the field at infinity implies that $D_o = 0$. Employing the boundary conditions at $\rho = a$ and $\rho = b$ (c.f. (3) and (4)) we arrive, after some algebraic manipulations, at the following expression for the potential inside the cable

$$\Phi_1(\rho,\phi) = \sum_{n=1}^{\infty} x_n (\rho/a)^n \cos n\phi + \sum_{n=1}^{\infty} y_n (\rho/a)^n \sin n\phi, \qquad \rho < a. \tag{44}$$

The coefficients x_n and y_n are given by the solutions of the following two dual series equations,

$$\sum_{n=0}^{\infty} x_n \cos n\phi = 0$$

$$\sum_{n=1}^{\infty} n(1 + k_n) x_n \cos n\phi = KE_o a \cos \phi_o \cos \phi \tag{45}$$

and

$$\sum_{n=1}^{\infty} y_n \sin n\phi = 0$$

$$\sum_{n=1}^{\infty} n(1 + k_n) y_n \sin n\phi = KE_o a \sin \phi_o \sin \phi, \tag{46}$$

where

$$k_n = \frac{1}{\varepsilon_1+\varepsilon_2}\left\{\varepsilon_1 + \varepsilon_2\left[1 - \frac{\varepsilon_2-1}{\varepsilon_2+1}\left(\frac{a}{b}\right)^{2n}\right]\left[1 + \frac{\varepsilon_2-1}{\varepsilon_2+1}\left(\frac{a}{b}\right)^{2n}\right]\right\} - 1$$

and (47)

$$K = \frac{\varepsilon_2+1}{2(\varepsilon_1+\varepsilon_2)}\left\{1 - \frac{\varepsilon_2-1}{\varepsilon_2+1}\left(\frac{b}{a}\right)^2 + \left[1 - \frac{\varepsilon_2-1}{\varepsilon_2+1}\left(\frac{a}{b}\right)^2\right]\left[1 + \frac{\varepsilon_2-1}{\varepsilon_2+1}\left(\frac{a}{b}\right)^2\right]^{-1}\left[1 + \frac{\varepsilon_2-1}{\varepsilon_2+1}\left(\frac{b}{a}\right)^2\right]\right\}$$

The dipole moment, $\underline{p}'$ per unit length of the slot is given by[6]

$$\underline{p}' = a\varepsilon_1\varepsilon_o\hat{x}\int_{-\phi_1}^{\phi_1}\Phi_1(a,\phi)\,d\phi = 2a\varepsilon_1\varepsilon_o\hat{x}\sum_{n=1}^{\infty}\frac{x_n\sin n\phi_1}{n}\ . \qquad (48)$$

To find this dipole moment one first has to solve the set of equations (45) and in the general case it is not possible to find a closed form solution of (45). Numerical means must therefore be applied to find the x_n. However, in the special case where $b >> a$ or $b = a$ it is possible to solve (45) with analytical means. In this case we have[8]

$$x_n = \frac{4a\varepsilon_2 E_o'\cos\phi_o}{\varepsilon_1+\varepsilon_2}\,\xi_n$$

(49)

$$\xi_n = \frac{1}{2}\int_{\cos\phi_1}^{1}[P_n(x) - P_{n-1}(x)]\,dx$$

so that the polarizability of the aperture is given by

$$\alpha_e = \frac{p}{E_o'} = \frac{\varepsilon_1\varepsilon_2\varepsilon_o a^2}{\varepsilon_1+\varepsilon_2}\,G(\phi_1) \qquad (50)$$

where

$$G(\phi_1) = 4\sum_{n=1}^{\infty}\left[\frac{\sin n\phi_1}{n} - \frac{\sin(n+1)\phi_1}{n+1}\right]I_n - I_o\sin\phi_1 \qquad (51)$$

and

$$I_n = \int_{\cos\phi_1}^{1} P_n(x)dx. \tag{52}$$

Here, E_o' is the electric field in the medium surrounding the cable, i.e., $E_o' = E_o$ and $\varepsilon_2 = 1$ when $b = a$ and $E_o' = E_o/(1 + \varepsilon_2)$ when $b >> a$.

The normalized factor $G(\phi_1)$ of the slot is graphed in Figure 5. In order to understand the effect of the radius of curvature on the polarizability we have also graphed in broken lines in Figure 5 the polarizability of a slot of width d in a planar sheet, which is given by[6]

$$\alpha_e = \pi d^2/4. \tag{53}$$

The equivalent opening angle of this planar slot is defined as $\phi_1 = d/a$; so the function plotted is

$$G_1(\phi_1) = \pi\phi_1^2/4. \tag{54}$$

It can be observed from Figure 5 that the effect of the radius of curvature on the polarizability is negligible when the total slot width is less than or equal to the radius of curvature.

Before concluding this section some comments about the results presented in this note are in order. Two isolated effects on the polarizability of an aperture have been studied namely (1) the dielectric effects and (2) the effects of the radius of curvature of the shield. Although the combined effect can be obtained by solving (45) (which is a rather cumbersome task) it is felt that this effect can be obtained by judiously combing the results of Sections IV and V.

Finally, most cable jackets have appreciable conductivity. The presence of such conductivity will affect not only the electric polarizability but also the magnetic polarizability, the latter being left out of consideration in the present note. One might think that an approximate way to account for the conductivity effect on the electric polarizability is to replace the dielectric constant of the jacket ε_2 by $\varepsilon_2(1+i\sigma/\omega\varepsilon_2)$ in the results presented throughout this note, ε_2/σ being the relaxation time of the jacket. Unfortunately, such simple replacement will lead to many inconsistencies, as can be seen from

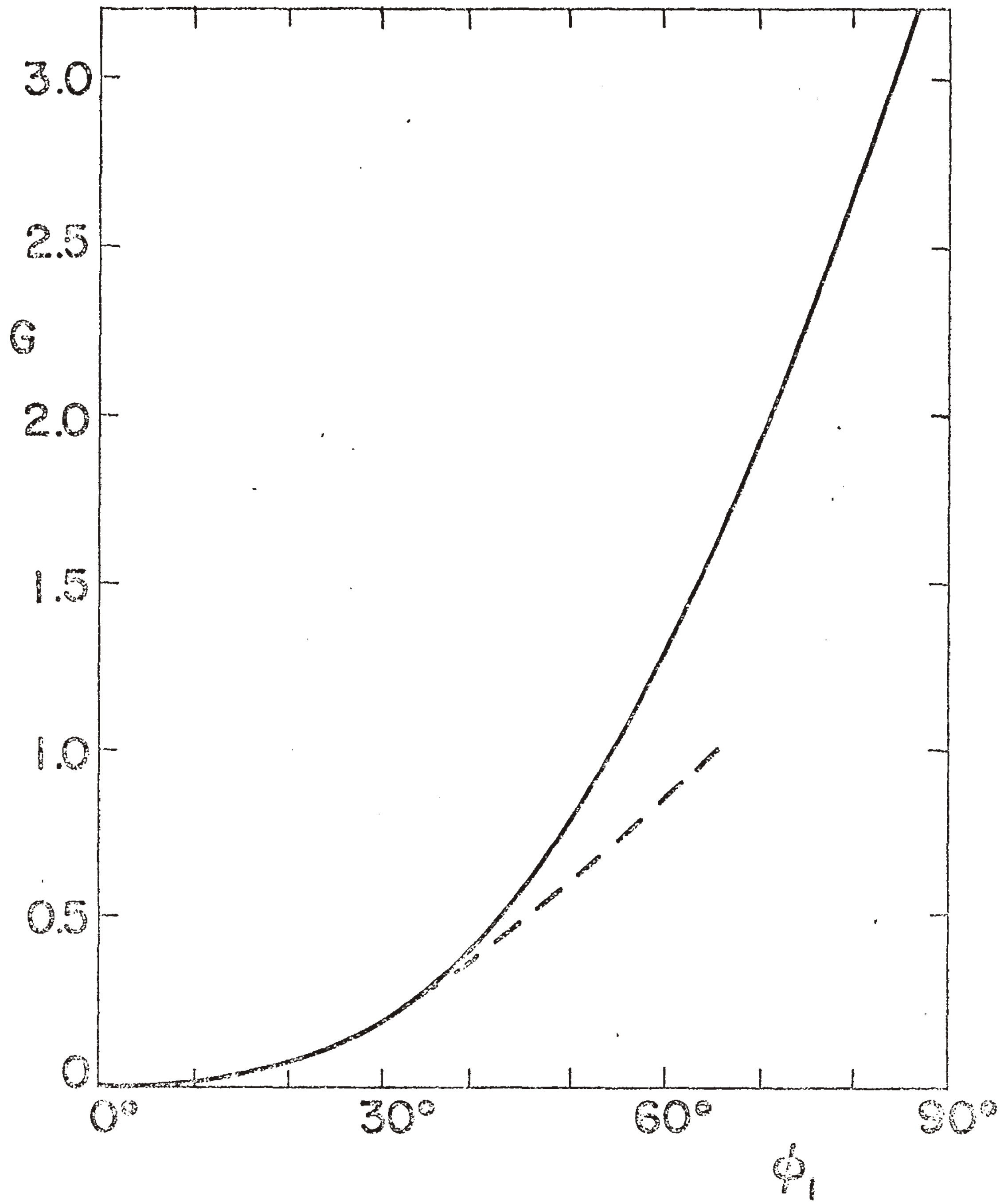
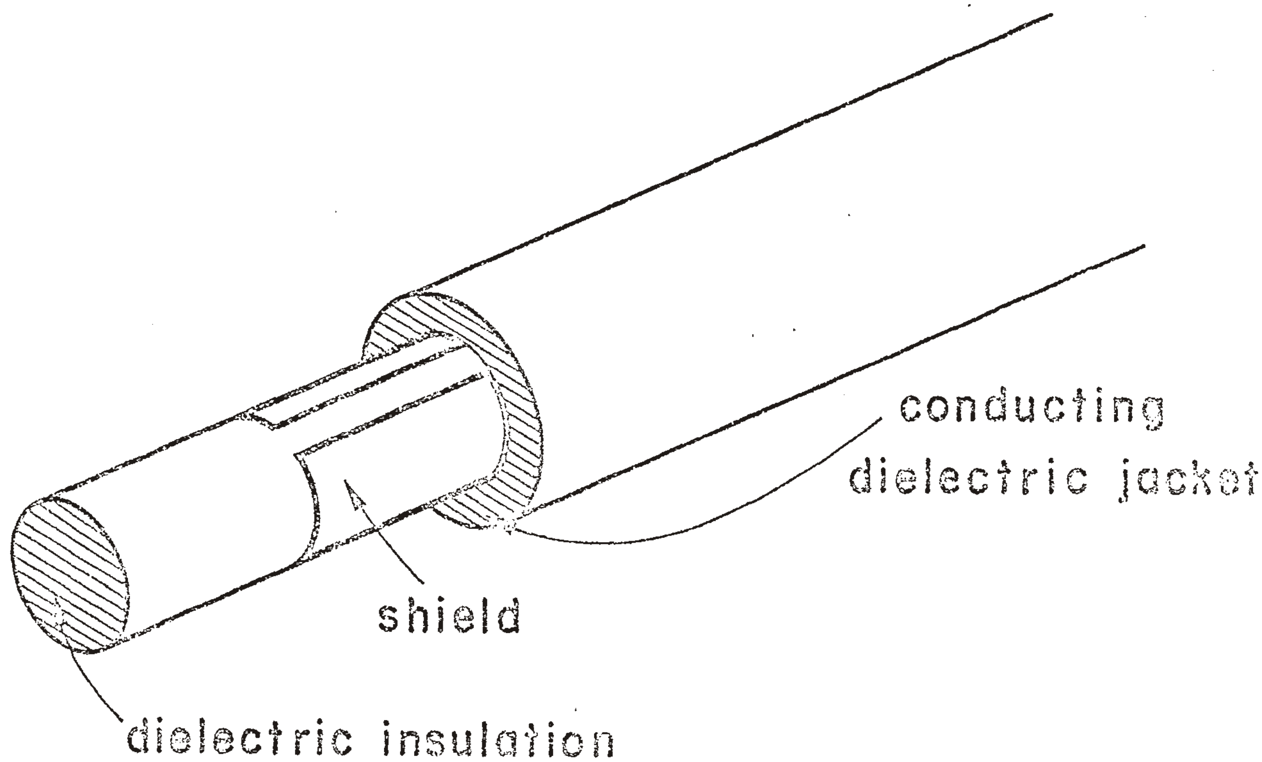


Figure 5. The variation with the slot angle of the polarizability per unit length of a slot in a circular cylinder.

Maxwell's equations. Nevertheless, conductivity is present in all cable jackets and therefore its effect on EMP coupling with cable systems merits immediate attention. Figure 6 shows a relevent, well-posed problem the solution of which will reveal what one desires to know about the jacket's conductivity.



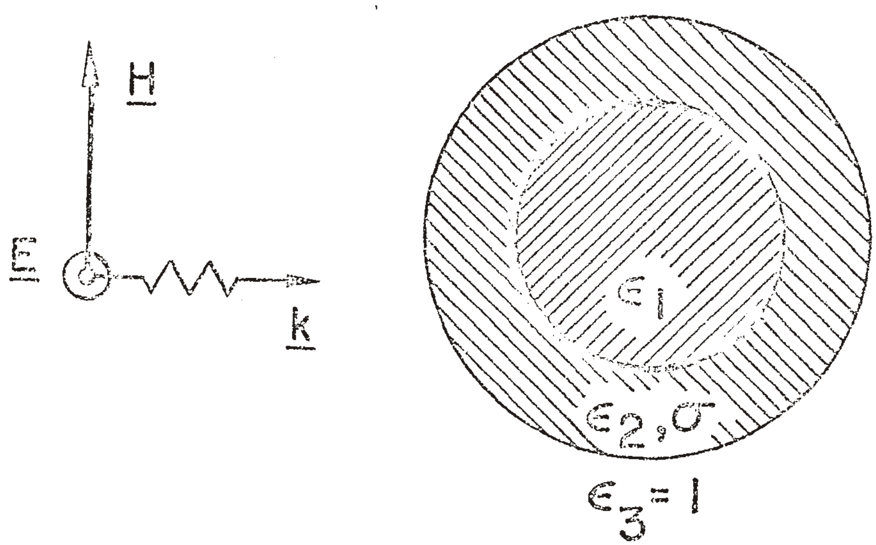


Figure 6. A cable shield with conducting jacket and a longitudinal slit.

Acknowledgment

We had many enlightening discussions with Dr. K. S. H. Lee and helpful comments from Dr. C. E. Baum. This project has been fully supported by the Defense Nuclear Agency (DNA) through the Air Force Weapons Laboratory (AFWL).

References

1. H. Kaden, Wirbelströme und Schirmung in der Nachrichtentechnik, Springer Verlag, Berlin, 1959.

2. R. W. Latham, "An Approach to Certain Cable Shielding Calculations," Interaction Note 90, January 1972.

3. R. W. Latham, "Small Holes in Cable Shield," Interaction Note 118, September 1972.

4. E. F. Vance, "Shielding Effectiveness of Braided-Wire Shields," Interaction Note 172, April 1974.

5. S. Fraenkel, "Terminal Response of Braided-Shield Cables to External Monochromatic Electromagnetic Fields," Interaction Note 124, July 1972.

6. K. S. H. Lee and C. E. Baum, "Application of Modal Analysis to Braided-Shield Cables," Interaction Note 132, January 1973.

7. K. S. H. Lee, "Shields with Periodic Apertures," Interaction Note 89, January 1972.

8. I. N. Sneddon, Mixed Boundary Value Problems in Potential Theory, North-Holland Publishing Company, Amsterdam, 1966.

9. W. Magnus, F. Oberhettinger, and R. P. Soni, Formulas and Theorems for the Special Functions of Mathematical Physics, Springer Verlag, New York, 1966.