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CALCULATION OF THE PER-UNIT-LENGTH CAPACITANCE MATRIX FOR SHIELDED INSULATED WIRES

by

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ABSTRACT

In this report, the per-unit-length capacitance matrix of a multiconductor transmission line enclosed in a shielding tube is calculated using the method of multiseries expansion. Each individual wire may be dielectric coated.

The methods of evaluating the corresponding per-unit-length inductance matrix and the propagation matrix are also presented.

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SECTION I

INTRODUCTION

Cables containing many wires have been widely used for signal transmission within large systems. Recently, the effects of electromagnetic pulse (EMP) have drawn much attention to the studies of effective shielding techniques for such multiconductor cables. A practical shielding structure for these cables is a grounded conducting tube, as shown in Figure 1. In order to study the mutual coupling of the wires in the presence of the grounded shield, the per-unit-length capacitance matrix, which determines the "quasi-TEM" transmission line solutions (ref. 1, 2, 3), must be obtained.

The purpose of this report is to demonstrate a numerical method which calculates the per-unit-length capacitance matrix

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1. F.Y. Chang, "Transient Analysis of Lossless Coupled Transmission Lines in a Nonhomogeneous Dielectric Medium," IEEE Trans. on Microwave Theory and Techniques, Vol. MTT-18, No. 9, pp. 616-626, September 1970.

2. C.R. Paul, "On Uniform Multimode Transmission Lines," IEEE Trans. on Microwave Theory and Techniques, Vol. MTT-21, No. 8, pp. 556-558, August 1973.

3. C.R. Paul, "Useful Matrix Chain Parameter Identities for the Analysis of Multiconductor Transmission Lines," IEEE Trans. on Microwave Theory and Techniques, Vol. MTT-23, No. 9, pp. 756-760, September 1975.

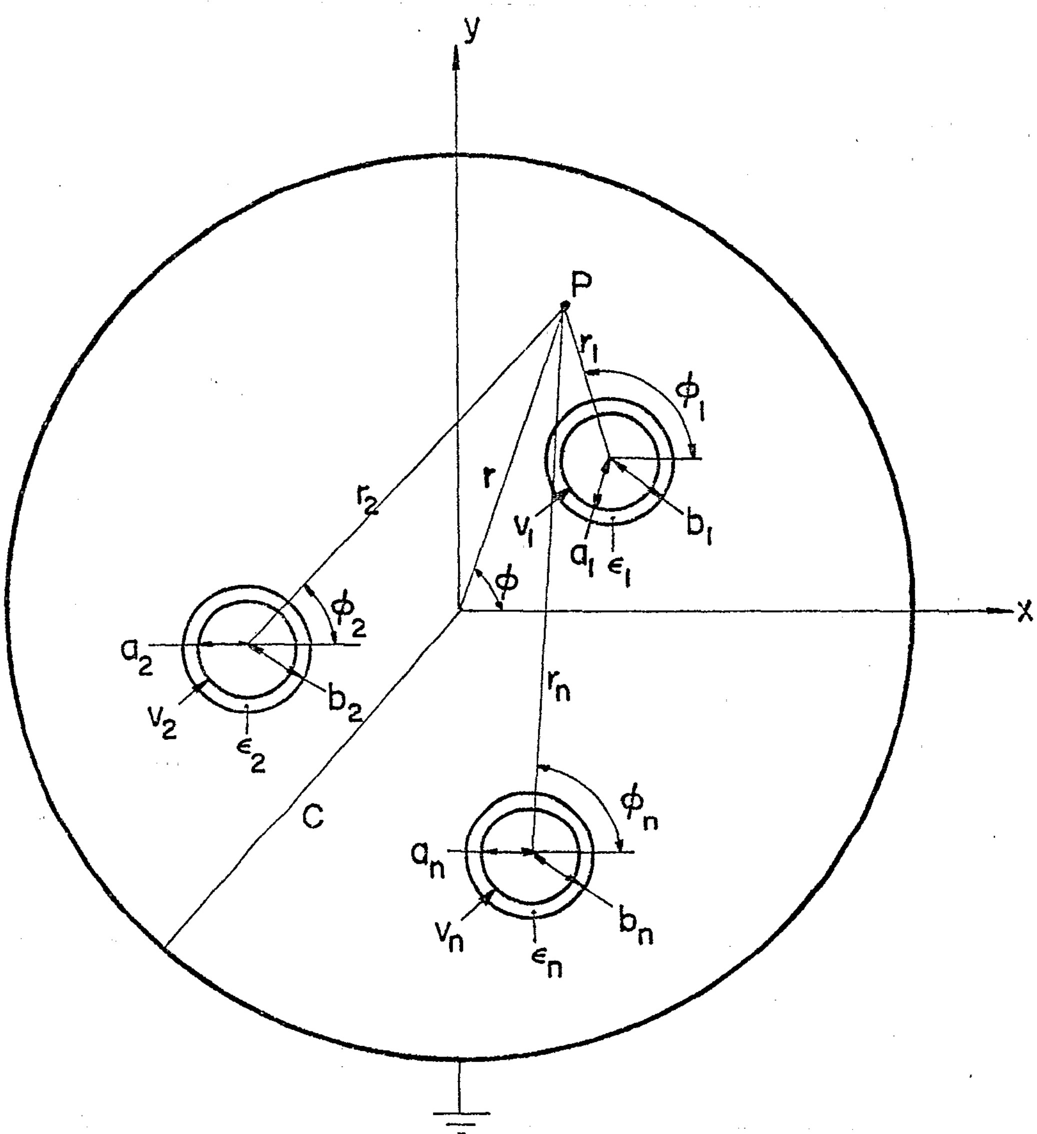

Figure 1. Configuration of the Shielded, Insulated Wires

(ref. 4, 5, 6) of the dielectric-coated wires in Figure 1. The i^{th} wire has a conducting core of radius a_i, the outer radius of the dielectric layer is b_i, the relative dielectric constant is ε_i, with the value of $i = 1, 2, \ldots, n$. The total number of wires is n. The center of the i^{th} wire is (x_{ci}, y_{ci}). The shielding tube has its center located at the origin $(x_c = 0, y_c = 0)$ with a radius of c.

Theoretically, the problem can be solved by the conformal mapping technique (ref. 4, 5). However, the dielectric layer surrounding the wires makes the mapping very complicated. It is also possible to solve the problem using the method of successive images (ref. 4, 7). Again the process of images involving the dielectric coatings is not simple, and the imaging is very tedious if many wires are involved.

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4. R. Plonsey and R. Collin, Principles and Applications of Electromagnetic Fields, McGraw-Hill Co., New York (1961), p. 71.

5. E. Weber, Electromagnetic Fields - Theory and Applications, Vol. 1 - Mapping of Fields, John Wiley and Sons, New York (1950), pp. 139, 223, 226, 379.

6. J.C. Clements, C.R. Paul, A.T. Adams, "Computation of the Capacitance Matrix for Systems of Dielectric-Coated Conductors," IEEE Trans. on Electromagnetic Compatibility, Vol. EMC-17, No. 4, pp. 238-248, November 1975.

7. M.P. Sarma and W. Janischewskj, "Electrostatic Field of a System of Parallel Cylindrical Conductors," IEEE Trans. on Power Apparatus and Systems, Vol. PAS-88, No. 7, pp. 1069-1079, July 1969.

Recently, the integral equations (ref. 8) and the method of moments (ref. 6, 9, 10, 11, 12) have been applied to solve the capacitance matrix for a system of multiconductor wires. The accuracy and numerical stability of the point-matching technique employed in references (6) and (12) are sensitive to the points chosen for matching the boundary conditions. In addition, there is no systematic way of selecting the match points when the wires are randomly scattered.

In this report, we shall provide a theoretical background for the "multi-series expansion" of the potential exterior to the dielectric coatings. The coefficients in this expansion will be solved by the "weighted least-square method" and the per-unit-length capacitance matrix is then obtained directly from the expansion coefficients.

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8. P.C. Chestnut, "On Determining the Capacitances of Shielded Multiconductor Transmission Lines," IEEE Trans. on Microwave Theory and Techniques, Vol. MTT-17, No. 10, pp. 734-745, October 1969.

9. R.F. Harrington, Field Computation by Moment Methods, MacMillan Co., New York (1968).

10. A.T. Adams, Electromagnetics for Engineers, Ronald Press Co., New York (1972).

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12. A.E. Feather and C.R. Paul, Computation of the Capacitance Matrices for Ribbon Cables, RADC-TR-76-101, Vol. II, Rome Air Development Center, NY, April 1976.

SECTION II

THEORY

In this section we shall describe the field expansions in all regions of the multiconductor cable. The "multi-series expansion" will be derived to represent the potential exterior to the dielectric coatings. The per-unit-length capacitance matrix will be directly related to the expansion coefficients.

1. Single Wire Solutions

Consider the conductor i, as in Figure 2a. The region R_i ($a_i < r_i < b_i$) is filled with dielectric material with dielectric constant ε_i. Outside the coating is the region R_o ($r_i > b_i$) with dielectric constant ε_o. According to reference (4), a complete solution of the electrostatic potential ψ_i in region R_i is

$$\psi_i = \sum_{m=1}^{\infty} (\alpha_{im} \cos m\phi_i + \beta_{im} \sin m\phi_i)(r_i^m + \gamma_{im} r_i^{-m})$$

$$+ \alpha_{i0} \ln \frac{r_i}{a_i} + \beta_{i0} \qquad a_i < r_i < b_i \qquad (1)$$

In Figure 2b, the solution of the region R_i', which has dielectric constant ε_o, is

$$\psi_i' = \sum_{m=1}^{\infty} (\alpha_{im}' \cos m\phi_i + \beta_{im}' \sin m\phi_i) r_i^m + \beta_{i0}' \qquad r_i < b_i \quad (2)$$

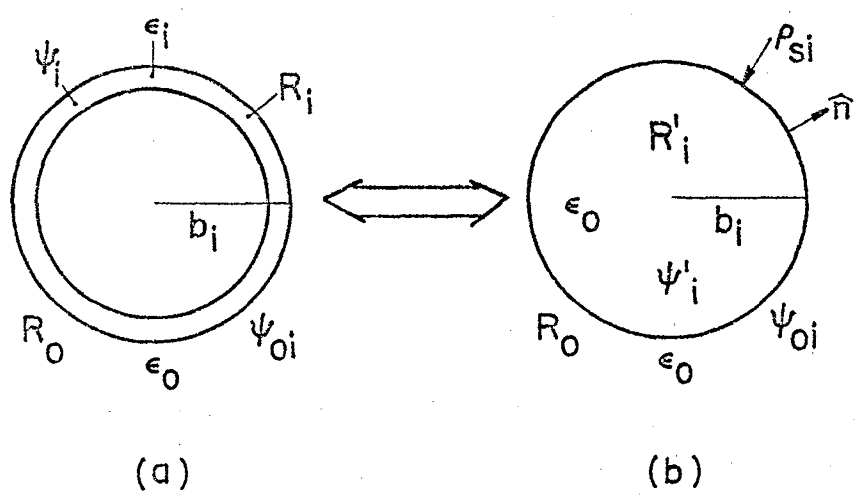

Figure 2. Configuration for the Induction Theorem

Hence the solution ψ_{oi} of region R_o, due to the i^{th} conductor, in both Figure 2a and Figure 2b is the same if the equivalent charge density

$$\rho_{si} = \frac{\partial}{\partial n} [\psi_i - \psi_i'] \, r_i = b_i \tag{3}$$

The general expression of ψ_{oi} is

$$\psi_{oi} = \sum_{m=1}^{\infty} (\eta_{im} \cos m\phi_i + \xi_{im} \sin m\phi_i) \, r_i^{-m}$$

$$+ \eta_{i0} \, \ln \frac{r_i}{b_i} + \xi_{i0} \tag{4}$$

where η_{im} and ξ_{im} are the potential expansion coefficients for the exterior region, R_o. The equivalence of ψ_{oi} in both Figures 2a and 2b induced by the surface charge density ρ_{si} is due to the induction theorem as described in ref. (13).

2. The Multi-series Expansions

If the induction theorem discussed above is applied for the conductors i, $i = 1, 2, \ldots, n$, the solution exterior to the dielectric coatings of all the wires of Figure 1 is equivalent to that of Figure 3. The equivalent surface charge densities per-unit-length are $\rho_{s1}, \rho_{s2}, \ldots, \rho_{sn}$ on each dielectric surface and ρ_{s0} on the surface of the grounded tube. Hence, by using the superposition theorem (ref. 4), the complete expansion of the potential is

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13. R.F. Harrington, Time-Harmonic Electromagnetic Fields, McGraw-Hill, New York (1961).

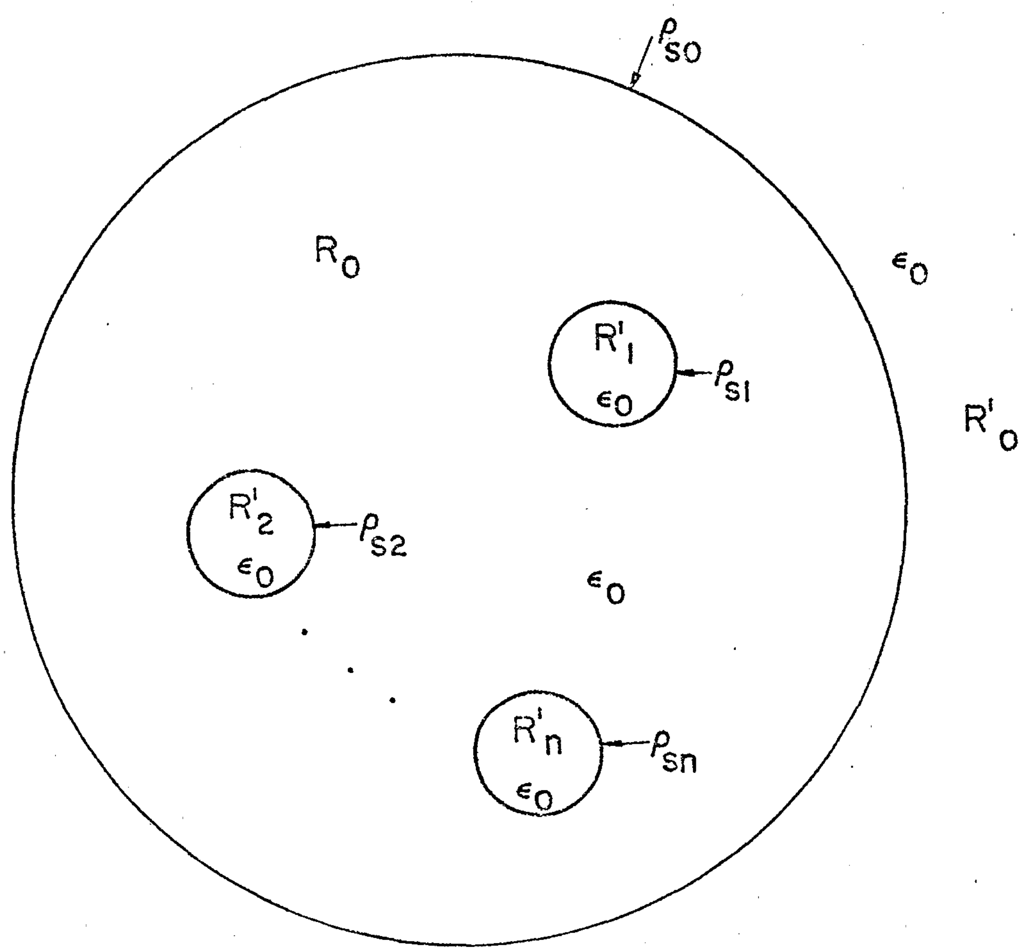

Figure 3. The Equivalent System of Figure 1 after Using the Induction Theorem

$$\psi = \sum_{i=1}^{n} \left[\sum_{m=1}^{\infty} (\eta_{im} \cos m\phi_i + \xi_{im} \sin m\phi_i) \, r_i^{-m} \right.$$

$$\left. + \eta_{i0} \, \ell n \frac{r_i}{b_i} \right] + \sum_{m=1}^{\infty} (\eta_{om} \cos m\phi + \xi_{om} \sin m\phi) \, r^m$$

$$+ \eta_{o0} \, \ell n \frac{r}{c} + \xi_{o0} \qquad r_i > b_i , \quad r < c \qquad (5)$$

where the constants ξ_{i0} are all absorbed in ξ_{o0} ; and $r_i > b_i$ for all $i = 1, 2, \ldots, n$; and $r < c$.

In equation (5) the potential due to the first summation is associated with all the interior conductors, whereas that due to the second sum is associated with the shielding tube.

In addition, the potential in the dielectric coating region R_i of conductor i is given in equation (1) as

$$\psi_i = \sum_{m=1}^{\infty} (\alpha_{im} \cos m\phi_i + \beta_{im} \sin m\phi_i)(r_i^m + \gamma_{im} \, r_i^{-m})$$

$$+ \alpha_{i0} \, \ell n \frac{r_i}{a_i} + \beta_{i0} \qquad (6)$$

where $a_i < r_i < b_i$, $i = 1, 2, \ldots, n$.

3. Boundary Conditions

On each conducting surface, $r_j = a_j$, the potential is V_j . That is

$$[\psi_j]_{r_j=a_j} = \sum_{m=1}^{\infty} (\alpha_{jm}\cos m\phi_j + \beta_{jm}\sin m\phi_j)(a_j^m + \gamma_{jm}\, a_j^{-m}) + \beta_{j0}$$

$$= V_j \qquad\qquad \text{for} \quad 0 \le \phi_j \le 2\pi \qquad\qquad (7)$$

Since equation (7) must hold for all $0 \le \phi_j \le 2\pi$, we have

$$\gamma_{jm} = -a_j^{2m} \quad ; \quad m = 1,\ 2,\ 3,\ \ldots. \quad \text{and } j = 1,\ 2,\ \ldots,\ n \qquad (8)$$

and

$$\beta_{j0} = V_j \quad ; \quad j = 1,\ 2,\ \ldots,\ n \qquad (9)$$

On each dielectric surface, $r_j = b_j$, the potential is continuous and the free charge is zero. Hence,

$$[\psi_j - \psi]_{r_j=b_j} = 0 \qquad (10)$$

and

$$\left[\varepsilon_j \frac{\partial\psi_j}{\partial r_j} - \varepsilon_o \frac{\partial\psi}{\partial r_j}\right]_{r_j=b_j} = 0 \qquad (11)$$

where ψ and ψ_j are given in equations (5) and (6) respectively.

On the grounded tube, the potential is grounded to zero,

$$[\psi]_{r=c} = 0 \tag{12}$$

One further condition is required to furnish the solution of the coefficients. This condition is obtained from the conservation of electric charges. That is

$$\int_0^{2\pi} \varepsilon_o \left[\frac{\partial\psi}{\partial r}\right]_{r=c} d\phi = \sum_{i=1}^{n} \int_o^{2\pi} \left[\varepsilon_i \frac{\partial\psi_i}{\partial r_i}\right]_{r_i=a_i} d\phi_i \tag{13}$$

4. The Per-Unit-Length Capacitance Matrix

Let the total free charge per-unit-length on the conductor i be Q_i'. Then

$$Q_i' = - \int_o^{2\pi} \varepsilon_i \left[\frac{\partial\psi}{\partial r_i}\right]_{r_i=a_i} d\phi_i$$

$$= -2\pi\, a_i\, \alpha_{i0}\, \varepsilon_i \frac{1}{a_i} = -2\pi\, \varepsilon_i\, \alpha_{i0} \tag{14}$$

The per-unit-length capacitance matrix is defined as

$$\begin{bmatrix} Q_1' \\ Q_2' \\ \vdots \\ Q_3' \end{bmatrix} = \begin{bmatrix} C_{11}' & C_{12}' \cdots & C_{1n}' \\ C_{21}' & C_{22}' \cdots & C_{2n}' \\ \vdots & \vdots & \vdots \\ C_{n1}' & C_{n2}' \cdots & C_{nn}' \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} \tag{15}$$

Taking $\quad V_i = \begin{cases} 1 \text{ volt} & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$

we have

$$\begin{bmatrix} C'_{1j} \\ C'_{2j} \\ \vdots \\ C'_{nj} \end{bmatrix} = \begin{bmatrix} Q'_1 \\ Q'_2 \\ \vdots \\ Q'_n \end{bmatrix}_{V_j=1 \text{ volt}} \tag{16}$$

The complete per-unit-length capacitance matrix is obtained by taking $j = 1, 2, \ldots, n$.

SECTION III

CALCULATION OF THE EXPANSION COEFFICIENTS

Plug γ_{im} and β_{i0}, which are given in equations (8) and (9), into equation (6), and truncate the expansion series into M_i finite terms. We then have

$$\psi_i = \sum_{m=1}^{M_i} (\alpha_{im} \cos m\phi_i + \beta_{im} \sin m\phi_i)\left(r_i^m - \frac{a_i^{2m}}{r_i^m}\right)$$

$$+ \alpha_{i0} \ln \frac{r_i}{a_i} + V_i \quad , \quad i = 1, 2, \ldots, n \qquad (17)$$

Similarly, summing the expansion series to M_o terms in equation (5)

$$\psi = \sum_{i=1}^{n} \sum_{m=1}^{M_i} (\eta_{im} \cos m\phi_i + \xi_{im} \sin m\phi_i)\ r_i^{-m}$$

$$+ \eta_{i0} \ln \frac{r_i}{b_i} + \sum_{m=1}^{M_0} (\eta_{om} \cos m\phi + \xi_{om} \sin m\phi)\ r^m$$

$$+ \eta_{o0} \ln \frac{r}{c} + \xi_{o0} \qquad (18)$$

The coefficients α_{im}, β_{im}, η_{im}, ξ_{im}, η_{om}, ξ_{om} in equations (17) and (18) are unknown constants which remain to be solved. The total number of unknowns N is

$$N = 4 \sum_{i=1}^{n} M_i + 2M_0 + 2n + 2 \tag{19}$$

These unknown coefficients can be solved by using the boundary conditions given in equations (10), (11), (12), and (13).

It is possible to use the point matching technique (ref. 12) to set up the linear equations for these coefficients. However, the stability of the linear equations is very sensitive to the matching points selected. For general problems, it is necessary to employ a systematic way of establishing a stable system of linear equations.

From equation (10), multiply both sides by 1, $\cos m\phi_j$, $\sin m\phi_j$, $m = 1, 2, \ldots, M_j$ successively, and integrate from $\phi_j = 0$ to 2π, we have

$$\int_0^{2\pi} [\psi_j - \psi]_{r_j=b_j} \, d\phi_j = 0 \tag{20a}$$

$$\int_0^{2\pi} [\psi_j - \psi]_{r_j=b_j} \cos m\phi_j \, d\phi_j = 0 \tag{20b}$$

$$\int_0^{2\pi} [\psi_j - \psi]_{r_j=b_j} \sin m\phi_j \, d\phi_j = 0 \tag{20c}$$

where $m = 1, 2, \ldots, M_j$ and $j = 1, 2, \ldots, n$. This establishes $2 \sum_{j=1}^{n} M_j + n$ equations.

Similarly, the condition given in equation (11) suffices for another $2\sum_{j=1}^{n} M_j + n$ linear equations. By multiplying equation (12) by 1, cos $m\phi$, sin $m\phi$, $m = 1, 2, \ldots, M_0$ successively, we have $2M_0 + 1$ more equations. Adding the condition of equation (13), we thus have a total of

$$N = 4\sum_{i=1}^{n} M_i + 2M_0 + 2n + 2$$

equations. The coefficients are then solved by the Gaussian elimination algorithms of the system of linear equations. Once the coefficients are solved, the per-unit-length capacitance matrix is obtained via Section II-4.

SECTION IV

NUMERICAL RESULTS

A FORTRAN program has been written to implement the techniques described in this report. The program is verified by the results of the three cases given in Figures 4a, 4b, and 4c. The first case to check the program is the coaxial cable with a layer of dielectric constant ε. The exact per-unit-length capacitance is

$$C' = \frac{2\pi}{\frac{1}{\varepsilon} \ln \frac{b}{a} + \frac{1}{\varepsilon_o} \ln \frac{c}{b}}$$

The numerical results of all the values of a, b, c, and ε used in the computations all agree with the exact per-unit-length capacitance to five-digit accuracies.

The per-unit-length capacitance matrix of the two conductors of Figures 4b and 4c are also compared against the exact per-unit-length capacitances (ref. 14). Five-digit accuracies are obtained for all the cases tested. The computation time on a CDC 7600 computer for the two conductors is about 0.4 second.

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14. ITT Reference Data for Radio Engineers, Chapter 24, Howard W. Sams & Co., Inc., Indianapolis (1975).

a) A Coaxial Cable

b) Two Conducting Wires

c) Two Conducting Wires

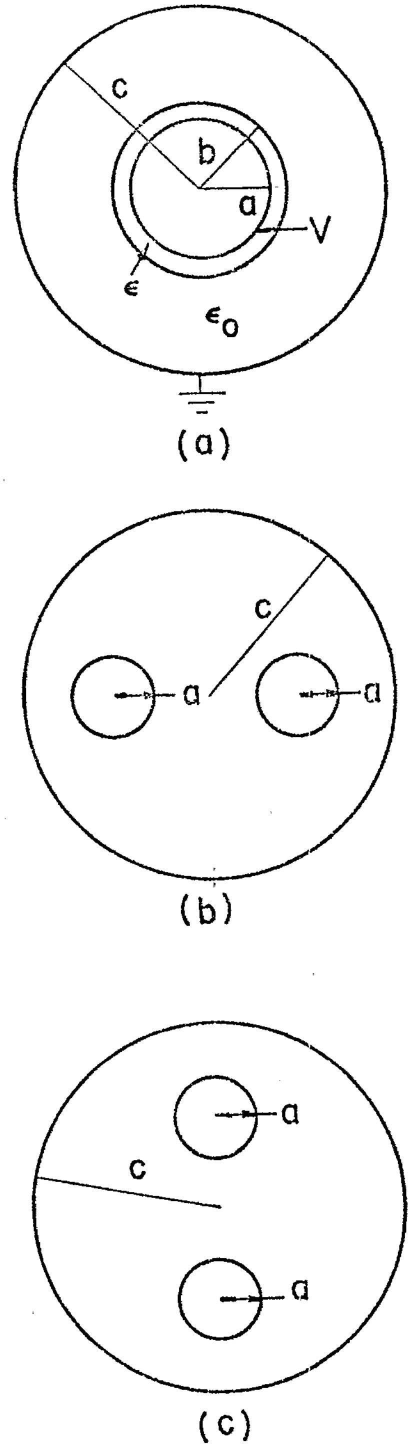

Figure 4.

The characteristic numbers of Figure 5 are given below

a_i = 1 unit length , $i = 1, 2, \ldots, 7$.

b_i = 2 unit lengths , $i = 1, 2, \ldots, 7$.

c = 10 unit lengths

$(x_{c1}, y_{c1}) = (0,0)$

$(x_{c2}, y_{c2}) = (5,0)$

$(x_{c3}, y_{c3}) = (2.5,4.3)$

$(x_{c4}, y_{c4}) = (-2.5,4.3)$

$(x_{c5}, y_{c5}) = (-5,0)$

$(x_{c6}, y_{c6}) = (-2.5,-4.3)$

$(x_{c7}, y_{c7}) = (2.5,-4.3)$

$M_i = 6$, $i = 0, 1, 2, \ldots, 7$.

For these values and a dielectric constant $\varepsilon_i = 2\,\varepsilon_o$, $i = 1, 2, \ldots 7$, the resulting per-unit-length capacitance matrix (in picofarad/meter) is

$$\bar{\bar{C}}' = \begin{bmatrix} 55.40 & -9.106 & -9.106 & -9.106 & -9.106 & -9.106 & -9.106 \\ -9.106 & 48.35 & -10.75 & -0.2226 & -0.0336 & -0.2226 & -10.75 \\ -9.106 & -10.75 & 48.35 & -10.75 & -0.2226 & -0.0336 & -0.2226 \\ -9.106 & -0.2226 & -10.75 & 48.35 & -10.75 & -0.2226 & -0.0336 \\ -9.106 & -0.0336 & -0.2226 & -10.75 & 48.35 & -10.75 & -0.2226 \\ -9.106 & -0.2226 & -0.0336 & -0.2226 & -10.75 & 48.35 & -10.75 \\ -9.106 & -10.75 & -0.2226 & -0.0336 & -0.2226 & -10.75 & 48.35 \end{bmatrix},$$

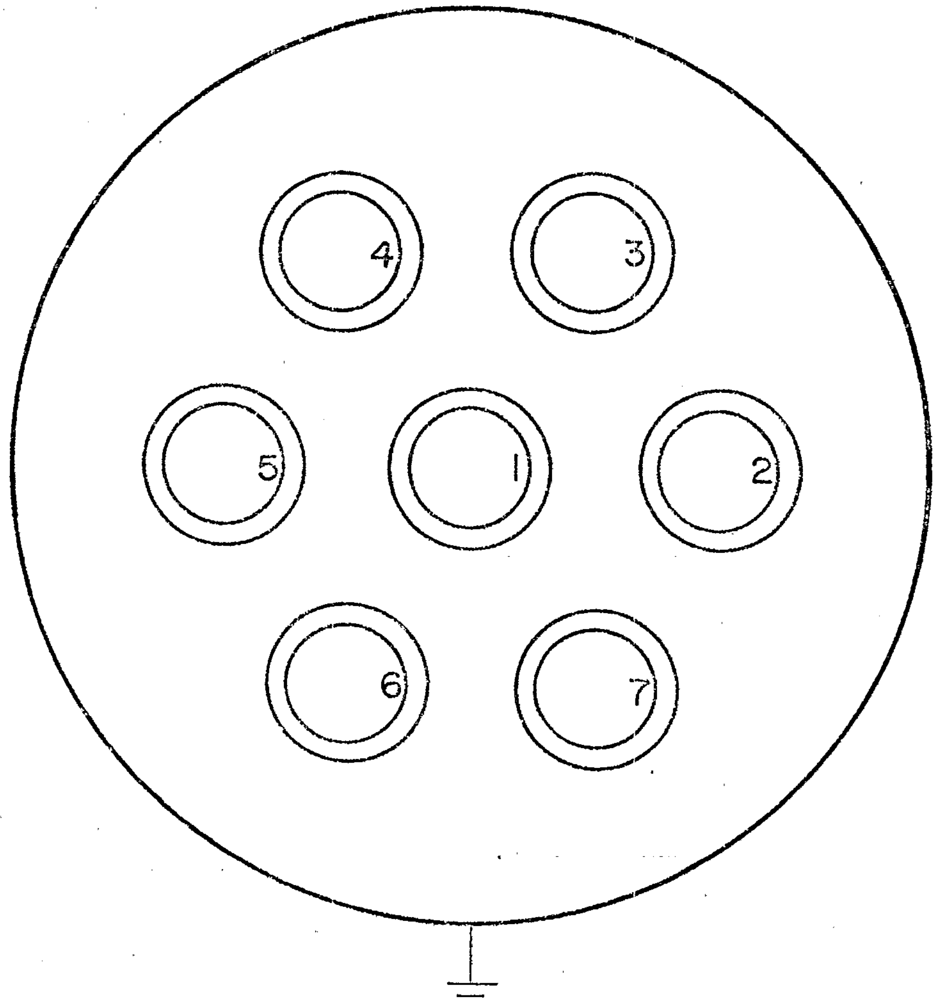

Figure 5. Dielectric Coated Wires in a Grounded Tube

Similarly, for $\varepsilon_i = 4\,\varepsilon_o$ for all wires, the per-unit-length matrix (in picofarad/meter) is found to be

$$\bar{\bar{C}}' = \begin{bmatrix} 74.55 & -12.35 & -12.35 & -12.35 & -12.35 & -12.35 & -12.35 \\ -12.35 & 61.47 & -14.83 & -0.168 & -0.0212 & -0.168 & -14.83 \\ -12.35 & -14.83 & 61.47 & -14.83 & -0.168 & -0.0212 & -0.168 \\ -12.35 & -0.168 & -14.83 & 61.47 & -14.83 & -0.168 & -0.0212 \\ -12.35 & -0.0212 & -0.168 & -14.83 & 61.47 & -14.83 & -0.168 \\ -12.35 & -0.168 & -0.0212 & -0.168 & -14.83 & 61.47 & -14.83 \\ -12.35 & -14.83 & -0.168 & -0.0212 & -0.168 & -14.83 & 61.47 \end{bmatrix}$$

The computation time for this 7-wire problem is about 7 seconds, which is reasonably fast for most practical computations.

The inductance per-unit-length matrix $\bar{\bar{L}}'$ can be readily obtained. Elements of $\bar{\bar{L}}'$ are independent of the dielectric materials, and hence for the configuration of Figure 5 with $\varepsilon_i = \varepsilon_o$, $i = 1, \ldots, 7$, we have

$$\bar{\bar{C}}_o' \, \bar{\bar{L}}' = \frac{1}{v^2} \bar{\bar{U}} \tag{21}$$

where $\bar{\bar{C}}_o'$ is the per-unit-length capacitance matrix of this configuration computed by using the computer code. $\bar{\bar{U}}$ is the 7×7 unity matrix and v is the speed of light inside the tube. For the geometry previously used, the per-unit-length capacitance matrix (in picofara/meter) is

$$\overline{\overline{C}}_o' = \begin{bmatrix} 37.0 & -5.96 & -5.96 & -5.96 & -5.96 & -5.96 & -5.96 \\ -5.96 & 34.7 & -6.76 & -0.258 & -0.0516 & -0.258 & -6.76 \\ -5.96 & -6.76 & 34.7 & -6.76 & -0.258 & -0.0516 & -0.258 \\ -5.96 & -0.258 & -6.76 & 34.7 & -6.76 & -0.258 & -0.0516 \\ -5.96 & -0.0516 & -0.258 & -6.76 & 34.7 & -6.76 & -0.258 \\ -5.96 & -0.258 & -0.0516 & -0.258 & -6.76 & 34.7 & -6.76 \\ -5.96 & -6.76 & -0.258 & -0.0516 & -0.258 & -6.76 & 34.7 \end{bmatrix}$$

The per-unit-length inductance matrix (microhenry/meter), using equation (21) is

$$\overline{\overline{L}}' = \begin{bmatrix} 304 & 105 & 105 & 105 & 105 & 105 & 105 \\ 105 & 293 & 97 & 53 & 44 & 53 & 97 \\ 105 & 97 & 293 & 97 & 53 & 44 & 53 \\ 105 & 53 & 97 & 293 & 97 & 53 & 44 \\ 105 & 44 & 53 & 97 & 293 & 97 & 53 \\ 105 & 53 & 44 & 53 & 97 & 293 & 97 \\ 105 & 97 & 53 & 44 & 53 & 97 & 293 \end{bmatrix}$$

This method of evaluating $\overline{\overline{L}}'$ enables one to compute the propagation matrix $\overline{\overline{\gamma}}$ of a multiconductor cable with dielectric coating, as

$$\overline{\overline{\gamma}}^2 = \overline{\overline{C}}'\overline{\overline{L}}'$$

for the current mode (ref. 15).

15. F.M. Tesche and T.K. Liu, Selected Topics in Transmission Line Theory, Interaction Notes, Note , Air Force Weapons Laboratory, Kirtland Air Force Base, NM, April 1977.

SECTION V

CONCLUSIONS

The per-unit-length capacitance matrix of a multiconductor transmission line enclosed in a shielding tube is evaluated by the method of multi-series expansion. Numerical values of typical examples are given.

The per-unit-length inductance matrix can be obtained by finding the inverse of the corresponding per-unit-length capacitance matrix with all dielectric coatings of the conductors removed. The propagation matrix of the configuration is then given by this inductance matrix and the per-unit-length capacitance matrix computed for the system with dielectric coatings.

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