Interaction Notes

Note 219

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Categorization of the Types of Apertures

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Abstract

Depending on various physical characteristics of practical apertures their electromagnetic responses can exhibit different characteristics. This note considers various types of apertures encountered and differentiates them into several classes based on their topological properties assuming the apertures are located in perfectly conducting planes. Impedance loading is also considered.

I. Introduction

There has been a considerable amount of work done in the area of coupling through apertures.[1-26] In particular, numerous cases of coupling through apertures in an infinite plane have been studied. The preliminary work concerning large apertures in an infinite plane was due to Kirchhoff.[1] Subsequent work by Bethe[2] has led to the understanding of small apertures through the introduction of equivalent electric and magnetic dipole moments. Booker's[4] extension of Babinet's principle gave further theoretical insight into the aperture problem.

Present interest in apertures arises from the need to understand EMP coupling through windows, doors, hatches, etc., into various objects. In some cases, as in certain cockpit windows, the aperture region is loaded by a resistive sheet. In cases such as hatches in aircraft, missiles, etc., the gasket surrounding the door could be resistive. In most of the instances involving aircraft or missiles, the region behind the window, door or hatch is small. As a consequence, it is more correctly treated as a cavity backed aperture.[10]

In considering the aperture coupling problem one then notes that the term aperture covers a wide variety of shapes and impedance loadings. Depending on the specifics of geometry and loading one then expects various types of electromagnetic responses. One expects objects with similar shapes and impedance distributions to have similar responses. Let us then categorize the aperture types according to their topology, both in a geometrical sense (general shape) and with regard to the location (in a topological sense) and general magnitude of any significant impedances. Topological concepts are useful for decomposing the EMP interaction problem into smaller problems.[27] This note uses topological concepts to distinguish between important variations in one kind of EMP interaction problem.

The apertures as discussed above are generally in a curved body. Hence the curvature of this body as well as the loading in the aperture (for sheet impedance loaded apertures) would have an influence on the aperture distribution and the distribution in the region into which the aperture couples.[12] Because of these complexities, even the problem of electromagnetic coupling through an aperture in a body of rotation is quite difficult. As a result, if the curvature of the body near the aperture is small, it is generally assumed to be zero. As an approximation one can often assume that the aperture is in an infinite plane sheet. The results obtained can then often be used as a step in the solution of the aperture in the more complicated structure.

Using the planar approximation the aperture problem can be split into three parts:

1. The short circuit surface current and charge densities can be obtained on the complicated object of interest.

2. The aperture fields are calculated using these short circuit surface current and charge densities. This analysis can be aided by the use of Babinet's principle.

3. The coupling into the interior of the complicated object is calculated by matching appropriate cavity fields to the aperture fields. Near resonance the cavity fields can be used to correct the short circuit surface current and charge densities in part 2.

If the wavelength of the external field is large compared to the largest dimension of the aperture, Bethe's quasistatic approximation based on the equivalent polarizabilities of the aperture can be used. This further simplifies the use of the 3 part decomposition of the aperture problem on a complicated object.

II. Classification of Aperture Problems

In principle, apertures can be categorized by their topology and physical composition. By their topology they can be classified as simple apertures, hatches, and gratings, and by their physical composition they can be separated into loaded and unloaded types.

A. Simple Apertures

A simple aperture is defined as a domain S_a in a perfectly conducting infinite (less S_a) plane S, such that S_a is simply connected with no zero sheet impedance regions allowed in S_a (except in the zero measure sense) as shown in figure 1. This can be expressed as

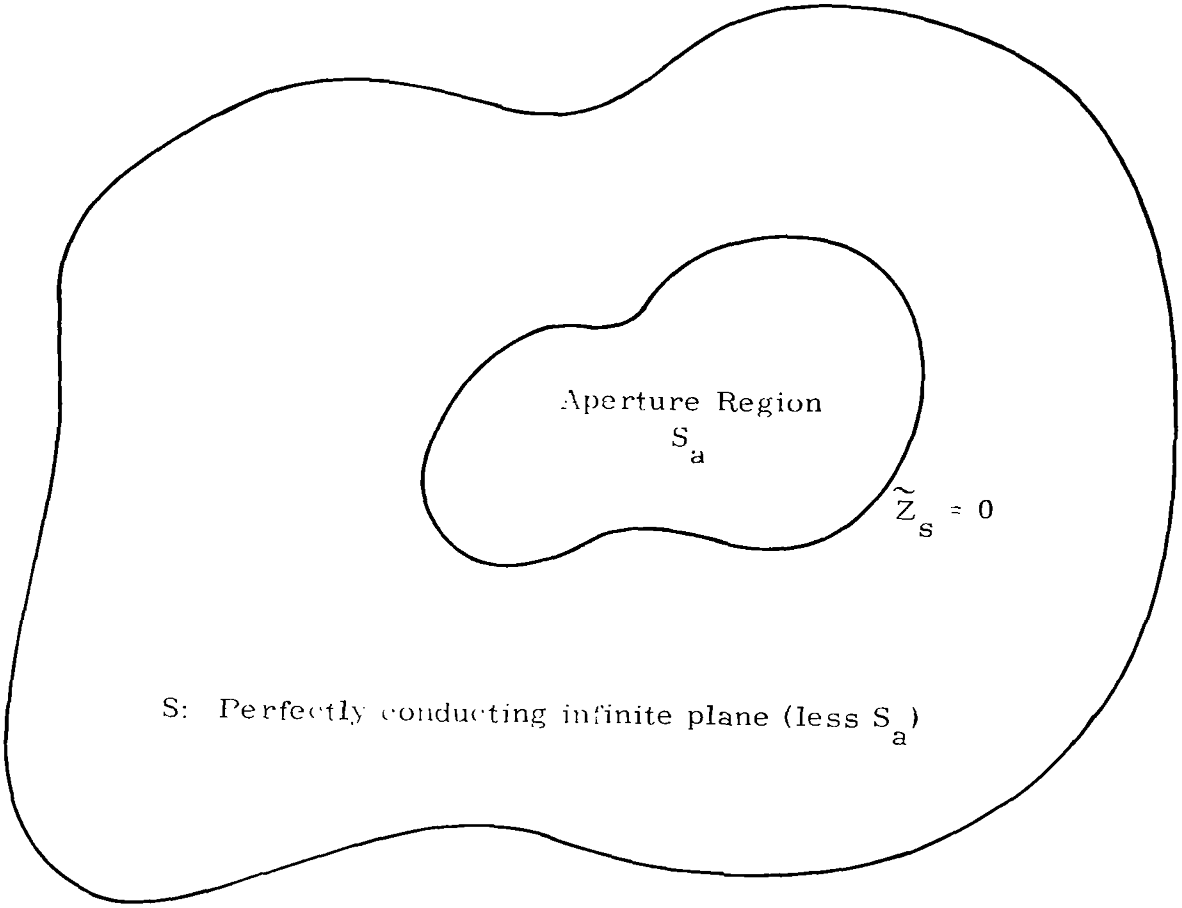
$$S_a \cup S = P \text{ (the entire plane)}$$

$$\text{sheet impedance} \begin{cases} \neq 0 & \text{on } S_a \text{ (simply connected)} \\ = 0 & \text{on } S \text{ (an infinite surface)} \end{cases}$$

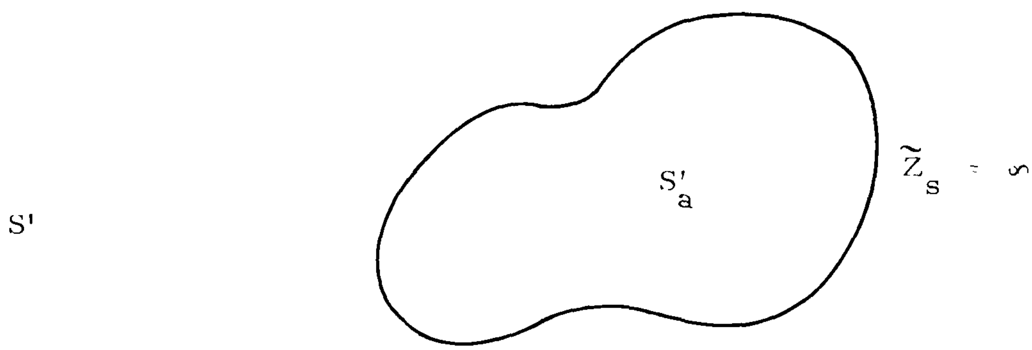
The aperture region S_a can be loaded or unloaded. If the aperture region is unloaded, the Babinet equivalent aperture (the complementary scatterer) is as shown in figure 2. If S_a is unloaded and S is perfectly conducting, the complementary scatterer S'_a is perfectly conducting and the surrounding region S' is free space. However, if S_a is loaded by an admittance sheet $\tilde{\overleftrightarrow{Y}}_s$, with the region S being perfectly conducting, the complementary scatterer S'_a has surface admittance $\tilde{\overleftrightarrow{Y}}'_s$ given by[17]

$$\tilde{\overleftrightarrow{Y}}'_s = \frac{4}{Z_o^2} \overleftrightarrow{\tau}' \cdot \left(\tilde{\overleftrightarrow{Y}}_s^{-1}\right)_2 \cdot \overleftrightarrow{\tau}' \qquad (1)$$

where $\left(\tilde{\overleftrightarrow{Y}}_s^{-1}\right)_2$ is defined as the two dimensional inverse of $\tilde{\overleftrightarrow{Y}}_s$ and $\overleftrightarrow{\tau}'$ is given by



Simple aperture



Complementary simple disc

Figure 1. Simple Aperture and its Complementary Disc

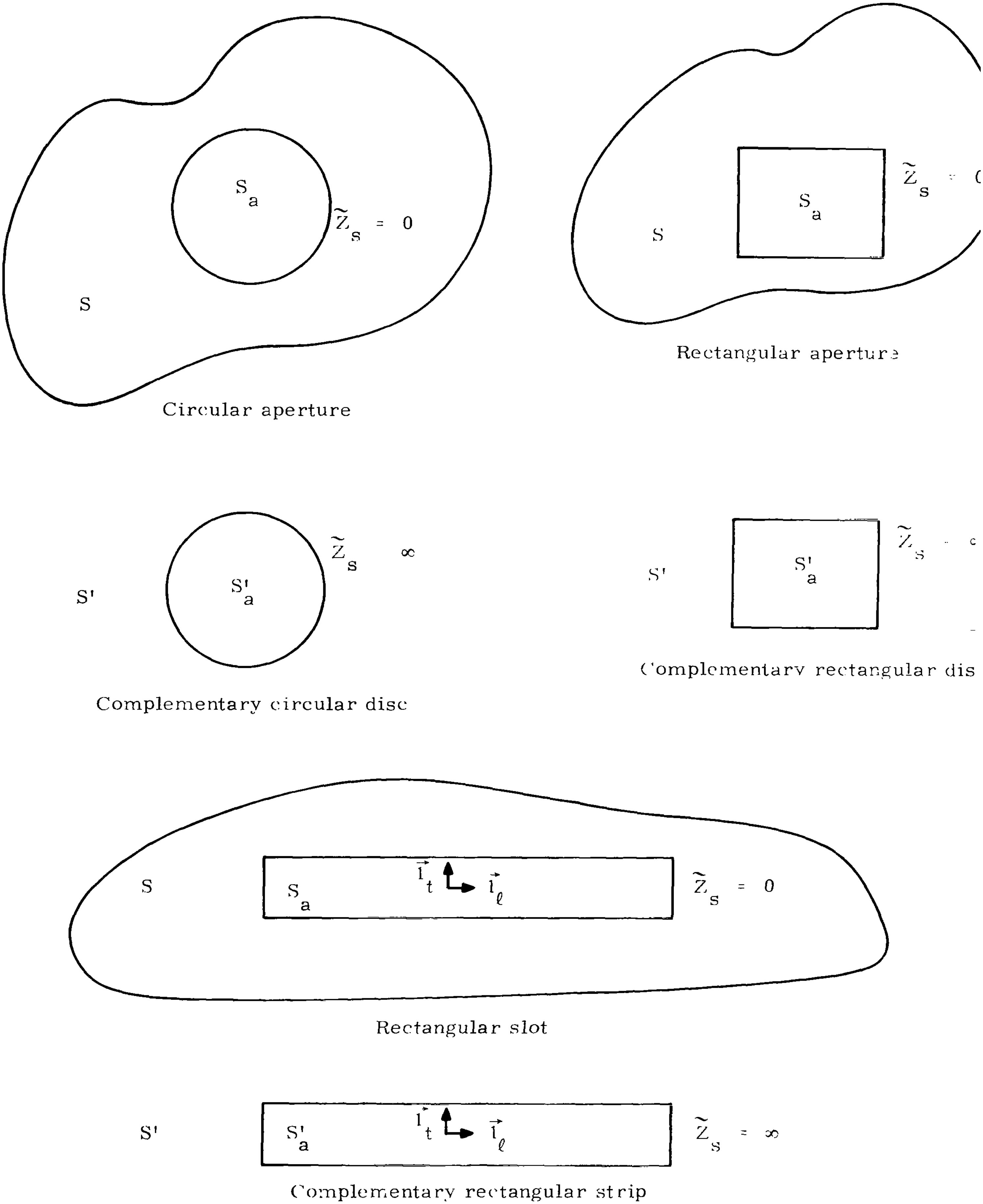


Figure 2. Typical Simple Apertures and Their Complementary Discs

$$\overset{\rightarrow}{\overset{\rightarrow}{\tau}}' = \begin{bmatrix} 0 & -qi & 0 \\ qi & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad q = \pm 1 \tag{2}$$

In the case of small apertures, approximate formulae for the polarizability of equivalent dipoles are available.[2] For small unloaded apertures these characteristics are independent of frequency, while, for the loaded case, they are frequency dependent. For the case of large apertures, the current on the complementary disc is calculated which in turn yields the aperture fields.

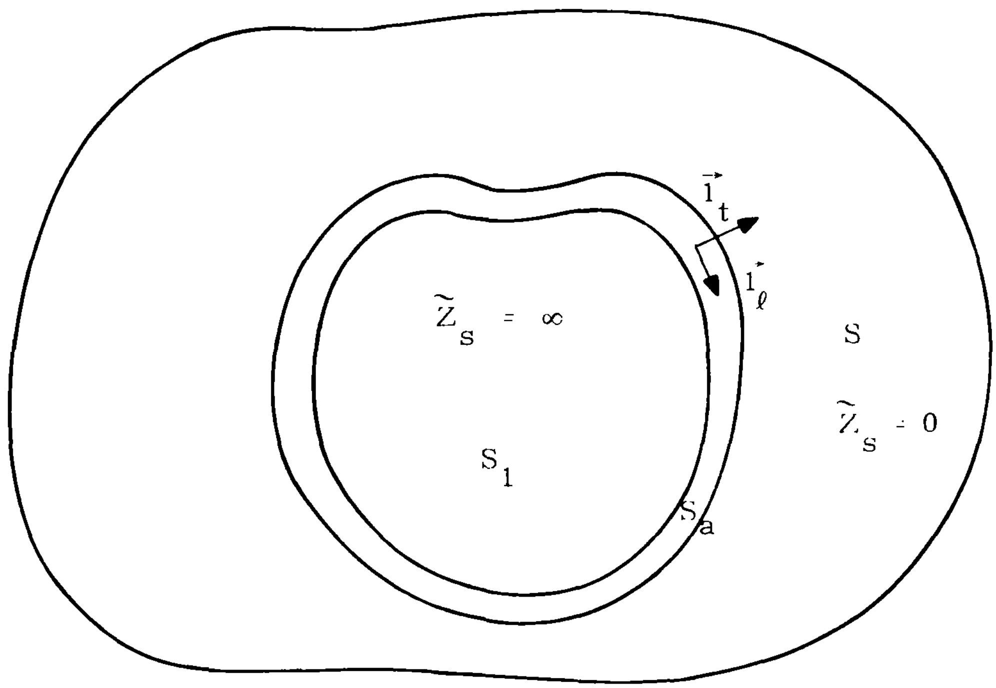
B. Hatches

A hatch is defined in part as a multiply connected domain S_a in an infinite perfectly conducting plane (less S_a) such that, on S_a, any zero surface impedance region has zero measure as shown in figures 3 and 4. The perfectly conducting plane is designated as S for regions exterior to S_a and $S_1, S_2, \cdots, S_N$ for regions interior to S_a. Mathematically this can be expressed as

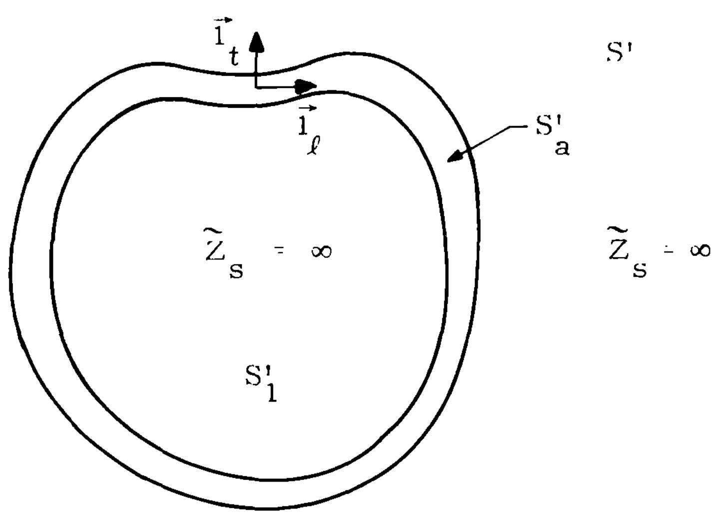
$$S_a \cup S \bigcup_{n=1}^{N} S_n = P \text{ (the entire plane)}$$

$$\text{sheet impedance} \begin{cases} \neq 0 & \text{on } S_a \text{ (multiply connected)} \\ = 0 & \text{on } S, S_1, S_2, \cdots, S_N \end{cases}$$

However not all domains satisfying the above conditions are hatches. For it to be a hatch, it has to satisfy certain additional conditions. For instance, considering an annular slot as shown in figure 5, it is not a hatch unless $\delta << r$. Mathematically, $\delta << r_1 < r_2$. This definition can simply be extended to more complicated hatches. The complement of the

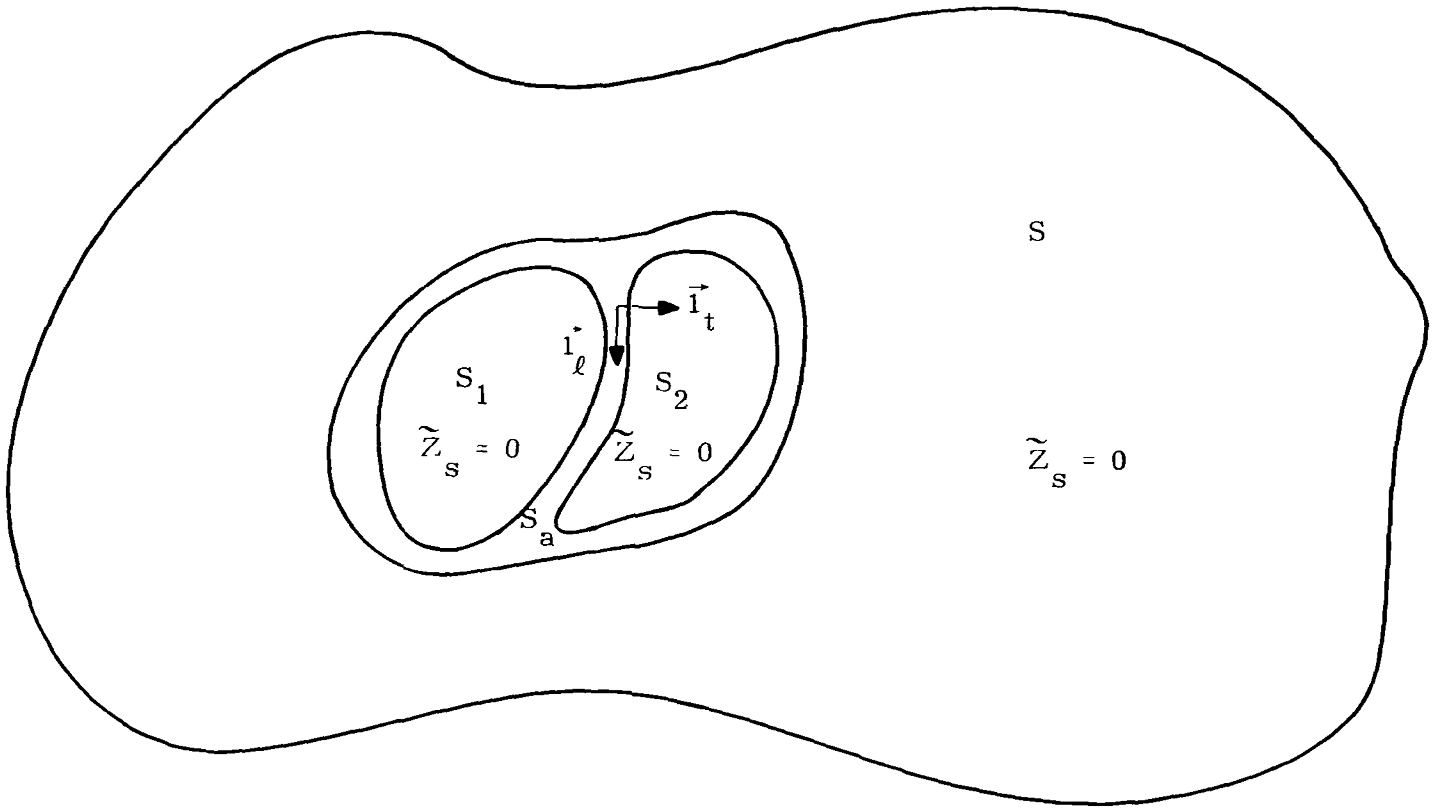


A simple hatch

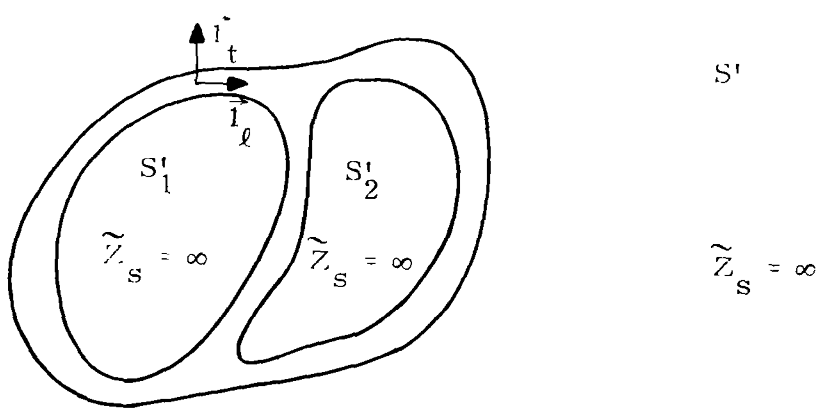


Complementary loop

Figure 3. A Simple Hatch

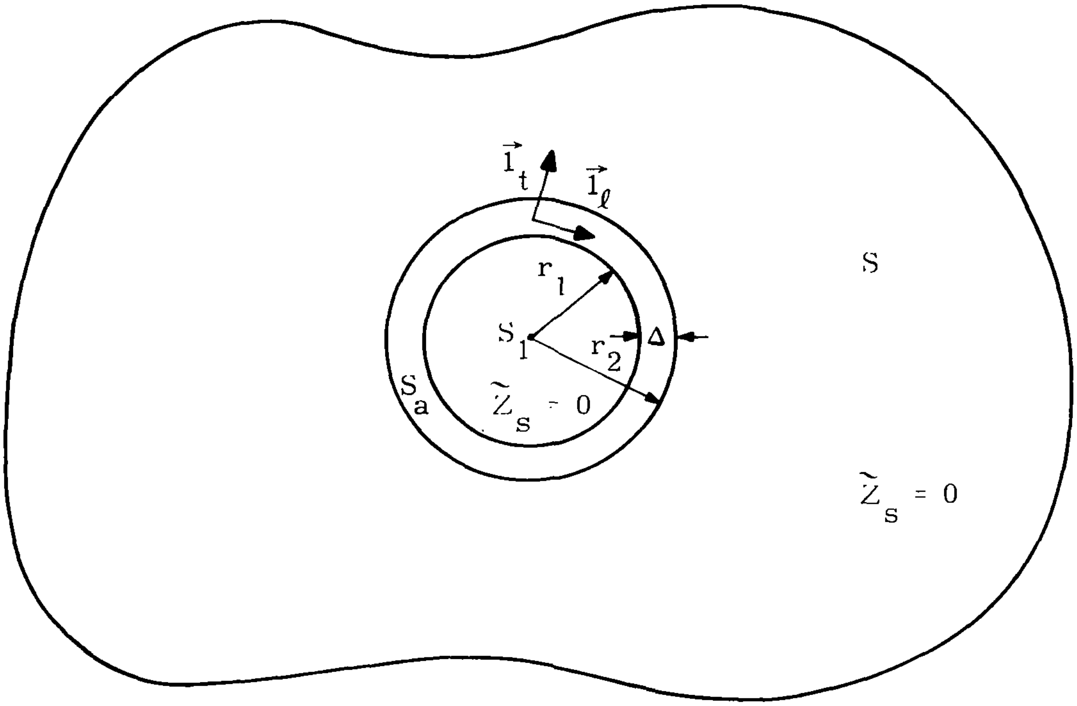


A multiple hatch

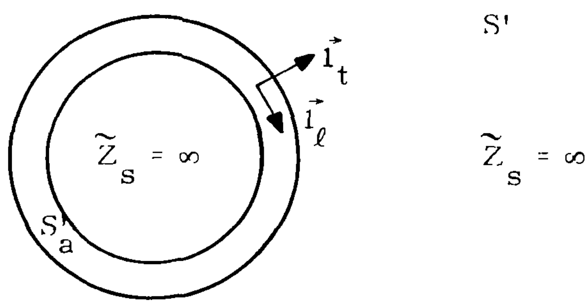


Complementary multiple loop

Figure 4. A Multiple Hatch



Annular slot



Complementary annular ring

Figure 5. Annular Slot

hatch resembles a thin wire loop. As a consequence, results from thin wire loops can be used to solve the problems of hatches. In the case of aircraft, for example, a closed door is a hatch.

A brief discussion on loaded hatches is in order. Let the aperture S_a be covered by a dyadic admittance $\tilde{\vec{\vec{Y}}}_s$. The question which naturally arises is which dyadic components are important? In the case of the rectangular slit as shown in figure 2, and hatches as shown in figures 3, 4, and 5, the component of the admittance $(\tilde{\vec{\vec{Y}}}_s)_{tt}$ along $\vec{1}_t$, the transverse direction, is important. However, in the complementary strip as shown in figure 2, and the complementary loops as shown in figures 3, 4, and 5, the component of the admittance $(\tilde{\vec{\vec{Y}}}'_s)_{\ell\ell}$ along $\vec{1}_\ell$, the longitudinal direction, is important.

In practice, the transverse admittance $(\tilde{\vec{\vec{y}}}_s)_{tt}$ per unit length is preferred. It is defined as

$$(\tilde{\vec{\vec{y}}}_s)_{tt} \equiv \frac{(\tilde{\vec{\vec{Y}}}_s)_{tt}}{\Delta} \text{ (semens/meter)}$$

where Δ is the width of the slot. The complementary admittance-length $(\tilde{\vec{\vec{y}}}'_s)_{\ell\ell}$ is

$$(\tilde{\vec{\vec{y}}}'_s)_{\ell\ell} \equiv (\tilde{\vec{\vec{Y}}}'_s)_{\ell\ell} \; \Delta \text{ (semen-meters)}$$

Similarly, the transverse impedance-length $(\tilde{\vec{\vec{z}}}_s)_{tt}$ is given by

$$(\tilde{\vec{\vec{z}}}_s)_{tt} = (\tilde{\vec{\vec{Z}}}_s)_{tt} \; \Delta \text{ (ohm-meters)}$$

while the complementary impedance per unit length $(\tilde{\vec{\vec{z}}}'_s)_{\ell\ell}$ is

$$(\tilde{\vec{z}}'_s)_{\ell\ell} = \frac{(\tilde{\vec{Z}}'_s)_{\ell\ell}}{\Delta} \text{ (ohms/meter)}$$

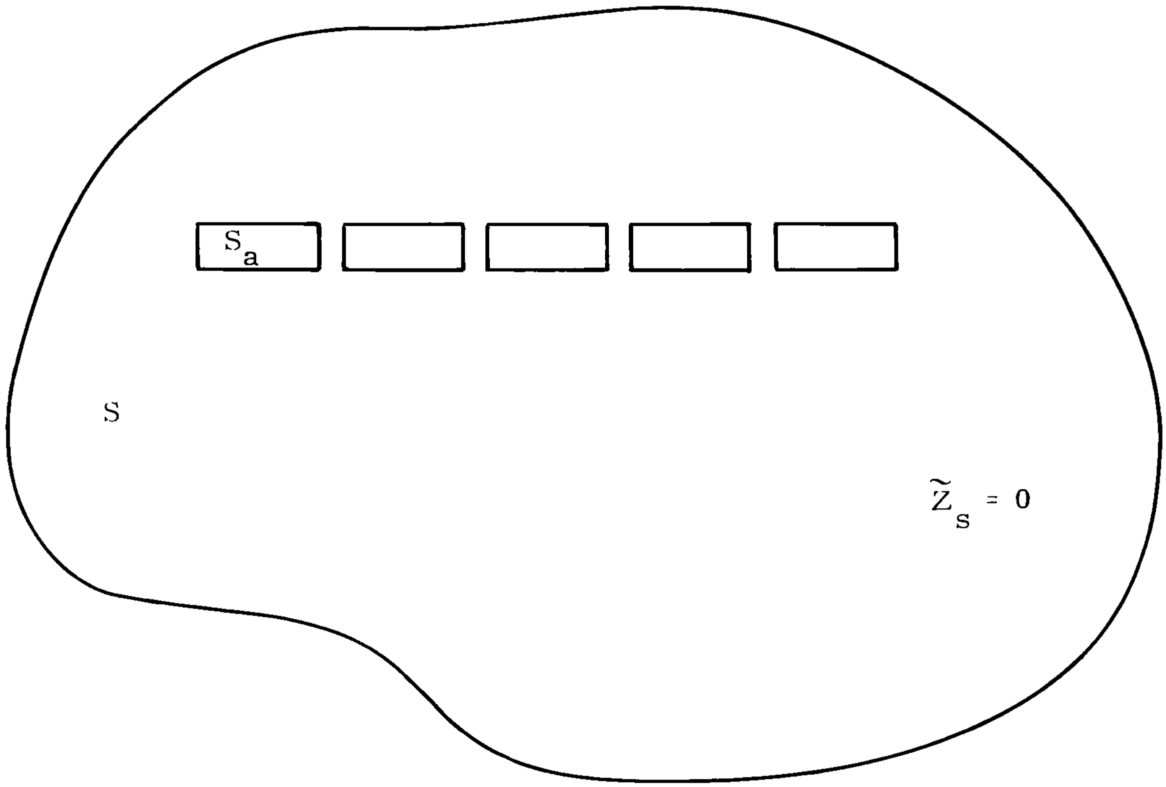
C. Gratings

A grating is defined as a periodic array of identical apertures (typically finite in number) with narrow separations between the apertures. The apertures may be impedance loaded. The narrow separations between the apertures can be referred to as "bars." These bars might also be impedance loaded except that the bar impedance should be small compared to the aperture impedance for such a structure to be thought of as a grating.

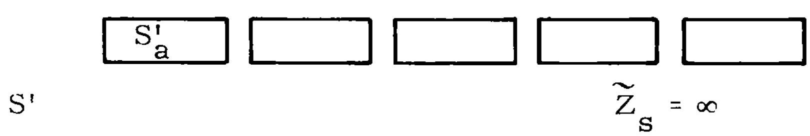
Some typical gratings are shown in figures 6 and 7. For example, in some cases aircraft windows can be thought of as forming a grating. In certain aircraft the windows are loaded. For these cases the complementary impedance can be obtained using the generalized Babinet's principle.

Very little if anything is known regarding numerical solutions for gratings. If the apertures are in the form of slots as shown in figure 8 with $b \ll a$ and $d_1, d_2 \gg a$ then dipole approximations can be used at low frequencies.[4,6] At the resonant frequency of the slots, the grating is almost transparent if the magnetic field is parallel to the slots and is highly reflective for other polarizations.

The problem of interest to EMP is when $d_1, d_2 \ll a, b$ with a and b of the same order. Because of the interaction between apertures, the dipole approximation is not valid for the individual apertures. However equivalent dipole moments and polarizabilities can still be defined for the entire grating for application to distant observers at sufficiently low frequencies. The computation of such polarizabilities, near fields, resonant penetration, etc., of the entire grating is somewhat complicated.

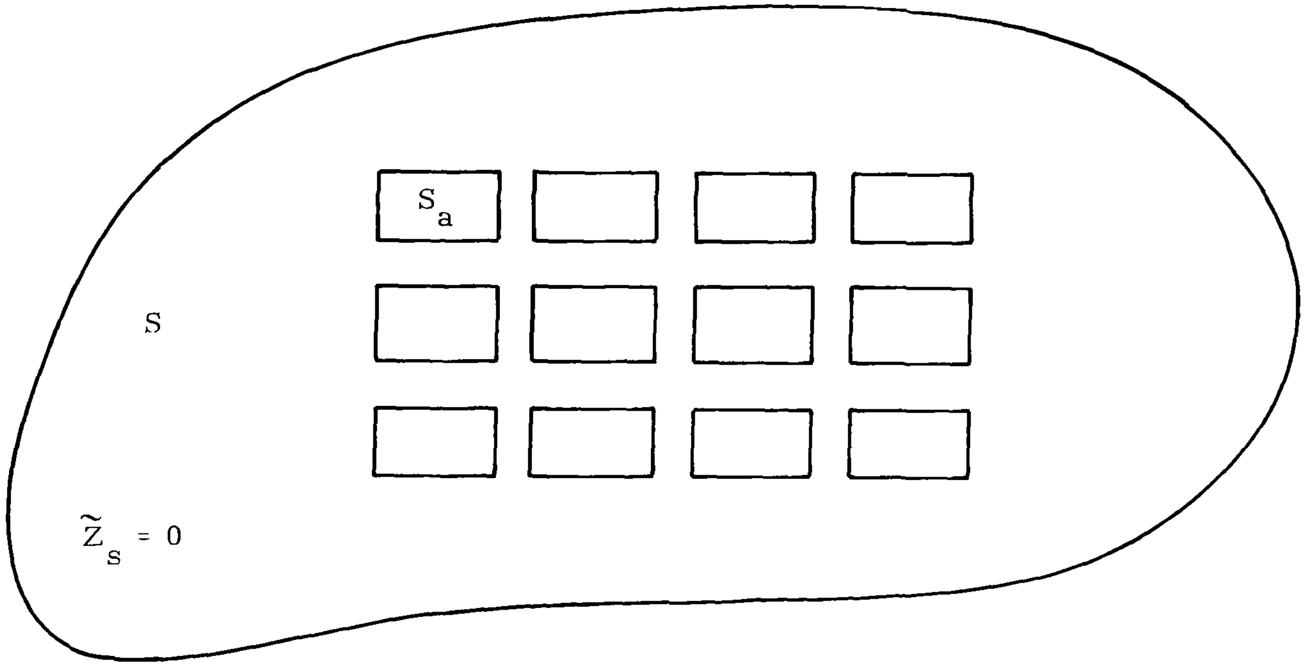


One dimensional grating

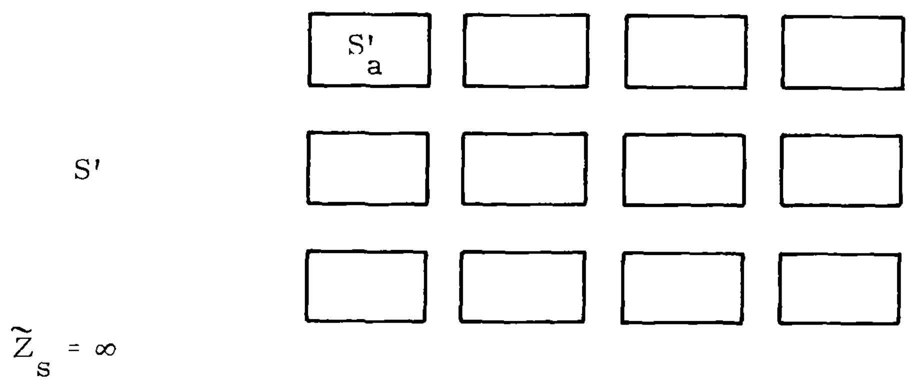


One dimensional complementary disc array

Figure 6. One Dimensional Grating

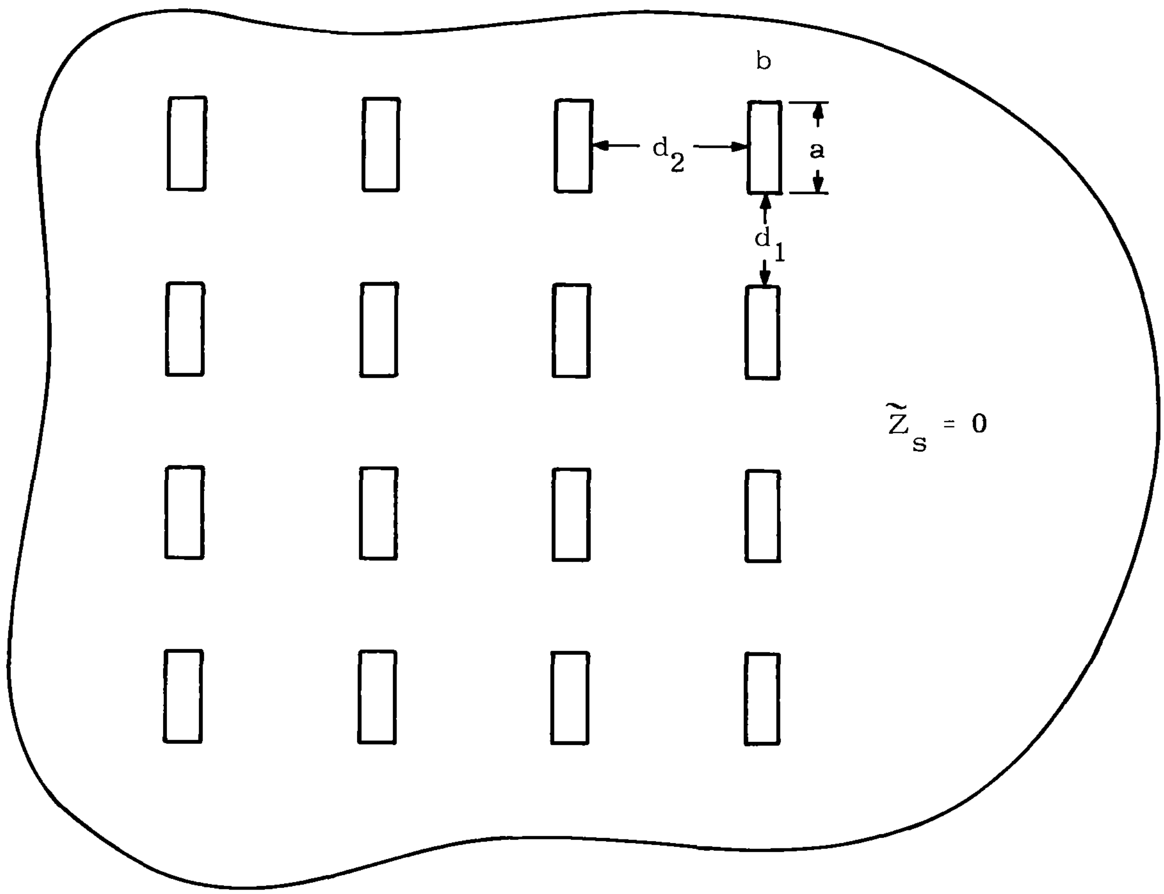


Two dimensional grating

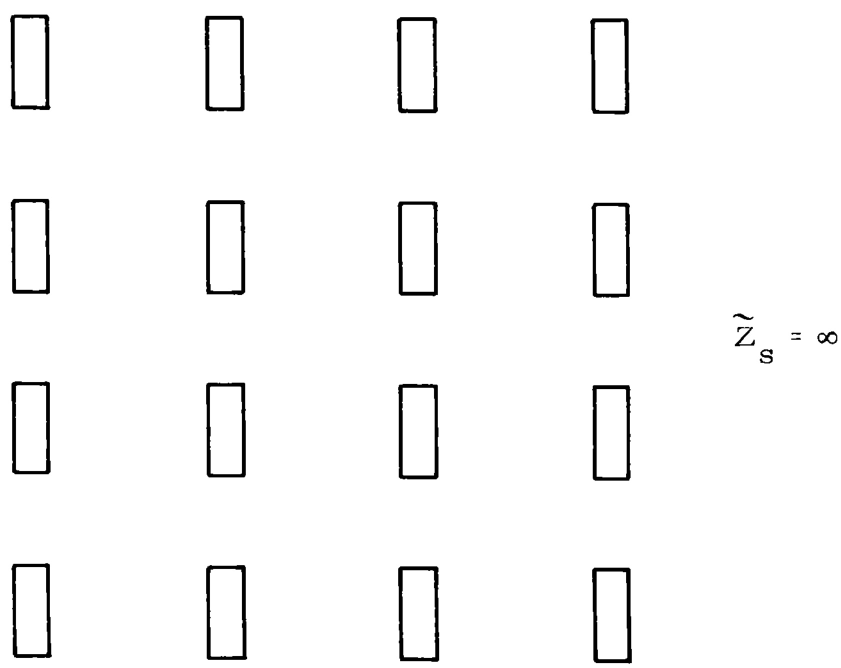


Two dimensional complementary disc array

Figure 7. Two Dimensional Grating



Two dimensional slot array



Two dimensional strip array

Figure 8. Periodic Array of Small Slots and Its Complement

III. Some Comments on Aperture Integral Equations

While this note is not intended as an exposition on aperture integral equations, still there are some observations that can be made which result from the aperture topology. For convenience let us discuss the apertures from the point of view of their equivalent discs (including impedance loading).

A. Simple Apertures

As in figures 1 and 2 the complement of a simple aperture is a simple disc, perhaps impedance loaded. Unless it is thin such as a slit then we are presented with a general two dimensional surface integral equation. For long thin slots as in figure 2 a thin-wire integral equation can be used. Such cases reduce to one dimensional integral equations, at least as an approximation. Thus for even a simple aperture what might be considered a topological property (thin vs. wide) has important consequences.

B. Hatches

As indicated in figures 3, 4, and 5 one of the essential features of a hatch is that the "joint" or "gasket" which is the aperture region (perhaps impedance loaded) is thin compared to the dimensions of the regions enclosed by the aperture. For such an aperture an approximation as a thin strip or wire for the complementary impedance loaded disc is quite appropriate. This leads to a thin wire integral equation on a planar loop or multiple loop structure.

Note the similarity between the thin wire hatch integral equation and the thin wire simple aperture integral equation for a thin slot. The difference is in the aperture topology. A simple (non closed) slot has a complement which behaves as an electric dipole. A hatch slot (closed) has a complement which is a loop and thus has a significant induced magnetic dipole moment (on the complement).

C. Gratings

In a sense a grating as in figures 6 and 7 is the inverse of a hatch in that the principal aperture regions are large compared to the regions (bars) between the apertures. The complement of a grating is a disc array where the discs are large compared to the spacing between them. One might attempt to solve then a surface integral equation on the complementary disc array.

An alternate approach would be to solve for the fields due to a point electric dipole in a single large aperture formed by both the aperture array and the bars separating them. This problem can be converted to that of an equivalent magnetic current element on the complementary large disc. This procedure (perhaps itself involving a numerical solution) would find the large aperture dyadic Green's function. This Green's function could be applied in an integral equation over the bars by setting tangential electric field equal to zero or equal to an impedance times a surface current density. This would then be a two step solution procedure.

IV. Summary

Since the topological properties of apertures have implications concerning the aperture response then such topological properties can be used as a basis for the decomposition of the general class of planar apertures into several categories. The aperture topology in a somewhat generalized sense can also be used to determine whether one or two dimensional integration is required in the integral equations describing the aperture response. The presence of any aperture impedance loading also affects the aperture classification and integral equations.

In this note simple apertures, hatches, and gratings have been discussed. While such aperture types have some practical importance they are not the only types one might define using topological concepts. As more experience is gained in this area the aperture decomposition will likely be extended and refined.

V. References

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