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A Comparison of Stick Model Skin Current Predictions with Scale Model Measurements for the E-4 and EC-135 Aircraft

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Abstract

Scale-model measurements of the axial skin current density on the E-4 and EC-135 aircraft are compared with predictions calculated using the six-length stick model. The stick-model predictions tend to be somewhat smaller in magnitude than the scale-model measurements except near aircraft resonances, but are otherwise in good agreement for frequencies less than or approximately equal to the second aircraft resonance and in fairly good agreement for frequencies less than the fourth aircraft resonance.

Acknowledgment

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A COMPARISON OF STICK MODEL SKIN CURRENT PREDICTIONS
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ABSTRACT

Scale-model measurements of the axial skin current density on the E-4 and EC-135 aircraft are compared with predictions calculated using the six-length stick model. The stick-model predictions tend to be somewhat smaller in magnitude than the scale-model measurements except near aircraft resonances, but are otherwise in good agreement for frequencies less than or approximately equal to the second aircraft resonance and in fairly good agreement for frequencies less than the fourth aircraft resonance.
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SECTION I
INTRODUCTION

The six-length stick model of an aircraft (refs. 1 and 2) provides a convenient method for obtaining the complex natural resonance frequencies and the natural current and charge modes on the aircraft surface, from which explicit expressions can be developed to predict the total current and total linear charge density at any position on the important aircraft sections, in either the frequency or time domain. In order to do this, it is necessary to ignore the detailed geometry of the aircraft surface and characterize the aircraft by its global length parameters, that is to say the lengths of the important aircraft sections and an overall average radius of curvature for the aircraft surface. Once the global parameters have been estimated for a particular aircraft, it is then a simple matter to predict the external coupling. The stick-model calculations assume that a linearly-polarized plane wave electromagnetic pulse (EMP), with the magnetic field along the wing axis, is incident on the aircraft in free space at some angle, θ, with respect to the fuselage. For this reason, comparisons of stick-model predictions with full scale measurements made at ground-based EMP simulators are inappropriate, since the presence of a finitely-conducting ground and other objects will change the electromagnetic characteristics of the aircraft surface. However, scale-model measurements made in the University of Michigan anechoic chamber closely simulate the same conditions that the stick-model calculations assume (refs. 3 and 4). For this reason, comparison with scale-model measurements is the best test of the accuracy of the six-length stick model.

The stick-model predictions do not give directly the current densities at the top and bottom of the aircraft sections, quantities which are measured in the scale-model experiments. On the other hand, the scale-model measurements do not give directly the total current, which is the quantity predicted by the stick model. To compare the results it is therefore necessary to invoke assumptions concerning the distribution of current around the circumference of an aircraft section, based on the magnetostatic solution
for an infinite cylinder. The validity of these assumptions can be tested using the scale-model measurements.

Because of time limitations, attention has been restricted to the current midway along the forward fuselage on the EC-135 and E-4 aircraft for $\theta = 90^\circ$ (topside) incidence. It will be seen that the agreement between calculation and measurement is good for frequencies less than or approximately equal to the second aircraft resonance frequency, and fairly good for frequencies as high as the fourth aircraft resonance frequency. Since the bulk of the energy in a typical EMP is contained in the frequency region below the second aircraft resonance, it is to be expected that the time-domain external coupling predictions based on the stick model will be accurate for all but the very earliest times. These concepts will be quantified in the discussion to follow.
SECTION II
STICK MODEL PARAMETERS

It is essential to choose the stick-model length parameters carefully so that the stick-model external coupling predictions will be as accurate as possible. As presented in references 1 and 2, there are seven length parameters in the stick model of an aircraft. The first six are the lengths of the important aircraft sections: the forward fuselage, the wings, the aft fuselage, the vertical stabilizer (bottom segment), the horizontal stabilizers, and the vertical stabilizer (top segment). On many aircraft, such as the E-4 and EC-135, the bottom segment of the vertical stabilizer has a length of zero, which simply means that the horizontal stabilizers are connected to the fuselage rather than to the vertical stabilizer. The seventh parameter, $\Omega$, is a dimensionless quantity which is a measure of the overall radius of curvature of the aircraft surface. Since $\Omega$ depends logarithmically on the radius, it is not a critical parameter in the stick model. For most aircraft, $6 \leq \Omega \leq 7$. As $\Omega \rightarrow \infty$, the stick model becomes more accurate.

The forward fuselage length ($l_1$) plus the aft fuselage length ($l_3$) should be approximately the distance from just aft of the nose to the point where the vertical stabilizer joins the fuselage. The wing length ($l_2$) is the distance from the wing root to the end of the wing, minus a few meters in case the wing has appreciable tapering. The vertical stabilizer lengths ($l_4$ and $l_5$) and the horizontal stabilizer length ($l_5$) are measured from the fuselage or the junction of the vertical and horizontal stabilizers to the end of the stabilizer. Finally, the wing root must be located for the stick model, which is to say that $l_1$ and $l_3$ must be chosen with the constraint that $l_1 + l_3$ should remain constant. In general, increasing $l_1$ will increase the (real) fundamental aircraft resonance frequency and decrease the (real) second aircraft resonance frequency. The exact choices for $l_1$ through $l_6$ are obviously somewhat arbitrary because a real aircraft is not a conjunction of very thin sticks, but a complicated structure which is only very approximately represented as thin sticks. In order to get the
best possible agreement between the stick-model predictions and the scale-model measurements, it would be desirable to make successive adjustments of $\lambda_1$ through $\lambda_6$ after comparison of the predictions with the measured data, although that was not done here.

Once the stick-model parameters are fixed, it is then a straightforward procedure to find the (complex) natural frequencies and then to estimate the total current at the same position on the forward fuselage that was used in the scale-model measurements. The mathematical details of this procedure are presented in references 1 and 2, and so will not be included here. Table 1 summarizes the stick-model parameters for the E-4 and EC-135 aircraft.

**Table 1**

**STICK MODEL PARAMETERS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>E-4</th>
<th>EC-135</th>
</tr>
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<tbody>
<tr>
<td>Forward Fuselage, $\lambda_1$</td>
<td>26 m</td>
<td>21 m</td>
</tr>
<tr>
<td>Wing, $\lambda_2$</td>
<td>29 m</td>
<td>17.5 m</td>
</tr>
<tr>
<td>Aft Fuselage, $\lambda_3$</td>
<td>27 m</td>
<td>14.5 m</td>
</tr>
<tr>
<td>Vertical Stabilizer (Bottom), $\lambda_4$</td>
<td>0 m</td>
<td>0 m</td>
</tr>
<tr>
<td>Horizontal Stabilizer, $\lambda_5$</td>
<td>11.5 m</td>
<td>7 m</td>
</tr>
<tr>
<td>Vertical Stabilizer (Top), $\lambda_6$</td>
<td>14 m</td>
<td>8.5 m</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>6.2</td>
<td>6.5</td>
</tr>
<tr>
<td>Fundamental Resonance</td>
<td>1.6 MHz</td>
<td>2.7 MHz</td>
</tr>
<tr>
<td>Second Resonance</td>
<td>2.8 MHz</td>
<td>3.8 MHz</td>
</tr>
<tr>
<td>Third Resonance</td>
<td>4.6 MHz</td>
<td>7.5 MHz</td>
</tr>
<tr>
<td>Fourth Resonance</td>
<td>5.7 MHz</td>
<td>9.3 MHz</td>
</tr>
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</table>
SECTION III
COMPARISON

The stick model predicts the total axial current, $I$, while the scale-model measurements give the axial current density $J(\phi)$ as a function of position along the fuselage circumference $\phi$. The point $\phi = 0$ corresponds to the top of the fuselage, $\phi = \pi$ to the bottom. In particular, the scale-model measurements give $J(0)$ and $J(\pi)$. It is therefore necessary to develop relationships between $I$ and $J(\phi)$ in order to make comparisons.

Due to the symmetries of the aircraft and the exciting field, it is to be expected that the following series representation is generally valid:

$$J(\phi) = J_0 + \sum_{n=0}^{\infty} J_{2n+1} \cos \left[ (2n+1)\phi \right]$$  \hspace{1cm} (1)

Assuming that the fuselage is roughly a circular cylinder, a good estimate of the total current is given by

$$I = \int_{0}^{2\pi} J(\phi) a(\phi) d\phi = 2\pi a_{\text{eff}} J_0 \equiv I_{\text{est}}$$ \hspace{1cm} (2)

where $a_{\text{eff}}$ is the effective fuselage radius.

Using equation (1),

$$I_{\text{est}} = \pi a_{\text{eff}} \left[ J(0) + J(\pi) \right]$$ \hspace{1cm} (3)

In this way, the estimated total current, $I_{\text{est}}$, as measured for scale models can be compared with the total current predicted by the stick model.

A more restrictive assumption must be invoked to estimate the current density predicted by the stick model. If the magnetostatic circular cylinder solution is valid for the aircraft fuselage, then there is only one term in the infinite series of equation (1) which is non-zero:
\[ J(\phi) = J_0 + 2H_0 \cos \phi \]  \hspace{1cm} (4)

where \( H_0 \) (or \( E_0 \)) is the strength of the incident magnetic (electric) field. In this case,

\[ \Delta H = \left[ J(0) - J(\pi) \right] = 4H_0 \]  \hspace{1cm} (5)

Equation (5) can be checked against the scale-model data as a test of the magnetostatic assumption. As a direct consequence of equations (2) and (4),

\[ J_{est}(0) = 2H_0 + I/(2\pi a_{eff}) \]  \hspace{1cm} (6)

\[ J_{est}(\pi) = -2H_0 + I/(2\pi a_{eff}) \]  \hspace{1cm} (7)

In this way, the estimated top and bottom current densities, \( J_{est}(0) \) and \( J_{est}(\pi) \), as predicted by the stick model can be compared with the measured top and bottom current densities, \( J(0) \) and \( J(\pi) \).

Figures 1 through 5 use E-4 scale-model measurements which were made at the University of Michigan between August 1975 and March 1977 (ref. 3). The measurement location is STA600, roughly midway on the forward fuselage. The effective radius used for comparison is 3.5 meters. Figures 1 and 2 are tests of equation (5) in magnitude and phase. Figure 3 is a comparison of measured and predicted total current using equation (3), while figures 4 and 5 are comparisons of measured and predicted current density using equations (6) and (7).

Figures 6 through 10 are analogous to figures 1 through 5, using EC-135 scale-model measurements which were made at the University of Michigan in 1977 (ref. 3). The measurement location is STA550, again about midway on the forward fuselage. The effective radius is 2.1 meters. The EC-135 models were equipped with model high frequency (HF) antennas, which appreciably affected the measurements at STA550 only in a narrow frequency band near 3.5 MHz (full scale). The resonance region of the model HF antennas can be clearly seen in figures 6 through 10.
Figure 1. Comparison of Measured Data for E-4 with Magnetostatic Solution.
Figure 2. Comparison of Measured Data for E-4 with Magnetostatic Solution.
Figure 3. Comparison of Total Current on E-4 Forward Fuselage.
Figure 4. Comparison of Current Density on E-4 Forward Fuselage (Top).
Figure 5. Comparison of Current Density on E-4 Forward Fuselage (Bottom).
Figure 6. Comparison of Measured Data for EC-135 with Magnetostatic Solution.
Figure 7. Comparison of Measured Data for EC-135 with Magnetostatic Solution.
Figure 8. Comparison of Total Current on EC-135 Forward Fuselage.
Figure 9. Comparison of Current Density on EC-135 Forward Fuselage (Top).
Figure 10. Comparison of Current Density on EC-135 Forward Fuselage (Bottom).
SECTION IV

CONCLUSION

Several features of interest are evident in figures 1 through 10. The measured data for the E-4 deviates appreciably from the magnetostatic prediction (figures 1 and 2), while the data for the EC-135 are in closer agreement (figures 6 and 7). Consequently, the stick-model predictions for the E-4 (figures 3 through 5) are not as close to the measured data as the predictions for the EC-135 (figures 8 through 10). There are basically two reasons for this. The E-4 forward fuselage is not as nearly a circular cylinder as is the EC-135 forward fuselage. In addition, the E-4 data exhibit wide variation for different sized scale models, in the frequency regions which overlap. The differences between the stick-model predictions and measured data for the E-4 are not very much greater than the differences between different E-4 scale models. The EC-135 data show greater consistency between different frequency regions and a closer resemblance to stick-model predictions.

There are also common points of difference between the scale-model measurements and the stick-model predictions for both aircraft. The values of the first two resonance frequencies agree reasonably well, but the predicted values of the peaks are higher than the measured values, especially for the second resonance, while the results between resonance frequencies are lower than the measured values. After the second resonance, both the E-4 and EC-135 predictions become less similar to the measured values.

Although the stick-model predictions are obviously not identical to the scale-model measurements, it is encouraging that representing the aircraft by only seven parameters ($l_1$ through $l_6$ and $\Omega$) can give results that are generally very reasonable. The stick-model expressions for the total current (and charge density) are simple enough to be evaluated on a programmable desk calculator (which was done to obtain the stick-model predictions used here), and are readily transformed analytically to the time domain in case time-domain results are required. Since scale-model results are not available in the frequency region below 1 MHz, which constitutes an important part of
the spectrum of a typical EMP, the stick-model predictions provide a valuable supplement to the data. As long as the spectrum of the EMP does not have important components beyond the second aircraft resonance, which is the case for a typical EMP and typical aircraft, the time domain stick-model prediction can be expected to be a good approximation of the transient response.
REFERENCES


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4. Liepa, V. V., "Surface Field Measurements on Scale Model E-4 Aircraft," Interaction Application Memo 17, March 1977