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ON THE EFFECTIVE TRANSFER IMPEDANCE OF THIN COAXIAL CABLE SHIELDS

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ABSTRACT

It is shown that the effective transfer impedance per unit length of a thin coaxial cable shield is given by $Z_s + h^2/Y_s$, where Z_s and Y_s are respectively the cable's inductive transfer impedance per unit length and capacitive transfer admittance per unit length, and h is the axial propagation constant. This general result is illustrated by consideration of a specific shield model, the M-filar filamentary helix.

I. INTRODUCTION

Coaxial cables have been used for the transmission of electromagnetic energy for many years and the basic analysis of such cables is familiar to most electrical engineers. The study of coupling of electromagnetic fields between the interior and exterior of such cables, motivated principally by an interest in understanding the phenomenon of crosstalk between adjacent cables, was begun many years ago; a fundamental paper dealing with this problem is that by Schelkunoff [1] published in 1934. More recently, Kaden [2] considered the problem of perforations in the shield, and attempts have been made to model realistic braided-wire shields [3-6].

A useful descriptor of the coaxial-cable shield is the "transfer impedance," defined as the ratio of the rate of change of open-circuit voltage between the center conductor and the shield to the total shield current:

$$Z_T \equiv \frac{1}{I_s} \frac{dV}{dz}\bigg|_{I=0}$$

I is the center-conductor current, and I_s denotes the shield current. If Z_T is not zero, a term involving Z_T is introduced as a driving function into the voltage-change transmission-line equation.

In 1972, following a conjecture of C. E. Baum's, Latham [4] showed that a driving term also appears in the current-change transmission-line equation for coaxial cables with periodic shields. Thus two parameters describe the exterior-to-interior coupling on coaxial cables: an inductive transfer impedance per unit length, and a capacitive transfer admittance per unit length. These quantities are discussed more fully in the following sections of this paper.

It is our purpose in this paper to show that the effective shield transfer impedance per unit length, defined as the ratio of average axial electric field at the shield to the total shield current, actually involves both the inductive and capacitive transfer immittances and also depends upon the axial propagation constant. This concept is used in formulating an external coupling impedance for the cable, defined as the ratio of the average axial electric field at the cable surface to the total cable current. This external coupling impedance has proven to be useful in the study of certain "leaky-feeder" communication systems in mine tunnels [7], in which a leaky coaxial cable is used as a passive transmission medium.

In Section II of this paper, the transmission-line theory is reviewed and some necessary definitions given. In Section III we derive the effective shield transfer impedance per unit length and the external coupling impedance per unit length for thinly shielded cables. A specific cable model is analyzed for illustrative purposes in Section IV; and Section V concludes the paper.

II. GENERAL CONSIDERATIONS

Let it be assumed that all source and field quantities vary with time as $\exp(j\omega t)$; this time dependence is suppressed in what follows. The transmission-line equations are written*

$$\frac{dV}{dz} = -ZI + Z_s I_t \tag{1a}$$

$$\frac{dI}{dz} = -YV - j\omega \frac{Y}{Y_s} Q_t \tag{1b}$$

in which z is the axial coordinate of the cylindrical system (ρ,ϕ,z), and V and I denote the line voltage and current respectively. I is the current on the center conductor and is taken to be positive in the +z-direction; and V is the potential of the center conductor with respect to the shield. Z and Y denote respectively the series impedance and the shunt admittance per unit length of the cable; Z_s is the inductive transfer impedance per unit length, and Y_s is the capacitive transfer admittance per unit length. I_t and Q_t are respectively the total current (i.e., the sum of the currents carried by the center conductor and the shield) and the total charge per unit length on the cable; these quantities are related by the continuity equation

$$j\omega Q_t = -\frac{dI_t}{dz} \tag{2}$$

The parameters Z and Y are conveniently expressed as

$$Z = Z_c + Z_s \tag{3a}$$

$$\frac{1}{Y} = \frac{1}{Y_c} + \frac{1}{Y_s} \tag{3b}$$

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*With some changes in notation from [4].

Z_c and Y_c are respectively the series impedance and the shunt admittance per unit length associated principally with the cable interior and the center conductor. For a coaxial cable whose shield is confined to an electrically thin layer at $\rho = b$ and whose center conductor has radius $\rho = a$, it is easy to show that

$$Z_c = Z_i + \frac{j\omega\mu}{2\pi} \ln \frac{b}{a} \tag{4a}$$

$$Y_c = \frac{2\pi j\omega\varepsilon}{\ln \frac{b}{a}} \tag{4b}$$

in which μ and ε are respectively the permeability and permittivity of the medium filling the region $a < \rho < b$, and Z_i is the impedance per unit length of the center conductor.

If the cable is axially uniform or if it has a periodic shield whose axial period is much smaller than the axial wavelength, and if the excitation depends upon z as exp(-jhz), then it is convenient to write the transmission-line equations (1) in the form

$$-jh\hat{V} = -Z\hat{I} + Z_s\hat{I}_t \tag{5a}$$

$$-jh\hat{I} = -Y\hat{V} - jh\frac{Y}{Y_s}\hat{I}_t \tag{5b}$$

in which

$$\begin{pmatrix} V(z) \\ I(z) \\ I_t(z) \end{pmatrix} = \begin{pmatrix} \hat{V} \\ \hat{I} \\ \hat{I}_t \end{pmatrix} e^{-jhz}$$

and Q_t has been expressed in terms of I_t by using Eq. (2).

The transmission-line parameters Z_c, Z_s, Y_c, and Y_s may be determined once $\hat{I}/\hat{I}_t$ and $\hat{V}/\hat{I}_t$ are known as functions of h. Defining

$$r(h) = \hat{I}/\hat{I}_t \tag{7a}$$

$$hs(h) = \hat{V}/\hat{I}_t \tag{7b}$$

we obtain from Eqs. (5)

$$-jh^2 s = -rZ + Z_s \tag{8a}$$

$$-jhr = -Yhs - jhY/Y_s \tag{8b}$$

from which the transmission-line parameters may be found. The functions r(h) and s(h) are determined from the solution of an appropriate boundary value problem. Conversely, if the transmission-line parameters are known, r(h) and s(h) may be expressed in terms of these parameters as follows:

$$r(h) = \frac{Z_s + h^2/Y_s}{Z + h^2/Y} \tag{9a}$$

$$hs(h) = \frac{jh(Z_s/Y - Z/Y_s)}{Z + h^2/Y} \tag{9b}$$

It is therefore apparent that when the transmission-line parameters for a given cable are known, the induced current and voltage can be determined from a knowledge of the total current I_t. This current can be related to the axial component of the electric field at the outer surface of the cable by means of the external coupling impedance per unit length Z_{ec}, defined by

$$Z_{ec} \equiv \hat{E}_{za}/\hat{I}_t \tag{10}$$

E_{za} denotes the average value of E_z at the outer surface of the cable. It is assumed that in order for this concept to be useful, the circumference of the cable is small in comparison to the free space wavelength. The external coupling impedance may in general be determined from the solution of an appropriate boundary value problem.

In the next section of this paper, we shall show that under certain conditions the external coupling impedance may be directly evaluated in terms of the transmission-line parameters and the electrical properties of the cable jacket. It is also shown that the effective transfer impedance per unit length of the shield depends upon the axial propagation constant h and is given by $Z_s + h^2/Y_s$.

III. THINLY SHIELDED COAXIAL CABLES

We define a thin coaxial cable shield as one through which the tangential components of electric field are continuous. The thin surface-impedance layer considered by Wait [7] is such a shield, as is the multifilar filamentary helix considered in Section IV of this paper.

Consider a coaxial cable with a center conductor of radius a having impedance per unit length Z_i. The center conductor is surrounded by a dielectric layer of permittivity ε_1 extending to $\rho = b$. A thin shield is located at $\rho = b$ and is surrounded by an outer dielectric jacket of permittivity ε_2 extending to $\rho = c$. The permeability of both dielectrics is assumed to be equal to μ_o. The geometry of this general coaxial cable model is shown in Fig. (1).

The shield will in general be a structure periodic in both ϕ and z, and as a consequence doubly infinite sets of space harmonics of both TE and TM fields are necessary to describe the electromagnetic field of the cable. However, the currents $\hat{I}$ and $\hat{I}_t$ and the average electric field $\hat{E}_z$ involve only the TM_{00} space harmonic, and we shall focus attention on this component of the total field in what follows.

In the interior of the cable, we have

$$\hat{E}_{z00} = \frac{-\lambda_1^2}{j\omega\varepsilon_1} \hat{\Psi}_{00} \tag{11a}$$

$$\hat{H}_{\phi 00} = \frac{-d\hat{\Psi}_{00}}{d\rho} \tag{11b}$$

in which $\lambda_1^2 = h^2 - k_1^2$, $k_1^2 = \omega^2 \mu_0 \varepsilon_1$, and

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho \frac{d\hat{\Psi}_{00}}{d\rho}\right) - \lambda_1^2 \hat{\Psi}_{00} = 0 \tag{12}$$

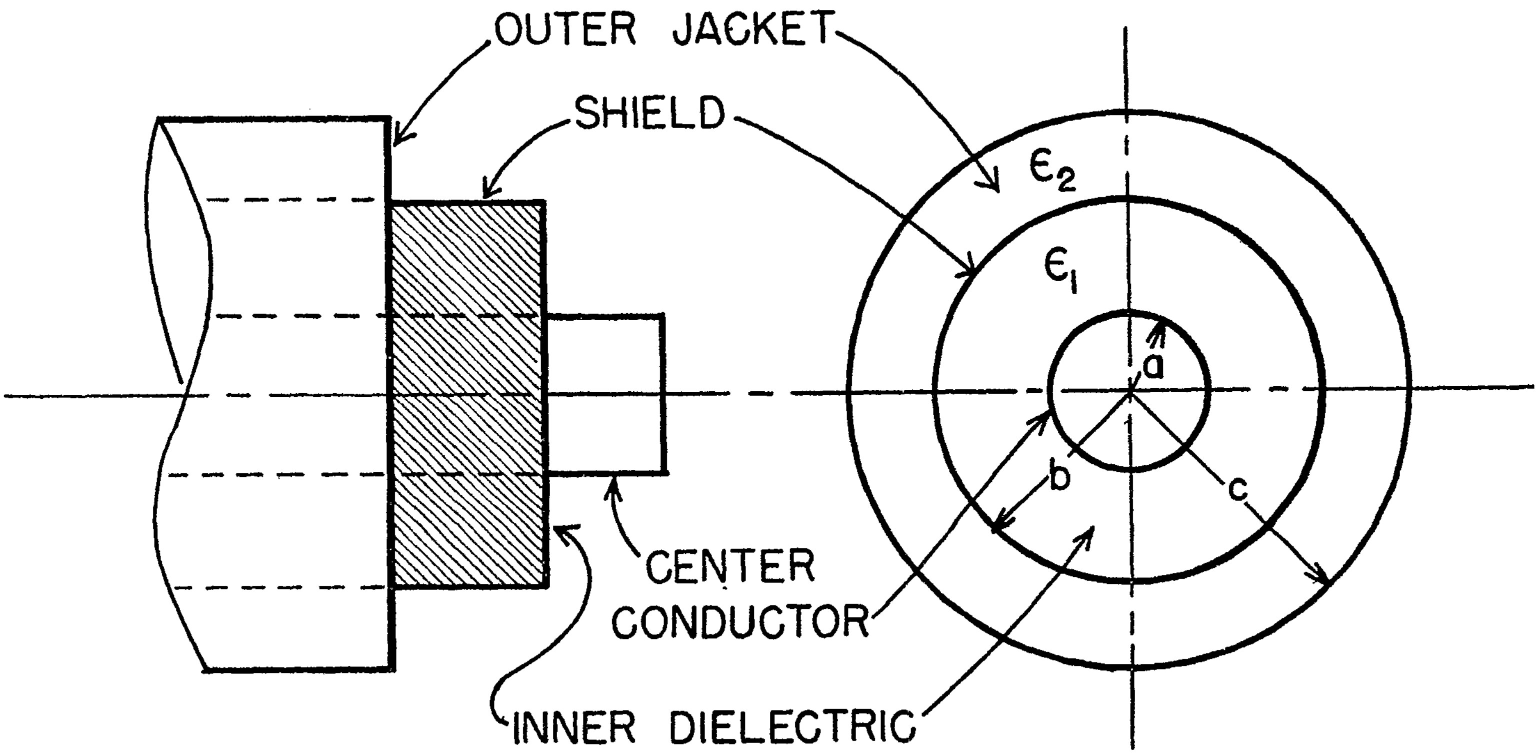

Figure 1. Geometry of the general coaxial cable model

Constructing a solution of Eq. (12) in the case $\lambda_1 b << 1$ and applying the condition

$$\hat{E}_{z00}(\rho=a) = 2\pi a Z_i \hat{H}_{\phi 00}(\rho=a) \tag{13}$$

yields the result that for $a \leq \rho \leq b$,

$$\hat{E}_{z00}(\rho) = \hat{I} \left(\frac{\lambda_1^2}{2\pi j\omega\varepsilon_1} \ln \frac{\rho}{a} + Z_i \right) \tag{14}$$

Assuming that $\hat{E}_{z00}$ is continuous through the shield at $\rho = b$ and similarly constructing a solution for $\hat{E}_{z00}$ in the outer jacket, we find readily that

$$\hat{E}_{z00}(\rho=c) = \frac{\lambda_2^2 \hat{I}_t}{2\pi j\omega\varepsilon_2} \ln \frac{c}{b} + \hat{I} \left(\frac{\lambda_1^2}{2\pi j\omega\varepsilon_1} \ln \frac{b}{a} + Z_i \right) \tag{15}$$

Now dividing Eq. (15) through by $\hat{I}_t$ and using Eqs. (3), (4), and (9a), we obtain an expression for Z_{ec}:

$$Z_{ec} = Z_2 + \frac{(Z_c + h^2/Y_c)(Z_s + h^2/Y_s)}{(Z_c + h^2/Y_c) + (Z_s + h^2/Y_s)} \tag{16}$$

in which

$$Z_2 \equiv \frac{\lambda_2^2}{2\pi j\omega\varepsilon_2} \ln \frac{c}{b} \tag{17}$$

A network representation of Z_{ec} is shown in Fig. (2). It will be noted that the shunt element representing the shield is $Z_s + h^2/Y_s$. This is the effective transfer impedance per unit length of the shield itself, as we can show directly by expressing $\hat{E}_{z00}(\rho=b)$ in terms of $\hat{I}_s$, the current on the shield. We have from Eq. (14)

$$\hat{E}_{z00}(\rho=b) = (Z_c + h^2/Y_c)\hat{I}_s \cdot \frac{\hat{I}}{\hat{I}_s} \tag{18}$$

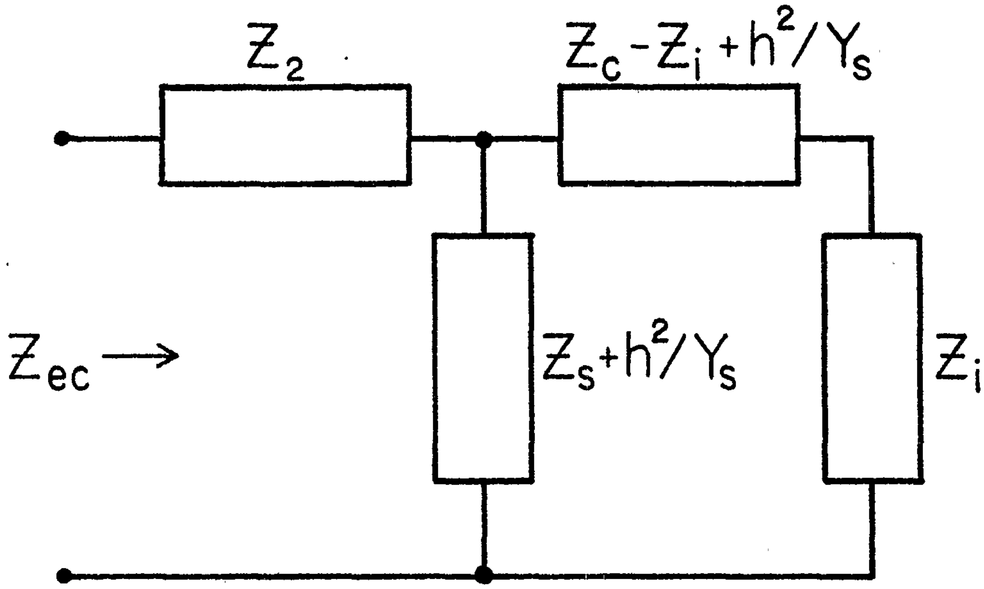

Figure 2. Network representation of the external coupling impedance per unit length

Since $\hat{I}_t = \hat{I} + \hat{I}_s$,

$$\frac{\hat{I}}{\hat{I}_s} = \frac{r(h)}{1 - r(h)} \tag{19}$$

and therefore, using Eq. (9a), we obtain

$$\hat{E}_{z00}(\rho=b) = (Z_s + h^2/Y_s)\hat{I}_s \tag{20a}$$

$$= Z_{Te}\hat{I}_s \tag{20b}$$

where $Z_{Te} \equiv Z_s + h^2/Y_s$ denotes the effective transfer impedance per unit length of the shield.

It is apparent that the effect of capacitive coupling of the electromagnetic field between the interior and the exterior of the cable is to make the shield's effective transfer impedance spatially dispersive. This result appears to be new.

In the following section of this paper, we shall consider a specific coaxial cable model in some detail, in order to illustrate the results which have been presented above.

IV. A MULTIFILAR-HELIX SHIELDED CABLE

We consider a coaxial cable model whose shield is an M-filar filamentary helix of pitch angle ψ (measured with respect to the z-axis). For simplicity, the shield wires and the center conductor are assumed to be perfect, and dielectrics are absent (i.e., $\varepsilon_1 = \varepsilon_2 = \varepsilon_o$). The shield wires have radius r, and r is assumed to be small in comparison to the separation of the wires from each other and to their distance from the center conductor. The geometry of this cable model is shown in Fig. (3).

The transmission-line parameters for this cable model have been found by Latham [4] using electrostatic and magnetostatic considerations. We shall reconsider the model from an electromagnetic viewpoint, assuming that the transverse cable dimensions are small in comparison to the free-space wavelength (i.e., $k_o b \ll 1$) and that the axial period of the shield is small in comparison to the axial wavelength ($bh \ll M \tan\psi$). The purpose of our analysis is to verify the utility of the transmission-line formulation in this low-frequency case and to demonstrate, for this model at least, the assertions made in the previous section regarding the external coupling impedance and the effective transfer impedance per unit length of the shield.

The electric and magnetic fields are expressed in terms of two functions Ψ and Φ which satisfy the scalar Helmholtz equation as follows:

$$\bar{E} = \nabla \times \Phi\bar{a}_z + \frac{1}{j\omega\varepsilon_o} \nabla \times \nabla \times \Psi\bar{a}_z \quad (21a)$$

$$\bar{H} = \nabla \times \Psi\bar{a}_z - \frac{1}{j\omega\mu_o} \nabla \times \nabla \times \Phi\bar{a}_z \quad (21b)$$

Ψ yields the TM part of the electromagnetic field and Φ the TE part.

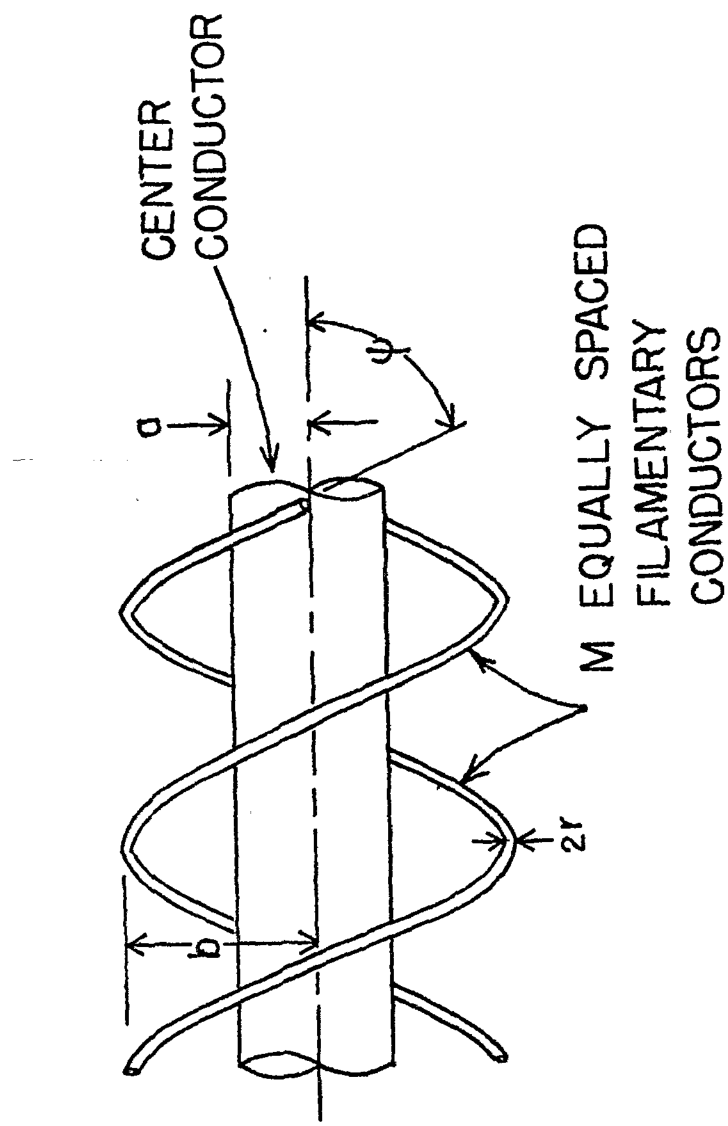

Figure 3. Geometry of the multifilar-helix shielded cable

The shield is periodic in both ϕ (period $2\pi/M$) and z (period $2\pi b/M \cot\psi$), and the appropriate Floquet forms for Ψ and Φ are

$$\begin{pmatrix}\Psi\\ \Phi\end{pmatrix} = \sum_{\substack{\ell=-\infty\\ n=\ell M}}^{\infty} \begin{pmatrix}\hat{\Psi}_n\\ \hat{\Phi}_n\end{pmatrix} e^{jn\phi - jh_n z} \tag{22}$$

in which $\hat{\Psi}_n$ and $\hat{\Phi}_n$ depend only upon ρ, and

$$h_n = h + \frac{n \tan\psi}{b} \tag{23}$$

We shall consider that the coaxial cable is illuminated by a TM-polarized plane wave of frequency such that $k_o b << 1$. Appropriate forms for $\hat{\Psi}_n$ and $\hat{\Phi}_n$ in each of the two regions of the problem are therefore

$$a \leq \rho < b: \quad \hat{\Psi}_{ni} = A_n[I_n(\lambda_n\rho)K_n(\lambda_n a) - I_n(\lambda_n a)K_n(\lambda_n\rho)] \tag{24a}$$

$$\hat{\Phi}_{ni} = B_n[I_n(\lambda_n\rho)K_n'(\lambda_n a) - I_n'(\lambda_n a)K_n(\lambda_n\rho)] \tag{24b}$$

$$\rho > b: \quad \hat{\Psi}_{no} = C_n K_n(\lambda_n\rho) - \frac{j\omega\varepsilon_o}{\lambda_o^2}\hat{E}_{zinc} I_o(\lambda_o\rho)\delta_{no} \tag{24c}$$

$$\hat{\Phi}_{no} = D_n K_n(\lambda_n\rho) \tag{24d}$$

in which $\hat{E}_{zinc}$ is the amplitude of the z-component of the incident electric field, δ_{no} is the Kronecker delta-function, and

$$\lambda_n^2 = h_n^2 - k_o^2 \tag{25}$$

The currents on the shield filaments are initially unknown, but are assumed to be identical because of the azimuthal uniformity of the excitation. The space-harmonic components of the shield surface current density at $\rho = b$ are

$$\hat{J}_{s\phi n} = \frac{\hat{I}_s \tan\psi}{2\pi b} \tag{26a}$$

$$\hat{J}_{szn} = \frac{\hat{I}_s}{2\pi b} \tag{26b}$$

in which the total shield current I_s is simply (M cosψ) times the current on a single filament.

The boundary conditions to be applied at $\rho = b$ are

$$\hat{E}_{zni} = \hat{E}_{zno} \tag{27a}$$

$$\hat{E}_{\phi ni} = \hat{E}_{\phi no} \tag{27b}$$

$$\hat{H}_{\phi no} - \hat{H}_{\phi ni} = \hat{J}_{szn} \tag{27c}$$

$$\hat{H}_{zno} - \hat{H}_{zni} = -\hat{J}_{s\phi n} \tag{27d}$$

These conditions suffice to determine the unknown coefficients $A_n - D_n$ in terms of the shield current $\hat{I}_s$ and the incident field $\hat{E}_{zinc}$. The additional boundary condition to be imposed is that, since the shield wires are assumed to be perfectly conducting, the electric field parallel to the shield wires vanishes on the wire surfaces:

$$\sin\psi \, E_\phi + \cos\psi \, E_z \Big|_{\substack{\text{on shield} \\ \text{filament}}} = 0 \tag{28}$$

The coefficients $A_n - D_n$ are readily determined using Eqs. (21)-(27). We obtain

$$A_n = \frac{\hat{I}_s}{2\pi} \frac{K_n(\lambda_n b)}{K_n(\lambda_n a)} \left(1 - \frac{nh_n \tan\psi}{\lambda_n^2 b}\right) - \frac{j\omega\varepsilon_o \hat{E}_{zinc}}{\lambda_0^2 K_o(\lambda_o a)} \delta_{no} \tag{29a}$$

$$B_n = \frac{-j\omega\mu_o \hat{I}_s \tan\psi}{2\pi\lambda_n} \frac{K_n'(\lambda_n b)}{K_n'(\lambda_n a)} \tag{29b}$$

$$C_n = \frac{\hat{I}_s}{2\pi} \frac{1}{K_n(\lambda_n a)} [I_n(\lambda_n b)K_n(\lambda_n a) - I_n(\lambda_n a)K_n(\lambda_n b)]$$

$$+ \frac{j\omega\varepsilon_o \hat{E}_{zinc}}{\lambda_o^2 K_o(\lambda_o a)} I_o(\lambda_o a)\delta_{no} \tag{29c}$$

$$D_n = \frac{-j\omega\mu_o \hat{I}_s \tan\psi}{2\pi\lambda_n} \frac{1}{K_n'(\lambda_n a)} [I_n'(\lambda_n b)K_n'(\lambda_n a) - I_n'(\lambda_n a)K_n'(\lambda_n b)] \tag{29d}$$

Now apply the wire boundary condition of Eq. (28) at $\rho = b$, $z = b\phi\cot\psi + r\csc\psi$. After some algebraic simplification, the result is the relation

$$\hat{I}_s(Z_p + Z_q) = \frac{\hat{E}_{zi}}{K_o(\lambda_o a)} [I_o(\lambda_o b)K_o(\lambda_o a) - I_o(\lambda_o a)K_o(\lambda_o b)] \tag{30}$$

in which

$$Z_p = \frac{\lambda_o^2}{2\pi j\omega\varepsilon_o} \frac{K_o(\lambda_o b)}{K_o(\lambda_o a)} [I_o(\lambda_o b)K_o(\lambda_o a) - I_o(\lambda_o a)K_o(\lambda_o b)] \tag{31a}$$

$$Z_q = \frac{-j\omega\mu_o \tan^2\psi}{2\pi} \sum_{\substack{\ell=-\infty \\ n=\ell M}}^{\infty} e^{-jnr\sec\psi/b} \frac{K_n'(\lambda_n b)}{K_n'(\lambda_n a)} [I_n'(\lambda_n b)K_n'(\lambda_n a) - I_n'(\lambda_n a)K_n'(\lambda_n b)]$$

$$+ \frac{1}{2\pi j\omega\varepsilon_o} \sum_{\substack{\ell=-\infty \\ \ell\neq 0 \\ n=\ell M}}^{\infty} e^{-jnr\sec\psi/b} \frac{K_n(\lambda_n b)}{K_n(\lambda_n a)} [I_n(\lambda_n b)K_n(\lambda_n a) - I_n(\lambda_n a)K_n(\lambda_n b)]$$

$$\cdot \left(\lambda_n - \frac{nh_n \tan\psi}{\lambda_n b}\right)^2 \tag{31b}$$

In the limiting cases $k_o b \ll 1$, $bh \ll M \tan\psi$, Eqs. (30) and (31) simplify to

$$\hat{I}_s(Z_p + Z_q) = \frac{\hat{E}_{zi}}{K_o(\lambda_o a)} \ln\frac{b}{a} \tag{32}$$

$$Z_p = \frac{\lambda_o^2}{2\pi j\omega\varepsilon_o} \ln\frac{b}{a} \tag{33a}$$

$$Z_q = \frac{j\omega\mu_o \tan^2\psi}{4\pi} (1 - a^2/b^2)$$

$$- \frac{j\omega\mu_o \tan^2\psi}{\pi} \sum_{\substack{\ell=1 \\ n=\ell M}}^{\infty} \cos(\frac{nr}{b} \sec\psi) G_n^{(1)}(\tan\psi, \frac{a}{b} \tan\psi)$$

$$+ \frac{h^2}{\pi j\omega\varepsilon_o} \sum_{\substack{\ell=1 \\ n=\ell M}}^{\infty} \cos(\frac{nr}{b} \sec\psi) G_n^{(2)}(\tan\psi, \frac{a}{b} \tan\psi) \qquad (33b)$$

where

$$G_n^{(1)}(x,y) \equiv \frac{K_n'(nx)}{K_n'(ny)} [I_n'(nx)K_n'(ny) - I_n'(ny)K_n'(nx)] \qquad (34a)$$

$$G_n^{(2)}(x,y) \equiv \frac{K_n(nx)}{K_n(ny)} [I_n(nx)K_n(ny) - I_n(ny)K_n(nx)] \qquad (34b)$$

Comparison of Eq. (33b) with Latham's results indicates that $Z_q = Z_s + h^2/Y_s$; however, we shall make this identification only tentatively and write

$$Z_q = Z_s' + h^2/Y_s' \qquad (35)$$

in which

$$Z_s' = \frac{j\omega\mu_o}{4\pi} \tan^2\psi(1 - \frac{a^2}{b^2})$$

$$- \frac{j\omega\mu_o \tan^2\psi}{\pi} \sum_{\substack{\ell=1 \\ n=\ell M}}^{\infty} \cos(\frac{nr}{b} \sec\psi) G_n^{(1)}(\tan\psi, \frac{a}{b} \tan\psi) \qquad (36a)$$

$$\frac{1}{Y_s'} = \frac{1}{\pi j\omega\varepsilon_o} \sum_{\substack{\ell=1 \\ n=\ell M}}^{\infty} \cos(\frac{nr}{b} \sec\psi) G_n^{(2)}(\tan\psi, \frac{a}{b} \tan\psi) \qquad (36b)$$

Furthermore, we shall write

$$Z_p = Z'_c + \frac{h^2}{Y'_c} \tag{37}$$

where

$$Z'_c = \frac{j\omega\mu_o}{2\pi} \ln \frac{b}{a} \tag{38a}$$

$$\frac{1}{Y'_c} = \frac{1}{2\pi j\omega\varepsilon_o} \ln \frac{b}{a} \tag{38b}$$

Now consider the total current on the cable and the current on the center conductor. We have

$$\hat{I}_t = -2\pi b \left.\frac{d\hat{\Psi}_{oo}}{d\rho}\right)_{\rho=b} \tag{39a}$$

$$\hat{I} = -2\pi a \left.\frac{d\hat{\Psi}_{oi}}{d\rho}\right)_{\rho=a} \tag{39b}$$

and it is readily found from Eqs. (24a), (29a), and (32) that

$$\hat{I}_t = \frac{2\pi j\omega\varepsilon_o \hat{E}_{zinc}}{\lambda_o^2 K_o(\lambda_o a)} \tag{40a}$$

$$\hat{I} = \hat{I}_t - \hat{I}_s \tag{40b}$$

in the case $k_o b << 1$. Now it is easy to show using Eqs. (32), (35), (37), and (40b) that

$$\frac{\hat{I}}{\hat{I}_t} = r(h) = \frac{Z'_s + h^2/Y'_s}{Z' + h^2/Y'} \tag{41}$$

where

$$Z' = Z'_c + Z'_s \tag{42a}$$

$$\frac{1}{Y'} = \frac{1}{Y'_c} + \frac{1}{Y'_s} \tag{42b}$$

The average electric field at ρ=b is given by

$$\hat{E}_{za} = \frac{-\lambda_o^2}{j\omega\varepsilon_o}\hat{\Psi}_{oo} = \frac{-\lambda_o^2}{j\omega\varepsilon_o}\hat{\Psi}_{oi} \tag{43a}$$

$$= \frac{\lambda_o^2\hat{I}}{2\pi j\omega\varepsilon_o}\ell n\frac{b}{a} \tag{43b}$$

$$= (Z'_c + h^2/Y'_c)\hat{I} \tag{43c}$$

and the external coupling impedance is therefore, from Eqs. (41) and (43c),

$$Z_{ec} = \frac{(Z'_c + h^2/Y'_c)(Z'_s + h^2/Y'_s)}{Z' + h^2/Y'} \tag{44}$$

Furthermore, it is easy to show using the above results that

$$\frac{\hat{E}_{za}}{\hat{I}_s} = Z'_s + h^2/Y'_s \tag{45}$$

so that the effective transfer impedance per unit length of the shield is given by

$$Z_{Te} = Z'_s + h^2/Y'_s \tag{46}$$

It is apparent from Eqs. (44) and (46) that the external coupling impedance and the effective shield transfer impedance per unit length have the forms which were discussed in the previous section of this paper. It remains to show that $Z'_s = Z_s$, $Y'_s = Y_s$, $Z'_c = Z_c$, and $Y'_c = Y_c$, i.e., that the primed quantities are actually identical to the transmission-line parameters for this cable. We have already derived in Eq. (41) an expression for r(h); we now obtain an expression for s(h) as follows. The potential of the center conductor with respect to the shield is defined to be the integral of the radial electric field from the center conductor to a point on one of the shield filaments, the integral being taken at constant ϕ and z. In the low-frequency case under consideration, the TE part of the radial electric field does not contribute to V, and as a consequence

$$V = \frac{1}{j\omega\varepsilon_o} \frac{\partial \Psi_i}{\partial z} \Bigg|_{\substack{\text{filament} \\ \text{surface}}} \tag{47}$$

From Eqs. (24a) and (29a), in the limits $k_o b \ll 1$, $bh \ll M\tan\psi$, we find

$$\hat{V} = \frac{-h\hat{I}_s}{2\pi\omega\varepsilon_o} \ln \frac{b}{a} - \frac{h\hat{I}_s}{\pi\omega\varepsilon_o} \sum_{\substack{\ell=1 \\ n=\ell M}}^{\infty} \cos\left(\frac{nr}{b} \sec\psi\right) G_n^{(2)}\left(\tan\psi, \frac{a}{b}\tan\psi\right) + \frac{jh\hat{E}_{zinc}}{\lambda_o^2 K_o(\lambda_o a)} \ln \frac{b}{a} \tag{48}$$

and using Eqs. (36b), (38b), and (40a), we have

$$\hat{V} = \frac{-jh\hat{I}_s}{Y'} + \frac{jh\hat{I}_t}{Y'_s} \tag{49}$$

so that division by $\hat{I}_t$ and use of Eqs. (40b) and (41) yields

$$hs(h) = \frac{jh(Z'_s/Y' - Z'/Y'_s)}{Z' + h^2/Y'} \tag{50}$$

It is now necessary only to compare Eq. (41) with Eq. (9a) and Eq. (50) with Eq. (9b), and it is apparent that

$$Z'_c = Z_c = \frac{j\omega\mu_o}{2\pi} \ln \frac{b}{a} \tag{51a}$$

$$\frac{1}{Y_c'} = \frac{1}{Y_c} = \frac{1}{2\pi j\omega\varepsilon_o} \ln\frac{b}{a} \tag{51a}$$

$$Z_s' = Z_s = \frac{j\omega\mu_o \tan^2\psi}{4\pi}\left(1 - \frac{a^2}{b^2}\right) + \frac{j\omega\mu_o \tan^2\psi}{\pi}\sum_{\substack{\ell=1\\ n=\ell M}}^{\infty} \cos(n\pi c)G_n^{(1)}(\tan\psi, \frac{a}{b}\tan\psi) \tag{51c}$$

$$\frac{1}{Y_s'} = \frac{1}{Y_s} = \frac{1}{\pi j\omega\varepsilon_o}\sum_{\substack{\ell=1\\ n=\ell M}}^{\infty} \cos(n\pi c)G_n^{(2)}(\tan\psi, \frac{a}{b}\tan\psi) \tag{51d}$$

in which we have introduced the expression for the optical coverage of the shield

$$c = \frac{Mr\ \sec\psi}{\pi b} \tag{52}$$

Therefore, for this particular cable model in the low-frequency case $k_o b \ll 1$, $bh \ll M\tan\psi$, the effective transfer impedance per unit length of the shield and the external coupling impedance per unit length are given by the expressions derived in Section III of this paper. They can therefore be expressed directly in terms of the transmission-line parameters of the cable.

It is of interest to express the infinite series in Eqs. (51c) and (51d) above in closed form if possible. Latham has presented several expressions for these, applicable in various special cases. The one which may be of most interest is the case in which M is sufficiently large to permit the replacement of the modified Bessel functions in the series expressions by their uniform asymptotic forms [8]. The series may then be summed without difficulty, and we find that if $M \ln(b/a) \gg 1$†, then

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†This assumption means physically that the $n \neq 0$ space harmonics do not penetrate into the cable interior to the depth of the center conductor.

$$Z_s \simeq \frac{j\omega\mu_o \tan^2\psi}{4\pi}\left(1 - \frac{a^2}{b^2}\right) - \frac{j\omega\mu_o \sec\psi}{2\pi M} \ln\left(2\sin\frac{\pi c}{2}\right) \tag{53a}$$

$$\frac{1}{Y_s} \simeq \frac{-\cos\psi}{2\pi M j\omega\varepsilon_o} \ln\left(2\sin\frac{\pi c}{2}\right) \tag{53b}$$

the expressions being valid when $c \leq 1$.

A special feature of the multifilar helix shielded cable studied in this section is the angular asymmetry of the shield. A consequence of this asymmetry is that the $n = 0$ space harmonics for TE and TM fields are coupled, so that the scattering characteristics of this cable are insufficiently described by the external impedance alone. We therefore augment the description of this cable as a scatterer by calculating the total magnetic current carried by the cable. Defining this magnetic current

$$I_{tm} = -\int_0^{2\pi} bE_\phi(\rho=b)\,d\phi \tag{54}$$

we readily obtain

$$I_{tm} = \frac{j\omega\mu_o b\, I_s \tan\psi}{2}\left(1 - \frac{a^2}{b^2}\right) \tag{55}$$

or equivalently

$$\hat{I}_{tm} = \frac{j\omega\mu_o b \tan\psi}{2Z_{Te}}\left(1 - \frac{a^2}{b^2}\right)\hat{E}_{za} \tag{56}$$

It will be noted that $\hat{I}_{tm} = 0$ when $\psi = 0$ and the angular asymmetry disappears. In a cable whose shield comprised two counterwound helices (i.e., pitch angles $\pm\psi$) the angular asymmetry would not be present and the total magnetic current would be zero; and there would be no "macroscopic" coupling between TE and TM parts of the external field due to the presence of the cable.

V. CONCLUDING REMARKS

We have defined a "thin" coaxial cable shield and have shown that for such a shield, the external coupling impedance per unit length is related in a relatively simple way to the transmission-line parameters of the cable itself and to the properties of the outer dielectric jacket. The result indicates that the effective transfer impedance per unit length of the shield is not simply its inductive transfer impedance per unit length, but it also contains a contribution due to capacitive coupling. Thus the external behavior of the cable cannot completely be described by the inductive transfer impedance alone; capacitive effects must also be considered.

It is emphasized once again that the transmission-line formulation is useful only when the cable is small compared to wavelength, both in transverse and axial (period) dimensions. Otherwise a complete electrodynamic treatment of the coupling problem is necessary and the simplifications afforded by the use of the transmission-line parameters will lead to incorrect results. However, in the many practical cases in which the approximations are valid, the transmission-line formulation of the external impedance provides a simple means of relating the external behavior of a leaky coaxial cable to its properties as a transmission-line.

In the event that the tangential components of the electric field cannot be considered continuous through the shield, the simple relation $Z_{Te} = Z_s + h^2/Y_s$ may no longer apply, although we suspect that the capacitive transfer admittance will still influence Z_{Te}. This problem is presently being studied, and the results will be reported in a future paper.

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