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BOUNDING SIGNAL LEVELS AT WIRE TERMINATIONS BEHIND APERTURES

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ABSTRACT

This report develops techniques for bounding the voltages and currents at terminations on a wire which is excited by incident electromagnetic energy coupled through an aperture. The theory of aperture coupling for low frequencies is reviewed and the quasistatic aperture problem is modeled by dipole moments and the corresponding polarizabilities. Bounding methods are considered and the bound of a circumscribing ellipse is chosen. The interaction with a wire and modifications to the coupling are developed using spatial approximations. The analysis identifies a new capacitive term in the aperture loading. Bounds are developed for the power waves launched on the wire structure and the termination signal levels bound with a term included for multiple reflections. Tighter bounds are obtained by separating the incident field into individual parts typically characterized by poles in the complex frequency domain.

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I. INTRODUCTION:

The past two decades have involved a substantial effort in the analysis and measurement of electromagnetic pulse (EMP) effects. The interest has centered on the electromagnetic effects resulting from nuclear explosions, particularly high altitude blasts, with a lesser effort in lightening strike effects. The Air Force is primarily interested in the survivability of aircraft and weapon systems when exposed to an electromagnetic pulse environment.

An EMP problem is typically broken into three separate problems: external interaction, coupling, and internal interaction. Since coupling effects are often minimal, it is not uncommon to neglect the coupling effect on external interaction and also to neglect the internal interaction effect on coupling. The knowledge of currents coupled to equipment inside of aircraft due to incident electromagnetic energy is of vital importance to assessing the survivability of aircraft exposed to high level electromagnetic energy. The coupling problem is further broken into three classes: direct coupling to antennas and similar structures, aperture coupling, and diffusion through the skin. The objective of this investigation was to develop a method for obtaining bounds on the signal levels at terminations resulting from energy coupled to a wire behind an aperture. A more difinitive statement of the objectives is given in Section II.

Numerous authors have contributed a wealth of information to the subject of interaction and coupling of electromagnetic energy with structures. The external interaction problem often involves computation or measurement of the currents and charges on an approximate structure geometry. [1,2] This often involves simple stick or pipe models of aircraft. [3] The resultant current and charge is the short-circuit current and charge used in the aperture coupling problem. The theory of coupling by small apertures [4-7] is reviewed in Section III. The intent was to provide a complete development with standard notation for a small aperture in a plane. The well-known solutions for the circle and ellipse are given which provide the basis for bounds obtained in the following sections.

Section IV provides the development for the interaction of a wire with the aperture. For a thin wire at least one aperture dimension from the aperture, it is shown that the interaction can be modeled by a voltage and a current source on a transmission line. A new capacitive term arising from the discharge of the aperture region by outgoing currents has been identified for a wire close to the aperture. Comparison of the sources is made to those obtained for aperture coupling through a coaxial sheath. This comparison suggests the planar problem bounds above the coupling to a wire by an aperture in a shield given the same short circuit aperture fields from the external interaction problem.

Using the planar problem as a bound for aperture coupling, bounds are developed in Section V for the planar problem and thus for shield problems. The planar bounds involve two steps: First, current methods of bounding the aperture polarizabilities are reviewed and a bound for the aperture polarizabilities is given. Secondly, bounds for signals launched on the wire structure are developed along with bounds for the resultant currents and voltages. The results are generalized with the power wave concept to account for geometrical variations along the wire. These bounds are applied to a simple problem in Section VI. The results of the example suggest the separation of transient incident fields into several pieces as might be characterized by a series of complex exponentials; summing the bounds obtained for each separate part of the incident field. Measured data are also considered.

Section VII summarizes the results and provides recommendations for further work.

II. OBJECTIVES:

The prime objective of this project was to develop a technique for bounding above the magnitudes of currents and voltages in terminations of wires. In particular, these wires are located behind an aperture in a perfectly electric conducting plane. The following sub-objectives were chosen to accomplish the task:

(1) To review quasi-static theory for coupling by small apertures.

(2) To bound the aperture polarizabilities above.

(3) To develop the spatial approximation theory for aperture coupling to a thin wire.

(4) To develop upper bounds for the power or voltage waves launched on the wire and for the resultant voltages and currents at the terminations.

Due to the nature of EMP, the objectives were restricted to small apertures. To make the problem tractable for the given time period, the wire was assumed to be thin and to be sufficently far from the aperture for the aperture polarizability approximation to be valid. The sponsor expressed minimal concern for cavity effects, resulting in its exclusion from the theoretical treatment of such as part of the task.

III. APERTURE COUPLING:

Coupling through an aperture in an infinite plane is generally cast in the context of a diffraction problem. To determine the coupled or diffracted fields, we must determine the perturbed fields in the aperture and on the plane from which the diffracted fields may be computed. For the problem at hand, the computation may be limited to the electric field intensity in the aperture of the plane by image theory or an appropriate dyadic Green's function. The diffracted fields of a small aperture are typically computed from two equivalent dipoles which approximate the aperture field expansion. [8]

Following the development of Butler, et al [7], we consider the aperture shown in Fig. 1 cut in a perfect electric conductor (PEC) with sources on both sides of the plane. We write the total fields

$$\bar{E}^{\pm} = \bar{E}^{SC\pm} + \bar{E}^{D\pm} \tag{1}$$

and

$$\bar{H}^{\pm} = \bar{H}^{SC\pm} + \bar{H}^{D\pm} \tag{2}$$

where $\pm$ designates $z \gtrless 0$, SC designates the fields with the aperture A shorted, and D designates the diffracted fields. Incorporating image theory in the basic boundary value expressions for the electromagnetic fields we may write [9] ($e^{j\omega t}$ convention)

$$\bar{E}^{D\pm} = \pm 2 \int_A [\nabla' G(\bar{r},\bar{r}') \times (-\hat{z} \times \bar{E}(\bar{r}))] \, ds' \tag{3}$$

and

$$\bar{H}^{D\pm} = \pm \frac{2}{j\omega\mu} \int_A [k^2 G(-\hat{z} \times \bar{E}) - \nabla_s' \cdot (-\hat{z} \times \bar{E}) \, \nabla' G] \, ds' \tag{4}$$

where G is the free space scalar Green's function

$$G(\bar{r},\bar{r}') = \frac{e^{-jk|\bar{r}-\bar{r}'|}}{4\pi|\bar{r}-\bar{r}'|} \tag{5}$$

with $k = \omega\sqrt{\mu\varepsilon}$. Since $\hat{z} \times \bar{E}^{SC\pm} = 0$ at the PEC, $\bar{E}$ may be replaced by $\bar{E}^{D+}$ or $\bar{E}^{D-}$ in the integrals of (3) and (4) if desired.

The boundary conditions at the aperture require the continuity of

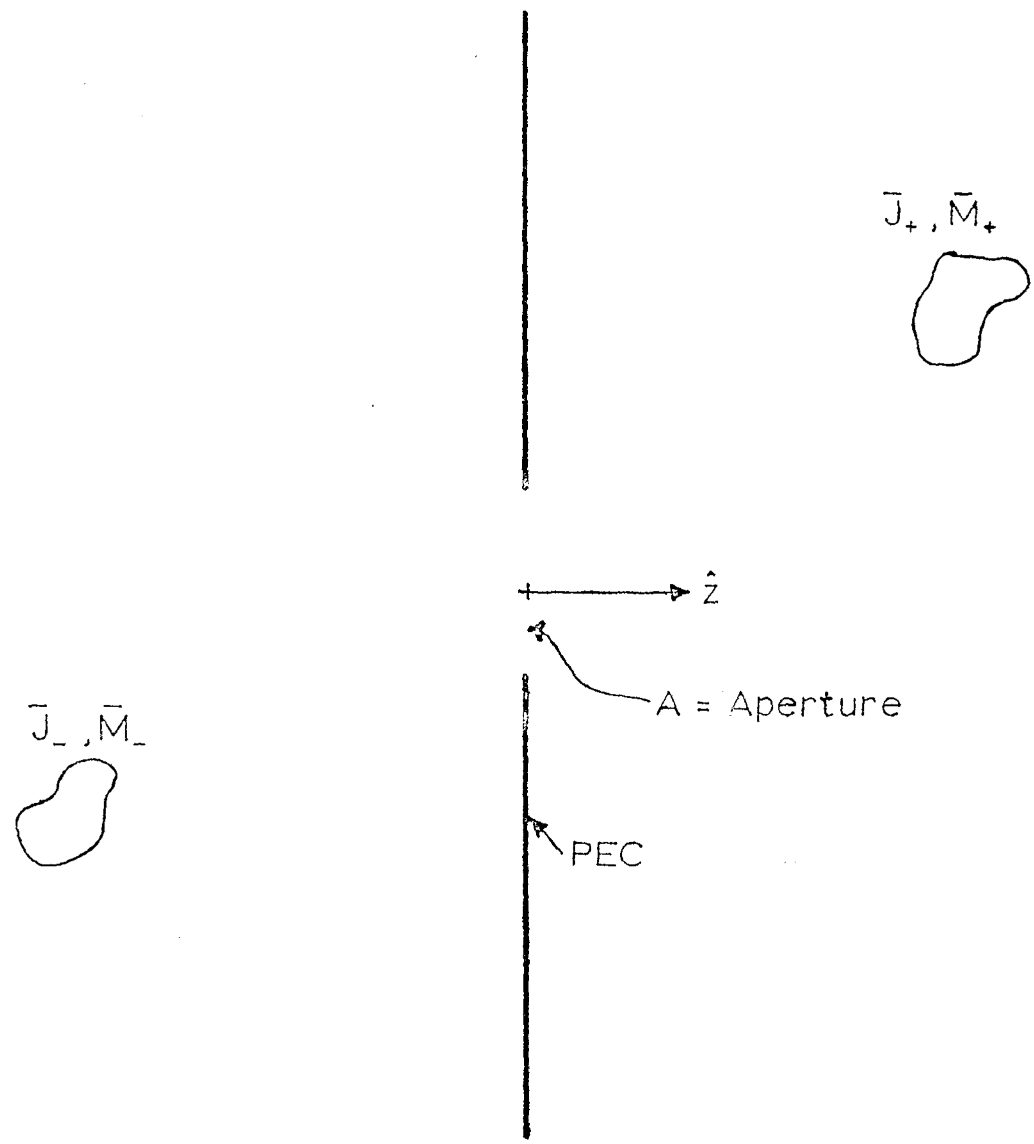


Figure 1. Source locations for aperture coupling through a planar perfect electric conductor.

$\bar{E}$ and $\bar{H}$. Taking the limits of (3) and (4) from both the left and right and imposing field continuity in the aperture we obtain

$$\hat{z} \cdot (\bar{E}^{SC-} - \bar{E}^{SC+}) = 4\hat{z} \cdot \fint_A [\nabla' G \times \bar{M}] \, ds' \tag{6}$$

and

$$\hat{z} \times (\bar{H}^{SC-} - \bar{H}^{SC+}) = \frac{4}{j\omega\mu} \hat{z} \times \fint_A [k^2 G\bar{M} - (\nabla' \cdot \bar{M}) \nabla' G] \, ds' \tag{7}$$

where

$$\bar{M} = -\hat{z} \times \bar{E}(\bar{r}), \quad \bar{r} \in A$$

is the equivalent magnetic surface current in A, the – through the integral denotes the Cauchy principal value, and the surface ∇'_s is replaced by ∇' without ambiguity. Since the tangential components of $\bar{E}^{SC\pm}$ and the normal components of $\bar{H}^{SC\pm}$ are zero by interaction with the PEC, the tangential $\bar{E}$ and normal $\bar{H}$ continuity of Eqs. (1) thru (4) are automatically satisfied.

Two observations may be make about (6) and (7) as they stand. First we note that (6) may be obtained from the divergence of (7), implying that (7) is sufficient for non-zero frequencies. Second, due to the existence of edges with an aperture, we must impose an edge constraint on $\bar{M}$. From energy considerations this constraint may be written in two parts as

$$\hat{n}_e \cdot \bar{M} = O(\rho^{1/2}), \quad \rho \to 0 \tag{8a}$$

and

$$\hat{n}_e \times \bar{M} = O(\rho^{-1/2}), \quad \rho \to 0 \tag{8b}$$

for $\hat{n}_e$ the normal to the edge in the plane, ρ the distance to the edge, and $O(x)$ read order of x. Eq. (7) has been solved numerically by Graves, et al [10]. A modified form of these equations for the dual problem of the disk were proposed by Mittra, et al [11], and subsequently solved numerically by Rahmat-Samii [12] for apertures up to about three wavelength dimensions. However, our interest is in expanding (6) and (7) for low frequency coupling rather than obtain a complete current description.

Lord Rayleigh [13] proposed the use of a series in k to obtain equations for the dominant quasi-static terms in aperture coupling. Bouwkamp [4] used this idea to solve some canonical apertures analytically in the low frequency region. More recently, De Meulenaere and Van Bladel [14] have treated the quasi-static problem of several shapes numerically. We now proceed with a review of this quasi-static theory.

Using a Rayliegh series, we expand all field quantities $\bar{F}(\bar{r},k)$ as

$$\bar{F} = \bar{F}_0 + jk\bar{F}_1 + \ldots + (jk)^n\bar{F}_n + \ldots \tag{9}$$

where we have assumed analytic fields in k for the low frequency region. We also expand the free space Green's function as

$$G(\bar{r},\bar{r}') = G_0 + jkG_1 + \ldots + (jk)^n G_n + \ldots \tag{10}$$

where $G_0 = 1/4\pi R$, the static free space Green's function, and $R = |r-r'|$. Substituting (9) and (10) into (6) and (7) we obtain

$$\hat{z} \cdot (\bar{E}_N^{SC-} - \bar{E}_N^{SC+}) = 4\hat{z} \cdot \sum_{n=0}^{N} \int_A [\nabla' G_n \times \bar{M}_{N-n}]\, ds' \tag{11}$$

and

$$\hat{z} \times (\bar{H}_N^{SC-} - \bar{H}_N^{SC+}) = \frac{4}{\eta}\, \hat{z} \times \sum_{n=0}^{N+1} \int_A [-G_n \bar{M}_{N-n-1} - (\nabla' \cdot \bar{M}_{N+1-n})\, \nabla' G_n] ds' \tag{12}$$

where $\bar{M}_{-1} = \bar{M}_{-2} = 0$, $\eta = \sqrt{\mu/\varepsilon}$, and

$$\int_A [\nabla' G_0\, (\nabla' \cdot \bar{M}_0)]\, ds' = 0. \tag{13}$$

Integrating (13) we have

$$\int_A [G_0(\nabla' \cdot \bar{M}_0)]\, ds' = A = \text{constant} \tag{14}$$

with a solution [12] for a circle of radius a given by

$$\nabla' \cdot \bar{M}_0 = A/(\pi\sqrt{a^2 - \rho'^2}). \tag{15}$$

The surface integral of (15) is

$$2\pi a M_{0\rho}(a) = 2aA$$

or

$$A = \pi M_{0\rho}(a)$$

which requires A to be zero upon imposing the edge constrint of (8a). Assuming that we may generalize this result for the circle to arbitrary apertures, we take the solution to (13) to be

$$\nabla' \cdot \bar{M}_0 = 0. \tag{16}$$

It is interesting to note that if (13) had been interpreted as a finite part integral, the solution in (15) would have contained terms $O[(a - \rho')^{-n/2}]$ corresponding to the multiplicity of solutions to Maxwell's equations in the vicinity of an edge with no physical edge constraints imposed [15].

It is straight forward to show that the divergence of (12) for N gives (11) for N-1. However our general interest is in N=0 for both (11) and (12) to solve for $\bar{M}_0$ and the divergence of $\bar{M}_1$ or m_0, the zeroth order magnetic charge. The use of (12) for N=1 would provide the further complication of adding the unknown divergence of $\bar{M}_2$ and still require the solution of (11).

To summarize, the quasi-static equations for the magnetic current and charge in the aperture are

$$(E_{0z}^{SC+} - E_{0z}^{SC-}) = 4\hat{z} \cdot \nabla \times \int_A \bar{M}_0 G_0 \, ds', \tag{17}$$

$$\hat{z} \times (\bar{H}_0^{SC+} - \bar{H}_0^{SC-}) = \frac{4}{\mu} \hat{z} \times \nabla \int_A m_0 G_0 \, ds', \tag{18}$$

and

$$\nabla' \cdot \bar{M}_0 = 0 \tag{16}$$

where m_0 is given by $(-\nabla' \cdot \bar{M}_1)\sqrt{\mu\varepsilon}$. The importance of using both $\bar{M}_0$ and m_0 may be observed by rewriting (3) and (4) as

$$\bar{E}^{D\pm} = \pm 2\nabla \times (-\int_A \bar{M}G \, ds') \tag{19}$$

and

$$\bar{H}^{D\pm} = \pm 2 \left[-\frac{jk}{\eta} \int_A \bar{M}G \, ds' - \frac{1}{\mu} \nabla \int_A mG \, ds'\right] \tag{20}$$

where $\eta = \sqrt{\mu/\varepsilon}$. The near electric fields are dominated by the zero order magnetic current whereas the near magnetic fields are dominated by the zero order magnetic charge requiring both terms for distances less than one wavelength. This requirement can also be shown to be valid in the far field of the aperture (distances greater than one wavelength).

It is more common to describe the fields in the distant region of the aperture (distances > maximum aperture dimension) which extends the far field expansion into the near field region. From (19) we define the magnetic vector potential by

$$\bar{F}(\bar{r}) = \int_A \bar{M}(\bar{r}')G(\bar{r},\bar{r}')\, ds'. \tag{21}$$

For $r \gg r'$, we may expand G to obtain

$$G(\bar{r},\bar{r}') \sim G(\bar{r},0)\ [1 + (\hat{r}\cdot\bar{r}')\,\frac{1 + jkr}{r}\,].$$

Substituting into (21) we have

$$\bar{F}(\bar{r}) \sim G(\bar{r},0) \int_A \bar{M}(\bar{r}')\ [1 + (\hat{r}\cdot\bar{r}')\,\frac{1 + jkr}{r}\,]\, ds'. \tag{22}$$

The first integral is given by

$$\int_A \bar{M}\, ds' = \int_A [\bar{M} - \nabla' \cdot (\bar{M}\bar{r}')]\, ds'$$

using the constraint (8a) and the surface form of the divergence theorem. Expanding the dyadic divergence we obtain

$$\begin{aligned} \int_A \bar{M}\, ds' &= - \int_A \bar{r}'(\nabla' \cdot \bar{M})\, ds' \\ &= j\omega \int_A \bar{r}'m\, ds'. \end{aligned} \tag{23}$$

The second integral may be expanded as

$$\int_A (\hat{r}\cdot\bar{r}')\bar{M}\, ds' = \frac{1}{2}\int_A \{\hat{r} \times (\bar{M} \times \bar{r}') + [(\hat{r}\cdot\bar{r}')\bar{M} + \bar{r}'(\hat{r}\cdot\bar{M})]\}\, ds'$$

The last two terms may be written in dyadic form as

$$\bar{M} \cdot \nabla'(\bar{r}'(\hat{r}\cdot\bar{r}')).$$

Subtracting the divergence of $(\bar{M}\bar{r}'(\hat{r}\cdot\bar{r}'))$ which integrates to zero due to (8a), we obtain

$$\int_A (\hat{r}\cdot\bar{r}')\bar{M}\, ds' = \frac{1}{2}\int_A [\hat{r} \times (\bar{M} \times \bar{r}') + j\omega(\hat{r}\cdot\bar{r}')\bar{r}'m]\, ds' \tag{24}$$

Substituting (23) and (24) into (22)

$$\bar{F} \sim j\omega G(\bar{r},0) \int_A \bar{r}'m\, ds'$$
$$- \nabla G(\bar{r},0) \times \frac{1}{2} \int_A (\bar{M} \times \bar{r}')\, ds'$$
$$- j\omega\nabla G(\bar{r},0) \cdot \frac{1}{2} \int_A \bar{r}'\bar{r}'m\, ds' \tag{25}$$

with the magnetic vector potential composed of a magnetic dipole, an electric dipole, and a magnetic quadrupole given respectively by

$$\bar{p}_m^{im} = \frac{1}{\mu} \int_A \bar{r}'m\, ds' \tag{26a}$$

$$\bar{p}_e^{im} = \frac{\varepsilon}{2} \int_A (\bar{M} \times \bar{r}')\, ds' \tag{26b}$$

and

$$\bar{\bar{q}}_m^{im} = \frac{1}{\mu} \int_A \bar{r}'\bar{r}'m\, ds'. \tag{26c}$$

Adding these dipole moments and their images, the diffracted field may be written

$$\bar{E}^{D\pm} = \pm[- \frac{2}{\varepsilon} \nabla \times (\bar{p}_e^{im} \times \nabla G) + 2j\omega\mu\bar{p}_m^{im} \times \nabla G + j\omega\mu\nabla \times (\bar{\bar{q}}_m^{im} \times \nabla G)] \tag{27}$$

and

$$\bar{H}^{D\pm} = \pm[- 2 \nabla \times (\bar{p}_m^{im} \times \nabla G) - 2j\omega\bar{p}_e^{im} \times \nabla G - \nabla \times \nabla \times (\bar{\bar{q}}_m^{im} \times G)]. \tag{28}$$

Usually the quadrupole term is small and therefore neglected. In fact $\bar{\bar{q}}_{m0}^{im}$ is zero for a circular aperture in the quasi-static problem.

The short-circuited fields in (17) and (18) are nearly constant for small apertures sufficiently far from the sources such that

$$\bar{p}_e^{im} = \varepsilon\alpha_e^{im}\hat{z}\hat{z} \cdot (\bar{E}^{SC-} - \bar{E}^{SC+}) \tag{29a}$$

and

$$\bar{p}_m^{im} = - \bar{\bar{\alpha}}_m^{im} \cdot (\bar{H}^{SC-} - \bar{H}^{SC+}) \tag{29b}$$

where $\bar{\bar{\alpha}}_m^{im}$ has no z components and $\bar{p}_e^{im}$ and $\bar{p}_m^{im}$ have been approximated by the zero order current and charge. The neglected quadrupole term would be represented by a triad term if it were to be used. The quantities α_e^{im} and $\bar{\bar{\alpha}}_m^{im}$ are called the aperture electric and magnetic polarizabilities respectively.

Equations (16) to (18) have been solved analytically for the circle and ellipse [4,6,16] with the resultant polarizabilities of Table I. K and E represent the complete elliptic integrals of the first and second kind respectively and the ellipse major axis of length ℓ is along x. The polarizabilities of several other shapes have been obtained numerically by De Meulenaere and Van Bladel [14].

To use these results for aperture coupling into aircraft cavities, one obviously becomes concerned with the effects of nearby conductors and surface curvature on the coupling dipole moments. Latham [17] has shown that surface radii of curvature and distance to nearby conductors of at least the linear dimensions of the aperture cause less that one percent variation in the aperture polarizabilities. I defer the discussion on the dipole moment effects to Section IV.

Table I

APERTURE POLARIZABILITIES

Shape	α_{e}^{im}	α_{mxx}^{im}	α_{myy}^{im}
Circle (a = radius)	$\frac{2}{3}a^3$	$\frac{4}{3}a^3$	$\frac{4}{3}a^3$
Ellipse (e = eccentricity)	$\frac{\pi\ell^3(1-e^2)}{3E(e)}$	$\frac{\pi\ell^3e^2}{3[K(e)-E(e)]}$	$\frac{\pi\ell^3e^2}{3[E(e)/(1-e\ )-K(e)]}$
Narrow Ellipse ($e \simeq 1$)	$\frac{\pi\ell^3(1-e^2)}{3}$	$\frac{2\pi\ell^3}{3[\ell n(16/(1-e^2))-2]}$	$\frac{\pi\ell^3(1-e^2)}{3}$

$$e = \left[1 - \left(\frac{\text{major axis}}{\text{minor axis}}\right)^2\right]^{\frac{1}{2}}$$

IV. WIRE BEHIND APERTURE:

Coupling to a wire behind an aperture has been treated in a variety of ways. Kajfez [18] derived the equivalent sources using both mode-matching and reciprocity, but neglected the loading effects of the aperture on the wire. Lee and Yang [19] developed the same sources using transform approximations and added the effects of the loading in order to discuss the effects of a wire close to the aperture. The problem has also been cast in numerical form by Butler and Umashankar [20]. The following development is spatially equivalent to the transform method of Lee and Yang, but identifies an additional capacitance not included in the previous development. Under the problem constraints, it will be shown that only the sources are needed and are equivalent to those obtained by Kajfez. If the wire approaches the aperture, then the lumped elements must be included and the new capacitance becomes of importance.

The geometry of interest is shown in Fig. 2 with the constraints as follows: the wire of radius a is considered to be thin and the distance ρ_0 is greater than or equal to the maximum aperture dimension. Two simultaneous boundary value problems are involved in this development. The aperture problem may be considered solved in terms of the polarizabilities of the aperture once the short circuit fields of the incident field and wire have been computed. The remaining problem is to determine the wire current from the dipole moments by requiring the electric field on the wire to be zero.

The electric field along the wire due to the aperture is obtained from (27) and (28) as

$$E_x^A = 2\left[\frac{p_e^{im}}{\varepsilon}\frac{\partial^2 G}{\partial x \partial z} + j\omega\mu p_{my}^{im}\frac{\partial G}{\partial z}\right]. \tag{30}$$

The electric field due to the x-directed wire current and its image is

$$E_x^W = \frac{1}{j\omega\varepsilon}\left(k^2 + \frac{\partial^2}{\partial x^2}\right)\int_{-\infty}^{\infty} I(x')\left[\frac{e^{-jk\sqrt{(x-x')^2+a^2}}}{4\pi\sqrt{(x-x')^2+a^2}} - \frac{e^{-jk\sqrt{(x-x')^2+4d^2}}}{4\pi\sqrt{(x-x')^2+4d^2}}\right]dx'. \tag{31}$$

Since the kernel of (31) is approximately zero for $|x-x'| > 2d$, we integrate (31) as though $I(x')$ were constant to obtain

$$E_x^W \simeq \frac{Z_0}{jk}\left(k^2 + \frac{d^2}{dx^2}\right) I(x) \tag{32}$$

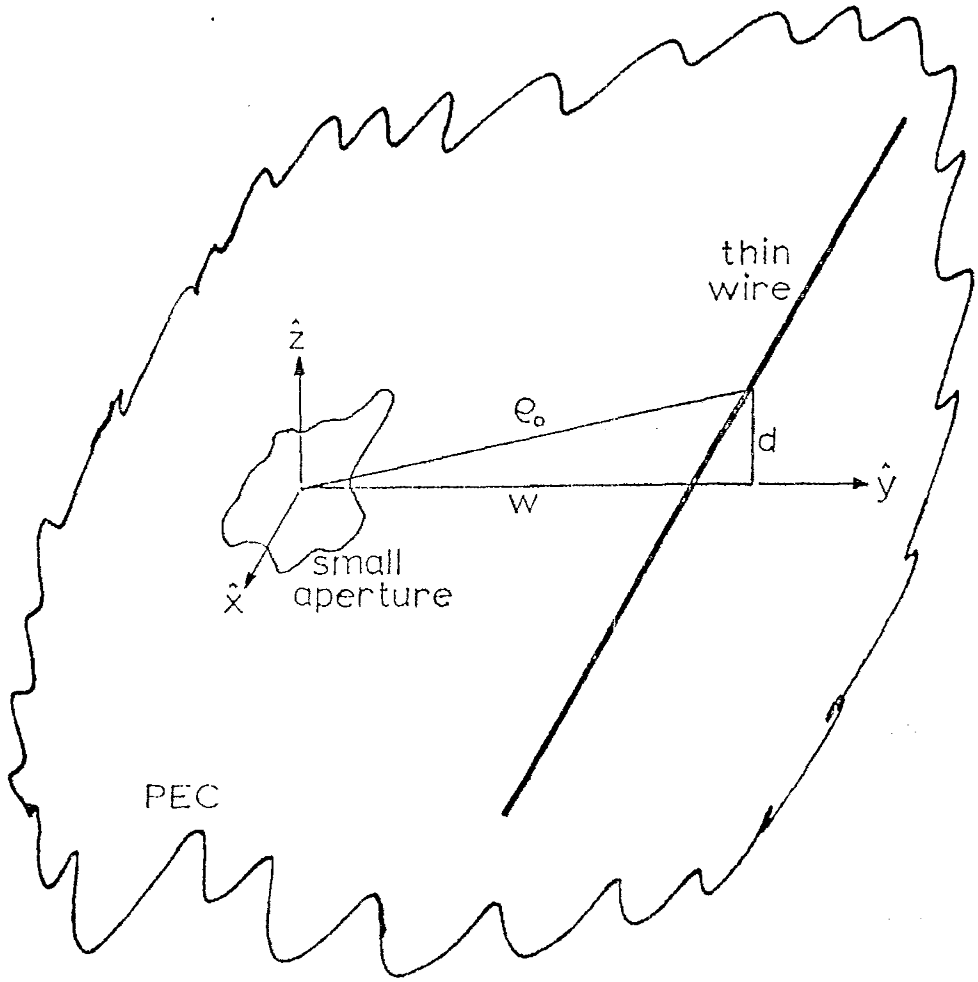


Figure 2. Wire geometry for aperture coupling through a planar perfect electric conductor.

where the impedance of a thin wire over a conducting plane Z_0 given by $\eta\ \ln(2d/a)/2\pi$ has been used.

To obtain the current, we set (32) equal to the negative of (30) and invert the differential operator to obtain

$$I(x) = \frac{1}{Z_0}\int_{-\infty}^{\infty} e^{-jk|x-x'|}\ [p_e^{im}/\varepsilon\ \frac{\partial^2 G}{\partial x'\partial z'} + j\omega\mu p_{my}^{im}\frac{\partial G}{\partial z'}]\ dx'$$

$$+ A\ e^{-jkx} + B\ e^{jkx}$$

which may be approximated due to the peaked nature of $\frac{\partial G}{\partial z'}$ by

$$I(x) \approx \frac{e^{-jk|x|}}{Z_0}\ [jkp_e^{im}/\varepsilon\ \mathrm{sgn}(x) - j\omega\mu p_{my}^{im}]\ (\frac{d}{2\pi\rho_0^2})$$

$$+ A\ e^{-jkx} + B\ e^{jkx}$$

$$= A\ e^{-jkx} + B\ e^{jkx} + C\mathrm{sgn}(x)e^{-jk|x|} + D\ e^{-jk|x|} \tag{33}$$

Computing the average short circuit fields in the aperture we obtain

$$p_{my}^{im} = -[\hat{y}\cdot\bar{\bar{\alpha}}_m^{im}\cdot\bar{H}^{SC-} - \frac{d\alpha_{myy}^{im}}{\pi\rho_0^2}\ (A + B + D)] \tag{34a}$$

and

$$p_e^{im} = \varepsilon\alpha_e^{im}[\hat{z}\cdot\bar{E}^{SC-} + \frac{\eta\ d}{\pi\rho_0^2}\ (A - B + C - \frac{C}{jk\rho_0})]. \tag{34b}$$

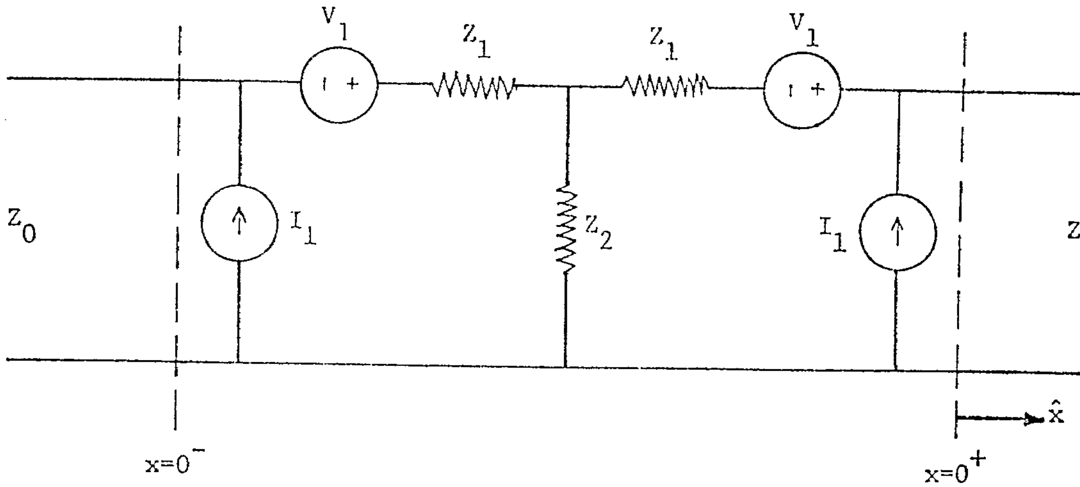
The last term in (34b) accounting for the capacitive discharge of the aperture region by the outgoing current was not obtained by Lee and Yang. As ρ_0 becomes small, this new capacitive term will dominate the capacitance of the aperture region. Substituting (34) into (33) and solving for C and D we obtain

$$C = j\omega\varepsilon\alpha_e^{im}\frac{\eta}{Z_0}\ (\frac{d}{2\pi\rho_0^2})\ \frac{[(\hat{z}\cdot\bar{E}^{SC-}) + \eta(\frac{d}{\pi\rho_0^2})(A-B)]}{1 + \frac{jk\eta\alpha_e^{im}}{2Z_0}\ (\frac{d}{\pi\rho_0^2})^2(\frac{1}{jk\rho_0} - 1)} \tag{35a}$$

and

$$D = \frac{j\omega\mu}{Z_0}\ (\frac{d}{2\pi\rho_0^2})\ \frac{[\hat{y}\cdot\bar{\bar{\alpha}}_m^{im}\bar{H}^{SC-} - \alpha_{myy}^{im}(\frac{d}{\pi\rho_0^2})(A + B)]}{1 + \frac{jk\eta\alpha_{myy}^{im}}{2Z_0}\ (\frac{d}{\pi\rho_0^2})^2}. \tag{35b}$$

We may model the equations of (33) and (35) by the transmission line model of Fig. 3. For the dimensional constraints chosen, it is



$$V_1 = j\omega\mu \left(\frac{d}{2\pi\rho_0^2}\right) \left(\hat{y} \cdot \bar{\bar{\alpha}}_m^{im} \cdot \bar{H}^{SC-}\right)$$

$$I_1 = j\omega\varepsilon \frac{\eta}{Z_0} \left(\frac{d}{2\pi\rho_0^2}\right) \alpha_e^{im} \left(\hat{z} \cdot \bar{E}^{SC-}\right) \left[1 + \frac{\eta\alpha_e^{im}}{2Z_0\rho_0} \left(\frac{d}{\pi\rho_0^2}\right)^2\right]^{-1}$$

$$Z_1 = j\omega L_1 = j\omega \left[\mu \frac{\alpha_{\mu yy}^{im}}{2} \left(\frac{d}{\pi\rho_0^2}\right)^2\right]$$

$$Z_2 = 1/j\omega C_2 + j\omega L_2$$

$$= - \frac{Z_0}{j\omega\varepsilon\eta} \left(\frac{1}{2\rho_0} + \frac{Z_0}{\eta\alpha_e^{im}\left(\frac{d}{\pi\rho_0^2}\right)^2}\right) - j\omega L_1/2$$

Figure 3. General source and impedance model for the aperture region of a wire behind a planar conductor.

easily shown that Z_1 and Z_2 may be neglected compared to Z_0 as may the braket in the I_1 expression. What remains is the two source model of Kajfez [18]. We observe that as long as the conductor is at least an aperture dimension away, the dipole moments depend only on the exterior short-circuited fields. We would suspect this to also be true for a coax.

We may obtain the sources for a coax given by Latham [17] simply by replacing Z_0 by the coax impedance $\eta\ \ell n(b/a)/2\pi$ [21] and $(2d/\rho_0^2)$ by $1/b$ where b is the radius of the coaxial outer sheath. From the general form of Latham and the results presented, one might hypothosize that V_1 and I_1 may be obtained for any concave geometry by letting Z_0 be the line impedance and $d/\pi\rho_0^2$ be replaced by the ratio of the short-circuit aperture current density to the total current on the line. For b=d, the coaxial sources are approximately one-half those for the planar case. This is reasonable in the sense that the more confined coaxial region causes fewer coupled magnetic field lines to cut the wire and fewer coupled electric flux lines to interact with the line charge. This suggests a bound similar to that of Harrison [22] for the exterior problem. It is reasonable to claim that in the time domain, the signals coupled to wires behind apertures irrepective of cavities are bound by the signals coupled to a wire behind an aperture in a plane given the same dipole moments determined from the exterior interaction problem.

Taking the planar problem as bounding aperture coupling, we model the equivalent transmission problem as shown in Fig. 4. In the Laplacian frequency domain we may write the voltage and current at Z_4 as

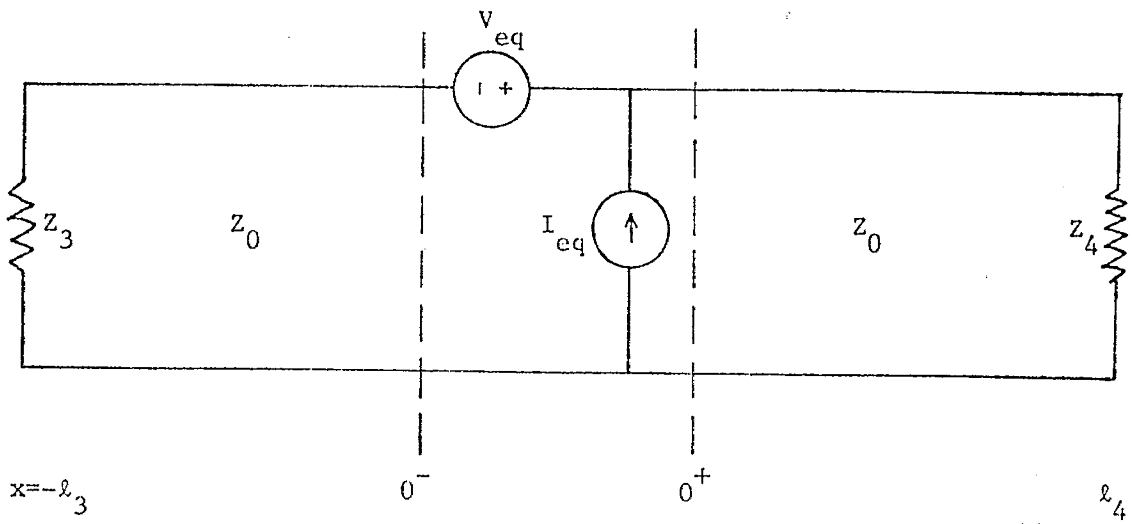
$$V_4 = \frac{-e^{-s\tau_4}}{1-\Gamma_3\Gamma_4\ e^{-2s\tau_T}} \left[(I_{eq}Z_0+V_{eq}) + \Gamma_3(I_{eq}Z_0-V_{eq})e^{-2s\tau_3}\right]\frac{Z_4}{Z_0+Z_4} \qquad (36)$$

and

$$I_4 = V_4/Z_4 \qquad (37)$$

where τ_3, τ_4, and τ_T represent the time delays of ℓ_3/c, ℓ_4/c, and $(\ell_3+\ell_4)/c$ respectively (c - the speed of light). These equations will be the basis for our discussion of the bounds in the next section.

Before we proceed with the bounding problem, it is interesting to



$$V_{eq} = j\omega\mu b\,(\hat{y} \cdot \bar{\bar{\alpha}}^{im}_{m} \cdot \bar{H}^{SC-})$$

$$Z_0 I_{eq} = j\omega\varepsilon\eta b\,\alpha_e^{im}(\hat{z} \cdot \bar{E}^{SC-})$$

$$Z_0 = \eta \ln(2d/a)/2\pi$$

$$b = \frac{J^{SC}_{Aperture}}{I(0)} = \frac{d}{\pi\rho_0^2}$$

Figure 4. Simplified transmission line model for aperture coupling to a wire behind a planar conductor.

note some power relationships for aperture coupling. The average power transmitted by a small aperture to a half-space is given by

$$P_{\frac{1}{2}} = \frac{k^4}{6\pi\eta} \left[\left|\frac{p_e^{im}}{\varepsilon}\right|^2 + \left|\eta p_m^{im}\right|^2 \right].$$

The average power launched on the wire structure is

$$P_W = \frac{k^2}{Z_0} \left(\frac{d}{\pi\rho_0^2}\right) \left[\left|\frac{p_e^{im}}{\varepsilon}\right|^2 + \left|\eta p_{my}^{im}\right|^2 \right].$$

Neglecting p_{mx}^{im}, which is unrelated to the wire problem, we have

$$P_W/P_{\frac{1}{2}} = \frac{3}{\pi^2 \, \ell n(2d/a)} \left(\frac{\lambda d}{\rho_0^2}\right)^2 .$$

For many problems of interest, d is on the order of ρ_0. Since ρ_0 has been required to be much less than λ, one wavelength, the power launched on the wire is much greater than that transmitted into the half-space. This increase in power is due to the higher field strength in the region of the aperture due to the presence of the wire. This is easier to visualize for the coaxial structure in which power is coupled to the TEM mode of the coax. Without the wire, the structure is simply a cutoff waveguide.

V. BOUNDS:

In the previous sections we have developed the relations for low frequency aperture coupling to a wire. We have suggested that such coupling may be bounded above by a wire behind an aperture in a plane. This suggestion was based on comparison of the planar and coaxial problems and the physical mechanisms bounding the coaxial problem above by the planar problem. Hence, the first step in bounding the signals at terminations is to replace the given wire structure with a wire situated behind an aperture in a planar perfect electric conductor.

The second step is to bound the aperture polarizabilities and thus the sources in the transmission line model. For circles and ellipses, no bounds are required since exact formulae are available. However, other structures, which in general must be treated numerically, are not readily amenable to analysis and thus suggest the use of a bound.

Fikhmanas and Fridberg [23] have developed variational methods for bounding the polarizabilities. Unfortunately these methods still require a fair amount of computation. Papas and Jaggard [24,25,26] have considered bounds which depend only on the area and perimeter of the aperture and are based on symmetrization of isoperimetric variational analysis. The primary disadvantage of their results is exclusion of the aperture eccentricity which causes the magnetic polarizabilities not to be bounded as given, but which requires an averaging of the components of the magnetic polarizability. In fact, the bounds that are presented may be classed more as estimates since they do reasonably estimate the polarizabilities in many cases and do not bound the polarizabilities in several other cases.

Since our prime interest is in an absolute upper bound, I suggest that the aperture of interest be bounded by an elliptical aperture of minimum area circumscribing the given aperture. This bound would exceed that of constant perimeter for concave objects as is suggested by Papas, [24], but is less than the bound of Papas for many convex objects. Comparing this bound to the results of De Meulenaere and Van Bladel [14] for the polarizabilities of several shapes, we find the ellipse to bound the rectangle and diamond by a factor of approximately 1.8. The rounded-off rectangle is also bounded by about 1.8 for eccentricities approaching unity and is equal to the ellipse for small eccentricity. In a similar manner, the circle bounds the polarizabilities of the cross.

In those cases when it is undesirable to compute the ellipse dimensions and bounds, a less tight bound may be obtained with a circle circumscribing the aperture. In this instance the polarizabilities are

$$\alpha_e^{im} = \frac{2}{3}\,(\text{radius})^3$$

and

$$\alpha_{mxx}^{im} = \alpha_{myy}^{im} = \frac{4}{3}\,(\text{radius})^3 .$$

These polarizabilities and those of an ellipse with an x-directed major axis are tabulated in Table I of Section III.

Having bound the original problem by the planar problem and developed bounds for the aperture polarizability, we may complete the problem by determining the termination signal levels from the model of Fig. 4. The appropriate equations for this problem are (36) and (37). In many instances the geometry is known, but Z_3 and Z_4 are not. Let us consider several cases of interest. If the terminations are matched, then Γ_3 and Γ_4 are zero and V_4 and I_4 are

$$\begin{aligned}|V_4| &= |I_{eq}Z_0 + V_{eq}| \;/\; 2 \\ &\le (|I_{eq}Z_0| + |V_{eq}|) \;/\; 2\end{aligned}$$

and

$$|I_4| = |V_4| \;/\; Z_0$$

where we neglect phase cancellations that may occuer between I_{eq} and V_{eq} represented by the previously discussed bounds.

Absolute bounds may be obtained by considering open and short circuit terminations along with a multiple reflection decay constant. In the time domain, (36) may be written for resistive terminations as

$$\begin{aligned}v_4(t) = -\frac{Z_4}{Z_0+Z_4}\sum_{n=0}^{\infty}\Gamma_3^n\Gamma_4^n\,[Z_0 i_{eq}(t-\tau_4-2n\tau_T)+v_{eq}(t-\tau_4-2n\tau_T) \\ +\Gamma_3 Z_0 i_{eq}(t+\tau_4-2(n+1)\tau_T)-\Gamma_3 v_{eq}(t+\tau_4-2(n+1)\tau_T)] .\end{aligned} \qquad (38)$$

The simplest bound of (38) is to neglect all multiple-reflection phase cancellations, set Z_4 to infinity and let ℓ_3 tend to zero to obtain

$$|v_4(t)| \leq \sum_{n=0}^{\infty} 2e^{-\sigma n}\max[Z_0|i_{eq}(t-2n\tau_T)|,|v_{eq}(t-2n\tau_T)|]$$

where the overall time delay has been neglected and σ represents the attenuation of the reflections and the power loss of the line if any. This may be written as a bound on $|v_4|$

$$|v_4|_{max} \leq 2\,\frac{\max[Z_0|i_{eq}|_{max},|v_{eq}|_{max}]}{1 - e^{-(\sigma+2\alpha\tau_T)}} \tag{39}$$

where α is the exponential decay bound on the sources.

To bound the current, the only modification is to let Z_3 and Z_4 be short circuits to obtain

$$|i_4|_{max} \leq |v_4|_{max} \;/\; Z_0. \tag{40}$$

In the rare case of oscillations in the exterior fields at multiple frequencies, it might possibly be required to remove the maximum operator in the bounds and replace it by a sum. We shall neglect this case here and consider an alternative in the next section. If the phase delay of Γ_3 and Γ_4 are known to combine with twice the line length delay to cancel over the frequency range of interest, the denominators of (39) and (40) may be set to unity giving the bounds in terms of the maximum time derivative of the incident signal.

Current transmission line measurement and analysis techniques make use of power waves [27]. $Z_0 i_{eq}$ and v_{eq} need only be divided by $2\sqrt{Z_0}$ to be normalized in the power wave sense. Thus we may bound the voltage and current at terminations with a local geometry characterized by Z_L as

$$|v_4|_{max} \leq 2\sqrt{\frac{Z_L}{Z_0}}\;\frac{\max[Z_0|i_{eq}|_{max},|v_{eq}|_{max}]}{1 - e^{-(\sigma+2\alpha\tau_T)}} \tag{41a}$$

and

$$|i_4|_{max} \leq |v_4|_{max} \;/\; Z_L \tag{41b}$$

where τ_T is proportional to the minimum distance between significant obstructions on the line one might encounter, such as the ribbing inside an aircraft.

VI. EXAMPLES:

To obtain a feeling for the capabilities of the developed bounds we consider the case analyzed by Kajfez [18]. He considered a circular aperture problem with the following parameters:

aperture radius	10 mm
w (y distance to wire)	20 mm
a (wire radius)	1 mm
d (wire height)	10 mm
ℓ_4	2.1 m
ℓ_3	3.0 m
ϕ } TM incident wave direction	120°
θ	90°

and an incident time behavior of a double exponential

$$F(t) = A_0 \, (e^{-\alpha t} - e^{-\beta t}) \, u(t).$$

The incident parameters used were

$$A_0 = 100 \text{ kV/m}$$
$$\alpha = 3 \times 10^6 \text{ s}^{-1}$$
$$\beta = 10^8 \text{ s}^{-1}.$$

The time domain equivalent sources may be obtained from Fig. 4 as

$$Z_0 i_{eq} = 2.001 \times 10^{-14} \times \frac{\partial F}{\partial t}$$

and

$$v_{eq} = 4.902 \times 10^{-14} \times \frac{\partial F}{\partial t}.$$

The maximum of $\frac{\partial F}{\partial t}$ occurs at t=0 and is 10^{13} V/m-s. For $Z_L = Z_0$ and the α of (41) the same as in F(t), we have

$$\begin{aligned} |v_4|_{max} &\leq .98 \; / \; (1 - .903) \text{ V} \\ &= 10.1 \text{ V}. \end{aligned} \qquad (42)$$

Kajfez chose Z_4 as 10 kΩ and Z_3 as 10Ω to obtain a peak voltage of approximately 0.5 volts occuring before reflections. Later voltages were of lesser value. Noticing that Γ_3 and Γ_4 provide phase cancellation with the line length for the frequencies of interest, we may neglect the denominator in (42) to obtain

$$|v_4|_{max} \leq .98 \text{ V}$$

which is a reasonable bound.

It may be of interest to obtain a better bound without neglecting the denominator. In the problem given, the early time behavior has a decay constant of β and not α. One method of improving the bound is to bound the response to each pole term of the incident field separately and add the results. To do this we write

$$F(t) = A_0 \, [(1 - e^{-\beta t}) - (1 - e^{-\alpha t})] \, u(t)$$

to obtain the bounds

$$|v_4^{\alpha}|_{max} = .303 \text{ V}$$

and

$$|v_4^{\beta}|_{max} = 1.014 \text{ V}$$

which sum to bound v_4 at 1.317 V. By treating the poles separately we obtain a reasonable bound and do not have to be concerned with multiple modes requiring the maximum function of (41) to be replaced by a sum in some instances as suggested in Section V.

An interesting observation may be made by comparison with the measured data of Lin, et al [28], for a circular aperture of radius 18 in. with a wire centered 3.5 in. behind the aperture and 24 in. long. The low frequency data obtained for a parallel plate short circuit field incident from the x-direction has the same frequency behavior obtained from (36). (Constant for $Z_3 = \infty$ and proportional to f for $Z_3 = 0$) However, (36) overestimates the levels for Z_4 infinity and both open- and short-circuited Z_3 by approximately 20 dB. In addition, (36) has been used where the constraints on its derivation are no longer valid due to both the aperture interaction and the polarizability approximation. One might conjecture from these results that (36) and (37) bound problems for wires close to the aperture. Further study is required to support this conclusion.

VII. CONCLUSIONS AND RECOMMENDATIONS:

This report has presented the results of an investigation on bounding signal levels coupled to wire termination behind apertures. The theory of small aperture coupling has been reviewed and interaction with wires developed in the spatial domain. The latter development has identified a capacitance not found in previous developments. This capacitance results from the discharge of the aperture region by currents launched on the wire and is of importance if the wire is sufficiently close to the aperture.

Bounds were developed for the signal levels at terminations with the following results: A wire behind an aperture in a plane bounds the problem of a wire behind an aperture in an enclosing structure for the same external short circuit fields; the aperture polarizabilities may be bound above by the polarizabilities of either a circumscribing ellipse of minimum area or less tightly bound by an circumscribing circle; bounds on the power waves launched on the wire may be obtained from the aperture bounds using proportionalities related to the geometry in the vicinity of the aperture; Bounding multiple reflections with a decay constant, bounds on the termination voltages and currents are proportional to the power waves and a multiple reflection function; For phase cancelling multiple reflections, only the power waves need be considered, neglecting the multiple reflections; and To obtain a tight bound with multipe reflections, the bounds for the separate parts of the incident field should be summed if the field is separable into different time functions. These results follow a step-by-step form for obtaining bounds to the signals at desired terminations.

There are several recommendations for further work. It would be desireable to review the work of Lee and Yang [19] for wires close to an aperture to include the capacitive term that was not included in their work. It would also be desireable to extend this work to include larger apertures for applications in conformal antenna design. In some instances, the currents on wires in a bundle have been found to be larger than expected (greater than the bulk current). Hence it is desireable to consider the coupling to thick wires, wire bundles, and the associated differential mode currents in wire bundles.

Though the bounds appear to be reasonable when compared to computation, it would be worthwhile to set up measurements of several canonical problems for both aperture polarizability and wire coupling. With the importance of direct coupling, it would also be useful to develop the theory of coupling by wires passing through apertures between the interior and exterior regions. This coupling might involve either antenna structures or control cables, the latter ofter covered by composite panels and excited by diffusion. From a theory for such coupling, similar bounds may be developed as in this report for apertures.

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