Interaction Notes

Note 326

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Stick-Model Characterization of the Natural Frequencies and Natural Modes of the Aircraft

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Abstract

A necessary first step in the characterization of the external EMP responses of an aircraft is the determination of the natural frequencies and natural modes of the aircraft. This report presents a simple method for estimating the natural frequencies and natural modes of an aircraft. The aircraft is modeled by eight thin, perfectly-conducting sticks representing the fuselage, wings, and stabilizers; the important global features of the external surface of the aircraft are thereby retained while, at the same time, the mathematical difficulty of the problem is greatly reduced.

General results are presented for the natural frequencies and natural modes in terms of the global lengths of the aircraft surface and the overall radius of curvature. Specific results for the B-1, E-4, and EC-135 aircraft are also presented.

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SECTION I

INTRODUCTION

It is desirable to formulate a general scheme whereby the natural frequencies and natural modes of an aircraft can be estimated without the necessity of obtaining numerical solutions of integral equations as has been done in the past. This report presents one such method, which will be called the six-length stick model. The six-length stick model sacrifices the details of the aircraft surface, such as dielectric windows, skin panel joints, and local variations in the radius of curvature, in exchange for a tractable mathematical formulation which preserves the important global features of the external surface of the aircraft: the lengths of the wings, fuselage, and stabilizers; and the overall radius of curvature as represented by the stick-model Ω-parameter [ref. 1]. Once the natural frequencies and natural modes have been determined, it is then a straightforward matter to calculate the external axial current and linear charge density induced by an EMP in either the frequency or time domain. This will be done in a later report.

The natural frequencies and natural modes are presented here for three aircraft of particular interest: the B-1 (wings swept forward), the E-4, and the EC-135. The six-length stick model is sufficiently general to be used easily for other aircraft.

SECTION II

THE SIX-LENGTH STICK MODEL

The problem of calculating the current induced on a perfectly-conducting "thin stick" by an incident plane wave in the absence of a ground plane involves the solution of a difficult integral equation which is beyond the scope of the present discussion. It suffices for the purposes of this report to use the frequency-domain result of Marin [ref. 2], which, neglecting end conditions, is

$$I_{ind}(k,x,\theta) = \frac{4\pi j E_o}{k Z_o \Omega \sin\theta} e^{jkx\cos\theta} \tag{1}$$

where (see figure 1) $\Omega = 2\ \ell n[2(\text{stick length})/(\text{stick radius})]$, k is the wavenumber of the incident field, x is the position along the stick, Z_o is the impedance of free space, E_o is the electric field strength of the incident wave, and θ is the angle between the propagation vector and the negative unit vector along the stick. The phase of the incident field is set to zero at $x = 0$ and $\underline{H}$ is polarized perpendicularly to the stick.

The six-length stick model is composed of eight sticks: the forward fuselage (ℓ_1), the port and starboard wings (ℓ_2), the aft fuselage (ℓ_3), the bottom section of the vertical stabilizer $(\ell_4$; $\ell_4 = 0$ on some aircraft), the port and starboard horizontal stabilizers (ℓ_5), and the top section of the vertical stabilizer (ℓ_6). (See figure 1). To simplify the analysis, the sections are assumed to join at right angles. This simplification has no zero-order effect on the real parts of the natural frequencies or on the natural modes, and only a slight effect on the imaginary (damping) parts of the natural frequencies due to small changes in the radiated energy at the resonances, as will be seen in the following calculations.

To first order in Ω^{-1}, the current on each stick can be written as a directly-induced component plus a sine and cosine component:

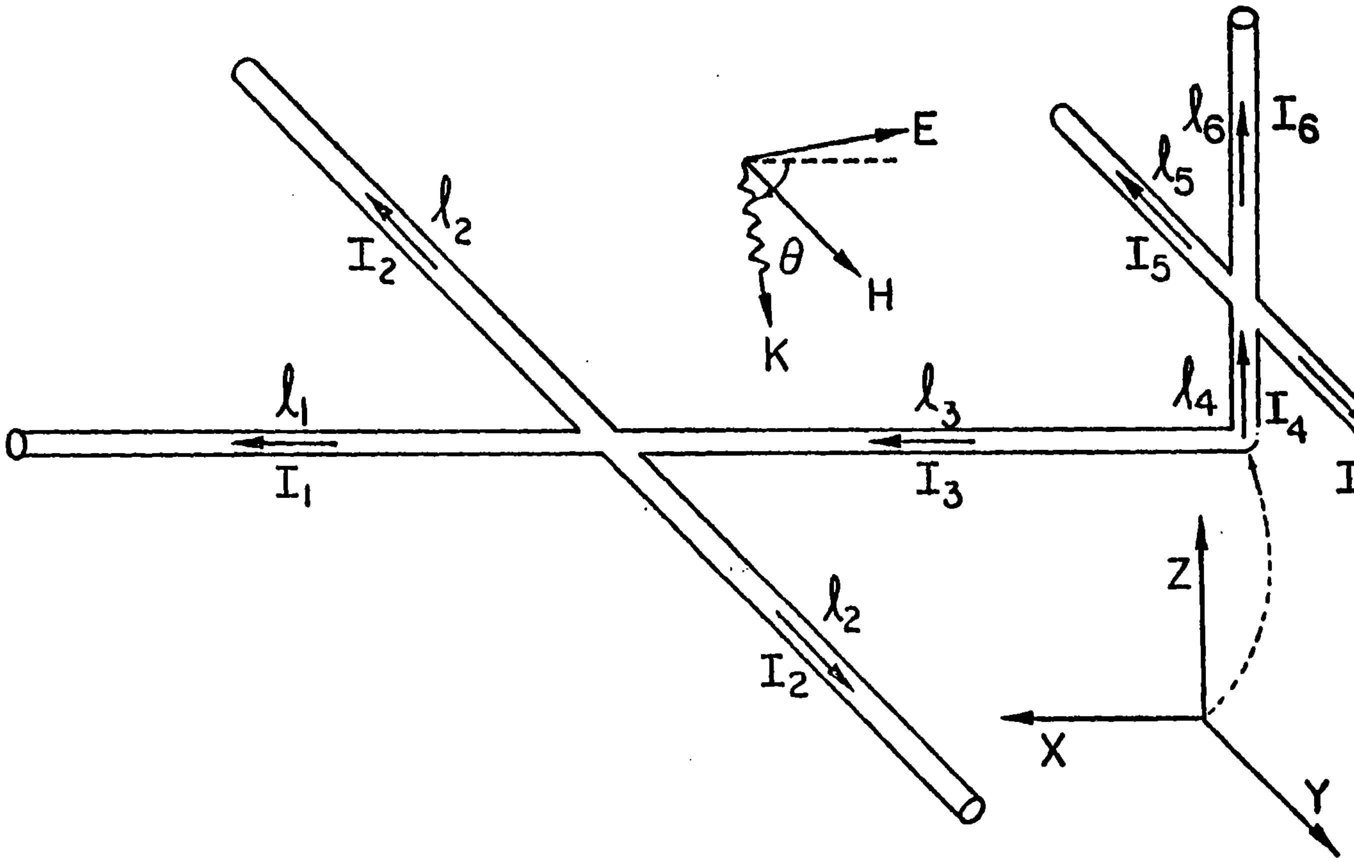

Figure 1. The six-length stick model.

$$I_n(\xi) = I_{ind,n}(k,\xi,\theta) + S_n \sin[k(\xi - \ell_n)] + C_n \cos[k(\xi - \ell_n)] \ . \qquad (2)$$

Although the natural frequencies and natural modes are found by searching for nontrivial solutions of $I_n(\xi)$ when there is no incident field $(E_o = 0)$, it will be helpful for later reports to consider a particular incident plane wave for which $\underline{E}$ is perpendicular to the wings and horizontal stabilizer. This case is illustrated in figure 1. The interesting question of what happens when the incident angle is such that $\underline{E}$ is parallel to the fuselage or vertical stabilizer is left to later reports.

S_n and C_n must be chosen to satisfy junction and end conditions. At the free ends of the sticks, the current must vanish: $I_n(\xi_{end}) = 0$. At the junctions, the current must be conserved and the quasi-static potential (charge density) must be continuous. The end conditions are immediately satisfied by letting

$$I_1(x) = I_{ind}(k,x,\theta) + S_1 \sin[k(x-\ell_1-\ell_3)] - I_{ind}(k, \ell_1+\ell_3, \theta)\cos[k(x-\ell_1-\ell_3)]$$

$$I_2(y) = S_2 \sin[k(y - \ell_2)] \qquad (y > 0)$$

$$I_3(x) = I_{ind}(k,x,\theta) + S_3 \sin[k(x - \ell_3)] + C_3 \cos[k(x - \ell_3)]$$

$$I_4(z) = - I_{ind}(k,z, \pi/2 - \theta) + S_4 \sin[k(z - \ell_4)] + C_4 \cos[k(z - \ell_4)] \qquad (3)$$

$$I_5(y) = S_5 \sin[k(y - \ell_5)] \qquad (y > 0)$$

$$I_6(z) = - I_{ind}(k,z, \pi/2 - \theta) + S_6 \sin[k(x-\ell_4-\ell_6)]$$

$$+ I_{ind}(k, \ell_4 + \ell_6, \pi/2 - \theta)\cos[k(z-\ell_4-\ell_6)]$$

It should be remarked that, due to the choice of polarization, the current on the port side of the aircraft $(y > 0)$ is a mirror image of the current on the starboard side $(y < 0)$ and the expressions for I_n are symmetric under the transformation $n \to 7 - n$, $\theta \to \pi/2 - \theta$, $E_o \to -E_o$.

The junction conditions lead to the following set of eight linear equations for the quantities S_1 through S_6 , C_3 , and C_4 :

(1). Conservation of current at $x = \ell_3$:

$$-S_1 \sin k\ell_1 - 2S_2 \sin k\ell_2 + I_{ind}(k,\ell_3,\theta) - I_{ind}(k, \ell_1 + \ell_3, \theta)\cos k\ell_1 = I_{ind}(k,\ell_3,\theta) + C_3 \tag{4a}$$

(2) and (3). Continuity of quasi-static potential at $x = \ell_3$:

$$\frac{1}{k}\frac{\partial}{\partial x} I_{ind}(k,x,\theta)\Big|_{\ell_3} + S_1 \cos k\ell_1 - I_{ind}(k, \ell_1 + \ell_3, \theta)\sin k\ell_1 = \frac{1}{k}\frac{\partial}{\partial x} I_{ind}(k,x,\theta)\Big|_{\ell_3} + S_3 \tag{4b}$$

$$S_2 \cos k\ell_2 = \frac{1}{k}\frac{\partial}{\partial x} I_{ind}(k,x,\theta)\Big|_{\ell_3} + S_3 \tag{4c}$$

(4). Conservation of current at $x = z = 0$:

$$I_{ind}(k,0,\theta) - S_3 \sin k\ell_3 + C_3 \cos k\ell_3 -I_{ind}(k,0, \pi/2 - \theta) - S_4 \sin k\ell_4 + C_4 \cos k\ell_4 = 0 \tag{4d}$$

(5). Continuity of quasi-static potential at $x = z = 0$:

$$\frac{1}{k}\frac{\partial}{\partial x} I_{ind}(k,x,\theta)\Big|_0 + S_3 \cos k\ell_3 + C_3 \sin k\ell_3 = -\frac{1}{k}\frac{\partial}{\partial z} I_{ind}(k,z, \pi/2 - \theta)\Big|_0 + S_4 \cos k\ell_4 + C_4 \sin k\ell_4 \tag{4e}$$

(6) and (7). Continuity of quasi-static potential at $z = \ell_4$:

$$S_5 \cos k\ell_5 = -\frac{1}{k}\frac{\partial}{\partial x} I_{ind}(k,z,\ \pi/2-\theta)\Big|_{\ell_4} + S_4 \tag{4f}$$

$$-\frac{1}{k}\frac{\partial}{\partial z} I_{ind}(k,z,\ \pi/2-\theta)\Big|_{\ell_4} + S_6 \cos k\ell_6 + I_{ind}(k,\ \ell_4+\ell_6,\ \pi/2-\theta)\sin k\ell_6 =$$
$$-\frac{1}{k}\frac{\partial}{\partial z} I_{ind}(k,z,\ \pi/2-\theta)\Big|_{\ell_4} + S_4 \tag{4g}$$

(8). Conservation of current at $z = \ell_4$:

$$-S_6 \sin k\ell_6 - 2S_5 \sin k\ell_5 - I_{ind}(k,\ell_4,\ \pi/2-\theta) + I_{ind}(k,\ \ell_4+\ell_6,\ \pi/2-\theta)\cos k\ell_6$$
$$= -I_{ind}(k,\ell_4,\ \pi/2-\theta) + C_4 \tag{4h}$$

These equations are listed in matrix form in table 1.

Table 1. Matrix Equation (4)

$$\begin{bmatrix} -\sin k\ell_1 & -2\sin k\ell_2 & 0 & -1 & 0 & 0 & 0 & 0 \\ \cos k\ell_1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos k\ell_2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\sin k\ell_3 & \cos k\ell_3 & \cos k\ell_4 & -\sin k\ell_4 & 0 & 0 \\ 0 & 0 & \cos k\ell_3 & \sin k\ell_3 & -\sin k\ell_4 & -\cos k\ell_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & \cos k\ell_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & \cos k\ell_6 \\ 0 & 0 & 0 & 0 & -1 & 0 & -2\sin k\ell_5 & -\sin k\ell_6 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ C_3 \\ C_4 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix} = \frac{4\pi j E_o}{k\Omega Z_o}$$

SECTION III

DETERMINATION OF NATURAL FREQUENCIES AND NATURAL MODES

The natural frequencies and natural modes are found by setting $E_o = 0$ and examining equations (4) for non-trivial solutions. Setting the determinant equal to zero, one obtains

$$0 = \cos k\ell_1 \cos k\ell_2 \cos k(\ell_3+\ell_4) \cos k\ell_5 \cos k\ell_6 \times$$
$$\{\tan k\ell_1 + 2\tan k\ell_2 + \tan k(\ell_3+\ell_4) + 2\tan k\ell_5 + \tan k\ell_6 \tag{5}$$
$$- \tan[k(\ell_3+\ell_4)](\tan k\ell_1 + 2\tan k\ell_2)(\tan k\ell_6 + 2\tan k\ell_5)\} .$$

Unless ℓ_1/ℓ_2 (or ℓ_5/ℓ_6) is equal to the ratio of two odd, positive integers, the only solutions of equation (5) are also solutions of

$$0 = \tan k\ell_1 + 2\tan k\ell_2 + \tan k(\ell_3+\ell_4) + 2\tan k\ell_5 + \tan k\ell_6 \tag{6}$$
$$- \tan[k(\ell_3+\ell_4)](\tan k\ell_1 + 2\tan k\ell_2)(\tan k\ell_6 + 2\tan k\ell_5) .$$

Values of k which satisfy equation (6) can be found numerically for any given set of ℓ_n. Only the positive roots are of interest because if k_m is a root of equation (6), then $-k_m$ is also a root. The real part of the $m^{\underline{th}}$ natural frequency, in hertz, is

$$f_m = ck_m/2\pi . \tag{7}$$

When $k = k_m$, non-trivial solutions for S_1 through S_6, C_3, and C_4 are found to be

$$S_1 = S_3/\cos k\ell_1 , \quad S_2 = S_3/\cos k\ell_2 , \quad S_3 = \text{arbitrary},$$

$$C_3 = -S_3(\tan k\ell_1 + 2\tan k\ell_2) , \quad C_4 = -S_4(\tan k\ell_6 + 2\tan k\ell_5) ,$$

$$S_6 = S_4/\cos k\ell_6\ , \quad S_5 = S_4/\cos k\ell_5\ ,$$

$$S_4 = S_3\,\frac{\cos k\ell_3 - \sin k\ell_3(\tan k\ell_1 + 2\tan k\ell_2)}{\cos k\ell_4 - \sin k\ell_4(\tan k\ell_6 + 2\tan k\ell_5)}\ . \tag{8}$$

The natural modes are then found by simple substitution in equations (3).

The final task is to find the imaginary components of the natural frequencies (damping constants). To do this, one must find for each resonance the ratio of the average radiated power, P_m , to the stored magnetic energy per cycle, W_m . The temporal damping constant, α_m , is then given by

$$\alpha_m = \frac{P_m}{4W_m} = \frac{ck_m P'_m}{\Omega\ W'_m}\ , \tag{9}$$

where

$$W'_m = \int_{\text{sticks}} I^2(\xi)d\xi \tag{10}$$

$$P'_m = \int_{\text{sticks}'}\int_{\text{sticks}''}\Bigg[p(','')I(\xi')I(\xi'') - \frac{1}{k_m^2}\left(\frac{\partial}{\partial\xi'}\ I(\xi')\right)\left(\frac{\partial}{\partial\xi''}\ I(\xi'')\right)\Bigg]\frac{\sin[k_m|\underline{r}' - \underline{r}''|]}{|\underline{r}' - \underline{r}''|}\,d\xi'd\xi'' \tag{11}$$

$$p(','') = \begin{cases} 0 & \text{if stick}' \text{ perpendicular to stick}'' \\ 1 & \text{if stick}' \text{ parallel to stick}'' \end{cases}$$

The integral expression for W'_m can be evaluated explicitly, while the expression for P'_m must be evaluated numerically. A more detailed development of these integral expressions is available in ref. 2.

SECTION IV
RESULTS AND CONCLUSIONS

The six-length stick model has been used to calculate the first several natural frequencies and natural modes for the B-1, E-4, and EC-135. The natural frequencies are listed in tables 2, 3, and 4, while the natural modes are illustrated in figures 2, 3, and 4. Figure 5 illustrates the resonances for the three aircraft in s-space.

It is of interest to notice that the three aircraft have very similar first and second natural modes. The first, or fundamental, mode can be characterized by currents flowing down the forward fuselage and in the wings, joining at the wing root, continuing down the aft fuselage (then up the vertical stabilizer bottom portion, if any), and finally branching and flowing up the vertical stabilizer (top portion) and out the horizontal stabilizers. This mode utilizes the entire global length of the aircraft and is therefore the lowest resonance. In the second mode, the foward fuselage and wings are dominant. Current flows down the forward fuselage and out the wings with little coupling to the aft portion of the aircraft. There is approximately one half-wavelength along the forward fuselage and one wing, which accounts for the fact that this resonance is the next-to-lowest. The higher-order resonances are more complicated and do not tend to obey any general rules.

It is to be hoped that the six-length stick model will serve as a useful tool in the analysis of aircraft resonances. The simplicity of the mathematical formulation and the ease of obtaining numerical results for any particular aircraft suggest that this model should at least be applied as a first approximation and check, which can be made more precise by the application of other, more complicated, models.

Later reports will show how equations (4) can be solved to find the axial current and linear charge density at any point on the aircraft surface in either the frequency or time domain.

Table 3. Resonances for the E-4

Lengths (meters)

1.	Forward fuselage................	25.0
2.	Wing............................	36.0
3.	Aft fuselage....................	36.0
4.	Vertical stabilizer (bottom)....	0.0
5.	Horizontal stabilizer...........	12.0
6.	Vertical stabilizer (top).......	16.0

$\Omega = 6.2$

Frequency (MHz)	Decay time (μ sec)
1.3	1.27
2.6	0.57
3.9	0.50
5.0	0.37
5.3	0.23

Table 4. Resonances for the EC-135

Lengths (meters)

1. Forward fuselage................ 13.0
2. Wing............................ 20.0
3. Aft fuselage.................... 23.0
4. Vertical stabilizer (bottom)..... 0.0
5. Horizontal stabilizer............ 7.0
6. Vertical stabilizer (top)....... 11.0

$\Omega = 6.5$

Frequency (MHz)	Decay time (μ sec)
2.1	0.78
4.8	0.38
6.4	0.26
7.8	0.20
9.0	0.17

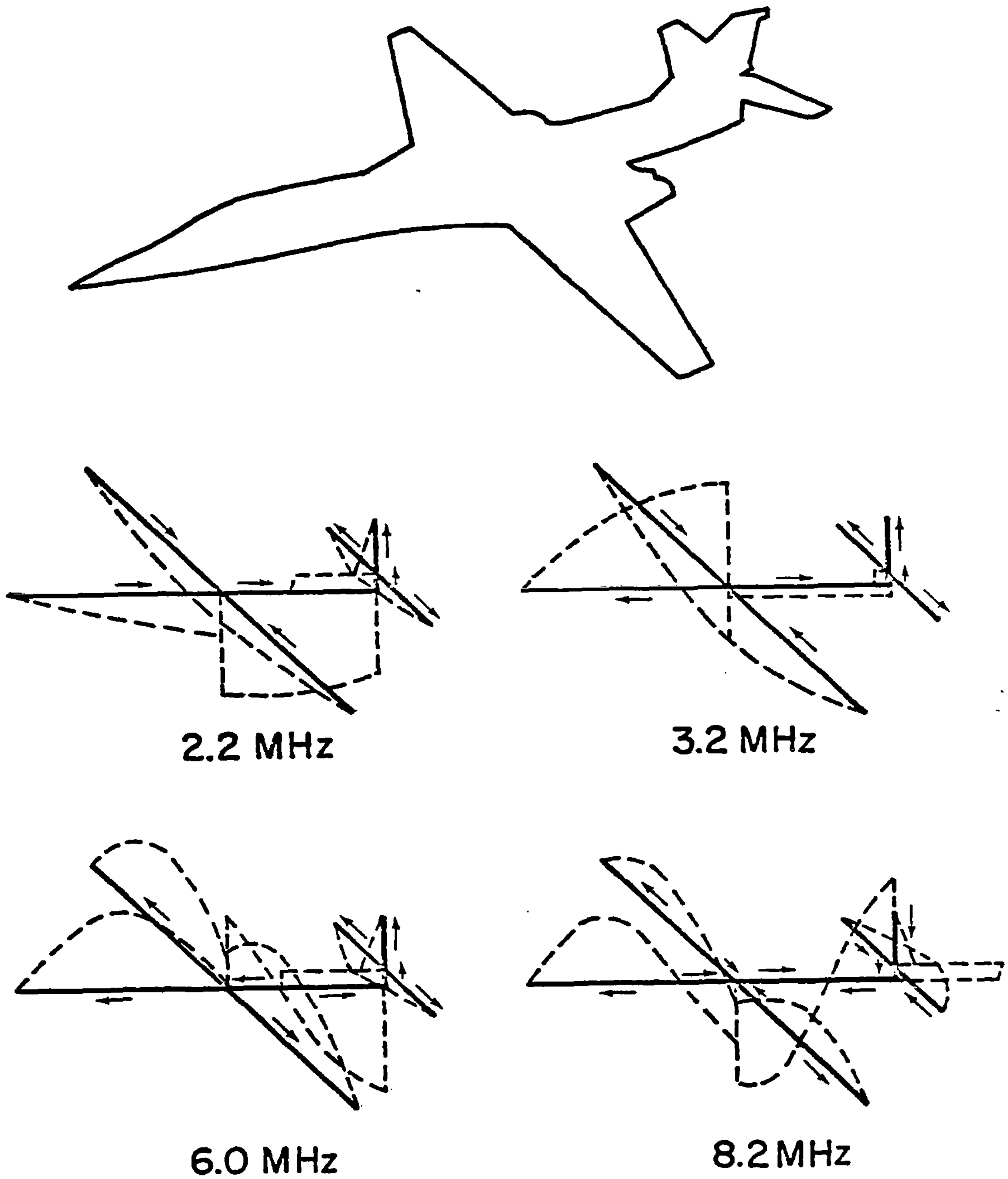

Figure 2. B-1 natural modes. The dashed lines represent the current distribution on the aircraft segments at resonance, while the arrows indicate direction.

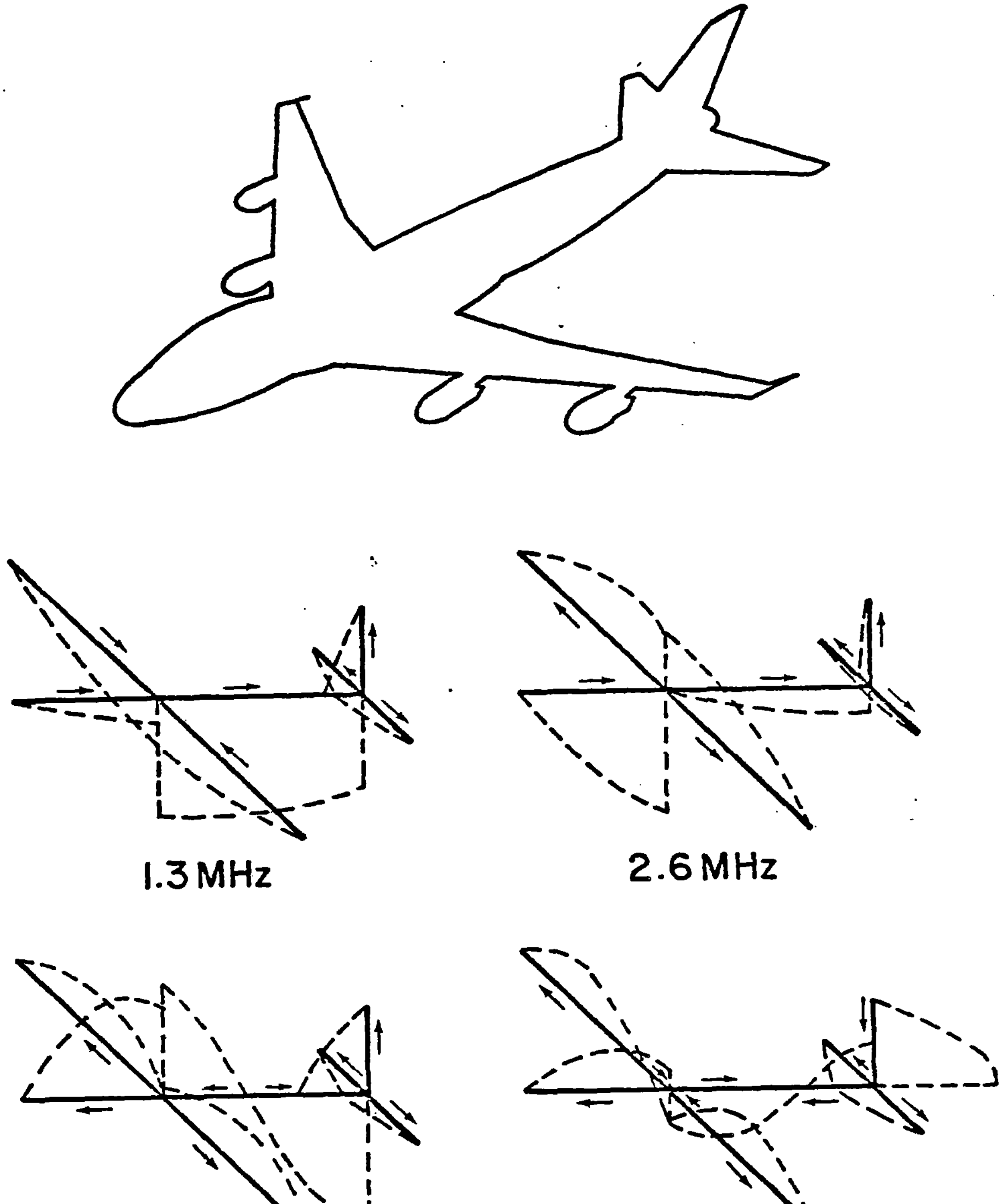

Figure 3. E-4 natural modes. The dashed lines represent the current distribution on the aircraft segments at resonance, while the arrows indicate direction.

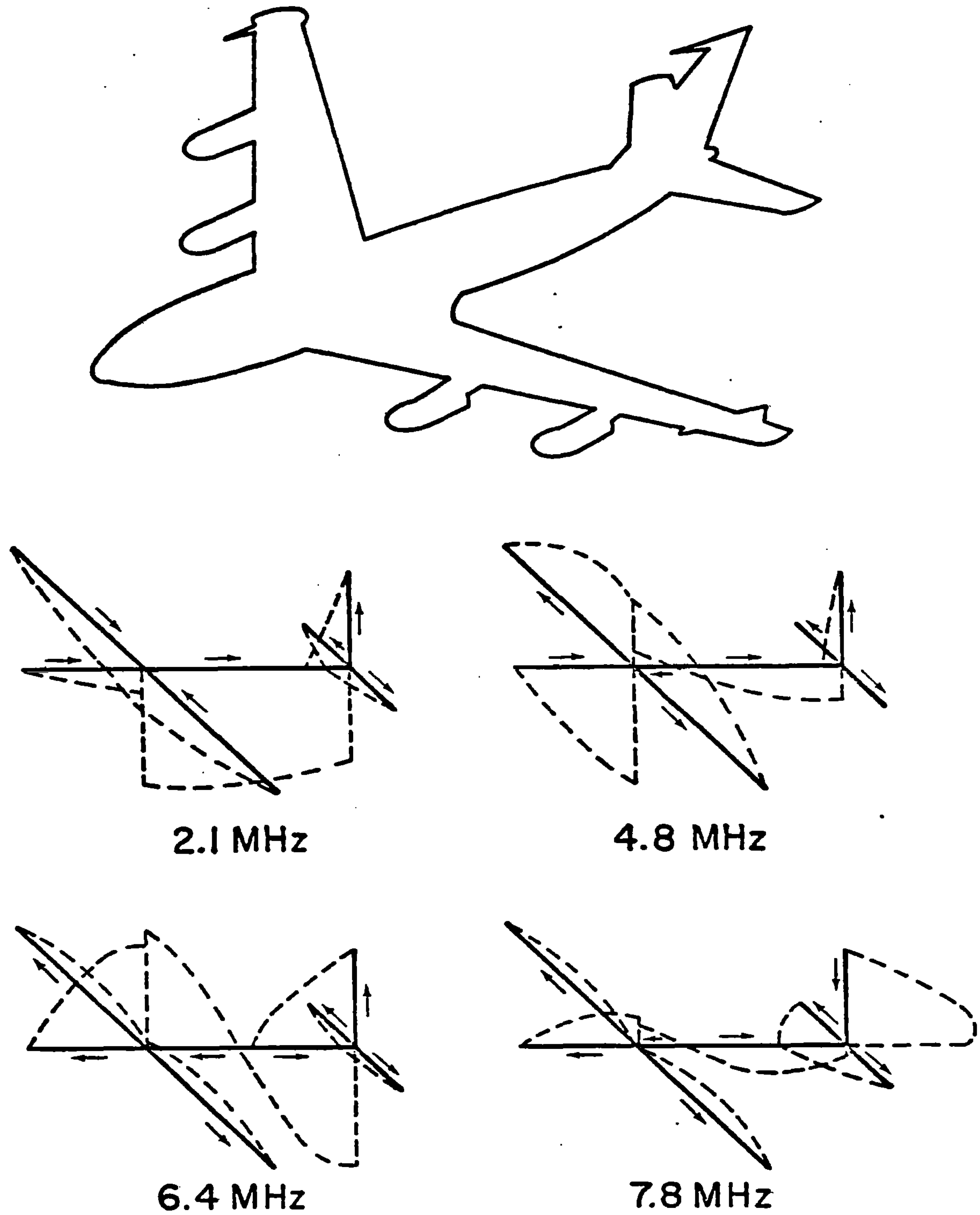

Figure 4. EC-135 natural modes. The dashed lines represent the current distribution on the aircraft segments at resonance, while the arrows indicate direction.

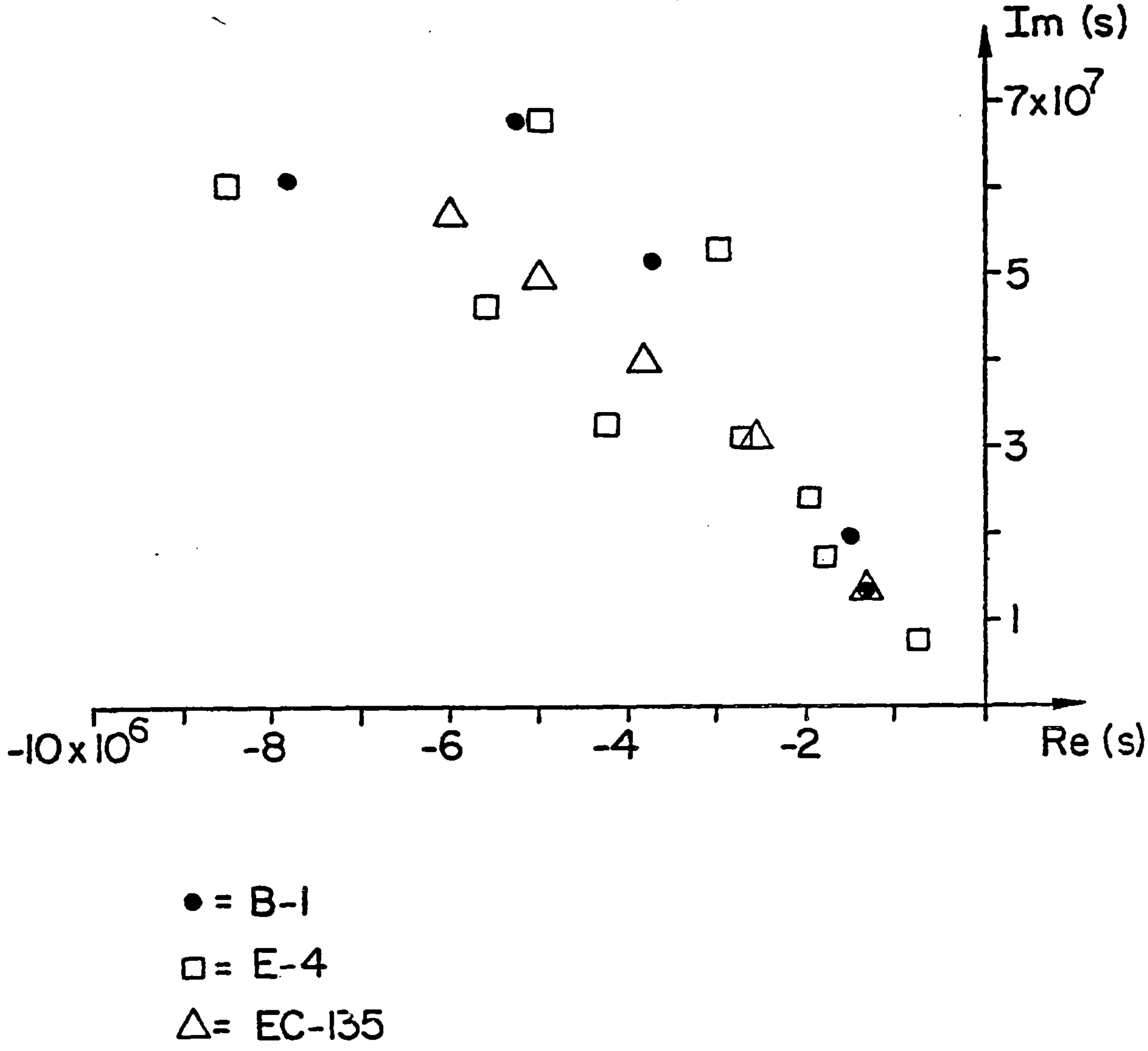

Figure 5. Aircraft natural frequencies (Re and Im denote, respectively, real and imaginary part.)

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