SC-CR-71 5110

SIMPLIFICATIONS IN THE STUDY OF GROUNDED-SHIELD, "BALANCED" TWIN-LEAD CABLES

Sidney Frankel and Associates
P. O. Box 126
Menlo Park, California, 94025

for
Sandia Laboratories, Albuquerque

July 1971

Prepared Under Contract No. DO AF(29-601)-64-4457

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ILLUSTRATIONS

1. Introduction

Typical aircraft or missile cabling involves the use of bulk cables containing a variety of conductor systems, - single conductors, shielded or unshielded (within the cable shield); and twisted pairs, also shielded or unshielded. With the usual assumptions made throughout the present study, such physical systems may be treated as multiconductor TEM lines.[1]

The purpose of this memorandum is to discuss the response of twisted shielded pairs to a signal leaking into the system through a break in the bulk shield. Although the purpose of using the shielded twisted-pair configuration is to afford protection against just such a contingency, the actual method of termination of the leads and grounding of the shields often negates the shielding inherent in the twin-lead arrangement.

The analytical solution to the general class of problems described in the opening paragraph above has been given in reference 1. Although explicit expressions for all terminal voltages and currents are given, actual computations, even for a single shielded pair, are so lengthy and tedious that automatic computation is a practical necessity.*

Unfortunately, inspection of the solution, either as a system of equations, or as computed data, generally yields minimal and generally unsatisfying insights into the physical phenomena actually taking place in the system. The specific purpose of this memorandum is to study the simplest problems with a view toward enhancing such insights.

2. Technical Discussion

In order to "explain" the dynamics to be investigated, this memorandum assumes that an explanation of obscure phenomenon has

*When the twin-lead shield is insulated from the cable bulk shield, a single shielded pair is a 3-line (N = 3 in reference 1).

been developed when that phenomenon can, by a brief sequence of logical steps, involving concrete physical ideas rather than abstract analytical manipulations, be related to ideas already well-accepted.

The following concepts will be assumed to be well-accepted ideas, although a brief review of some of them is in order:

1. The whole body of information relating to conventional 2-conductor (1-line) transmission-line theory, including properties of propagation, impedance, and cascading.

2. The behavior of TEM waves guided by multiple parallel conductor systems. The elementary concepts involved here have been available in the literature for at least a quarter of a century.[2]

3. The compensation theorem for investigating the behavior of networks. The compensating-voltage form of the theorem is the one usually given.[3] However, we will use it in the current-compensating form.[4] While perhaps not so familiar as the former, the latter is its exact circuit dual, both in form and in derivation.

All data related to conventional transmission line theory (1-line) were obtained from reference 5.

In addition to reference 2, a more detailed discussion of TEM waves on parallel wires is availabe in Chapters I and II of reference 6. Briefly, the fundamental phenomena are as follows:*

TEM waves guided by parallel lossless conductors of invariant cross-section travel at a single speed in either or both directions along the line. The behavior of the fields in a transverse plane is invariant with respect to time and to position along the line, except for a scale factor representing the travelling-wave nature of the field. In fact, the electric field in any transverse plane satisfies Laplace's equation for electrostatic fields, as well as

*Lossless lines assumed.

the boundary conditions at the conductor surfaces. Thus, the field is a unique quasi-static field for a specified set of potentials on the conductors. The magnetic field is uniquely specified at each point by the electric field and the electromagnetic constants of the medium.

As a consequence of these facts, for a given set of conductor potentials, only one wave is possible in the forward direction along the line. Similarly, only one wave is possible in the reverse direction, although the specifying potentials for the back wave are generally independent of those for the forward wave. For a forward wave, the current on any conductor at any cross section is related to the potentials by

$$I_j^f = \sum_{k=1}^{N} Y_{jk} V_k^f, \; j = 1, \ldots, N \tag{1}$$

for a system of N conductors above ground. In matrix notation,

$$\underline{I} = \underline{Y} \; \underline{V} \tag{2}$$

where the Y_{jk} are the line admittance coefficients. To exhibit behavior with distance along the line for monochromatic excitation, write

$$V_k^f = A_k \, e^{-j\psi}, \; k = 1, \ldots, N \tag{3}$$

where the A_k are determined by conductor potentials at some point along the line, while ψ measures their common phase variation along the line.

Similarly, for a back wave, with a set of potentials

$$V_k^b = B_k \, e^{j\psi} \tag{4}$$

is associated a set of currents

$$I_k^b = - \sum_{k=1}^{N} Y_{jk} V_k^b \tag{5}$$

The total picure is therefore

$$\left.\begin{aligned} \underline{V} &= \underline{A}\, e^{-j\psi} + \underline{B}\, e^{j\psi} \\ \underline{I} &= \underline{Y} \left\{\underline{A}\, e^{-j\psi} - \underline{B}\, e^{j\psi}\right\} \end{aligned}\right\} \tag{6}$$

Thus, Equation (1) (or Equation (5)) states that the current on any conductor can be visualized as the superposed effects induced by the potentials on <u>all</u> conductors. The magnitudes of these potentials (A_k or B_k) are determined by the line terminal conditions. If the line has two finite terminals, waves travel, generally, in both directions, and both sets of constants must be determined.

Interestingly, Equations (1), (2), and (5) may be solved for the potentials in terms of the currents, so that an equally valid formulation is

$$\left.\begin{aligned} \underline{I} &= \underline{G}\, e^{-j\psi} + \underline{H}\, e^{j\psi} \\ \underline{V} &= \underline{Z} \left\{\underline{G}\, e^{-j\psi} - \underline{H}\, e^{j\psi}\right\} \end{aligned}\right\} \tag{7}$$

where $\underline{Z} = \underline{Y}^{-1}$ is the line impedance matrix. The implication here is that, for a specified set of line currents, the potential on any conductor can be visualized as the superposed effects induced by the currents on all conductors.

The current-compensation theorem is discussed in reference 4. An elementary example will illustrate its use. Consider Figure 1a, which shows an impedanceless generator exciting a line of length, θ,

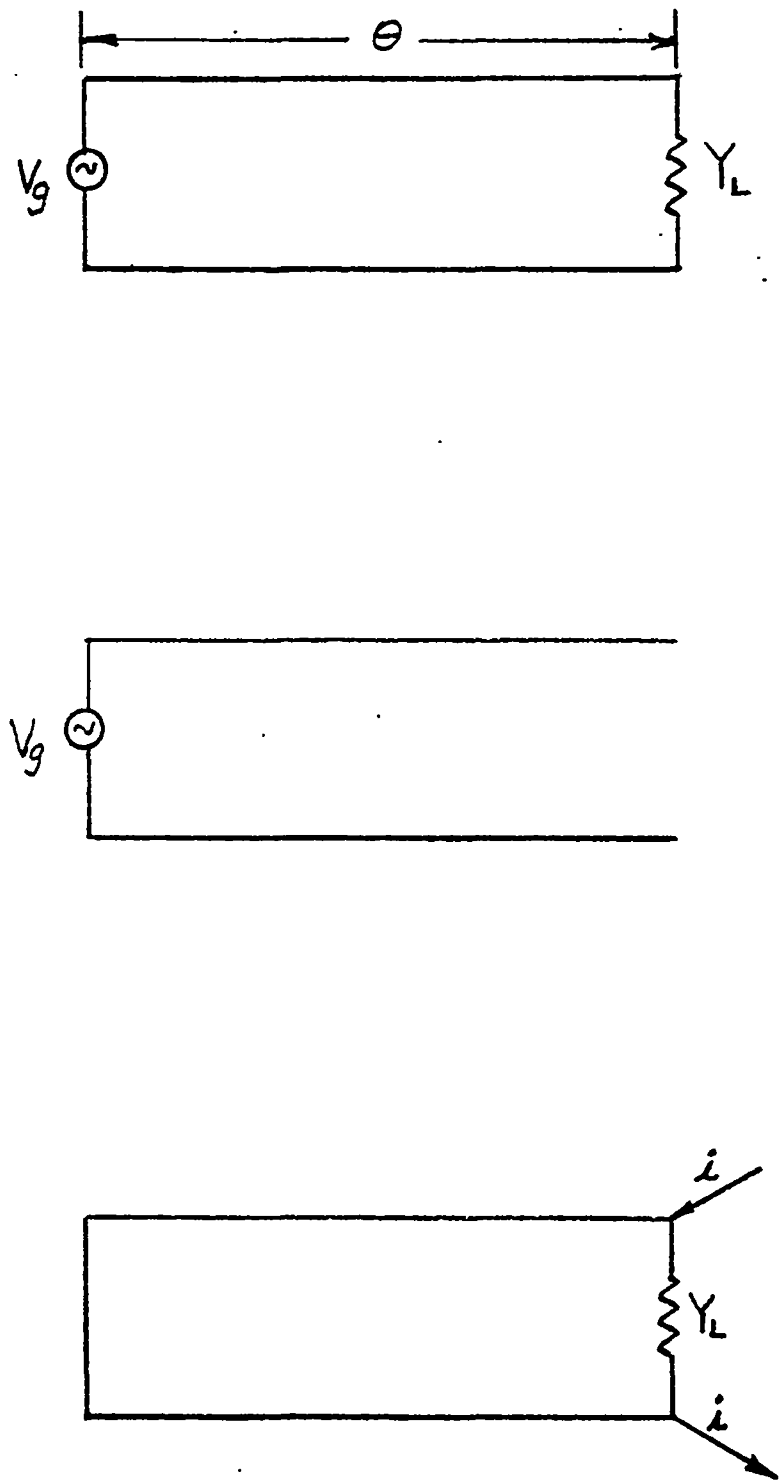


Fig.1. Current-compensation theorem illustrated.

terminated in an admittance, Y_L. To find the voltage, V_L, across this load, using the compensation theorem, first write the output voltage when $Y_L = 0$ (Figure 1b). This voltage is evidently

$$V_o^o = V_g \sec \theta$$

The compensation theorem now states that the effect of connecting Y_L to the output terminals can be determined by superposing, on the new circuit, the effect of a current generator

$$i = -Y_L V_o^o$$

applied between the terminals across which V_o^o was measured (Figure 1c). In the latter figure the admittance seen at the output terminals is

$$Y^o = Y_L - j Y_o \cot \theta$$

where Y_o is the line characteristic admittance. Therefore, the incremental voltage is

$$v^o = \frac{i}{Y^o} = - \frac{Y_L V_o^o}{Y^o}$$

The total voltage is

$$V^o = V_o^o + v^o = V_o^o\left(1 - \frac{Y_L}{Y^o}\right)$$

$$= V_g \sec \theta\left(\frac{-j Y_o \cot \theta}{Y^o}\right)$$

$$= - \frac{j Y_o V_g \csc \theta}{Y^o}$$

2.1 Single Twin Cable, Shield Grounded At One End

Consider the configuration of Figure 2 which shows a shielded twin pair inside a bulk shield, and terminated in loads R^i and R^o. A generator of voltage V_g simulates an external field penetrated through a break in the bulk shield, say through a terminal connector. Aside from the fact that somewhat simpler concepts are involved by placing the break at one end, the fact is that this is a worst case situation. A source located at the other end of the shield would look essentially into an open circuit at all frequencies.

The loads R^i and R^o in typical situations are one or two orders of magnitude greater than the line impedances, and therefore have little effect on line behavior except at resonant conditions, when they, along with cable losses, limit the cable response. However, we are not concerned here with actual peak values, which are readily determined by automatic computation, but, rather, with the general nature of the responses, and their explanations in terms of familiar ideas. Therefore, we simplify matters further by letting R^i, $R^o \to \infty$.

Inside the twin shield the potential is everywhere the same as on the shield at the same cross-section. Therefore, there is no potential difference across the twin leads. Thus there is no coupling to the loads.

We now note that, except at resonance, currents on conductors Nos. 1 and 2 are zero. To see this, use a current equation from Equations (7):

$$I_1 = G_1 e^{-j\psi} + H_1 e^{j\psi}$$

In the remainder of this memorandum we will simplify notation by writing

$$s = e^{j\psi}$$

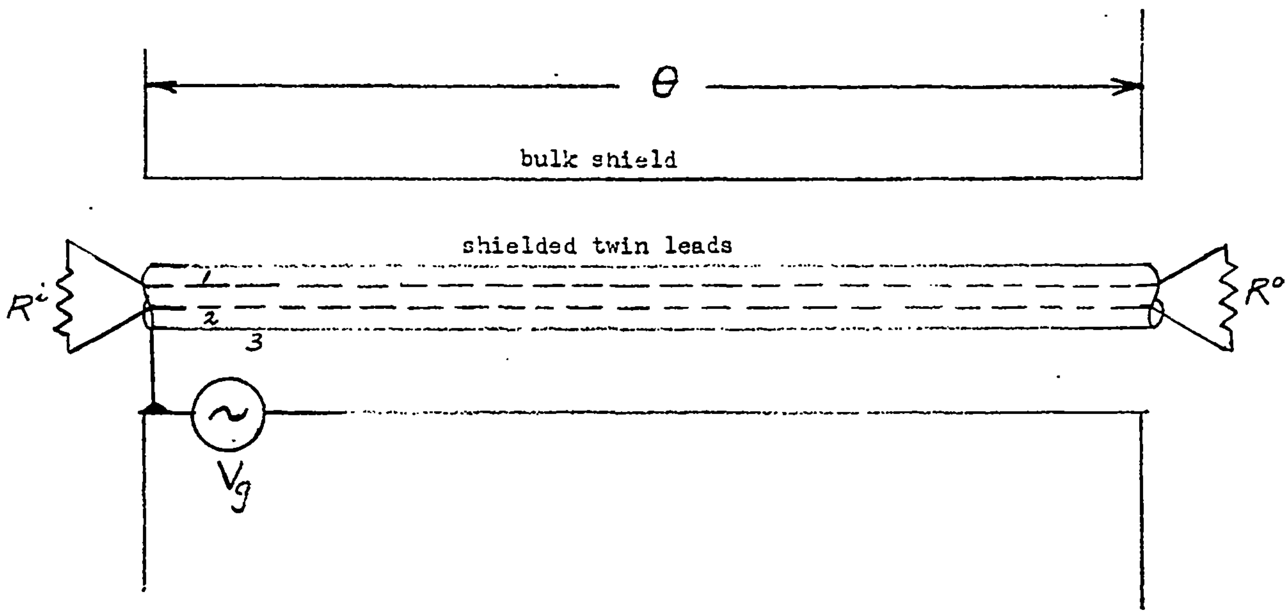


Fig. . Shielded twin pair inside bulk shield.

and for a line of finite length, θ,

$$S = e^{j\theta}$$

Thus, corresponding to $\psi = 0, \theta$, we have $s = 1, S$, respectively.

Back to the equation for I_1,

$$I_1 = s^{-1} G_1 + s H_1$$

The current is zero at the ends of the line.

$$\left.\begin{aligned} 0 &= G_1 + H_1 \\ 0 &= S^{-1} G_1 + S H_1 \end{aligned}\right\}$$

This pair of simultaneous homogeneous equations has only the solution

$$G_1 = H_1 = 0$$

unless the determinant of the system is zero:

$$\begin{vmatrix} 1 & 1 \\ S^{-1} & S \end{vmatrix} = S - S^{1} = 2j \sin\theta = 0$$

or $\theta = m\pi$, m integer.

The same argument, of course, applies to conductor No. 2.

This result does not imply that the potentials are zero. In fact, since the only current is

$$I_3 = s^{-1} A_3 + sB_3$$

The second of Equations (7) then yields

$$V_1 = Z_{13}(s^{-1} A_3 - sB_3)$$

$$V_2 = Z_{23}(s^{-1} A_3 - sB_3)$$

$$V_3 = Z_{33}(s^{-1} A_3 - sB_3)$$

As a matter of fact it is a straightforward matter to prove from elementary electrostatics that $Z_{13} = Z_{23} = Z_{33}$ for this configuration, so that, in fact, $V_1 = V_2 = V_3$, as expected.

Since conductor No. 3 potential is influenced only by its own current, we know that the potential at the open end of the line is

$$V_3^o = V_1^o = V_2^o = V_g \sec \theta \qquad (8)$$

The ideal conditions of Figure 2, leading to zero pickup on the loads, is not always realized. Frequently, a termination consists of a high impedance, R^o, as shown, one end of which is connected to a relatively large mass of metal, which, while not grounded, represents a large unbalancing capacitance (possibly several hundred pF) to ground. This places an admittance

$$Y_c = j\omega C \qquad (9)$$

between one of the twin lead terminals and the bulk shield, as in Figure 3.

The compensation theorem then states that the effect of this change is to introduce a current source

$$i = -Y_c V_1^o = -Y_c V_g \sec \theta \qquad (10)$$

as shown in Figure 4.

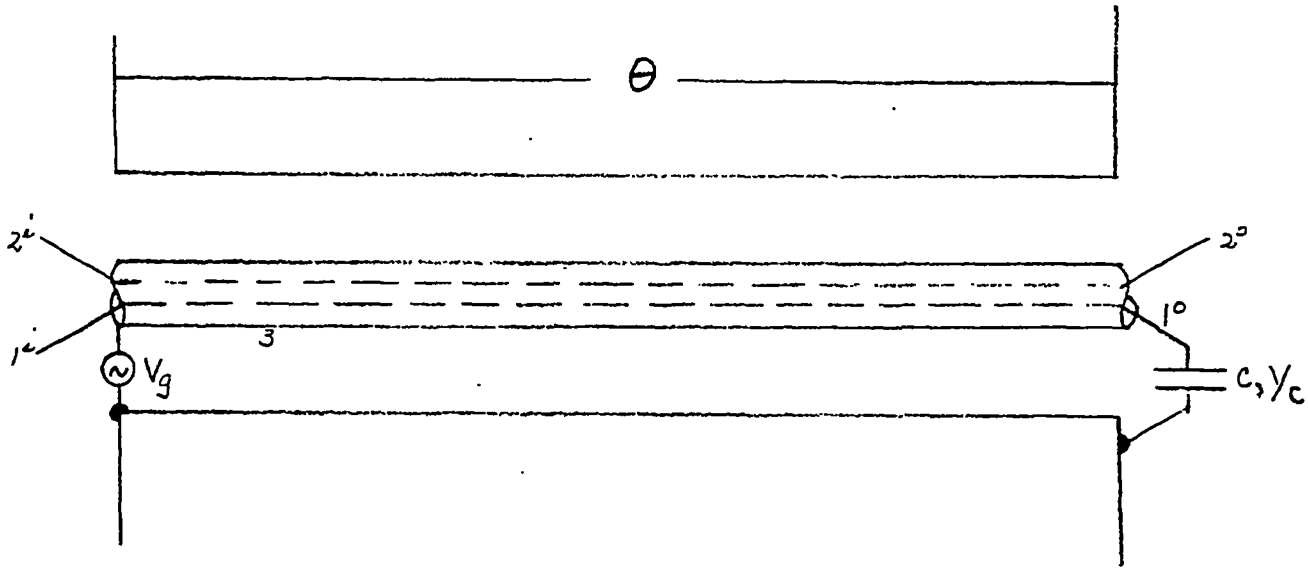


Fig.3. Shielded twin pair with unbalanced load.

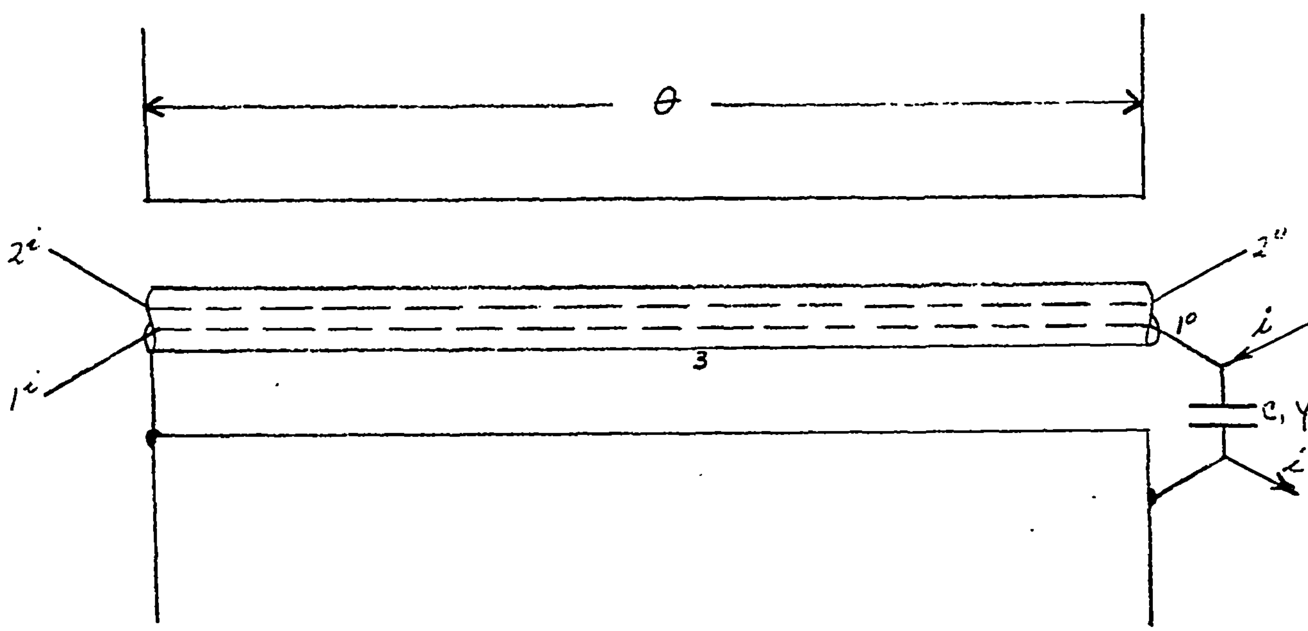


Fig.4. Compensation theorem applied to shielded pair with unbalanced load.

The effect of this current source in producing output voltage changes, v_1^o and v_2^o, on conductors No. 1 and 2 respectively, will now be investigated.

The first quantity to be determined is the admittance looking into the line at terminal 1^o, in order to determine v_1^o, and to determine what fraction of i flows into No. 1; since that current, and whatever current is induced on No. 3, will determine the potential on No. 2 by Equations (7).

At this point we digress in order to introduce an additional analytical tool not mentioned previously.

2.1.1 Equivalent Circuits

The effect of the equivalent circuits about to be discussed is to replace a number of continuously coupled lines by conventional 1-lines coupled only at their terminals by means of ideal 1:1 transformers.

In general there is a 1-line for every different line coefficient not equal to zero (Z_{ij} and Z_{ji} are treated as one coefficient). Thus, a 2-line is replaced by three 1-lines, a 3-line by six 1-lines, and, in general, an N-line by $\frac{1}{2}N(N + 1)$ 1-lines. However, in the present instance, because no current flows on No. 2, except for $\theta = m\pi$, we can ignore its presence, taking its effect into account only insofar as its presence affects the values of the other line coefficients.

We are therefore dealing with a 2-line. For forward waves only, the canonical equations of such a line are, from Equations (7),

$$\left.\begin{aligned} V_1 &= Z_{11}\, I_1 + Z_{13}\, I_3 \\ V_3 &= Z_{13}\, I_1 + Z_{33}\, I_3 \end{aligned}\right\} \qquad (11)$$

Following Uchida[7] we note that these also represent the equations of a two-mesh network, as in Figure 5a. Next, the individual impedances of that figure are replaced by infinite 1-lines of appropriate characteristic impedance, coupled to the circuit, for maximum flexibility, through ideal 1:1 transformers (Figure 5b). In the next step we make the following stipulations (Figure 5c):

1. We assume that V_1 and V_3 of Figure 5b are actually being applied at input terminals 1^i and 3^i of the line (left side of Figure 5c).

2. The left half of Figure 5c corresponds to Figure 5b if the 1-lines of 5c are assumed to be infinite.

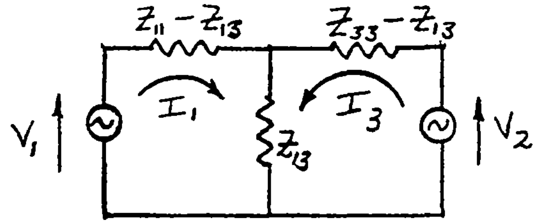
3. If now the infinite 1-lines are replaced by the set of 1:1 transformers on the right, interconnected in 1:1 correspondence with those on the left, and if terminals 1^o and 3^o are connected to a Y-network as shown in Figure 6, then it is an easy matter to show that each of the 1-lines sees its own characteristic impedance, regardless of the relative excitations at 1^i and 1^o.

Thus the arrangement of Figure 5c is externally equivalent to a 2-line for waves travelling in the positive direction. Obviously it is also equivalent for waves in the back direction. Since these comprise, in combination, all possible situations, the equivalence holds for all conditions of termination.

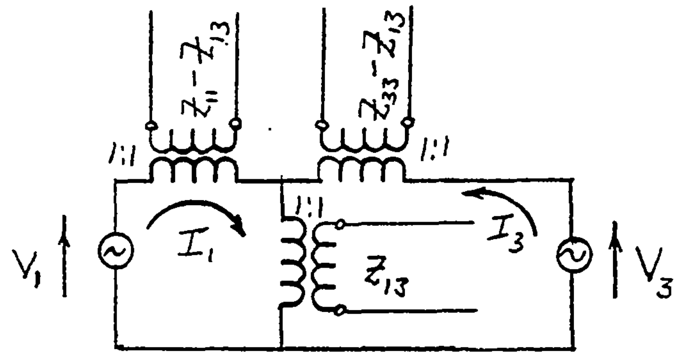
This equivalence does not always lead to useful results (see Uchida for other, more sophisticated arrangements). However, it is suitable for our purpose.

2.1.2 Equivalent Circuit for Determining Output Admittances

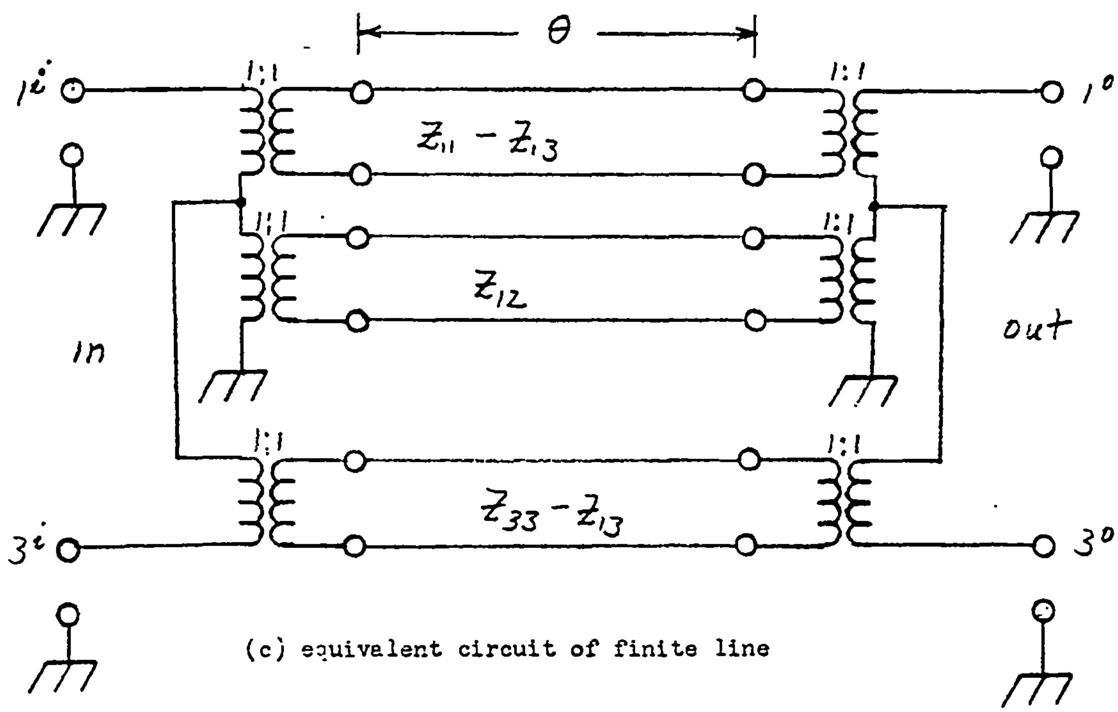
As stated in the preceding section, the conductor system is a 2-line except at $\theta = m\pi$. The situation is shown schematically in Figure 7. We wish to determine the impedance at the 1^o-terminal driving point. Note that terminals 1^i and 3^o are open-circuited



(a) two-mesh network analog



(b) equivalent circuit of infinite line



(c) equivalent circuit of finite line

Fig.5. Equivalent circuit of a 2-line.

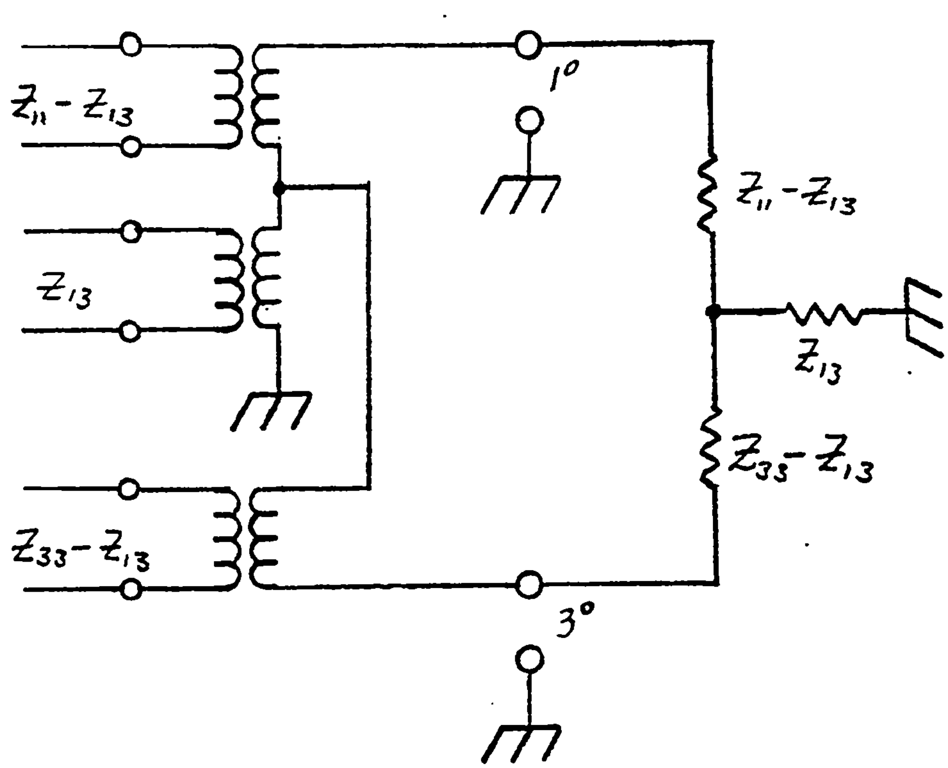


Fig.6. Match termination for finite 2-line

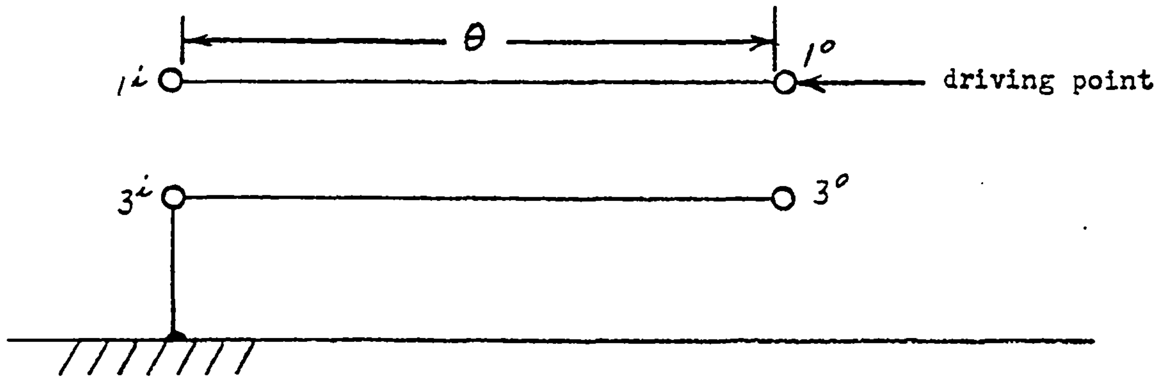


Fig.7. Schematic for determination of output admittance

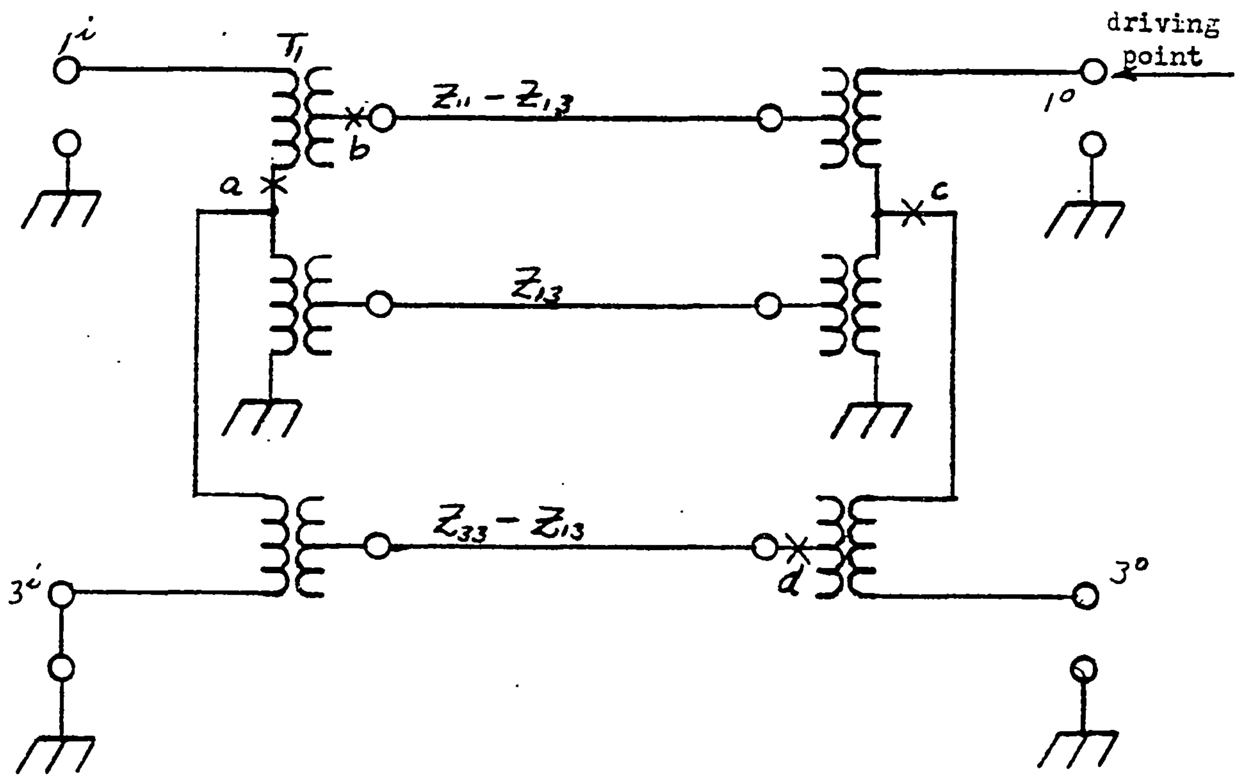


Fig.8. Equivalent circuit interconnections.

while 3^i is shorted to ground. Applying these terminations to Figure 5c yields Figure 8. We note the following consequences of this set of terminations.

1. The open circuit at 1^i means no current flows through T_1; the circuit may be cut open at points a and b.

2. Similarly the open circuit at 3^o means the circuit may be cut open at c and d.

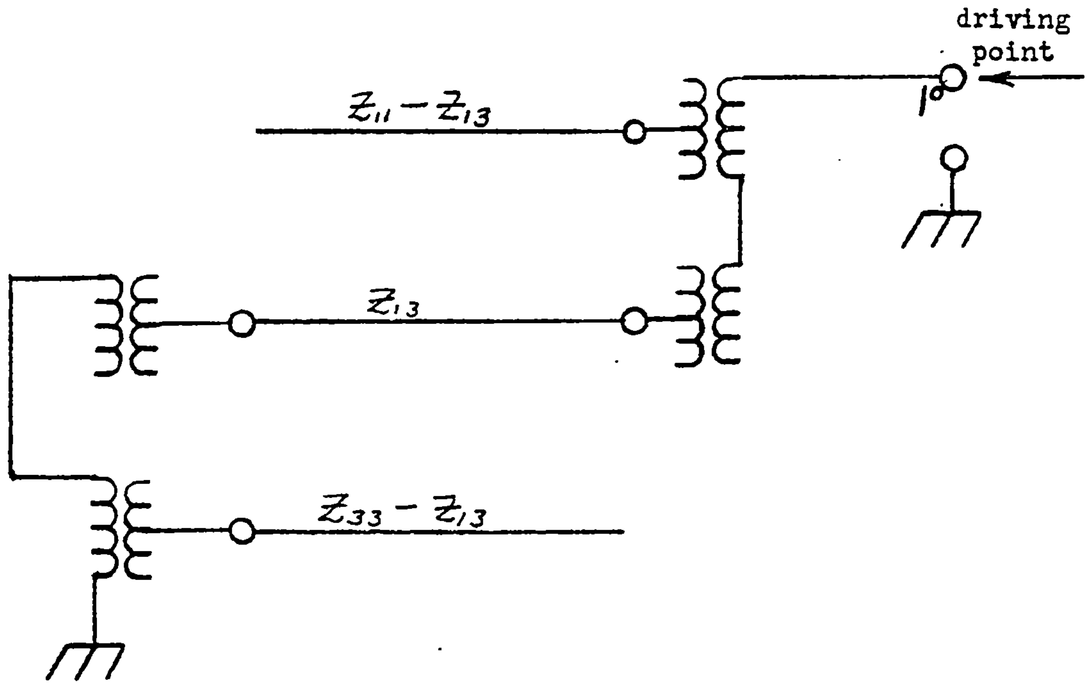
The resulting reduced circuit is shown in Figure 9a, with a further obvious reduction in 9b. It is now clear that terminal 1^o sees an impedance consisting of two impedances in series, - one resulting from an open circuited line of characteristic impedance $(Z_{11} - Z_{13})$, the other resulting from two lines in cascade, open-circuited at the far end. The result is displayed perhaps more advantageously in Figure 10. The circuit has been drawn in this way because, with terminal 3^i open, (corresponding to removing the short to ground on the left of Figure 10) the configuration is a two-port with known filter characteristics.[8] Thus, with the short in place the impedance seen at 1^o is the filter short-circuit impedance.

An interpretation of this result states that energy transfers from 1^o to 3^i by way of the coupling impedance, and travels through a phase lag of θ in the process. The driving point impedances ζ_1 and ζ_3 merely represent the impedances that would be seen at the terminals if the coupling impedance, Z_{13}, were zero.

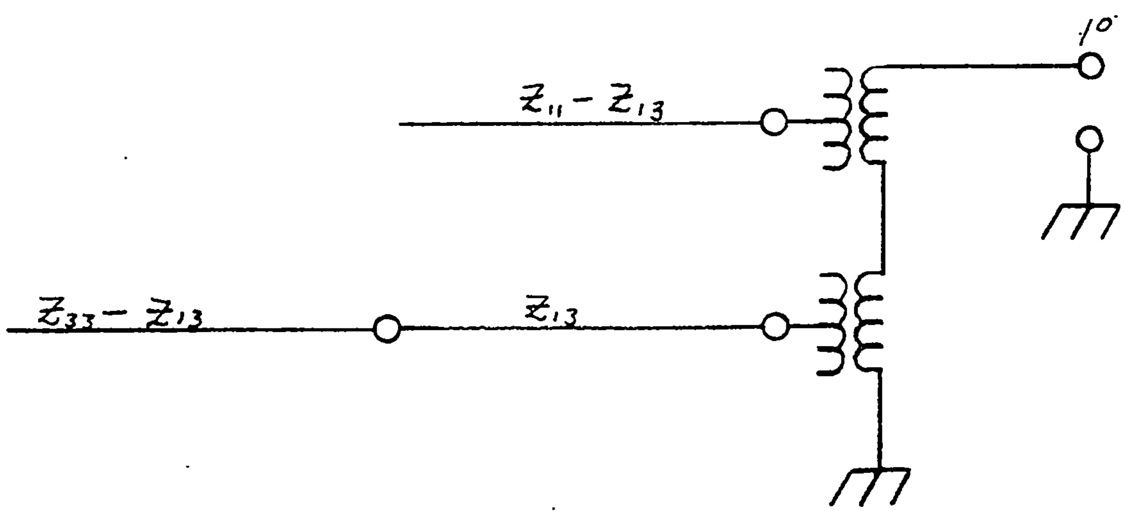
The driving point impedance at 1^o is now readily determined from elementary line theory. The terminating impedance at the left of the two-port of Figure 10 is

$$Z^o = a\zeta_3 \quad , \quad a = -j \cot \theta$$

The impedance looking into the right-hand port of the line of impedance Z_{13} is (see reference 5, p. 22-4)

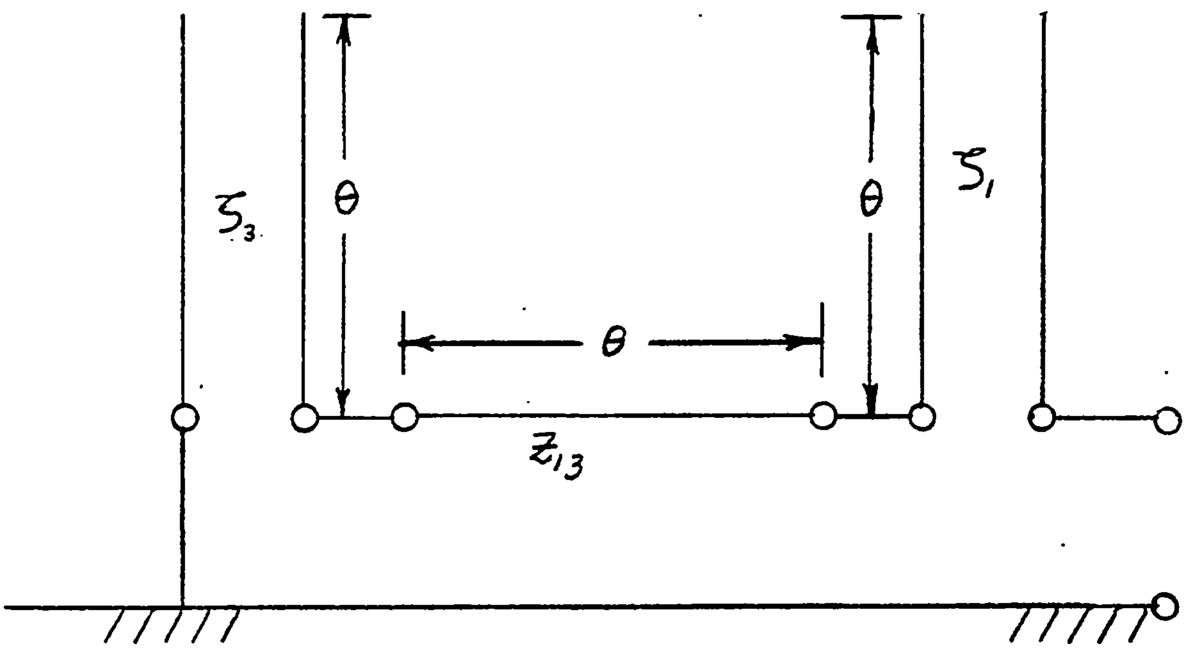


(a)



(b)

Fig.9. Equivalent circuit reductions.



$$\zeta_1 = Z_{11} - Z_{13}$$
$$\zeta_3 = Z_{33} - Z_{13}$$

Fig.10. Eqivalent circuit, final schematic

$$Z^{i} = Z_{13}\,\frac{a^2\zeta_3 + Z_{13}}{a\,Z_{33}}$$

The overall impedance at terminal 1^{o} is then

$$Z_1^{o} = Z^{i} + a\zeta_1 = \frac{a^2(Z_{11}\,Z_{33} - Z_{13}^2) + Z_{13}^2}{a\,Z_{33}} \qquad (12a)$$

$$= \frac{a}{Y_{11}} + \frac{Z_{13}^2}{a\,Z_{33}} \qquad (12b)$$

where we have written[6]

$$Y_{11} = \frac{Z_{33}}{Z_{11}\,Z_{33} - Z_{13}^2} \qquad (13)$$

The increment in voltage at 1^{o} terminal is

$$v_1^{o} = \frac{i}{Y_1^{o} + Y_c} \quad , \quad Y_1^{o} = \frac{1}{Z_1^{o}}$$

$$= -\,\frac{Y_c\,V_1^{o}}{Y_1^{o} + Y_c} \qquad (14)$$

by Equation (10).

The total voltage at 1^{o} is

$$V_1^{oT} = V_1^{o} - \frac{Y_c\,V_1^{o}}{Y_1^{o} + Y_c}$$

$$= \frac{Y_1^{o}\,V_1^{o}}{Y_1^{o} + Y_c} = \frac{V_1^{o}}{1 + Z_1^{o}\,Y_c} \qquad (15)$$

With the help of Equations (10) and (12b) this becomes

$$V_1^{OT} = \frac{V_g \csc\theta}{\cot\theta - \omega C(Z_{11} - \frac{1}{Y_{11}} \csc^2\theta)} \tag{16}$$

The resonant frequencies are given by

$$\theta = \cot^{-1}\left\{\frac{1}{2}\left[-\frac{Y_{11}}{\omega C} + \left(\frac{Y_{11}^2}{\omega^2 C^2} + 4\left[Z_{11}\ Y_{11} - 1\right]\right)^{\frac{1}{2}}\right]\right\} \tag{17}$$

To determine the voltage change, v_2^o at the output terminal of conductor No. 2, recall that such change must result from the change currents on conductors No. 1 and 3. Consulting Figure 7, note that in addition to the impressed current at terminal 1^o a reflected wave on No. 1 may be expected. In addition, waves of current in both directions may be expected on conductor No. 3. In fact, writing

$$\begin{aligned} i_1^o &= \text{output current on conductor No. 1} \\ &= -\frac{Y_1^o\ i}{Y_1^o + Y_c} \end{aligned} \tag{18}$$

we have, in addition,

$$\left.\begin{aligned} i_1^i &= i_1(s = 1) = 0 \\ v_3^i &= v_3(s = 1) = 0 \\ i_3^o &= i_3(s = S) = 0 \end{aligned}\right\} \tag{19}$$

The respective standing waves of current may be written (Equations (7))

$$\left.\begin{aligned} i_1 &= s^{-1} G_1 + sH_1 \\ i_3 &= s^{-1} G_3 + sH_3 \end{aligned}\right\} \qquad (20)$$

while the voltage on No. 3 is a linear combination of current-induced waves

$$v_3 = Z_{13}(s^{-1} G_1 - sH_1) + Z_{33}(s^{-1} G_3 - sH_3) \qquad (21)$$

The boundary conditions (18) and (19) then yield

$$\left.\begin{aligned} S^{-1} G_1 + SH_1 &= i_1^o \\ G_1 + H_1 &= 0 \end{aligned}\right\} \qquad (22a)$$

$$\left.\begin{aligned} S^{-1} G_3 + SH_3 &= 0 \\ Z_{13} G_1 - Z_{13} H_1 + Z_{33} G_3 - Z_{33} H_3 &= 0 \end{aligned}\right\} \qquad (22b)$$

Simultaneous solution of Equations (22a) yields

$$G_1 = - H_1 = j \tfrac{1}{2} i_1^o \csc \theta \qquad (23)$$

and the conventional 1-line current for conductor No. 1

$$i_1 = \frac{\sin \psi}{\sin \theta} i_1^o$$

Then Equations (22b) are

$$\left.\begin{aligned} S^{-1} G_3 + SH_3 &= 0 \\ G_3 - H_3 &= -j \frac{Z_{13}}{Z_{33}} i_1^o \csc \theta \end{aligned}\right\} \tag{24}$$

yielding

$$\left.\begin{aligned} G_3 &= -j \frac{Z_{13}}{Z_{33}} \frac{S}{\sin 2\theta} i_1^o \\ H_3 &= j \frac{Z_{13}}{Z_{33}} \frac{S^{-1}}{\sin 2\theta} i_1^o \end{aligned}\right\} \tag{25}$$

for the current-wave amplitudes on No. 3.

The voltage increment on conductor No. 2 is, therefore,

$$v_2 = Z_{12}(s^{-1} G_1 - sH_1) + Z_{23}(s^{-1} G_3 - sH_3) \tag{26}$$

At the 2^o terminal, s = S, and

$$v_2^o = Z_{12}(S^{-1} G_1 - SH_1) + Z_{23}(S^{-1} G_3 - SH_3)$$

which, with the help of Equations (23) and (25), reduces to

$$v_2^o = j\left(Z_{12} \cos \theta - \frac{Z_{13} Z_{23}}{Z_{33}} \sec \theta\right) i_1^o \csc \theta$$

Thus, the voltage induced across the output load is

$$
\left.\begin{aligned}
\Delta v &= v_1^o - v_2^o \\
&= \frac{i}{Y_1^o + Y_c} \left\{1 + j\, Y_1^o \left(Z_{12} \cos\theta + \frac{Z_{13}\, Z_{23}}{Z_{33}} \sec\theta\right) \csc\theta\right\}
\end{aligned}\right\} \quad (27)
$$

where i is given by Equation (10).

A simpler, though somewhat less accurate, picture is obtained as follows:

Knowing the current at both ends of line No. 1, and ignoring the reaction from No. 3 due to i_1, the current on No. 1 is definitely fixed as

$$
i_1 = \frac{\sin\psi}{\sin\theta}\, i_1^o
$$

while the voltages due to this current are

$$
\left.\begin{aligned}
v_1 &= j\, Z_{11}\, i_1^o \csc\theta \cdot \cos\psi \\
v_2 &= j\, Z_{12}\, i_1^o \csc\theta \cdot \cos\psi
\end{aligned}\right\}
$$

on lines No. 1 and 2 respectively. Their difference at the output terminals is

$$
\Delta v = v_1^o - v_2^o = j(Z_{11} - Z_{12})\, i_1^o \cot\theta \quad (28)
$$

which is the desired approximate result. That this checks (approximately) with the previous result can be seen as follows:

If, in Equation (27), we replace i by i_1^o via Equation (18), and, in Equation (12b) recognize that ignoring the reaction from No. 3 implies

$$Z_1^o \simeq \frac{a}{Y_{11}} \simeq a\ Z_{11} = -j\ Z_{11} \cot \theta,$$

then substitution of these approximations in Equation (27) yields Equation (28).

2.1.3 Physical Interpretation

The physical picture that emerges from this analysis is about as follows:

When the twin leads have balanced loads at both ends, the interference produced by leakage of an external field into one end of the cable is negligible. If, however, one of the leads is loaded at one end by a capacitance to ground, the effect is as though a current source had been impressed on the lead at that point. The (nearly) open circuit at the other end of the lead causes a large standing wave of current on that conductor.

Although the current source and reflected wave cause, generally, no current to flow on the other lead, they induce voltage waves both on the other leads and on the shield. Because of the nature of the shield termination, current does result on the shield, and consequently it, too, induces voltage waves on the second lead. Current and voltage waves on the unbalanced lead and on the shield adjust their amplitudes and phases to meet the boundary conditions at the terminals of these conductors. The total voltages on the twin leads are then simply related to the currents on all conductors by way of the line impedance coefficients, and with the usual adjustment of algebraic sign, depending on whether the current wave associated with a particular component of voltage is a forward wave or a back wave.

2.2 Two Shielded Twin Cables, Shields Grounded at Opposite Ends: Response of Shields Only

The arrangement to be discussed is shown in Figure 11. In this case two shielded pairs are contained within the bulk shield, and are grounded at opposite ends of the cable, left open at the other ends. Again, in the absence of the unbalancing capacitance, C_1, each lead of each pair is raised to the same potential as its outer shield, so that no potential appears across the pair. Of course, the potential of the unbalanced-pair shield may be expected to be different from the preceding case. Apart from that, the potential difference of the unbalanced pair is similar to that of the preceding case.

While it is true that the unbalanced current on No. 1 induces a voltage on No. 6 (the other pair's shield), the reaction back on No. 1 may be expected to be considerably smaller than that of the voltage induced on No. 3, as given by Equation (14) of the preceding case.

The schematic describing the situation to be studied is shown in Figure 12a. An equivalent circuit, derived from a circuit like Figure 5c, is shown at 12b. A study of the original diagram from which 12b is derived shows that the desired voltage, V_3^o, is the sum of two voltages

$$V_3^o = V_a + V_b \tag{29}$$

where V_a is the voltage at the open end of the ζ_3-line and V_b is the voltage at the output of the coupling line, Z_{36}. Evidently this says the voltage at the open end of No. 3 is its conventional line voltage, V_a, when the coupling is zero. When the coupling is not zero, an additional voltage, V_b, is added.

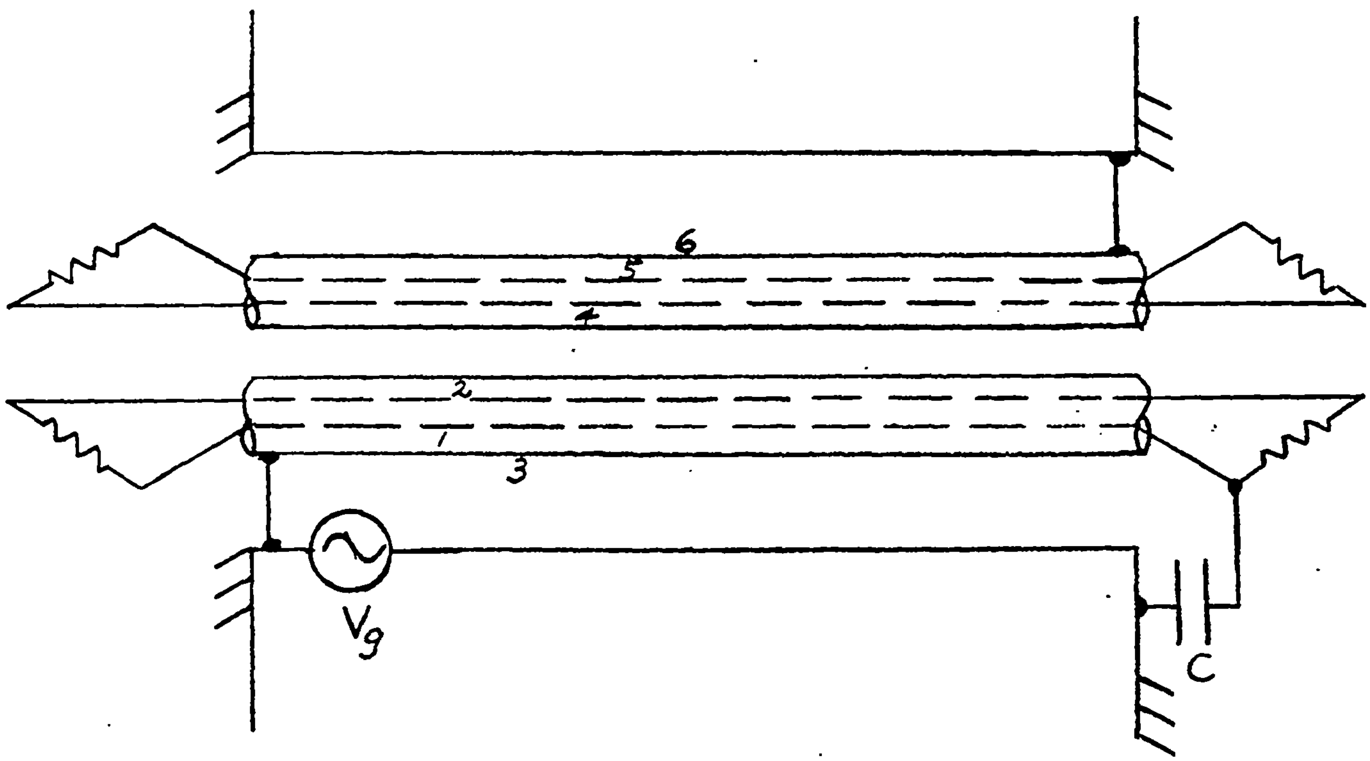
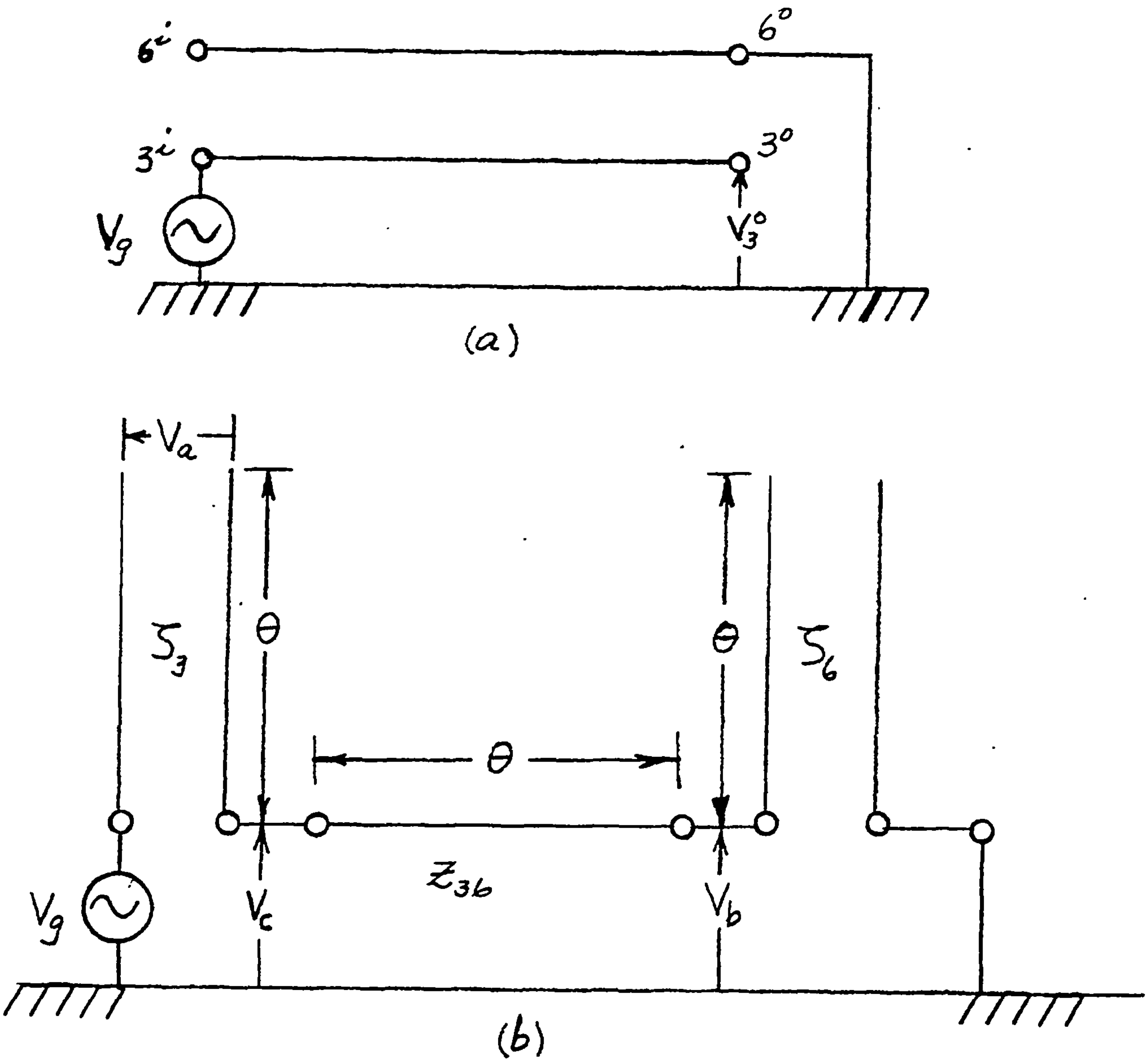


Fig.11. Schematic, two shielded twin cables grounded at opposite ends.



$$\zeta_3 = Z_{33} - Z_{36}$$

$$\zeta_6 = Z_{66} - Z_{36}$$

Fig.12. Equivalent circuit, two cable shields grounded at opposite ends.

Without spelling out the details, it is straightforward to show that near $\theta = \pi/2$ (say for $|\psi| = |\pi/2 - \theta| \ll 1$), the voltages at the ends of the coupling line are related as

$$\frac{V_b}{V_c} = \frac{\zeta_3\psi}{j_3\psi^2 - Z_{36}}$$

Furthermore, for $|\psi|$ small, the impedance of the ζ_3-line is small, while the input impedance to the coupling line is high. Thus, $V_c \simeq V_g$ and

$$\frac{V_b}{V_g} \simeq \frac{\zeta_3\psi}{\zeta_3\psi^2 - Z_{36}} \tag{30}$$

For the same reason, the input voltage to the ζ-3 line at $\psi = 0$ is zero, whence $V_a = 0$. Thus, V_3^o may be expected to resemble Equation (30) near $\theta = \pi/2$. Equation (30) is plotted qualitatively in Figure 13. The result shows two voltage poles separated by an amount proportional to the square root of the coupling impedance. The picture obviously is one of two over-coupled resonant circuits. The overcoupling is a consequence of the fact that the normal load has been replaced by a dead short at terminal 6^o.

Using Figure 12b and conventional 1-line formulas, one readily obtains the values of the terminal dynamic quantities:

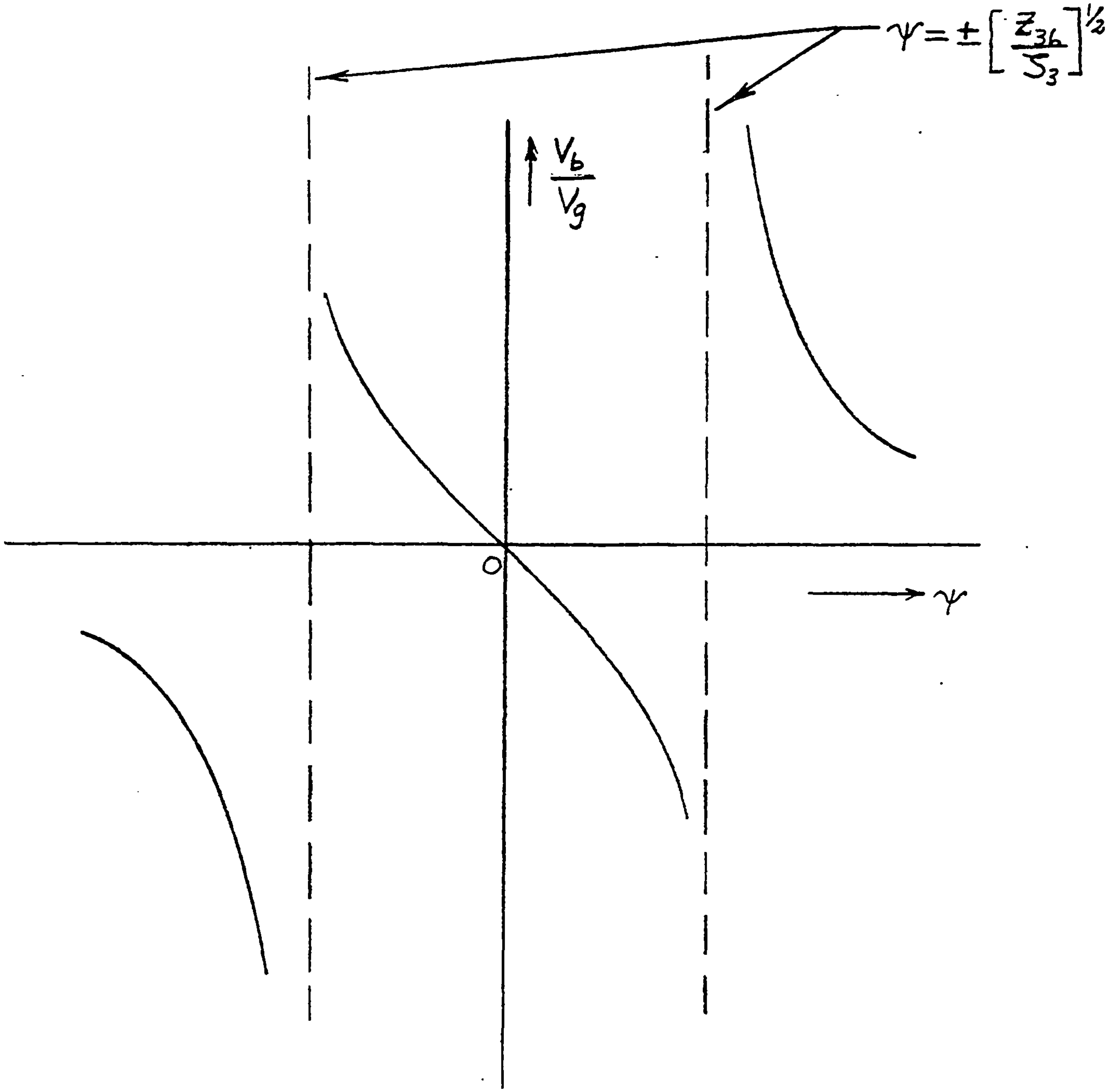


Fig.13. Approximate shield open-end voltage near $\theta = \frac{\pi}{2}$

$$
\left.
\begin{aligned}
I_3^i &= \frac{V_g}{Z^i} \\
V_6^i &= j\,\frac{Z_{36}}{Z^i}\tan\theta \cdot V_g \\
V_3^o &= -j\csc\theta\,\frac{V_g}{Z^i\,Y_{33}} \\
I_6^o &= \frac{Z_{36}\,V_g}{Z_{66}\,Z^i}\sec\theta
\end{aligned}
\right\} \qquad (31)
$$

where

$$
Z^i = -\frac{j\cot\theta}{Y_{33}} + j\,\frac{Z_{36}^2}{Z_{66}}\tan\theta \qquad (32)
$$

3. Conclusion

In attempting to clarify the nature of the dynamic behavior of multiconductor shielded-pair cables, we have found two devices useful for relating this behavior to that of conventional two-wire lines: (1) the analysis and subsequent synthesis of voltages and currents in terms of travelling induced waves (2) the substitution, in an equivalent circuit, of lumped couplings at the ends of conventional lines in place of the continuous coupling along the multiconductor line.

4. References

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