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MULTICONDUCTOR ANTENNA TRANSMISSION LINES WITH ARBITRARILY POSITIONED LOAD IMPEDANCES IN AN INCIDENT FIELD

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SUMMARY

The problem considered is that of a multiple conductor transmission line with arbitrarily positioned impedances in series with each wire. The objective is to determine the currents in each of the loads in terms of the amplitude of the incident electric field. The groundwork is laid in this paper for a very general theory for N wire structures when the impedances are situated at the centers of the conductors. If the load elements are in echelon rather than centrally positioned, it appears necessary to abandon the simultaneous integral equation approach employed and resort to the use of two network theorems: superposition and compensation. It is demonstrated that the results depend on a synthesis of antenna and transmission line behavior. This work is an extension to the theory of end-loaded conductors published previously in the G-EMC Transactions.
MULTICONDUCTOR ANTENNA TRANSMISSION LINES WITH ARBITRARILY POSITIONED LOAD IMPEDANCES IN AN INCIDENT FIELD

Introduction

This paper is a sequel to an earlier investigation undertaken to determine the currents in the load impedances of a multiconductor transmission line excited by an incident plane wave electromagnetic field. In the preceding paper the load elements—consisting of combinations of resistors, inductors, and capacitors—are assumed to be in series with the wires at their ends (see Figure 1, Reference 1). For this location of the lumped circuit elements in the structure, only transmission line currents flow in the impedances, because the antenna current vanishes at the ends of the wires. If now the loads are located elsewhere in the conductors, the impedances carry antenna as well as transmission line currents. This fact complicates the problem of calculating the total current in each load. It has been the experience of the writers that it is not too difficult an undertaking to obtain the currents flowing in N load impedances centrally oriented in the conductors of a multiple-wire antenna transmission line for parallel incidence of the electric field. However, for staggered loads the problem becomes more tenuous.

Some of the principles needed for the analysis of multiconductor antenna transmission lines in an incident field for arbitrarily positioned load impedances have been elucidated elsewhere. Familiarity of the reader with the contents of the referenced papers is assumed. In the present paper attention is directed, in the interest of brevity, toward finding the currents in the loads of two-conductor configurations excited by an incident plane wave electromagnetic field.


The Short-Circuit Current in a
Two-Conductor Antenna Transmission Line

Consider a two-wire transmission line with short-circuited terminations, as illustrated by Figure 1. The line lies in the yoz plane, with conductors parallel to the z axis. The wires are of length 2h and of radius a, and the spacing is b. The incident electric field $E_{z}^{inc}$ is polarized parallel to the axis of the wires and arrives at the azimuth angle $\phi$, measured from the positive x axis.

Following Reference 2, the equations for determining the currents in this circuit are as follows:

Conductor 1:

$$J_1(z) + I_1^E(z)\psi_a + I_2^E(z)\psi_b = -j\frac{4\pi}{\xi_0} \left[ C_1 \cos \beta z + \frac{E_{z}^{inc}}{\beta} \exp \left( j\frac{2h}{z} \sin \phi \right) \right].$$

Conductor 2:

$$J_2(z) + I_1^E(z)\psi_b + I_2^E(z)\psi_a = -j\frac{4\pi}{\xi_0} \left[ C_2 \cos \beta z + \frac{E_{z}^{inc}}{\beta} \exp \left(-j\frac{2b}{z} \sin \phi \right) \right].$$

Here $J_1(z)$ has the same significance as in Reference 2:

$$\psi_a = 2 \ln \left( \frac{d_1}{a} \right) ; \quad \psi_b = 2 \ln \left( \frac{d_2}{b} \right) ; \quad \beta = 2\pi/\lambda \quad (2)$$

is the radian wave number; $\lambda$ is the wavelength of the field $E_{z}^{inc}$; $C_1$ and $C_2$ are constants, and $\xi_0$ is the characteristic impedance of space. In the rationalized mks system of units used in this paper, $\xi_0 \approx 120\pi$ ohms.

The right-hand sides of (1) and (2) are proportional to the vector potentials on the surfaces of the conductors established by the currents flowing in them. These currents are, of course, excited by the incident field $E_{z}^{inc}$. The boundary condition at each end of the line is

$$\left[ \frac{\partial A_1(z)}{\partial z} - \frac{\partial A_2(z)}{\partial z} \right] = 0, \quad z = \pm h \quad (3)$$

where $A_1(z)$ and $A_2(z)$ are the vector potentials on wires 1 and 2, respectively.
By using (1) - (3), it is a simple matter to show that the short-circuit currents in conductors 1 and 2 of the transmission line pictured in Figure 1 are

\[
I_1^E(z) = \frac{E_1^L(z)}{2} + \frac{2 E_z^{inc} \sin \left( \frac{\theta_b}{2} \sin \varphi \right)}{\beta Z_c}
\]

\[
I_2^E(z) = \frac{E_2^L(z)}{2} - \frac{2 E_z^{inc} \sin \left( \frac{\theta_b}{2} \sin \varphi \right)}{\beta Z_c}
\]

Here \( Z_c \) is the characteristic impedance of the line;

\[
Z_c = \frac{\zeta_0}{\pi} \ln \left( \frac{b}{a} \right)
\]

Also, \( E_T^L(z) = I_1^E(z) + I_2^E(z) \) is the current flowing at the point \( z \) in an unloaded receiving dipole of effective radius \( d = \sqrt{ab} \) for parallel incidence of the electric field.

\[ \text{Figure 2 portrays a two-conductor antenna transmission line center driven by two impedanceless generators. The dimensions of this circuit are the same as those of the circuit pictured in Figure 1. In the previous section of the paper the currents } I_1^E(z) \text{ and } I_2^E(z) \text{ were found with } V_1 \text{ and } V_2 \text{ suppressed. In this section it is proposed to determine } I_1^V(z) \text{ and } I_2^V(z) \text{ with the incident field } E_z^{inc} \text{ suppressed. The simultaneous integral equations applicable to the circuit of Figure 2 are as follows:} \]

\[ J_a^V(z) + I_1^V(z) \psi_a + I_2^V(z) \psi_b = -j \frac{4\pi}{\zeta_0} \left( C_1 \cos \beta z + \frac{V_1}{2} \sin \beta |z| \right) \]

\[ J_b^V(z) + I_1^V(z) \psi_b + I_2^V(z) \psi_a = -j \frac{4\pi}{\zeta_0} \left( C_2 \cos \beta z + \frac{V_2}{2} \sin \beta |z| \right) \]

\[ J_d^V(z) \text{ is the same as in Reference 2 except that now the current } I_1^V(z) = I_1^V(z) + I_2^V(z) \text{ occurs under the integral sign instead of } I_T^E(z) = I_1^E(z) + I_2^E(z). \]
By adding (7) and (8) and setting $\psi_a = -\psi_b$ so that $d = \sqrt{a^2 + b^2}$, one obtains
\[ J_d(z) = -j \frac{4\pi}{c_0} \left[ \left( \frac{C_1 + C_2}{2} \right) \cos \beta z + \left( \frac{V_1 + V_2}{4} \right) \sin \beta |z| \right]. \] (9)

Equation (9) is in standard form; i.e.,
\[ J_d(z) = -j \frac{4\pi}{c_0} \left( C \cos \beta z + \frac{V_k}{2} \sin \beta |z| \right) \] (10)
inasmuch as $I_T^V(\pm h) = 0$.

The solution of (10) is
\[ I_T^V(0) = \frac{V_k}{2Z_d}. \] (11)

It follows from (9) - (11) that
\[ I_T^V(0) = \frac{V_1 + V_2}{2Z_d}. \] (12)

In (11) and (12), $Z_d$ is the driving point impedance of a symmetrical center-driven dipole of half-length $h$ and effective radius $d$. Tables for $Z_d$ are available in numerous places in the literature. 4,5 By subtracting (8) from (7) and applying (3), it is a simple matter to show that at $z = 0$,
\[ I_1^V(0) = \frac{I_T^V(0)}{2} - j \frac{1}{Z_c} \left( \frac{V_1 - V_2}{2} \right) \cot \beta h \] (13)
and
\[ I_2^V(0) = \frac{I_T^V(0)}{2} + j \frac{1}{Z_c} \left( \frac{V_1 - V_2}{2} \right) \cot \beta h, \] (14)
where $I_T^V(0)$ is given by (12).


The Currents in the Load Impedances of an Antenna Transmission Line When the Loads are at the Centers of the Conductors

The objective is to find the currents in the center-loading impedances caused by the incident field $E^{\text{inc}}_z$. Refer to Figure 3. By use of the compensation theorem, one may write

$$V_1 = -I_1(0)Z_{L1}$$  \hspace{1cm} (15)

and

$$V_2 = -I_2(0)Z_{L2},$$  \hspace{1cm} (16)

where $I_1(0)$ is the current in the lumped impedance $Z_{L1}$, and $I_2(0)$ is the current in the lumped impedance $Z_{L2}$. Also, by use of the superposition theorem,

$$I_1(0) = I_1^V(0) + I_1^E(0)$$ \hspace{1cm} (17)

$$I_2(0) = I_2^V(0) + I_2^E(0).$$ \hspace{1cm} (18)

Equations (12) - (18) are now solved for $I_1(0)$ and $I_2(0)$. The results are

$$I_1(0) = \frac{G + H}{D},$$ \hspace{1cm} (19)

$$I_2(0) = \frac{J + L}{D},$$ \hspace{1cm} (20)

where

$$G = 2Z_d \left[ I_1^E(0) + I_2^E(0) \right] \left[ Z_c \tan \beta h - jZ_{L2} \right],$$

$$H = Z_c \tan \beta h \left[ I_1^E(0) - I_2^E(0) \right] \left[ 2Z_d + Z_{L2} \right],$$

$$J = 2Z_d \left[ I_1^E(0) + I_2^E(0) \right] \left[ Z_c \tan \beta h - jZ_{L1} \right],$$

$$L = -Z_c \tan \beta h \left[ I_1^E(0) - I_2^E(0) \right] \left[ 2Z_d + Z_{L1} \right],$$

$$D = \left[ 2Z_d + Z_{L2} \right] \left[ Z_c \tan \beta h - jZ_{L1} \right] \hspace{1cm} \left[ \frac{2Z_d}{Z_{L1}} \right] \left[ Z_c \tan \beta h - jZ_{L2} \right].$$

(21)
Also, by using (4) and (5),

$$I_1^E(0) + I_2^E(0) = I_T^E(0) \tag{22}$$

$$I_1^E(0) - I_2^E(0) = \frac{4E_{inc}^E}{\beta^2 c} \sin \left(\frac{\beta b}{2} \sin \phi \right) \tag{23}$$

It is of interest to observe that when $Z_{L2} = 0$,

$$I_1(0) = \frac{2Z_cI_1^E(0)}{2Z_c + \left(\frac{Z_L1Z_c^*}{Z_d^*}\right) - jZ_{L1} \cot \beta h} \tag{24}$$

This result is easily verified. The short-circuit current multiplied by the impedance looking back into a network gives the open-circuit voltage $V_{0c}$. As illustrated by Figure 4, the open-circuit voltage drives a circuit consisting of the source impedance in series with the load impedance. Thus

$$V_{0c} = I_1^E(0)Z_{in1} \tag{25}$$

and

$$I_1(0) = \frac{V_{0c}}{Z_{in1} + Z_{L1}} = \frac{I_1^E(0)Z_{in1}}{Z_{in1} + Z_{L1}} \tag{26}$$

The well-known formula for the input admittance of a two-wire folded dipole is

$$Y_{in1} = \frac{1}{4Z_d} - j\frac{\cot \beta h}{2Z_c} \tag{27}$$

As before $Z_d$ is the driving point impedance of a symmetrical center-driven dipole of half-length $h$ and effective radius $d = \sqrt{ab}$. The source impedance is then

$$Z_{in1} = 1/Y_{in1} = \frac{4Z_cZ_d}{Z_c - j2Z_d \cot \beta h} \tag{28}$$

---

Substituting (28) into (26) yields (24) as anticipated.

The short-circuit currents $I_n^E(0)$ can be found for any $N$ conductor antenna transmission line by solving the applicable simultaneous integral equations. Also, the driving point currents can be found for the same structure by using similar techniques. It follows that the load currents, for center-loaded configurations, can be found for $N$ conductor circuits in terms of the incident field. It is assumed that transmission line theory is not violated; i.e., $\beta q << 1$, where $q$ is the maximum transverse dimension of the antenna transmission line. Also, it is required that the inequality $q << h$ be satisfied.

Staggered Load Impedances in a Two-Conductor Antenna Transmission Line Excited by an Incident Field

The circuit to be discussed (in a semiquantitative way because of its complexity) is illustrated by Figure 5f. The currents in the loads $Z_{L1}$ and $Z_{L2}$ excited by the incident field $E^\text{inc}$ are determined by the use of the superposition and compensation theorems. This requires introduction of circuits illustrated by 5a to 5f in Figure 5. Evidently, when the load impedances are in the center of the wires, as in Figure 3, the problem can be treated in the manner set forth here. Earlier in the paper the authors presented a technique of solution that may be applied to $N$ conductors, provided that the antenna transmission line is center loaded.

To obtain the currents $I_1$ and $I_2$ flowing in $Z_{L1}$ and $Z_{L2}$, respectively, requires complete analysis of the individual circuits a to e in Figure 5. To facilitate solution of these problems, it is convenient to introduce three coordinate systems. The load $Z_{L1}$ is located at $z = 0$, the load $Z_{L2}$ is located at $z' = 0$, and the center of the structure is located at $z'' = 0$. The distance between $Z_{L1}$ and $Z_{L2}$ is $l$. The relation between the coordinates is

$$z = z' - l$$
$$z = z'' + \left(\frac{h_2 - h_1}{2}\right) \tag{29}$$
$$z'' = z' - l - \left(\frac{h_2 - h_1}{2}\right).$$

The ends of the structure are located at $z = -h_1$, $z = h_2$; $z' = -(h_1 - l)$, $z' = h_2 + l$; and $z'' = -(h_1 + h_2)/2$; $z'' = (h_1 + h_2)/2$. These results follow from (29).

The assumed current directions are indicated in each of the drawings constituting Figure 5. It follows that the currents $I_1$ and $I_2$ are given by

$$I_1 = I_{1a}^E + I_{b}^V + I_{d}^V + I_{f}^V - I_{h}^V$$

$$I_2 = I_{2a}^E + I_{b}^V + I_{d}^V - I_{f}^V - I_{h}^V$$

$$13$$
and

\[ I_2 = I_{2a}^E + I_{1a}^E + I_{1}^V + I_{2}^V. \]  

(31)

\[ I_{1a}^E \text{ and } I_{2a}^E \text{ are functions of } I^{inc}_2, \text{ and are computed from such expressions as (4) and (5) for an} \]
\[ \text{antenna of effective radius } d = \sqrt{s}. \text{ All other currents appearing in (30) and (31) are functions of} \]
\[ V_1 \text{ or } V_2. \text{ These voltages are eliminated by use of the compensation theorem; i.e.,} \]

\[ V_1 = -I_1 Z_{L1} \]

(32)

\[ V_2 = -I_2 Z_{L2}. \]

(33)

Thus (30) and (31) become simultaneous equations involving the desired currents \( I_1 \) and \( I_2 \).

The authors now consider the circuits appearing in Figure 5 individually.

As indicated above, \( I_{1a}^{E} \) is obtained from (4), and \( I_{2a}^{E} \) from (5). The current \( I_{1}^{E} \) must be
\[ \text{known at } z'' = -(h_2 - h_1)/2 \text{ and at } z'' = -l - (h_2 - h_1)/2 \text{ for an unloaded receiving and scattering} \]
\[ \text{antenna of radius } d = \sqrt{s}. \]

Figure 5b represents an asymmetrical dipole. Note that the placement of shorting bars
\[ \text{across the top of the generators and across the bottom does not alter the circuit. The effective} \]
\[ \text{voltage is } V_1/2, \text{ and each generator carries half the current. Accordingly,} \]

\[ I_{b}^{V} = \frac{V_1}{4Z_{d1}}. \]

(34)

Here \( Z_{d1} \) is the impedance of an asymmetrical dipole\(^7\) of leg lengths \( h_1 \) and \( h_2 \) and effective
\[ \text{radius } d = \sqrt{s}. \]

The current \( I_{c}^{V} \) at \( z = -l \) in Figure 5b is obtained directly from the formula for the current
\[ \text{along an asymmetrical dipole.} \]
\[ \text{Note that } I_{c}^{V} \text{ is one-half the total current in the structure at } z = -l. \]

---


The circuit pictured in Figure 5c is a transmission line. From simple transmission line theory one obtains

\[
I_d^V = -\frac{jV_1}{Z_c (\tan \beta h_1 + \tan \beta h_2)},
\]

(35)

where \(Z_c\) is given by (6).

The current \(I_e^V\) in the circuit of Figure 5c is also obtained from simple transmission line considerations. The internal impedance of the generator of voltage \(V_1\) is

\[
Z_g = jZ_c \tan \beta h_2.
\]

(36)

The input impedance of the line looking down from the generator terminals is

\[
Z_{in} = jZ_c \tan \beta h_1.
\]

(37)

The equivalent circuit is shown in Figure 6.

Clearly,

\[
I_s = -\frac{-jV_1}{Z_c (\tan \beta h_1 + \tan \beta h_2)}
\]

(38)

\[
V_s = I_s Z_{in} = \frac{V_1 \tan \beta h_1}{\tan \beta h_1 + \tan \beta h_2}.
\]

(39)

The current \(I_e^V\) at distance \(l\) from the generator is

\[
I_e^V = I_s \cos \beta l - j \frac{V_s}{Z_c} \sin \beta l.
\]

(40)

Substituting (38) and (39) into (40) yields

\[
I_e^V = -j \frac{V_1}{Z_c} \left[ \frac{\cos \beta l + \sin \beta l \tan \beta h_1}{\tan \beta h_1 + \tan \beta h_2} \right].
\]

(41)

provided that the transmission line is dissipationless.
The current $I^v_g$ occurring in the circuit pictured in Figure 5d is given by

$$I^v_g = \frac{V_2}{4Z_{d2}},$$  \hspace{1cm} (42)

where $Z_{d2}$ is the driving point impedance of an asymmetrical dipole of effective radius $d = \sqrt{\frac{a}{b}}$ and leg lengths $h_2 + l$ and $h_1 - l$. The current $I^v_l$ at $z' = l$ is obtained from the formula for the current in an asymmetrical dipole. Again note that $I^v_l$ is one-half the total current in the structure at the cross section under consideration.

Again circuit e in Figure 5 is a transmission line. By analogy with circuit c in Figure 5, one obtains

$$I^v_l = -j\frac{V_2}{Z_c} \left[ \tan \beta(h_2 + l) + \tan \beta(h_1 - l) \right]$$  \hspace{1cm} (43)

and

$$I^v_h = -\frac{V_2}{Z_c} \left[ \frac{\cos \beta l + \sin \beta l \tan \beta(h_2 + l)}{\tan \beta(h_2 + l) + \tan \beta(h_1 - l)} \right].$$  \hspace{1cm} (44)

This completes determination of the individual currents constituting $I_1$ and $I_2$. Figure 5f. It is interesting to observe that among other factors the load currents depend on the input impedances of two asymmetricaly driven dipoles.

The evaluation of the ten component currents in the circuit would normally be done by use of a computer.

Conclusions

A general theory has been developed for the response of an N conductor impedance-loaded transmission line to an incident electric field when the loads are centrally positioned. Simultaneous integral equations for the currents appropriate to antennas driven by lumped generators, as well as antennas driven by incident fields, are employed. When the loads are in echelon, a general theory does not appear to be feasible. The solution of the problem is effected for a two-conductor configuration by using the superposition and compensation theorems. The modes of asymmetrical driven dipoles and transmission lines are involved. Extension of the theory to more than two conductors becomes a Herculean task.
APPENDIX

If \( h \leq \pi / 2 \) and \( \Omega = 2 \ln \left( \frac{2h}{d} \right) \geq 8 \), the current \( I_{T}^{E}(z) \) appearing in (4) and (5) may be computed from the formula

\[
I_{T}^{E}(z) = \frac{4 \pi}{\kappa_0} \frac{E_{z}}{\beta} \left[ \frac{\cos \beta z - \cos \beta h}{\psi_{du} \cos \beta h - \psi_{u}(h)} \right]^{1/3} \tag{45}
\]

where

\[
\psi_{du} = (1 - \cos \beta h)^{-1} \int_{-h}^{h} (\cos \beta z' - \cos \beta h) \left[ K(0, z') - K(h, z') \right] dz' \tag{46}
\]

\[
\psi_{u}(h) = \int_{-h}^{h} (\cos \beta z' - \cos \beta h) K(h, z') dz' \tag{47}
\]

with

\[
K(z, z') = \frac{\exp j \beta R}{R} \tag{48}
\]

\[
R = \sqrt{(z - z')^2 + d^2} \tag{49}
\]

The half-length of the antenna is \( h \) in the notation employed here. One should anticipate need for a computer to determine \( I_{T}^{E}(z) \). The impedance and current distribution along electrically short, moderately thin asymmetrical antennas driven by a generator (in contrast to an incident field) may be determined by reference to the literature.\(^9,\(^10\)

At high frequencies, when the antenna transmission line becomes electrically long, the brilliant work of Shen\(^7,\(^8\) should be utilized to find the currents along dipole receiving and scattering antennas,\(^7\) as well as the impedance and current distribution along asymmetrical dipoles.\(^8\)


The following remarks apply exclusively to Reference 7. The assumed, but suppressed, time dependence employed in Reference 7 is exp (-jωt). The writers prefer the time dependence exp (jωt). Accordingly, replace i by -j throughout the theory. For Equation (3) to be dimensionally correct, multiply it by $E_z^\text{inc} / \lambda$. Set $\theta_i = 90/2$ and note that log means in. Also, replace $z$ by $|z|$ in Equation (3), since $I_{s_1}^\text{e}(z) = I_{s_2}^\text{e}(-z)$. Using the notation employed in the present paper, Equation (8) of Reference 7 should be written

$$I_{s_1}^\text{e}(z^n) = I_{s_2}^\text{e}(z^n) + C_{s_1} I \cdot (z^n + h) + C_{s_2} I \cdot (z^n - h).$$

(50)

From (29) or Figure 5a of the present paper, it is noted that $h = -(h_1 + h_2)/2$ and $-h = -(h_1 + h_2)/2$. These values are used for $h$ in Equation (9), Reference 7. Interest centers in obtaining $I_{s_2}^\text{e}$ at the first position, $z^n = -(h_2 - h_1)/2$, and $I_{s_1}^\text{e}$ at the second position, $z^n = -l + (h_2 - h_1)/2$ (Figure 5a of the present paper). It follows that $z^n + h = h_1$ and $z^n - h = h_2$ for the first position. Also, for the second position, $z^n + h = -l + h_1$ and $z^n - h = -l - h_2$. Evidently, $z = z^n$ in Equation (3) of Reference 7.

The following remarks apply exclusively to Reference 8. Replace i by -j and multiply Equation (9) by $V_1$ or $V_2$ as appropriate. Also in Equation (6) replace $z$ by $|z|$ and note that $-\pi < \text{Im} [\ln f(z)] \leq \pi$. Again observe that log means in. The asymmetrical dipole pictured in Figure 5b of the present paper requires no modification in notation, and may be applied directly to obtain $Z_{d_1}$ and $I_{d_1}^\text{v}$.

To obtain $Z_{d_1}$ and $I_{d_1}^\text{v}$, note that $-h_1 = -(h_1 - l)$ and $h_2 = h_2 + l$. Refer to Figure 5d of the present paper. Hence Equation (11) in Reference 8 becomes

$$I_{d_1}^\text{v}(z) = I_0 (z) + C_d I \cdot (h_1 - l + z) + C_u I \cdot (h_2 + l - z).$$

(51)

$C_d$ and $C_u$ are obtained from Equations (14) and (15) of Reference 8 by replacing $h_1$ by $(h_1 - l)$ and $h_2$ by $(h_2 + l)$. Obviously, $z = z'$ in Equation (6), and the driving point is at $z' = 0$. The current $I_{d_1}^\text{v}$ is calculated at $z' = l$. This theory yields excellent results when a generator is not closer than 0.15$\lambda$ to either end of the dipole.
Figure 1. Diagram Used in Determining the Short-Circuit Current in a Two-Wire Antenna Transmission Line
Figure 2. Antenna Transmission Line Driven at the Center of Each Conductor by Impedanceless Generators
Figure 3. Center-Loaded Two-Conductor Antenna Transmission Line
Figure 4. The Equivalent Circuit of an Impedance-Loaded Folded Dipole
Figure 5. The Use of Superposition to Determine the Load Currents When the Impedances Are in Echelon.
Figure 6. Circuit for Determining the Current $I_e$ in Figure 5c