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The linear section of a cable is represented by self and transfer admittance functions. The functions are differential equations obtained from Prony analyses on the admittances. The linear section is coupled between two non-linear regions described by difference equations. Voltages and the quantity of primary and secondary breakdown are compared with the predictions of a standard code.

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Block 20.

The use of convolution integrals to represent the admittances is also discussed and presented.

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ILLUSTRATIONS

SECTION I

INTRODUCTION

1. BACKGROUND

This report discusses the adaptation of a linear section of transmission line to a time-domain code. The linear section is described by frequency-domain parameters appropriately transformed into the time domain. On both ends of the linear section, the time-domain code employs difference equations to describe regions which may exhibit a nonlinear response. The interfaces between the linear and potentially nonlinear sections of cable produce boundary conditions joining the variant mathematical descriptions of the regions.

2. OBJECTIVES

This document reports on the formulation (in the time domain) of a two-port representation for a linear transmission line, and the joining of this result to a nonlinear, time-domain, transmission-line code.

In general, the nonlinearities can occur in two regions: The first, or primary breakdown region, close to the burst, results from the high intensity fields associated with the nuclear device. The second, or secondary breakdown region, can occur at a discontinuity in the line far from the area of detonation. These are caused by signals propagated by the line and reflected at the discontinuity. These two regions are separated by a linear section of cable.

Of course, this central line section is quite compatible with the nonlinear codes; that is, the difference equations representing the transmission-line equations are compatible with linear, as well as nonlinear, line behavior. Unfortunately, however, the computational time, and thus the computer cost, is greatly increased, due to the large number of spacial increments which must be devoted to the linear section. The question naturally arises as to whether the difference equations for the linear section can be replaced by a less time-consuming scheme.

As we shall show, the linear section can be replaced by a two-port network section in the time domain. The most obvious formulation leads to convolution integrals relating the input and output voltages and currents. The convolution integrals, however, require an integration over all previous time at each time step.

The convolution integrals can be replaced by equivalent, ordinary differential equations. The advantage of this form is that the differential equation solution can be numerically stepped forward at each time step, based only on the value of the variables at the previous time step. Obviously, this procedure is less time consuming that that of convolution, particularly at long times.

In the following materials two approaches will be shown. The first derives the convolution integrals which relate the currents and voltages at the two ends of a linear transmission line, and the second approach derives the differential equations relating the same quantities. Because the differential equations are most easily formulated from rational polynomials in frequency, the following method is used to achieve the desired representation.

The network parameters are irrational functions of frequency; so, using Prony's method, we approximate the irrational functions by rational ones. Then, introducing the concept of network state, the rational frequency functions lead to a set of first-order, ordinary differential equations relating the two-port voltages and currents. These equations are in the desired form.

Finally, the state equations are integrated into the nonlinear difference equations to complete this goal.

To test the validity of the convolution integral and differential equation concepts, comparisons are presented between the unmodified time-domain code BLINE and the two approaches.

In section III, the convolution integrals for a lossless, linear line are incorporated into a modified time domain code and compared with the results from BLINE. In this example, field strengths are limited to produce a linear result.

The first example of the differential equation procedure is shown in section IV. Here, the transmission line is a coaxial cable driven from one end, and again, is contrained to be linear. Comparisons with the solution predicted by BLINE are shown.

The final example occurs in section V: it uses distributed sources and the differential-equation approach to propagate a signal inducing secondary breakdown. A comparison is made with the results predicted by BLINE.

All the examples use the same format of modifying BLINE. Specifically, the linear section appears as a single cell in the time-domain code inserted between two regions described by difference equations.

SECTION II

THEORY

1. THE TRANSMISSION LINE AS A TWO-PORT NETWORK

In this section, we consider the Laplace transform of the transmission line equations and their general solution in terms of the input voltages and currents.

The initial formulation is in terms of the two-port, general circuit parameters (ref. 1) which express the output voltage and current in terms of the input voltage and current. Since, for this problem, the admittance formulation (ref. 1) is more convenient, the equations are rearranged to this form. The admittance formulation expresses the input and output currents in terms of the input and output voltages.

Transmission Line Equations

The Laplace transform of the single transmission line equations is (ref. 2)

$$\frac{d}{dz}\begin{bmatrix} Y(z,s) \\ I(z,s) \end{bmatrix} = \begin{bmatrix} 0 & -Z(s) \\ -Y(s) & 0 \end{bmatrix} \begin{bmatrix} V(z,s) \\ I(z,s) \end{bmatrix} + \begin{bmatrix} e(z,s) \\ i(z,s) \end{bmatrix} \tag{1}$$

or, in more compact notation,

$$\frac{d}{dz}\ \vec{y}(z,s) = A\ \vec{y}(z,s) + \vec{g}(z,s) \tag{2}$$

where $$A = \begin{bmatrix} 0 & -Z(s) \\ -Y(s) & 0 \end{bmatrix}$$

Z(s) and Y(s) are the series impedance per unit length and the shunt impedance per unit length, respectively,

e(z,s) and i(z,s) are the distributed sources per unit length,

$\vec{g}(z,s) = [e(z,s),\ i(z,s)]$,

$\vec{y}(z,s) = [V(z,s),\ I(z,s)]$

s is the Laplace variable.

The general solution to equation (2) is (ref. 3)

$$\vec{y}(z,s) = e^{A(z-z_0)}\,\vec{y}(z_0,s) + \int_{z_0}^{z} e^{A(z-\zeta)}\,\vec{g}(\zeta,s)\,d\zeta \tag{3}$$

Here, $e^{A(z-z_0)}$ is an exponential matrix which will be determined shortly. Equation (3) is the two-port, general circuit-parameter matrix (active because of the source term) for the transmission line.

Since the eigenvalues of matrix A are distinct, the Sylvester expansion theorem (ref. 4) can be used to evaluate $e^{A(z-z_0)}$. The eigenvalues of A are determined by

$$\begin{vmatrix} -\lambda & -Z(s) \\ -Y(s) & -\lambda \end{vmatrix} = 0 \tag{4a}$$

The two eigenvalues of A are therefore

$$\begin{aligned} \lambda_1 &= \sqrt{Y(s)Z(s)} = \gamma \\ \lambda_2 &= -\sqrt{Y(s)Z(s)} = -\gamma \end{aligned} \tag{4b}$$

The Sylvester expansion theorm states that

$$e^{A(z-z_0)} = \sum_{i=1}^{2} f(\lambda_i)F(\lambda_i) \qquad (5)$$

where $f(\lambda_i) = e^{\lambda_i(z-z_0)}$

$$F(\lambda_i) = \prod_{\substack{j=1 \\ j\neq i}}^{2} \left[\frac{A-\lambda_1 II}{\lambda_i-\lambda_j}\right]$$

II is the identity matrix,

λ_i are the eigenvalues of A.

Substituting each equation (4b) into equation (5) yields

$$e^{A(z-z_0)} = \frac{e^{\gamma(z-z_0)}}{2}\begin{bmatrix} 1 & -Z_0 \\ -Y_0 & 1 \end{bmatrix} + \frac{e^{-\gamma(z-z_0)}}{2}\begin{bmatrix} 1 & Z_0 \\ Y_0 & 1 \end{bmatrix} \qquad (6)$$

$$= \begin{bmatrix} \dfrac{e^{\gamma(z-z_0)} + e^{-\gamma(z-z_0)}}{2} & \dfrac{e^{-\gamma(z-z_0)} - e^{\gamma(z-z_0)}}{2} Z_0 \\ \dfrac{e^{-\gamma(z-z_0)} - e^{\gamma(z-z_0)}}{2} Y_0 & \dfrac{e^{\gamma(z-z_0)} + e^{-\gamma(z-z_0)}}{2} \end{bmatrix}$$

where $Z_0 = \dfrac{1}{Y_0} = \sqrt{\dfrac{Z(s)}{Y(s)}}$

The elements of the matrix $e^{A(z-z_0)}$ are the general circuit parameters for a two-port representation of a transmission line of length $(z-z_0)$. For the problem at hand, the admittance formulation is more convenient, and can easily be obtained by algebraic manipulation of the variables in equation (3). Assuming the line is of length ℓ, the admittance form is

$$\begin{bmatrix} I_\ell(z_0,\ell,s) \\ I_{z_0}(z_0,\ell,s) \end{bmatrix} = [Y] \begin{bmatrix} V(\ell,s) \\ V(z_0,s) \end{bmatrix} + \begin{bmatrix} I_1(z_0,\ell,s) \\ I_2(z_0,\ell,s) \end{bmatrix} \tag{7}$$

where

$$[Y] = \begin{bmatrix} \dfrac{e^{\gamma\ell}+e^{-\gamma\ell}}{e^{-\gamma\ell}-e^{\gamma\ell}}\, Y_0 & \dfrac{-2}{e^{-\gamma\ell}-e^{\gamma\ell}}\, Y_0 \\ \dfrac{2}{e^{-\gamma\ell}-e^{-\gamma\ell}}\, Y_0 & -\dfrac{e^{\gamma\ell}+e^{-\gamma\ell}}{e^{-\gamma\ell}-e^{\gamma\ell}}\, Y_0 \end{bmatrix}$$

$$I_1(z_0,\ell,s) = -\frac{e^{\gamma\ell}+e^{-\gamma\ell}}{e^{-\gamma\ell}-e^{\gamma\ell}}\, Y_0\, G_1(z_0,\ell,s) + G_2(z_0,\ell,s)$$

$$I_2(z_0,\ell,s) = \frac{-2}{e^{-\gamma\ell}-e^{\gamma\ell}}\, Y_0\, G_1(z_0,\ell,s)$$

$$\begin{bmatrix} G_1(z_0,\ell,s) \\ G_2(z_0,\ell,s) \end{bmatrix} = \int_{z_0}^{z_0+\ell} e^{A(z_0+\ell-\zeta)}\vec{g}(\zeta,s)\,d\zeta$$

2. TRANSFORMATION OF THE ADMITTANCE FORMULATION TO THE TIME DOMAIN

The admittance formulation of equation (7) can be expressed as

$$
\begin{aligned}
I_\ell(z_0,\ell,s) &= Y_{11}(s)V(\ell,s) + Y_{12}(s)V(z_0,s) + I_1(z_0,\ell,s) \\
I_{z_0}(z_0,\ell,s) &= Y_{21}(s)V(\ell,s) + Y_{22}(s)V(z_0,s) + I_2(z_0,\ell,s)
\end{aligned}
\tag{8}
$$

Here Y_{11}, Y_{12}, Y_{21} and Y_{22} are the elements of matrix [Y].

Using the fact that the product of the Laplace transforms of two functions is equivalent to their convolution in the time domain, equation (7) can be directly converted to the time domain; that is,

$$
\begin{aligned}
i_\ell(z_0,\ell,t) &= \int_0^t y_{11}(t-t')v(\ell,t')dt' + \int_0^t y_{12}(t-t')v(z_0,t')dt' + i_1(z_0,\ell,t) \\
i_{z_0}(z_0,\ell,t) &= \int_0^t y_{21}(t-t')v(\ell,t')dt' + \int_0^t y_{22}(t-t')v(z_0,t')dt' + i_2(z_0,\ell,t)
\end{aligned}
\tag{9}
$$

Here, the lower-case letters stand for the inverse Laplace transforms of their upper-case counterparts. In general, the inverse transforms of Y_{11}, Y_{12}, Y_{21}, Y_{22}, I_1, and I_2 cannot be obtained analytically, and must be obtained numerically. The nature of the inverse transforms for the admittance parameters will be discussed more fully in section II.3. The inverse transforms of the equivalent current sources I_1 and I_2 are discussed more fully in section V.

Equation (9) is one form of the desired current-voltage relationship in the time domain. In a numerical context, however, the convolution integrals have a serious disadvantage; because, for each increment of time Δt, a numerical integration must be done over all previous time. Therefore, as time progresses, the integrations become longer and longer.

As is known from the theory of linear differential equations, a convolution integral can represent the solution of a differential equation (ref. 3). If the convolution integrals in equation (9) can be replaced by equivalent differential equations; then, from a numerical standpoint, the solution to the differential equations can be stepped forward in time, based only on the value of the variable at that time. Thus, the computational requirements at each time step are greatly reduced from those required for convolution.

As we shall see, if the admittances in equation (8) are rational functions of frequency (ratios of polynomials in frequency) then ordinary differential equations can be obtained which replace the convolution integrals (exactly) for all time. If these functions are irrational functions of frequency (as is true in the present case), differential equations can be found which replace the convolution integrals, arbitrarily closely, for any finite time. In the following material, we present a method to determine these differential equations.

Prior to further consideration, it is convenient to transfer the source terms, I_1 and I_2, to the left-hand side of equation (8), so that

$$\begin{aligned} I_{\ell}(z_0,\ell,s)-I_1(z_0,\ell,s) &= Y_{11}(s)V(\ell,s) + Y_{12}(s)V(z_0,s) \\ I_{z_0}(z_0,\ell,s)-I_2(z_0,\ell,s) &= Y_{12}(s)V(\ell,s) + Y_{22}(s)V(z_0,s) \end{aligned} \qquad (10)$$

The motivation for this change is that the self and transfer admittances of equation (10) describe the response of the system initially at rest. Therefore, the left-hand side currents are those which result from driving a system initially at rest with voltages $V(\ell,s)$ and $V(z_0,s)$.

Consider a typical term in equation (10), i.e.,

$$I(s) = Y(s)V(s) \qquad (11)$$

We assume initially that Y(s) is a rational fraction of the form

$$\frac{I(s)}{V(s)} = Y(s) = \frac{N(s)}{D(s)} = \frac{b_n s^n + b_{n-1} s^{n-1} + \ldots + b_o}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_o} \quad ; \; a_n \neq o \tag{12}$$

Note that the degree of the numerator can be at most equal to the degree of the denominator. If such is the case, then the impulse response y(t) corresponding to Y(s) contains an impulse of value b_n/a_n. Indeed, for the transmission line, this is the case. It is possible to extend this analysis to cases where the degree of the numerator can exceed the degree of the denominator; however, such an extension is not necessary here.

Following reference (3), we postulate that the differential equation relating i(t) and v(t), in equation (10), is

$$\begin{aligned} a_n \frac{d^n}{dt^n} i(t) + a_{n-1} \frac{d^{n-1}}{dt^{n-1}} i(t) + \cdots + a_o i(t) \\ = b_n \frac{d^n}{dt^n} v(t) + b_{n-1} \frac{d^{n-1}}{dt^{n-1}} v(t) + \cdots + b_o v(t) \end{aligned} \tag{13}$$

The Laplace transform of equation (13) should be reducable to equation (12). The Laplace transform of equation (13) is

$$\begin{aligned} & a_n \left\{ s^n I(s) - s^{n-1} i(o) - s^{n-2} \frac{d}{dt} i(o) - \cdots - \frac{d^{n-1}}{dt^{n-1}} i(o) \right. \\ & \quad + a_{n-1} \left\{ s^{n-1} I(s) - s^{n-2} i(o) - s^{n-3} \frac{d}{dt} i(o) \cdots - \frac{d^{n-2}}{dt^{n-2}} i(o) \right\} \\ & \quad + \cdots \end{aligned}$$

$$= b_n \left\{ s^n \; V(s) - s^{n-1} v(o) - s^{n-2} \frac{d}{dt} v(o) - \cdots \frac{-d^{n-1}}{dt^{n-1}} \; v(o) \right\}$$

$$+ b_{n-1} \left\{ s^{n-1} V(s) - s^{n-2} v(o) - s^{n-3} \frac{d}{dt} v(o) - \cdots \frac{-d^{n-2}}{dt^{n-2}} \; v(o) \right\} \tag{14}$$

$$+ \cdots$$

Solving equation (14) for I(s), the result is

$$I(s) = \frac{N(s)}{D(s)} V(s) + \frac{1}{D(s)} \Bigg\{ s^{n-1} \left[a_n i(o) - b_n v(o) \right]$$

$$+ s^{n-2} \left[a_n \frac{d}{dt} i(o) - b_n \frac{d}{dt} v(o) + a_{n-1} i(o) - b_{n-1} v(o) \right]$$

$$+ \cdots$$

$$+ \left[a_n \frac{d^{n-1}}{dt^{n-1}} i(o) - b_n \frac{d^{n-1}}{dt^{n-1}} v(o) + \cdots + a_1 i(o) - b_1 v(o) \right] \Bigg\} \tag{15}$$

Here N(s) and D(s) are defined in equation (12).

If the transfer function of the system is to be that of equation (12); i.e.,

$$I(s) = \frac{N(s)}{D(s)} V(s)$$

then all other terms in equation (15) must be equal to zero. These conditions form the interrelationship among the initial conditions on the variables. However, rather than using these initial conditions directly (v(t) is the source and i(t) the response), it is more convenient to make use of the concept of system state.

Basically, the state of a system is a property of the system which, together with system inputs, determine the future state of the system. For present purposes, the state can be a set of new variables defined as linear combinations of the original variables, $i(t)$ and $v(t)$, and their derivatives. Since the transmission line is initially at rest, and the fact that we wish equation (15) to follow from equation (14), a very convenient definition* of the state variables $x_i(t)$ is

$$\begin{aligned} x_1(t) &= a_n i(t) - b_n v(t) \\ x_2(t) &= a_n \frac{d}{dt} i(t) - b_n \frac{d}{dt} v(t) + a_{n-1} i(t) - b_{n-1} v(t) \\ &\vdots \\ x_n(t) &= a_n \frac{d^{n-1}}{dt^{n-1}} i(t) - b_n \frac{d^{n-1}}{dt^{n-1}} v(t) + \cdots + a_1 i(t) - b_1 v(t) \end{aligned} \tag{16}$$

It is easily seen that if the initial state $(x_1(o), x_2(o), \ldots, x_n(o))$ is zero, then equation (11) will result from equation (15). We have thus arrived at a set of variables whose initial value is zero, and which results in the desired relationship.

We now determine the differential equations which these new state variables must satisfy.

Equation (16) can be written

$$\begin{aligned} x_1(t) &= a_n i(t) - b_n v(t) \\ x_2(t) &= \frac{d}{dt} x_1(t) + a_{n-1} i(t) - b_{n-1} v(t) \\ &\vdots \\ x_n(t) &= \frac{d}{dt} x_{n-1}(t) + a_1 i(t) - b_1 v(t) \end{aligned} \tag{17}$$

* System state is not unique.

The last state variable $x_n(t)$ in equation (16) can be combined with equation (13) to yield

$$\frac{d}{dt} x_n(t) = - \left\{ a_o i(t) - b_o v(t) \right\} \tag{18}$$

Finally, solving equations (17) and (18) in terms of $\frac{d}{dt} x_k(t)$ and $i(t)$, and using the first relationship in equation (17); i.e.,

$$i(t) = \frac{1}{a_n} x_1(t) + \frac{b_n}{a_n} v(t),$$

leads to the result

$$\begin{aligned}
\frac{d}{dt} x_1(t) &= \frac{1}{a_n} \left\{ -a_{n-1}x_1(t)+a_n x_2(t)+(a_n b_{n-1}-a_{n-1}b_n)v(t) \right\} \\
\frac{d}{dt} x_2(t) &= \frac{1}{a_n} \left\{ -a_{n-2}x_1(t)+a_n x_3(t)+(a_n b_{n-2}-a_{n-2}b_n)v(t) \right\} \\
&\vdots \\
\frac{d}{dt} x_{n-1}(t) &= \frac{1}{a_n} \left\{ -a_1 x_1(t)+a_n x_n(t)+(a_n b_1-a_1 b_n)v(t) \right\} \\
\frac{d}{dt} x_n(t) &= \frac{1}{a_n} \left\{ -a_o x_1(t)+(a_n b_o-a_o b_n)v(t) \right\} \\
i(t) &= \frac{1}{a_n} \left\{ x_1(t)+b_n v(t) \right\}
\end{aligned} \tag{19}$$

Equation (19) is a set of first-order, ordinary, differential equations, which with zero initial conditions, describe the desired voltage current relationship between $i(t)$ and $v(t)$.

3. IRRATIONAL ADMITTANCE ELEMENTS

In the previous section, a technique is described which replaced a convolution integral with a set of ordinary differential equations. The method is useful, however, only if the individual admittance elements are rational functions of frequency.

For the transmission line, the admittance elements are irrational functions of frequency, so it is necessary to approximate them by rational ones, if this method is to be used. There are many ways to make this approximation (see, for example, reference 5). A particularly convenient method and the one that is used here, is Prony's method (ref. 6).

Using Prony's algorithm, it is possible to represent a time function, arbitrarily closely over any finite time, by a finite sum of decaying sinusoids*. Thus, the representation

$$f(t) \simeq \sum_{i=1}^{N} A_i \, e^{s_i t} \tag{20}$$

is obtained. The Laplace transform of this expression is

$$F(s) = L\{f(t)\} = \sum_{i=1}^{N} \frac{A_i}{s-s_i} = \frac{N(s)}{D(s)} \tag{21}$$

which is a rational function.

To use the Prony procedure it is necessary to inverse transform the admittance functions. For the transmission line, most of the admittance functions are sufficiently complex that the inversion must be done numerically. In addition, these inverse transforms contain impulses. The value of these impulses must be removed before numerical inversion.

* Simple decaying exponentials are included.

To illustrate the procedure, consider the characteristic admittance of a transmission line with lumped series resistance and inductance and only shunt capacitance (perfect dielectric). The characteristic admittance for this line is

$$Y_0(s) = \sqrt{\frac{sC}{R+sL}} \qquad (22)$$

We note that

$$\lim_{s \to \infty} Y(s) = \sqrt{\frac{C}{L}}$$

and since the Laplace transform of an impulse is a constant, the inverse transform of $Y_0(s)$ contains an impulse of value $\sqrt{\frac{C}{L}}$. Therefore, if we were to numerically inverse transform $Y_0(s)$, we would remove this impulse and inverse transform

$$Y_0'(s) = Y_0(s) - \sqrt{\frac{C}{L}} = \sqrt{\frac{sC}{R+sL}} - \sqrt{\frac{C}{L}} \qquad (23)$$

The prony procedure would then be applied to the result. Of course, an impulse of value $\sqrt{\frac{C}{L}}$ would then be added to the exponential approximation of equation (20).

It turns out that equation (22) can be analytically inverse transformed, however, and the result is

$$Y_0(t) = \sqrt{\frac{C}{L}} \frac{d}{dt} \left\{ \varepsilon^{-\frac{R}{2L} t} I_0\left(\frac{R}{2L} t\right) \right\} \qquad (24)$$

The impulse occurs in equation (24), because of the unit discontinuity of the expression in parenthesis at the origin.

Equation (24) cannot be entirely represented by a sum of simple exponentials. However, a Prony procedure applied to the first 30 microseconds of this function with the values

$$R = 1.3611 \times 10^{-2} \text{ ohms/meter}$$

$$L = 5.8322 \times 10^{-8} \text{ henry/meter}$$

$$C = 4.3878 \times 10^{-10} \text{ farad/meter}$$

indicates that two simple poles form a good approximation.

The following poles and residues result from the Prony analysis:

Pole	Residue
$-5.7362\text{x}10^4$	$1.826\text{x}10^3$
$-2.0056\text{x}10^5$	$8.298\text{x}10^3$

A comparison of the resulting approximate function with the original irrational function is shown in figure 1. Actually all values are within .1% except for the last several values which are within 1%.

The approximation to $Y_o(t)$ is then

$$Y_o(t) = \sqrt{\frac{C}{L}}\ \delta(t) + 1.826\text{x}10^3 \text{Exp}(-5.7362\text{x}10^4 t) + 8.298\text{x}10^3 \text{Exp}(-2.0056\text{x}10^5 t)$$

Here the impulse has been added to complete the expression. The Laplace transform of this function will form a rational approximation to $Y_o(s)$.

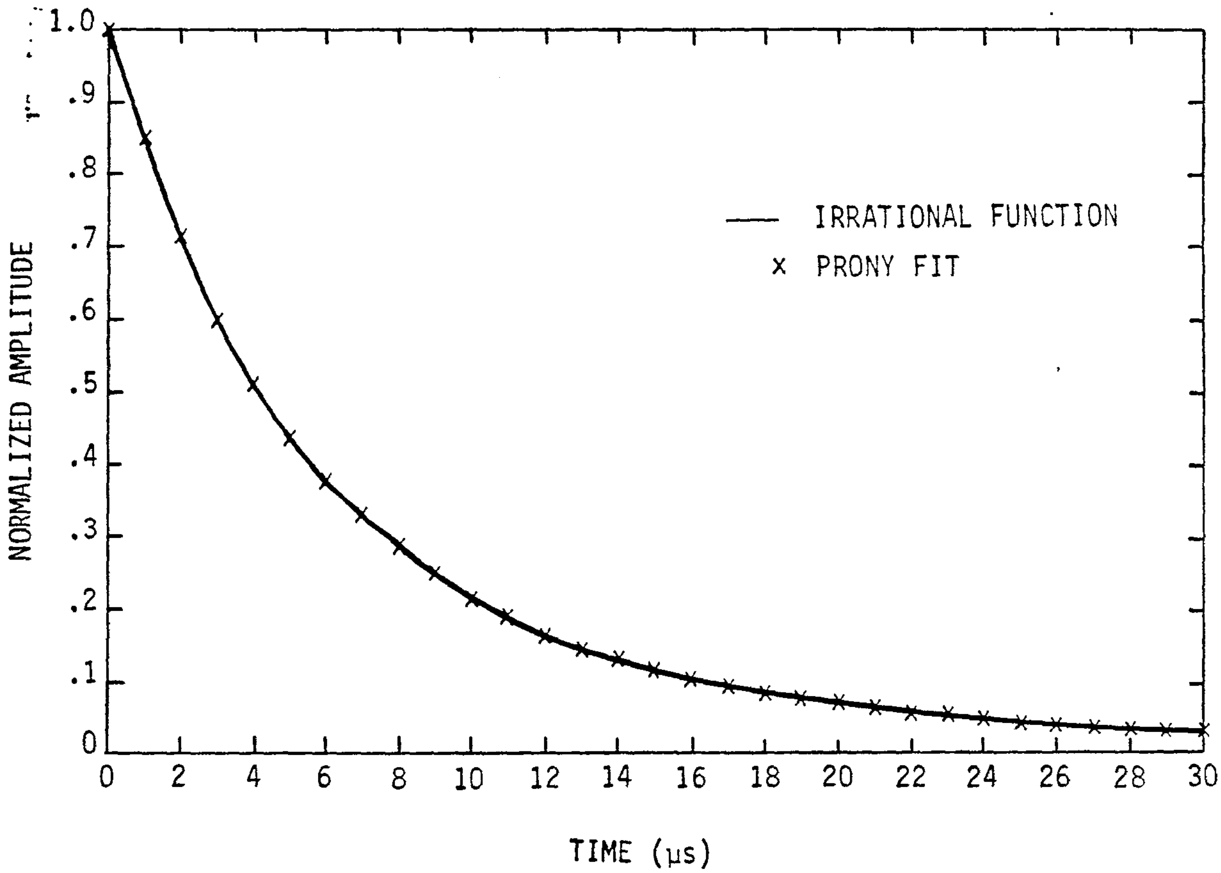


Figure 1. Comparison of Irrational Function With A Two Exponential Approximation.

4. ANALYSIS OF THE ADMITTANCE MATRIX

It is instructive to analyze the admittance matrix [Y] in equation (7) in more detail, by expanding the individual elements in the following way:

$$\frac{e^{\gamma\ell}+e^{-\gamma\ell}}{e^{-\gamma\ell}-e^{\gamma\ell}}\, Y_o = -\frac{1+e^{-2\gamma\ell}}{1-e^{-2\gamma\ell}}\, Y_o$$

$$= -(1+e^{-2\gamma\ell})(1+e^{-2\gamma\ell}+e^{-4\gamma\ell}+\cdots)\, Y_o \qquad (25a)$$

$$= -(1+2e^{-2\gamma\ell}+2e^{-4\gamma\ell}+\cdots)\, Y_o$$

$$\frac{2}{e^{-\gamma\ell}-e^{\gamma\ell}}\, Y_o = -2e^{-\gamma\ell}(1+e^{-2\gamma\ell}+e^{-4\gamma\ell}+\cdots)\, Y_o$$

$$= -2(e^{-\gamma\ell}+e^{-3\gamma\ell}+e^{-5\gamma\ell}+\cdots)\, Y_o \qquad (25b)$$

Therefore, the admittance matrix can be expressed as

$$[Y] = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} Y_o + 2e^{-\gamma\ell} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} Y_o + 2e^{-2\gamma\ell} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} Y_o + \cdots \qquad (25c)$$

We note that the multiplicative factors of 2 in this expression are due to the fact that this is a short-circuit admittance matrix, and the current reflection coefficient under short-circuit conditions is 2.

Due to the multiplicative factors of $e^{-n\gamma\ell}(n=0,1,2,\ldots)$ in equation (25), each matrix in the sum undergoes a delay (in the time domain) of multiples of a line length delay. If the line is lossy, this delay is of course dispersive; nevertheless, it is there. Therefore, the problem can be subdivided (time-wise) into a number of distinct parts, each of which is additive to the total result. Furthermore, since we are usually interested in results on the transmission line for a fairly short time, only the first several of the terms in equation (25) need be considered.

As an example to illustrate the time behavior of equation (25), we consider a lossless transmission line of length ℓ with parameters C and L. In this case,

$$Y_0 = \sqrt{\frac{C}{L}}$$

$$\gamma = \sqrt{s^2 LC} = s\sqrt{LC}$$

so that the exponential factors represent only time delay; i.e., multiplication of a Laplace transform by e^{-st_0} delays the ensuing time function by t_0.

Since $Y_0 = \sqrt{\frac{C}{L}}$ is constant, its inverse transform is an impulse of value $\sqrt{\frac{C}{L}}$. Therefore, the inverse transform of equation (25) is

$$\left[Y(t)\right] = \begin{bmatrix} -\delta(t) & 0 \\ 0 & \delta(t) \end{bmatrix}\sqrt{\frac{C}{L}} + 2\begin{bmatrix} 0 & \delta(t-t_0) \\ -\delta(t-t_0) & 0 \end{bmatrix}\sqrt{\frac{C}{L}} + 2\begin{bmatrix} -\delta(t-2t_0) & 0 \\ 0 & \delta(t-2t_0) \end{bmatrix}\sqrt{\frac{C}{L}} + \cdots \quad (26)$$

In this equation, $t_0 = \sqrt{LC}\,\ell$.

Each term in equation (26) is a simple impulse, so that the convolution integrals in equation (9) are easily evaluated. The result is

$$\begin{aligned} i(\ell,t) &= \sqrt{\frac{C}{L}}\left\{-v(\ell,t)+2v(z_0,t-t_0)-2v(\ell,t-2t_0) + \cdots\right\} + i_1(\ell,t) \\ i(z_0,t) &= \sqrt{\frac{C}{L}}\left\{v(z_0,t)-2v(\ell,t-t_0)+2v(z_0,t-2t_0) - \cdots\right\} + i_2(\ell,t) \end{aligned} \quad (27)$$

Equation (27), then, is the current-voltage relationship for a lossless line. In this expression all voltages are zero if $t-kt_0$ is negative. In section III a different derivation of the voltage and current relationship and numerical results are presented for a lossless transmission line.

When the line is not lossless, the inverse transform of equation (25c) is more complex. In general, each term contains an impulse plus a dispersive term, so that the convolution integrals must be numerically evaluated.

5. BURIED CABLE CONSIDERATIONS

The motivating problem for this study requires that we determine the voltage across the dielectric of a buried, insulated conductor; that is, we must determine the dielectric voltage to determine dielectric breakdown. In the previous sections, we have analyzed the behavior of a single transmission line, with no special consideration for the dielectric portion of the total line voltage. It is the purpose of this section to present the procedures to be followed to determine the dielectric voltage.

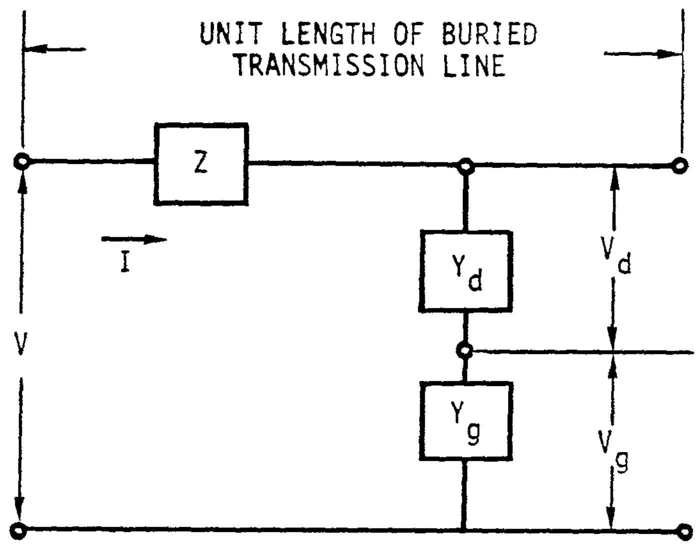
It is common practice in the analysis of buried cables to represent the dielectric and ground in the way illustrated in figure 2. Typically, Y_d and Y_g each consist of a parallel resistor-capacitor combination; so that, in the time domain, the transmission line equations can be written

$$\frac{\partial(v_d+v_g)}{\partial z} = - L \frac{\partial i}{\partial t} - Ri + E^{inc}(z,t)$$

$$\frac{\partial i}{\partial z} = - C_d \frac{\partial v_d}{\partial t} - G_d v_d \qquad (28)$$

$$\frac{\partial i}{\partial z} = - C_g \frac{\partial v_g}{\partial t} - G_g v_g$$

Here, the subscripts d and g refer to dielectric and soil, respectively.



Z = SERIES IMPEDANCE/UNIT LENGTH

Y_d = SHUNT ADMITTANCE/UNIT LENGTH OF THE DIELECTRIC

Y_g = SHUNT ADMITTANCE/UNIT LENGTH OF THE GROUND

Figure 2. Section of Buried, Insulated Conductor.

Indeed, the present nonlinear, time-domain, difference equations are representations of these partial differential equations.

To proceed, we again resort to the Laplace transform versions of the single transmission line equations which are

$$\frac{\partial V}{\partial z} = - ZI$$

$$\frac{\partial I}{\partial z} = - YV \tag{29}$$

Here, however, to conform to figure 2, the line parameter Y is the series combination of Y_d and Y_g; i.e.,

$$\frac{1}{Y} = \frac{1}{Y_d} + \frac{1}{Y_g}$$

$$\Rightarrow Y = \frac{Y_d Y_g}{Y_d + Y_g}$$

Now the dielectric voltage can be written in terms of line voltage as follows:

$$V_d = V \frac{\frac{1}{Y_d}}{\frac{1}{Y_d} + \frac{1}{Y_g}} = V \frac{Y_g}{Y_d + Y_g}$$

Therefore, in terms of dielectric voltage V_d, the total line voltage V is

$$V = \frac{Y_d + Y_g}{Y_g} V_d \tag{30}$$

Similarly, the line voltage V can be expressed in terms of ground voltage V_g; that is,

$$V = \frac{Y_d + Y_g}{Y_d} V_g \tag{31}$$

Expressions (29) and (30) are the desired relationships to convert from V_d or V_g to V and vice-versa. However, since $V = V_d + V_g$, it is only necessary to use one of these expressions. Both of these expressions are Laplace transforms, so that, in general, the methods of parts 3 and 4 of section must be used to obtain the corresponding time domain relationships.

We mention in passing that the transmission line equations can be written solely in terms of dielectric voltage V_d. This is accomplished by substituting equation (29) into the transmission line equation (28); the result is

$$\frac{\partial V_d}{\partial z} = \frac{-Y_g Z}{Y_d + Y_g} \; I = -Z'I$$

$$\frac{\partial I}{\partial z} = -Y_d V_d \tag{32}$$

where $Z' = \dfrac{Y_g Z}{Y_d + Y_g}$

Equation (31), while having different line parameters than equation (28), has the same propagation constant. Therefore, the linear section of transmission line can be modeled either with total voltage, or with dielectric voltage.

SECTION III

LOSSLESS LINEAR SECTION

1. LIMITATIONS OF THE APPLICATION

This section of the report uses the straightforward approach of convolution integrals (equation 9) to find the relationship between the currents and voltages at the ends of the linear insert. With no distributed sources, equation (9) becomes

$$i(z_0,t) = \int_0^t y_{11}(t-t')v(\ell,t')dt' + \int_0^t y_{12}(t-t')v(z_0,t)dt' \quad (33a)$$

$$i(\ell,t) = \int_0^t y_{21}(t-t')v(\ell,t')dt' + \int_0^t y_{22}(t-t')v(z_0,t)dt' \quad (33b)$$

To calculate the admittance matrix as a function of time, the inverse Fourier transform of the frequency representation must be found. As discussed in section II.3, both the self and transfer admittances contain impulses; as a result, if the impulses are not removed, the inverse Fourier transform is not defined. Another method to find the current-voltage relationship is the following. Examining only the first integral in equation (33a),

$$\int_0^t y_{11}(t-t')v(\ell,t')dt' = F^{-1}\Big(Y_{11}(s)V(\ell,s)\Big)$$
$$= F^{-1}\left[\left(\frac{Y_{11}(s)}{s}\right)\Big(sV(\ell,s)\Big)\right] = F^{-1}\left[\tilde{Y}_{11}(s)\tilde{V}(\ell,s)\right] \quad (34)$$

The quantity $\tilde{Y}_{11}(s)$, has no impulse behavior and therefore a well defined inverse Fourier transform; it contains instead, discontinuities in the time domain. The voltage part of the integral, $\tilde{V}(\ell,s)$, simply represents the time-domain derivative of the voltage. The equation now appears as

$$\int_0^t y_{11}(t-t')v(\ell,t')dt' = \int_0^t \tilde{y}_{11}(t-t') \frac{\partial}{\partial t} v(\ell,t')dt' \qquad (35)$$

The other elements of the admittance matrix, $\tilde{y}_{12}(t)$, $\tilde{y}_{21}(t)$, and $\tilde{y}_{22}(t)$ will have properties similar to $\tilde{y}_{11}(t)$; the following discussion of $\tilde{y}_{11}(t)$ can be applied to all other admittance terms and their contributions to the currents.

The time behavior of $\tilde{y}_{11}(t)$ consists of a discontinuity at $t = 0$ ($t = \gamma\ell$, one delay length for the transfer admittance terms) and at each subsequent $2n\gamma\ell$. Between the discontinuities at the reflection times, $\tilde{y}_{11}(t)$ is monotonically increasing. If the line is lossy, then in the limit as $t \to \infty$, $\tilde{y}_{11}(t)$ = constant. If the line is not lossy, the monotonic sections are flat, the discontinuities do not diminish with time, and there is no limit to $\tilde{y}_{11}(t)$.

The only case analyzed with the admittance matrix in this form was for a lossless line; the size of the discontinuity was then a constant equal to the characteristic admittance of the line (occuring at $t = 0$ and each two line delay lengths). In this form it was possible to include all reflection terms and signals propagating in both directions.

2. ADAPTATION TO BLINE

The junction of the linear section with the sections described by difference equations employs the solution of the dielectric and ground voltages at the junction points. The transmission-line equations (equations 28) employed by BLINE are

$$\frac{\partial}{\partial z}(v_d+v_g) = -L\,\frac{\partial i}{\partial t} - R_c i \tag{35a}$$

$$\frac{\partial i}{\partial z} = -C_d\,\frac{\partial v_d}{\partial t} - G_d v_d \tag{35b}$$

$$\frac{\partial i}{\partial z} = -C_g\,\frac{\partial v_g}{\partial t} - G_g v_g \tag{35c}$$

when no sources are included.

An examination of figure 3 shows the spatial difference equation for the current employing the self and transfer impedances is

$$\begin{aligned}\frac{\partial I}{\partial z} &= \frac{2}{\Delta z}\,(I_{j+2} - I_{j+1+\frac{1}{2}})\\ &= \frac{2}{\Delta z}\left(I_{j+2} - \int_0^t \tilde{y}_{21}(t-t')\,\frac{\partial v(\ell,t')}{\partial t'}\,dt'\right.\\ &\qquad \left. - \int_0^t \tilde{y}_{22}(t-t')\,\frac{\partial v(z_o,t')}{\partial t'}\,dt'\right)\end{aligned} \tag{36a}$$

on the right end of the linear section (j+2) and

$$\begin{aligned}\frac{\partial I}{\partial z} &= \frac{2}{\Delta z}\,(I_{j+\frac{1}{2}} - I_j)\\ &= \frac{2}{\Delta z}\left(\int_0^t \tilde{y}_{11}(t-t')\,\frac{\partial v(z_o,t')}{dt'}\,dt'\right.\\ &\qquad \left. + \int_0^t \tilde{y}_{12}(t-t')\,\frac{\partial v(\ell,t')}{\partial t'}\,dt' - I_j\right)\end{aligned} \tag{36b}$$

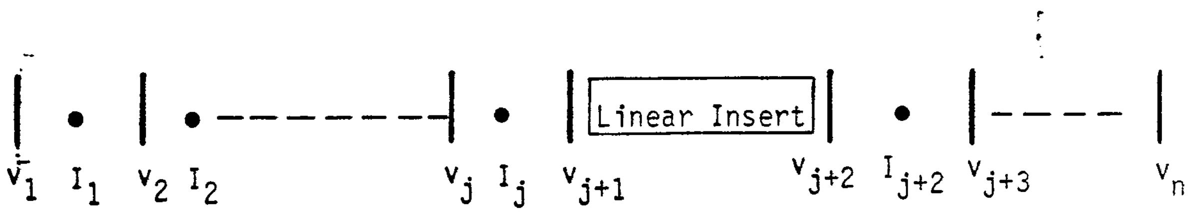


Figure 3. Placement of Current and Voltage Cells in BLINE and the Linear Insert in BLINE.

on the left end (j+1). In both cases v is the total voltage as given in section II.1.

Approximating the convolution integrals as

$$\int_0^t \tilde{y}_{11}(t-t')\,\frac{\partial v(z_o,t')}{\partial t'}\,dt' = \sum_{k=1}^{n} \tilde{y}_{11}^{n+2-k}\left(V_{j+1}^{k} - V_{j+1}^{i}\right) \tag{37}$$

where the k and n indices represent time; equations (36a), (35b), and (35c) then give the boundary conditions on the right side of the linear section

$$V_{d_{j+2}}^{n+1}\left(\frac{C_d}{\Delta t} + \frac{G_d}{2} - \frac{\tilde{y}_{22}^{1}}{\Delta z}\right) - V_{g_{j+2}}^{n+1}\,\frac{\tilde{y}_{22}^{1}}{\Delta z} \tag{38a}$$

$$= V_{d_{j+2}}^{n}\left(\frac{C_d}{\Delta t} - \frac{G_d}{2} - \frac{\tilde{y}_{22}^{1}}{\Delta z}\right) - V_{g_{j+2}}^{n}\,\frac{\tilde{y}_{22}^{1}}{\Delta z} + S_2$$

and

$$V_{g_{j+2}}^{n+1}\left(\frac{C_g}{\Delta t} + \frac{G_g}{2} - \frac{\tilde{y}_{22}^{1}}{\Delta z}\right) - V_{d_{j+2}}^{n+1}\,\frac{\tilde{y}_{22}^{1}}{\Delta z}$$

$$= V_{g_{j+2}}^{n}\left(\frac{C_g}{\Delta t} - \frac{G_g}{2} - \frac{\tilde{y}_{22}^{1}}{\Delta z}\right) - V_{d_{j+2}}^{n}\,\frac{\tilde{y}_{22}^{1}}{\Delta x} + S_2 \tag{38b}$$

where the quantities subscripted by d and g indicate capacitances, conductances and voltages across the dielectric and ground and

$$S_2 = \frac{2}{\Delta z}\left[\frac{1}{2}\tilde{y}_{21}^{n+1} V_{j+1}^{1} + \sum_{k=2}^{n} \tilde{y}_{21}^{n+2-k}\left(V_{j+1}^{k} - V_{j+1}^{k-1}\right)\right.$$

$$\left. + \frac{1}{2}\tilde{y}_{22}^{n+1} V_{j+2}^{1} + \sum_{k=2}^{n} \tilde{y}_{22}^{n+2-k}\left(V_{j+2}^{k} - V_{j+2}^{k-1}\right) - I_{j+2}^{n+1}\right] \tag{38c}$$

where $V = V_g + V_d$.

The equations (36b), (35b), and (35c) give the left end boundary conditions of the linear section,

$$V_{d_{j+1}}^{n+1}\left(\frac{C_d}{\Delta t} + \frac{G_d}{2} - \frac{\tilde{y}_{11}^{1}}{\Delta z}\right) - V_{g_{j+1}}^{n+1}\frac{\tilde{y}_{11}^{1}}{\Delta z}$$

$$= V_{d_{j+1}}^{n}\left(\frac{C_d}{\Delta t} + \frac{G_d}{2} - \frac{\tilde{y}_{11}^{1}}{\Delta z}\right) - V_{g_{j+1}}^{n}\frac{\tilde{y}_{11}^{1}}{\Delta z} + S_1 \tag{39a}$$

and

$$V_{g_{j+1}}^{n+1}\left(\frac{C_g}{\Delta t} + \frac{G_g}{2} - \frac{\tilde{y}_{11}^{1}}{\Delta z}\right) - V_{d_{j+1}}^{n+1}\frac{\tilde{y}_{11}^{1}}{\Delta z}$$

$$= V_{g_{j+1}}^{n}\left(\frac{C_g}{\Delta t} - \frac{G_g}{2} - \frac{\tilde{y}_{11}^{1}}{\Delta z}\right) - V_{d_{j+1}}^{n}\frac{\tilde{y}_{11}^{1}}{\Delta z} + S_1 \tag{39b}$$

where

$$S_1 = -\frac{2}{\Delta z}\left[\frac{1}{2}\,\tilde{y}_{12}^{\,n+1}\,V_{j+2}^{1} + \sum_{k=2}^{n}\tilde{y}_{12}^{\,n+2-k}\left(V_{j+2}^{k} - V_{j+2}^{k-1}\right)\right.$$

$$\left. + \frac{1}{2}\,\tilde{y}_{11}^{\,n+1}\,V_{j+1}^{1} + \sum_{k=2}^{n}\tilde{y}_{11}^{\,n+2-k}\left(V_{j+1}^{k} - V_{j+1}^{k-1}\right) - I_{j}^{n+1}\right] \tag{39c}$$

The resulting boundary conditions are simultaneous equations in V_d^{n+1} and V_g^{n+1}

3. COMPARISON WITH BLINE

The comparison problem run by BLINE and BLINE modified by the convolution-integral approach consisted of a 201 meter cable; the linear insert consisted of the center one meter. The other parameters are shown in in table 1.

TABLE 1

r_1	Core radius	2.5×10^{-2}m
r_2	Dielectric radius	2.627×10^{-2}m
ε_s	Soil dielectric constant	$10\ \varepsilon_0$ farads/meter
σ_s	Soil conductivity	10^{-3} mhos/meter
ε_d	Sheath dielectric constant	$2.3\ \varepsilon_0$ farads/meter

The observation points used in both analyses are at the two ends of the cable. The line was pulsed by a voltage excitation at the left end of the line with

$$V(t) = 1.0 - e^{-10^7 t} \tag{40}$$

Figure 4 shows the total voltage as a function of time predicted by BLINE and BLINE with the modification of the 1 meter linear section.

Figure 5 compares the total voltage at the end of the cable opposite from the source.

The voltage comparison at the source end of the cable shows that no reflection occurs at the interface between the linear and difference-equation described sections. The voltages at the far end of the cable show a lower amplitude after the pulse travels through the linear section when compared to the unmodified BLINE predictions; the cause of this discrepancy is presently unknown.

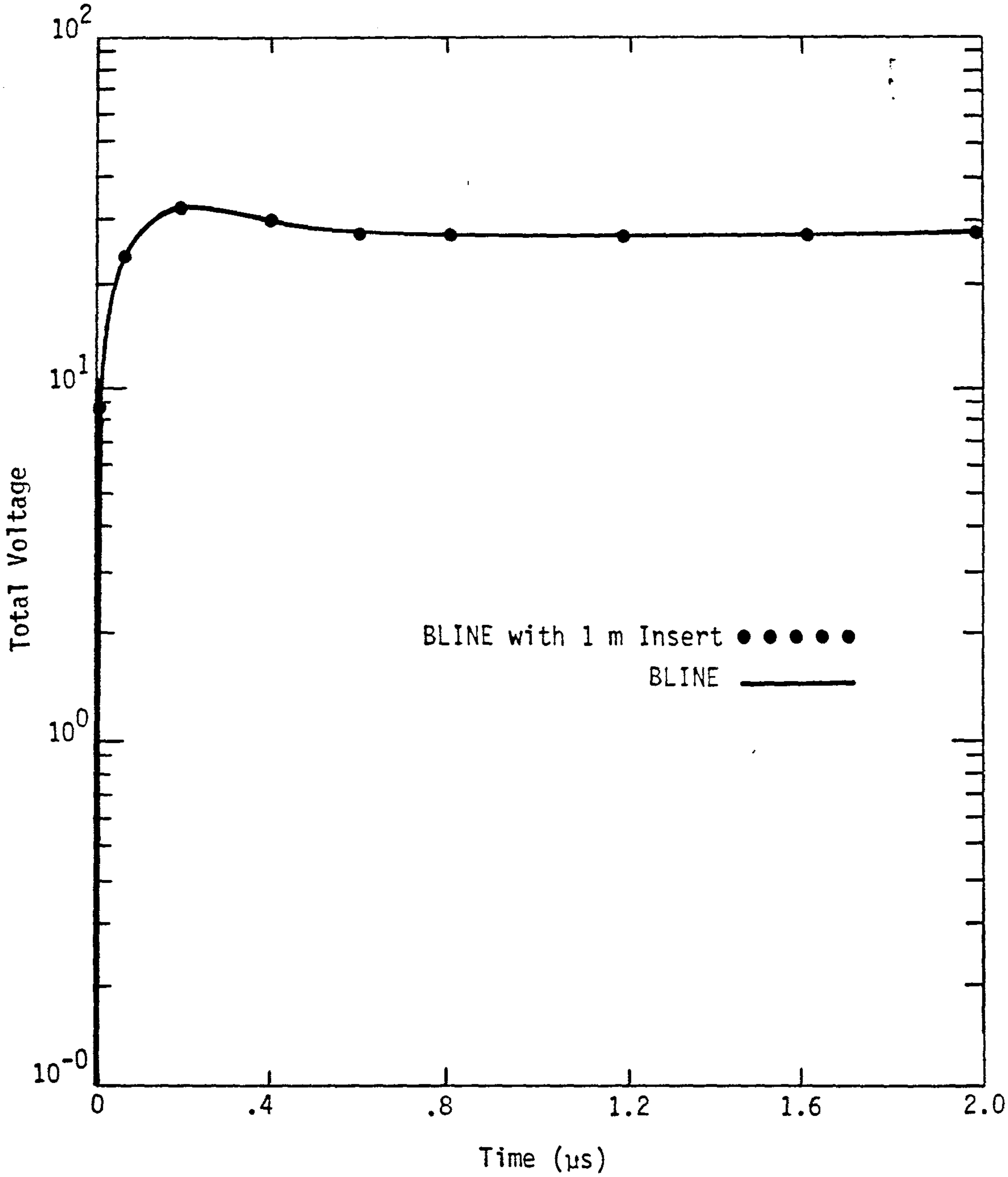


Figure 4. Predicted Voltage at the Source End of the Cable.

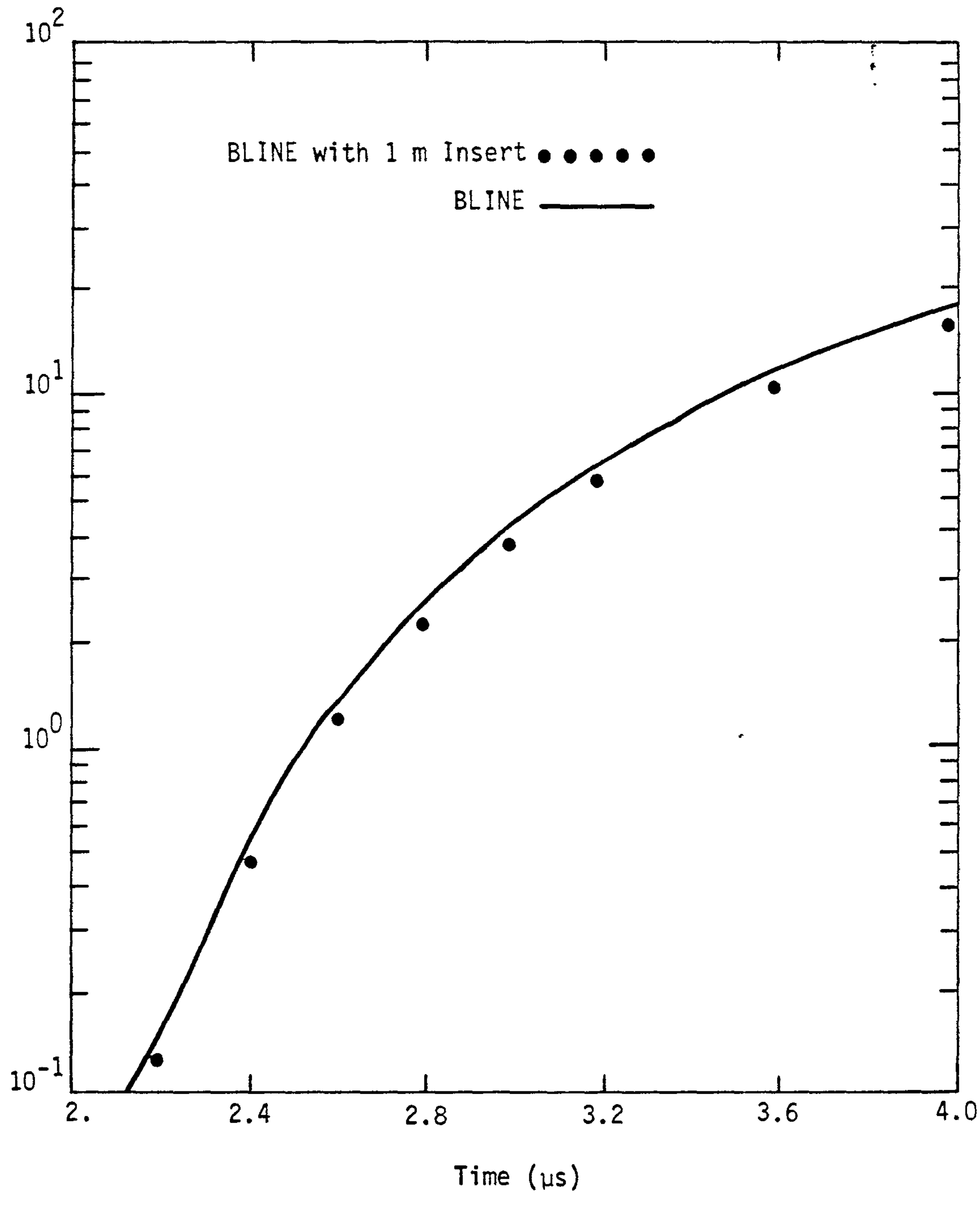


Figure 5. Predicted Voltage at the Load End of the Cable.

SECTION IV

LINEAR SECTION IN A COAXIAL LINE

1. LIMITATIONS OF THE APPLICATION

The reason for the insertion of a linear section into the analysis is to save the computer time necessary to propagate the signal in the linear section. In this region, the time domain analysis is not necessary since no breakdowns are expected; parameters linking the voltages and currents at the ends of the linear section are all that are necessary. Also, since the prediction of secondary breakdown phenomena is desired, one actually needs only the signal propagating from the source through the linear section. In the region of secondary breakdown, this signal is used to drive the nonlinear section and, analyzed by the time domain code, can reflect from the end of the line causing nonlinearities. This reflected signal, returning to the source region, cannot reasonably be expected to traverse the entire cable and add nonlinearities to the primary breakdown region; in this analysis its presence in the linear region is ignored.

Examining only the first pulse from the source end of the line, the current and voltage relationships from equation (25c) are

$$I(\ell,s) = -Y_o\, V(\ell,s) + 2e^{-\gamma\ell}\, Y_o\, V(z_o,s) \tag{41a}$$

$$I(z_o,s) = Y_o\, V(z_o,s) \tag{41b}$$

This form is used since all subsequent admittance terms in equation (25c) represent delay lengths associated with signals propagating toward the source and their reflections. The terms of the admittance matrix which are present are: the self admittance for both $I(\ell,s)$ and $I(z_o,s)$ and the transfer admittance delayed by one line length for $I(\ell,s)$.

The analysis of a coaxial line was chosen as the next step in order to introduce losses into the linear section via a nonzero resistance of the center conductor. The shield was chosen to have a zero resistance so the entire system would be represented by a single voltage; ordinarily, the calculation of two voltages are necessary, one across the dielectric and one across the soil. Inserting a large soil conductivity into the time domain code, resulted in the approximation of a coaxial cable with no shield resistance and no voltage drop across the soil.

Constant R, L, G, and C (section I.3) parameters were also used in the analysis to allow the subsequent steps in the development of the procedure to be more easily understood.

2. PRONY REPRESENTATION OF THE ADMITTANCES

The representation of Y_o by an impulse term and decaying sinusoids was shown in section I.3. The procedure to find the representation of the transfer admittance term is identical. First, the impulse must be identified and handled appropriately.

Assuming the coaxial cable contains only a shunt capacitance term, the characteristic admittance and propagation constant are

$$Y_o(s) = \sqrt{\frac{sC}{R+sL}} \tag{42a}$$

$$\gamma(s) = \sqrt{sC(R+sL)} \tag{42b}$$

At high frequencies, the limits of these are

$$\lim_{s\to\infty} Y_o(s) = \sqrt{\frac{C}{L}} \qquad (43a)$$

$$\lim_{s\to\infty} \gamma(s) = \frac{1}{2} R\sqrt{\frac{C}{L}} - s\sqrt{LC} \qquad (43b)$$

Thus the impulses of the admittances are

$$\text{Impulse } (Y_o) = \sqrt{\frac{C}{L}} \qquad (44a)$$

and

$$\text{Impulse } (2e^{-\gamma \ell} Y_o) = 2\sqrt{\frac{C}{L}} \exp\left[-\left(\frac{1}{2} R\sqrt{\frac{C}{L}} - s\sqrt{LC}\right)\ell\right] \qquad (44b)$$

Next, the Prony process is applied to

$$F^{-1}\left[Y_o(s) - \sqrt{\frac{C}{L}}\right] \qquad (45a)$$

and the transfer admittance advanced by one delay length,

$$F^{-1}\left\{\left[2e^{-\gamma \ell} Y_o - 2\sqrt{\frac{C}{L}} \exp\left(-\left\{\frac{1}{2} R\sqrt{\frac{C}{L}} - s\sqrt{LC}\right\}\ell\right)\right]e^{-s\sqrt{LC}\,\ell}\right\} \qquad (45b)$$

It is worthwhile to notice here that the Prony results of the self admittance term are independent of the line length, whereas the transfer admittance term is a function of the line length. The Prony analysis is done on the transfer admittance advanced by one delay length to reduce the number of poles necessary to achieve an adequate fit. The inverse transform of the transfer admittance is near zero until one delay length in time; at that time, the admittance is discontinuous to a finite value after which it decays. This type of time function can be represented by a sum of decaying exponentials, but far fewer poles are required if the discontinuity in the funciton is shifted back to the time origin.

Another difficulty arises in the inverse transform of a discontinuous function; Gibbs' phenomenon is present in the transform resulting in an inaccurate answer at the time origin. Two approaches are used to reduce this effect: the first is to calculate the frequency domain function to frequencies far higher than would appear necessary to resolve the actual behavior, $f \sim 10^{11}$ Hz. This increases the Gibbs' oscillation frequency and decreases the decay time. The second method is to analytically calculate the inverse transform at t = 0. This can be accomplished by taking the limit,

$$\lim_{s \to \infty} sF(s) = f(o^+) \tag{46}$$

where f(t) is the inverse transform of F(s). These limits for the admittances are

$$\lim_{s \to \infty} \left[s \left(-Y_o(s) + \sqrt{\frac{C}{L}} \right) \right] = \frac{R}{2L} \sqrt{\frac{C}{L}} \tag{47a}$$

and

$$\begin{aligned} &\lim_{s \to \infty} \left\{ s \left[-2e^{-\gamma \ell} Y_o(s) + 2\sqrt{\frac{C}{L}} \exp\left[-\left(\frac{1}{2} R \sqrt{\frac{C}{L}} - s\sqrt{LC}\ \ell \right) \right] \right] \right\} \\ &= \exp\left(s\sqrt{LC}\ \ell \right) \left[2\sqrt{\frac{C}{L}} \left(\frac{R}{2L} - \frac{\ell}{8} \left(\frac{R}{L} \right)^2 \sqrt{LC}\ \exp\left(-\frac{1}{2} R \sqrt{\frac{C}{L}}\ \ell \right) \right] \end{aligned} \tag{47b}$$

Again, the delay time, $\sqrt{LC}\ \ell$ shows in the limit of the transfer admittance.

Using the limit as s goes to infinity of sF(s) as the value of f(o), the two admittances were approximated by the Prony program as

$$F^{-1} \left[\sqrt{\frac{C}{L}} - Y_o \right] \approx a_1 e^{s_1 t} + a_2 e^{s_2 t} \tag{48a}$$

or

$$Y_o(s) \approx \sqrt{\frac{C}{L}} - \frac{a_1}{s-s_1} - \frac{a_2}{s-s_2} \tag{48b}$$

where $a_1 = 1.82602 \times 10^3$ $a_2 = 8.29867 \times 10^3$

$s_1 = -5.7362 \times 10^4$ $s_2 = -2.00559 \times 10^5$

and

$$F^{-1}\left\{-2e^{-\gamma\ell} Y_o(s) + 2\sqrt{\frac{C}{L}} \exp\left[-\left(\frac{1}{2} R\sqrt{\frac{C}{L}} - s\sqrt{LC}\right)\ell\right]\right\} \approx a_1 e^{s_1 t} + a_2 e^{s_2 t} \tag{49a}$$

or

$$2e^{-\gamma\ell} Y_o(s) = 2\sqrt{\frac{C}{L}} \exp\left[-\left(\frac{1}{2} R\sqrt{\frac{C}{L}} - s\sqrt{LC}\right)\ell\right] - \frac{a_1}{s-s_1} - \frac{a_2}{s-s_2} \tag{49b}$$

where

$a_1 = 1.9269 \times 10^3$ $a_2 = 5.98615 \times 10^3$

$s_1 = -4.68392 \times 10^4$ $s_2 = -1.93015 \times 10^5$.

The values of the two impulse terms were

$$\text{Impulse } (Y_o) = 8.6737 \times 10^{-2}$$

and

$$\text{Impulse}\left[2e^{-\gamma\ell} Y_o(s)\right] = 9.6134 \times 10^{-2}$$

In both cases, a fit using two poles and residues achieved an error of less than one percent at all except the last two time points.

The admittances can now be represented by ratios of polynomials in s,

$$Y_o(s) = \frac{N_{11}s^2 + N_{12}s + N_{13}}{D_{11}s^2 + D_{12}s + D_{13}} \tag{50a}$$

and

$$2e^{-\gamma\ell}Y_o(s) = \frac{N_{21}s^2 + N_{22}s + N_{23}}{D_{21}s^2 + D_{22}s + D_{23}} \tag{50b}$$

where the numerator and denominator in both cases are of the same order in s because of the impulses present (section II.2). The transfer admittance is understood to be delayed in time by one line length.

With the ratios of polynomials known for the two admittances, the state equations can be derived. From section II.2, the state equations and initial conditions are

$$x_{11}(0) = 0, \qquad x_{12}(0) = 0$$

$$x_{21}(0) = 0, \qquad x_{22}(0) = 0$$

$$\dot{x}_{11} = -\frac{D_{12}}{D_{11}} x_{11} + x_{12} + \left(N_{12} - N_{11}\frac{D_{12}}{D_{11}}\right) v \tag{60a}$$

$$\dot{x}_{12} = -\frac{D_{13}}{D_{11}} x_{11} + \left(N_{13} - N_{11}\frac{D_{13}}{D_{11}}\right) v \tag{60b}$$

$$\dot{x}_{21} = -\frac{D_{22}}{D_{21}} x_{21} + x_{22} + \left(N_{22} - N_{21}\frac{D_{22}}{D_{21}}\right) v \tag{60c}$$

$$\dot{x}_{22} = -\frac{D_{23}}{D_{21}} x_{21} + \left(N_{23} - N_{21}\frac{D_{23}}{D_{21}}\right) v \tag{60d}$$

where the first subscript refers to the self admittance (1) or the transfer admittance (2). The current as a function of the voltage is now given by

$$i = \frac{x_{11}}{D_{11}} + \frac{N_{11}}{D_{11}}\ v \qquad \text{(self admittance)} \tag{61a}$$

for the self admittance part and

$$i = \frac{x_{21}}{D_{21}} + \frac{N_{21}}{D_{21}}\ \tilde{v} \qquad \text{(transfer admittance)} \tag{61b}$$

for the transfer admittance portion. The quantity $\tilde{v}$ represents the voltage delayed by one propagation length of the linear section. Because there is interest in only the signal propagating from the source in the linear portion, the voltage-current relationship is given by only equation (61a) for the end of the linear section nearest the source. At the other end of the linear section, the voltage-current relationship is represented by the sum of the two current contributions.

3. ADAPTATION TO BLINE

The reason for the formulation of the equations giving current as a function of voltage was compatibility with BLINE. The boundary conditions in BLINE always employ a current as a function of voltage.

The solution of the state equations is easily realizable in BLINE using the formulation

$$x(t+\Delta t) = x(t) + \dot{x}(t)\Delta t. \tag{62a}$$

This gives

$$\begin{aligned} x_{11}(t+\Delta t) = {} & x_{11}(t)\left(1 - \frac{D_{12}}{D_{11}}\ \Delta t\right) + x_{12}(t)\ \Delta t \\ & + \left(N_{12} - N_{11}\ \frac{D_{12}}{D_{11}}\right)\Delta t\ \ v(t) \end{aligned} \tag{62b}$$

$$x_{12}(t+\Delta t) = x_{12}(t) - \frac{D_{13}}{D_{11}} x_{11}(t)\Delta t + \left(N_{13} - N_{11} \frac{D_{13}}{D_{11}}\right)\Delta t \; v(t) \tag{62c}$$

$$x_{21}(t+\Delta t) = x_{21}(t) \left(1 - \frac{D_{22}}{D_{21}} \Delta t\right) + x_{22}(t)\Delta t + \left(N_{22} - N_{21} \frac{D_{22}}{D_{21}}\right)\Delta t \; v(t) \tag{62d}$$

and

$$x_{22}(t+\Delta t) = x_{22}(t) - \frac{D_{23}}{D_{21}} x_{21}(t)\Delta t + \left(N_{23} - N_{21} \frac{D_{23}}{D_{21}}\right)\Delta t \; v(t) \tag{62e}$$

The current is now found from

$$i(t+\Delta t) = \frac{1}{D_{11}} x_{11}(t+\Delta t) + \frac{N_{11}}{D_{11}} v(t+\Delta t) \tag{62f}$$

for the self admittance current contribution and

$$i(t+\Delta t) = \frac{1}{D_{21}} x_{21}(t+\Delta t) + \frac{N_{21}}{D_{21}} \tilde{v}(t+\Delta t) \tag{62g}$$

for the transfer admittance current contribution.

In the case of a coaxial cable, the voltage across the shield is zero and only the dielectric voltage remains. The boundary condition present in BLINE can then be derived from equation (28b) of the transmission line equations,

$$\frac{\partial i}{\partial z} = -C_d \frac{\partial v_d}{\partial t} - G_d v_d \tag{63}$$

An examination of figure 3, which shows the relative placement of the current and voltage cells at the source end of the linear section in BLINE, indicates that equation (63) can be differenced as

$$\frac{2}{\Delta x}\left(I_j^{n+1} - I_{j+\frac{1}{2}}^{n+1}\right) = -\frac{C_d}{\Delta t}\left(V_{j+1}^{n+1} - V_{j+1}^{n}\right) - \frac{G_d}{2}\left(V_{j+1}^{n+1} + V_{j+1}^{n}\right) \tag{64}$$

where the superscript designates the time index, the subscript the position index and

$$I_{j+\frac{1}{2}}^{n+1} = \frac{1}{D_{11}} x_{11}^{n+1} + \frac{N_{11}}{D_{11}} V_{j+1}^{n+1} \tag{65}$$

is the current-voltage relationship at the source end of the linear section. This gives the voltage at the source end of the linear section as

$$V_{j+1}^{n+1}\left(\frac{C_d}{\Delta t} + \frac{G_d}{2} + \frac{2}{\Delta x}\frac{N_{11}}{D_{11}}\right) = V_{j+1}^{n}\left(\frac{C_d}{\Delta t} - \frac{G_d}{2}\right) + \frac{2}{\Delta x}\left(I_j^{n+1} - \frac{1}{D_{11}} x_{11}^{n+1}\right) \tag{66}$$

A similar analysis at the far end of the linear section shows

$$V_{j+2}^{n+1}\left(\frac{C_d}{\Delta t} + \frac{G_d}{2} - \frac{2}{\Delta x}\frac{N_{11}}{D_{11}}\right) = V_{j+2}^{n}\left(\frac{C_d}{\Delta t} - \frac{G_d}{2}\right) - \frac{2}{\Delta x}\left(I_{j+2}^{n+1} - \frac{1}{D_{11}} x_{11}^{n+1} - \frac{1}{D_{21}} x_{21}^{n+1} - \frac{N_{21}}{D_{21}} \tilde{V}_{j+1}^{n+1}\right) \tag{67}$$

which uses a current at the far end of the linear section of

$$I_{j+2-\frac{1}{2}}^{n+1} = \frac{1}{D_{11}} x_{11}^{n+1} + \frac{N_{11}}{D_{11}} V_{j+2}^{n+1} + \frac{1}{D_{21}} x_{21}^{n+1} + \frac{N_{21}}{D_{21}} \tilde{V}_{j+1}^{n+1} \tag{68}$$

The voltage term $\tilde{V}$ is the voltage occuring at the source end of the linear section. $\tilde{V}$ has been time delayed by the length of time it takes the signal to propagate along the linear section of the cable.

These voltage and current relations were then put into BLINE relating two adjacent voltages. The effect of this was to separate the two cells by the length of the inserted linear section.

4. COMPARISON WITH BLINE

To compare the approximation of a linear section by the differential equations, the following problem was run by the new method and BLINE. A coaxial cable was chosen with the parameters

$$R = .013611 \text{ ohms/meter}$$
$$L = 5.8327\times10^{-8} \text{ henries/meter}$$
$$G = 0.0$$
$$C = 4.3878\times10^{-10} \text{ farads/meter.}$$

These were derived from the physical parameters

$$r_1 = 6.35\times10^{-3} \text{ m}$$
$$r_2 = 8.5\times10^{-3} \text{ m}$$
$$\sigma_c = 5.8\times10^{5} \text{ mhos/meter}$$
$$\varepsilon_d = 2.3\ \varepsilon_o \text{ farads/meter}$$

where r_1 and r_2 are the radius of the core and dielectric, σ_c is the conductivity of the core, and ε_d is the dielectric constant of the core insulation.

A transmission line of 1.2 km length with these parameters was driven at one end with a triangular voltage pulse of one volt amplitude and 0.5 μsec duration. For the first comparison, the spatial cell size was chosen to be 10 m and observation points were made at distances of 100 m and 1100 m. The section of line represented by the differential equations was 1 km in length and was inserted between two 100 m sections. The observation points were taken at the two ends of the 1 km insert. The representation of this section used the poles and residues discussed in section III.3.

Figure 6 shows two voltages: one predicted by BLINE 100 m from the source end and the other predicted by the modified BLINE code at the same place. Figure 7 shows the voltages predicted by BLINE and the modified BLINE code at the junction of the insert 1.1 km from the source. It can be seen that this agreement is not good when BLINE uses a cell spacing of 10 m, but improves when the spacing is reduced to 5 m.

The discrepancy is probably due to the superior description of the transfer admittance by the differential-equation technique; agreement probably occurs in the limit as the cell spacing goes to zero. This is shown by the 5 m cell spacing resulting in a pulse more closely resembling the pulse of the differential-equation approach.

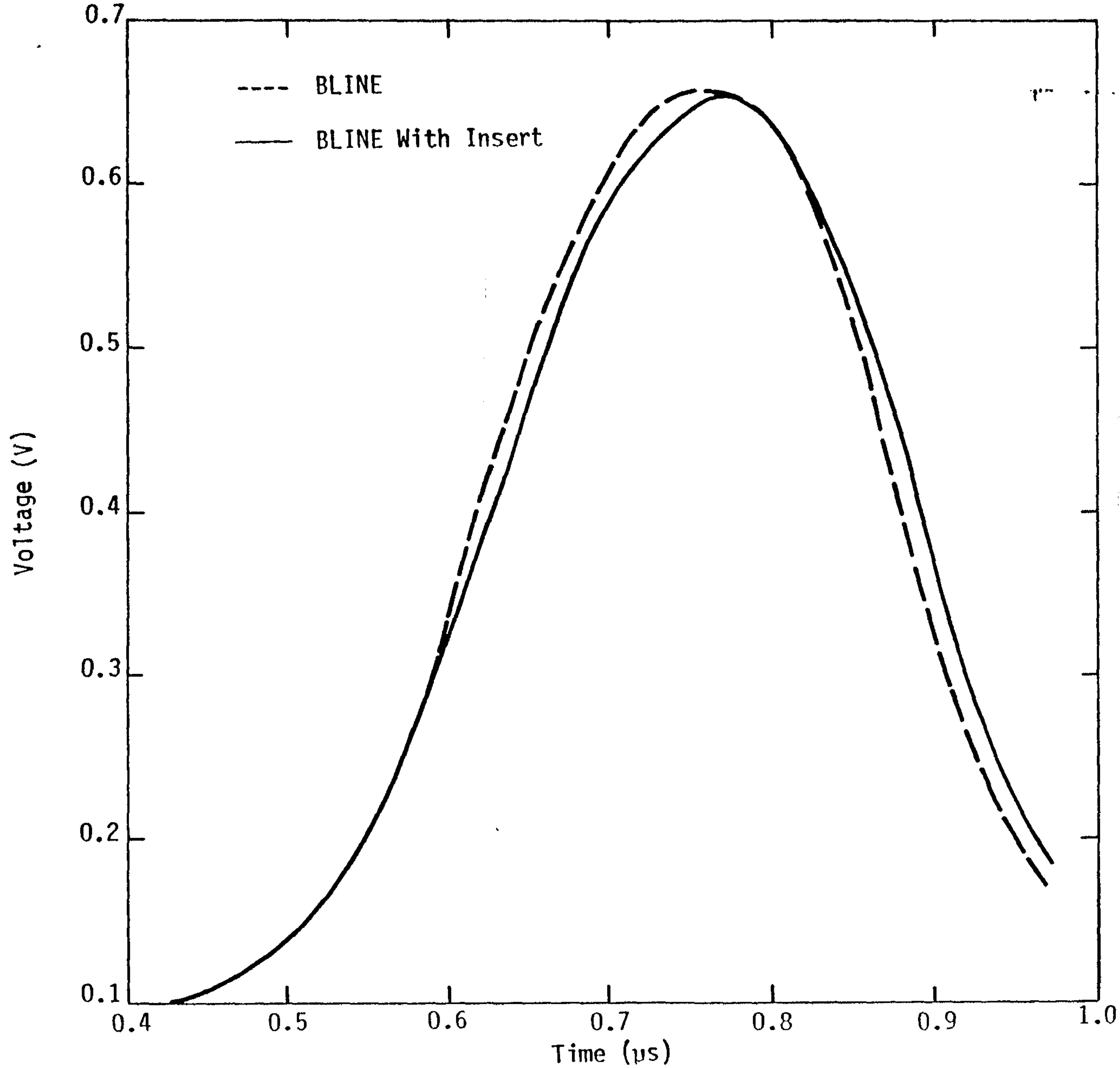


Figure 6. Comparison of BLINE and BLINE With 1 KM Insert at the Source End of the Insert.

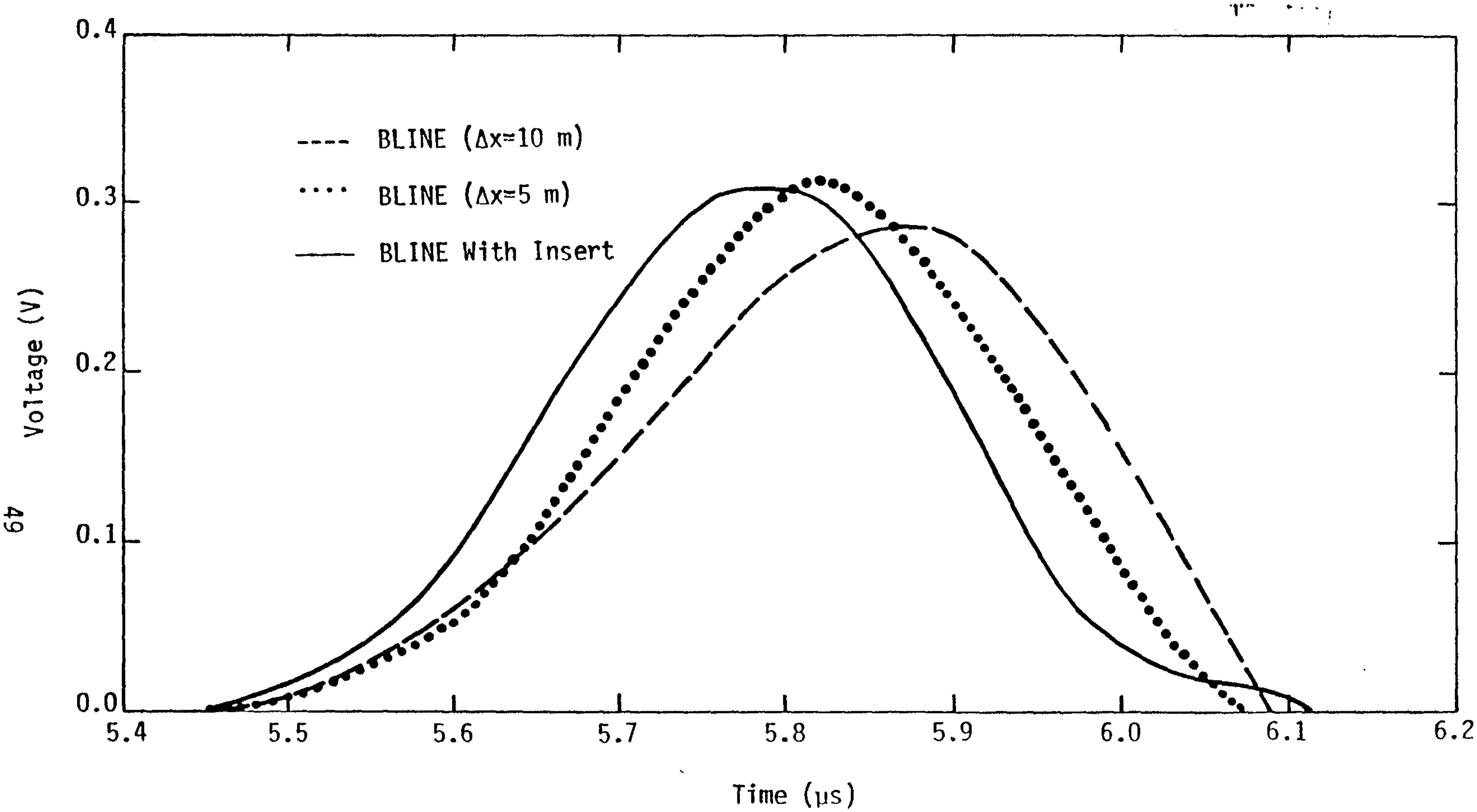


Figure 7. Comparison of BLINE With Two Cell Sizes and BLINE With 1 Km Insert at the Far End of The Insert.

SECTION V

LINEAR SECTION IN A BURIED CABLE

To model a realistic situation of cable breakdown, two more effects must be included. The first is to add a voltage drop across the soil so the entire voltage is not across the cable dielectric. The second addition is the effect of a distributed source along the linear insert. With these additions, the regions of primary and secondary breakdown separated by a linear length of cable can be described by the differential-equation technique and compared to the predictions of the unmodified time-domain code, BLINE.

1. DISTRIBUTED SOURCES

In the formulation described in section II, the effect of the distributed sources is to add currents at the ends of the linear insert (equation (7)). The solution of the differential equations then represents the sum of the currents induced by the distributed sources and the currents resulting from signals propagating to the linear section (equation (10)).

In the frequency domain, it is convenient to represent the E-field excitation of equation (2) as

$$e(z,s) = \sum_{n} A_n(s) e^{i\alpha_n(s)z} \tag{69}$$

The current sources will be assumed to be zero.

For a general E-field, as a function of time, the Fourier transform has to be accomplished numerically to achieve the frequency domain representation of equation (69). However, if the field as a time function is a sum of double exponentials, the Fourier transform can be found analytically; in this case,

$$E(z,t) = \sum_n c_n e^{-g_n(z-R_o)} \left(e^{-a_n(t-\tau_n)} - e^{-b_n(t-\tau_n)} \right) \tag{70}$$

where c_n is the amplitude, g_n is the spatial decay constant, τ_n is the time delay, R_o is the data range, and a_n and b_n are the temporal decay constants of the double exponential. The Fourier transform of this field gives

$$e(z,s) = \sum_n c_n e^{g_n R_o} e^{z(\frac{s}{c} - g_n)} e^{s\tau_n} \left(\frac{1}{a_n - s} - \frac{1}{b_n - s} \right) \tag{71}$$

which gives

$$A_n(s) = c_n e^{g_n R_o} e^{s\tau_n} \left(\frac{1}{a_n - s} - \frac{1}{b_n - s} \right) \tag{72a}$$

and

$$\alpha_n(s) = \frac{s}{ic} + ig_n \tag{72b}$$

by comparision with equation (69).

With the distributed source in the form of equation (69), the integration over the length of the linear insert can be easily accomplished; from equation (7), the integrals are

$$G_1(z_o,\ell,s) = \int_{z_o}^{z_o+\ell} \frac{e^{\gamma(z_o+\ell-\zeta)} + e^{-\gamma(z_o+\ell-\zeta)}}{2} \sum_n A_n e^{i\alpha_n \zeta} \, d\zeta \tag{73a}$$

or

$$G_1(z_o,\ell,s) = \sum_n A_n \frac{e^{i\alpha_n z_o}}{\alpha_n^2 + \gamma^2} \Big[-i\alpha_n e^{i\alpha_n \ell} + i\alpha_n \cosh(\gamma\ell) + \gamma \sinh(\gamma\ell) \Big] \tag{73b}$$

and

$$G_2(z_o,\ell,s) = \int_{z_o}^{z_o+\ell} Y_o \frac{e^{-\gamma(z_o+\ell-\zeta)} + e^{\gamma(z_o+\ell-\zeta)}}{2} \sum_n A_n e^{i\alpha_n\zeta} \, d\zeta \quad (74a)$$

or

$$G_2(z_o,\ell,s) = -\sum_n A_n Y_o \frac{e^{i\alpha_n z_o}}{\alpha_n^2 + \gamma^2} \Big[-\gamma e^{i\alpha_n\ell} + \gamma\cosh(\gamma\ell) + i\alpha_n \sinh(\gamma\ell)\Big] \quad (74b)$$

The currents which must be added to account for the distributed sources along the linear section are then

$$I_1(z_o,\ell,s) = -Y_o \left\{ \frac{e^{\gamma\ell} + e^{-\gamma\ell}}{e^{-\gamma\ell} - e^{\gamma\ell}} \left[\sum_n A_n \frac{e^{i\alpha_n z_o}}{\alpha_n^2+\gamma^2} \Big[-i\alpha_n e^{i\alpha_n\ell} + i\alpha_n\cosh(\gamma\ell) + \gamma\sinh(\gamma\ell)\Big]\right] + \sum_n A_n \frac{e^{i\alpha_n z_o}}{\alpha_n^2+\gamma^2}\Big[-\gamma e^{i\alpha_n\ell} + \gamma\cosh(\gamma\ell) + i\alpha_n\sinh(\gamma\ell)\Big]\right\} \quad (75a)$$

and

$$I_2(z_o,\ell,s) = \frac{2Y_o}{e^{-\gamma\ell} - e^{\gamma\ell}} \sum_n A_n \frac{e^{i\alpha_n z_o}}{\alpha_n^2+\gamma^2}\Big[-i\alpha_n e^{i\alpha_n\ell} + i\alpha_n\cosh(\gamma\ell) + \gamma\sinh(\gamma\ell)\Big] \quad (75b)$$

At this time it is useful to display the sources in a manner similar to the admittance formulation in equation (25); the terms as functions of the delay lengths are

$$I_1(z_o,\ell,s) = \sum_n \frac{A_n Y_o}{\alpha_n^2 + \gamma^2} \Bigg\{ e^{i\alpha_n(z_o+\ell)} \Big[\gamma - i\alpha_n \left(1 + 2e^{-2\gamma\ell} + 2e^{-4\gamma\ell} + \cdots\right)\Big]$$

$$+ 2i\alpha_n e^{i\alpha_n z_o - \gamma\ell} \left(1 + e^{-2\gamma\ell} + e^{-4\gamma\ell} + \cdots\right)\Bigg\} \tag{76a}$$

and

$$I_2(z_o,\ell,s) = \sum_n \frac{A_n Y_o e^{i\alpha_n z_o}}{\alpha_n^2 + \gamma^2} \Big[\gamma + i\alpha_n \left(1 + 2e^{-2\gamma\ell} + 2e^{-4\gamma\ell} + \cdots\right)$$

$$-2\alpha_n e^{(i\alpha_n - \gamma)\ell} \left(1 + 2e^{-2\gamma\ell} + 2e^{-4\gamma\ell} + \cdots\right) \tag{76b}$$

These show that the first term of I_1, the current addition at the end of the linear section farthest from the source, has a delay of $i\alpha_n(z_o+\ell)$, the light travel time from the source to the far end of the linear section. The next term of I_1 has a delay of $i\alpha_n z_o - \gamma\ell$, the light travel time to the near end of the linear section plus the propagation time along the linear section. All other terms are equivalent to reflections at the ends of the linear sections with subsequent additional delays of $2n\gamma\ell$.

The first term of I_2, the current addition at the end of the linear section closest to the source, has a delay of $i\alpha_n z_o$, the light travel time from the source to that end of the linear section. The next term of I_1 has an additional delay of $(i\alpha_n - \gamma)\ell$; this represents the delay of light traveling from the near to the far end of the linear section plus the propagation delay along the cable back to the near end.

As in section IV, the interest in predicting secondary breakdown only necessitates the use of those terms representing signals propagating away from the source. With this restriction, the sources can be approximated as

$$I_1(z_o,\ell,s) \approx \sum_n \frac{A_n Y_o}{\alpha_n^2 + \gamma^2} \left[e^{i\alpha_n(z_o+\ell)} (\gamma - i\alpha_n) + 2i\alpha_n e^{i\alpha z_o - \gamma\ell} \right] \quad (77a)$$

and

$$I_2(z_o,\ell,s) \approx \sum_n \frac{A_n Y_o e^{i\alpha_n z_o}}{\alpha_n^2 + \gamma^2} (\gamma + i\alpha_n) \quad (77b)$$

2. ADAPTATION TO BLINE

Two further modifications to the code used in the analysis of the previous section were necessary. The first involved the addition of the source terms I_1 and I_2 into the boundary conditions at the ends of the linear section. The second was the calculation of the dielectric and soil voltages at the ends of the linear section. The voltage propagated through the linear section is the total voltage, the sum of the dielectric and soil voltages, which necessitates the separation of the two at the boundaries of the linear and nonlinear sections.

Because of time limitations, a case was chosen where the soil conductivity was zero; the ultimate reality of this case was accomplished by choosing lumped parameters R, L, and C identical to those of a realistic case with frequency dependent parameters at a given frequency; the parameters were then assumed to be constant. The reason for choosing the soil conductance equal to zero was to allow a simple relationship between the dielectric and soil voltages. From figure 2, it can be seen that this relationship is

$$V_g = \frac{C_d}{C_g} V \quad (78a)$$

and

$$V_d = \frac{C_g}{C_g + C_d} V \quad (78b)$$

where V is the total voltage and the d and g subscripts refer to the dielectric and ground.

The capacitance across the ground, C_g, was calculated by assuming that the total capacitance, C, of the realistic calculation was the result of the soil and dielectric capacitances in series.

The boundary conditions at the junction of the linear and nonlinear sections are modifications of equations (66) and (67) including the current source and voltage fraction V_d/V. The dielectric and ground voltages were found from

$$V_{d_{j+1}}^{n+1}\left(\frac{C_d}{\Delta t}+\frac{2}{\Delta x}\ \frac{N_{11}}{D_{11}\ VF}\right)=V_{d_{j+1}}^{n}\left(\frac{C_d}{\Delta t}\right)+\frac{2}{\Delta x}\left(I_j^{n+1}-\frac{X_{11}^{n+1}}{D_{11}}-I_2^{n+1}-S_2^{n+1}\right) \tag{79a}$$

at the end of the linear section nearest the source and

$$V_{d_{j+2}}^{n+1}\left(\frac{C_d}{\Delta t}-\frac{2}{\Delta x}\ \frac{N_{11}}{D_{11}\ VF}\right)=V_{j+2}^{n}\left(\frac{C_d}{\Delta t}\right)-\frac{2}{\Delta x}\left(I_{j+2}^{n+1}-\frac{X_{11}^{n+1}}{D_{11}}-\frac{X_{21}^{n+1}}{D_{21}}-\frac{N_{21}}{D_{21}\ VF}\tilde{V}_{j+1}^{n+1}-I_1^{n+1}-S_1^{n+1}\right) \tag{79b}$$

at the other end of the linear section. In both cases

$$VF=\frac{C_g}{C_d+C_g} \tag{80a}$$

$$V_g=\frac{1-VF}{VF}V_d \tag{80b}$$

The source terms, S_1 and S_2, are the inverse Fourier transforms of equations (77a) and (77b). All other terms are identical to those of equations (66) and (67).

3. COMPARISON WITH BLINE

The comparison problem to be done by BLINE and the differential equation method was determined by first finding the lumped parameters of a buried cable at a frequency of 1.45×10^5 Hz. The frequency chosen was an estimate of the frequency of the pulse which is responsible for the generation of secondary breakdown. Even though the conductance was non-zero at this frequency, its effects were ignored. The input parameters were

$$\sigma_c = 5.8 \times 10^7 \text{ mhos/meter}$$

$$r_1 = 6.35 \times 10^{-3} \text{ m}$$

$$r_2 = 8.5 \times 10^{-3} \text{ m}$$

$$\varepsilon_d = 2.3\ \varepsilon_o \text{ farads/m}$$

% water content of the soil = 10%.

At a frequency of 1.45×10^5 Hz, the lumped transmission line parameters were

$$R = 1.4878 \times 10^{-1} \text{ ohms/m}$$

$$L = 1.4906 \times 10^{-6} \text{ henries/m}$$

$$C = 4.3379 \times 10^{-10} \text{ farads/m.}$$

These parameters were then used as constant R, L, and C for the linear section of the line to be represented by the differential-equation technique.

In the sections on either end of this linear region and the entire cable represented by BLINE, the voltage is divided into voltage across the dielectric and voltage across the ground; in this case the parameters are

$$R = 1.4878 \times 10^{-1} \text{ ohms/m}$$

$$L = 1.4906 \times 10^{-6} \text{ henries/m}$$

$$C_d = 4.3878 \times 10^{-10} \text{ farads/m}$$

$$C_g = 3.8175 \times 10^{-8} \text{ farads/m}$$

where

$$C_d = 2\pi \, \varepsilon_d \, / \, \ell n \, (r_1/r_2)$$

and C_g is calculated from

$$\frac{1}{C} = \frac{1}{C_d} + \frac{1}{C_g}$$

Using these parameters and a dielectric breakdown voltage of 10^7 v/m, BLINE was run to find the breakdown regions of a 4 km cable. The network was terminated at the source end by a 10 ohm resistor and at the far end by a 10^4 ohm resistor; this mismatch was purposely chosen to enhance secondary breakdown.

With this input, BLINE predicted a primary breakdown region of 2.6 km and a secondary breakdown region of 60 m, 4 km from the burst.

The region from 2.8 to 3.8 km remained linear. This was chosen as the section to be replaced by a line represented by the differential-equation technique.

To obtain the differential equations, the Prony analysis was done on the inverse Fourier transforms of the self and transfer admittances. As in the case discussed in section IV, the admittances were limited to the terms representing the signals traveling away from the source. The poles, residues, and impulse terms for the self admittance were

$a_1 = 1.63148 \times 10^2$ $\qquad a_2 = 6.88527 \times 10^2$

$s_1 = -2.5697 \times 10^4$ $\qquad s_2 = -8.64688 \times 10^4$

Impulse $= 1.7059 \times 10^{-2}$.

The terms representing the transfer admittance were

$a_1 = 1.14429 \times 10^2$ $\qquad a_2 = 6.04022 \times 10^1$

$s_1 = -8.20423 \times 10^4$ $\qquad s_2 = -1.421511 \times 10^4$

Impulse $= 9.5908 \times 10^{-3}$.

As in the previous case (section IV.2), a satisfactory fit was achieved for both admittances with two poles and residues. Because the breakdown phenomenon involves longer cables, the Prony fits were found to a time of 60 μs.

The computation of the source terms was done in the frequency domain and inverse Fourier transformed at times that could be directly utilized by the time-domain code at the two junction points.

To compare the two processes, observations were made of the dielectric voltage at the two junction points, 2.8 km and 3.8 km along the cable, and at the end of the line where secondary breakdown was expected. Because the primary breakdown region is essentially described by the same coding, no differences in this region were found between the two predictions.

Figure 8 shows the responses predicted at the junction of the primary breakdown region with the linear section of cable. The dotted line (at 70 μs), in this figure and the following two, is the limit to which the Prony approximations of the admittances were made. At larger times, an extrapolation is indicated. In figure 8, the variation between the predicted responses at 80 μsec, is the result of a reflection from the end of the cable; because only signals traveling from the source were handled by the differential equations, the reflection is not seen with this analysis. Figure 9 compares the response predicted by BLINE 3.8 km from the

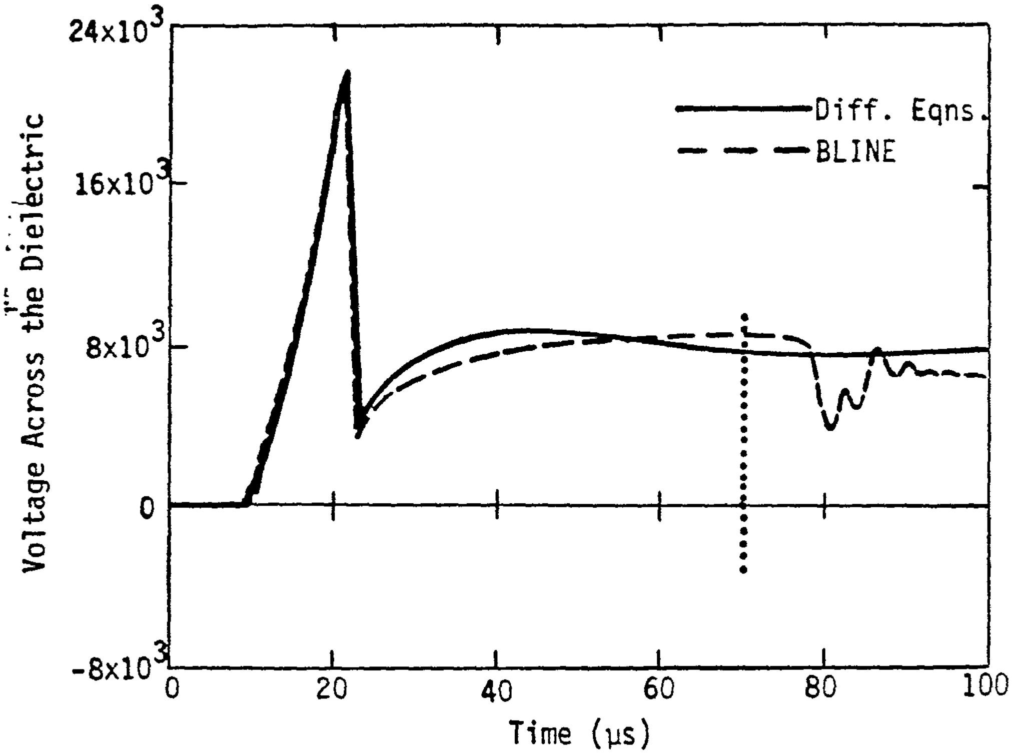


Figure 8. Dielectric Voltages Predicted by BLINE and the Differential-Equation Technique at the Linear-Nonlinear Interface Closest to the Source.

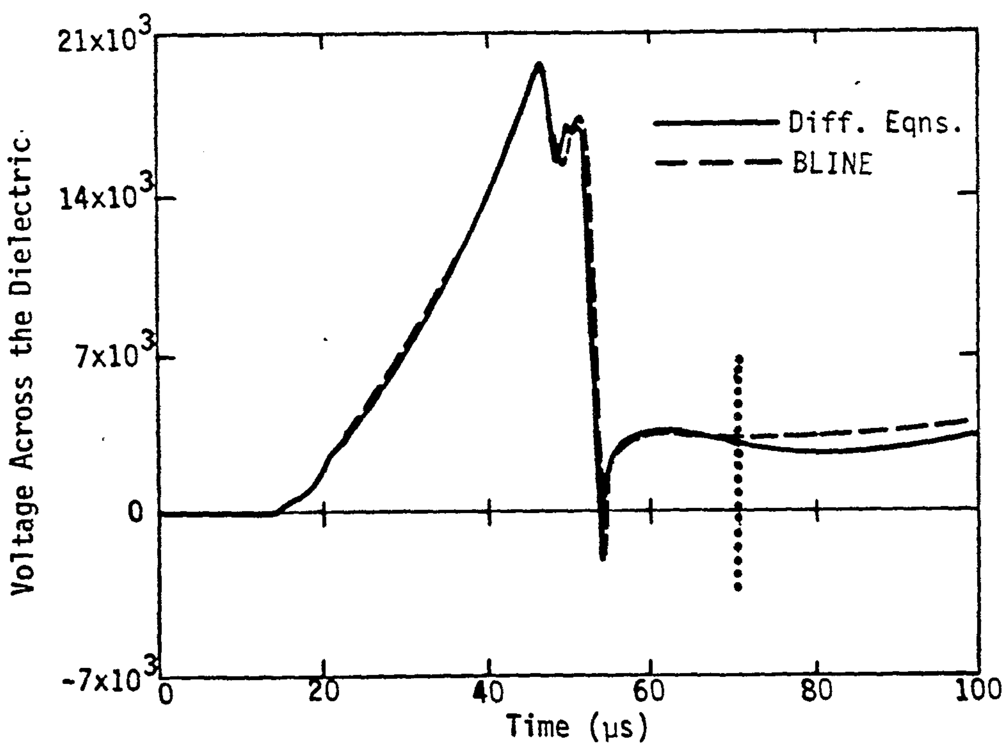


Figure 9. Dielectric Voltages Predicted by BLINE and the Differential-Equation Technique at the Linear-Nonlinear Interface Farthest from the Source.

source with the response generated by the differential equations. The last comparison between the two approaches is shown in figure 10. This shows the predicted responses at the end of the cable 4 km from the source. The oscillations are due to reflections off the end of the cable and the secondary breakdown region.

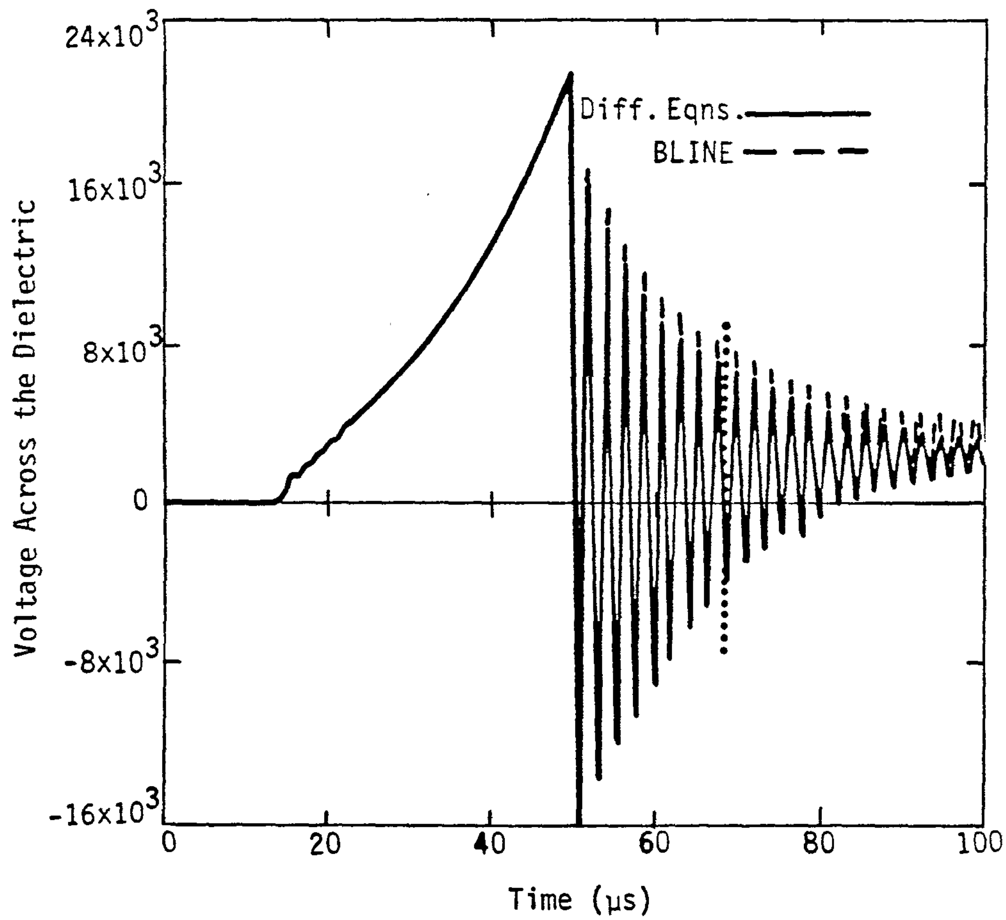


Figure 10. Dielectric Voltages Predicted by BLINE and the Differential Equation Technique at the Cable Termination.

The starting time and extent of secondary breakdown predicted by the two methods are

	Time	Extent
BLINE	49.54 μs	40 m
Diff. Eqn.	49.34 μs	60 m

The earlier time and greater extent of secondary breakdown predicted by the differential equation approach is probably a result of a more accurate representation of the propagation of the pulse through the 1 km linear region. A similar difference in pulse propagation was shown in the coaxial line comparison in section IV.4.

SECTION VI

CONCLUSIONS AND RECOMMENDATIONS

1. CONCLUSIONS

Two methods have been employed to represent a linear section of cable between the regions capable of describing nonlinear behavior.

The two methods employ a relationship between the currents and voltages at the ends of the linear section. The first uses convolution integrals to relate the frequency-domain response of current and voltage and presents the relationship in the time domain. The disadvantage of this approach is the necessity of the calculation of convolution integrals at each time step. The second approach recognizes that the convolution integrals in this case are the solutions of differential equations. Once the differential equations are obtained, each time step requires the addition of one increment to the differential-equation solution rather than the calculation of an entire convolution integral.

The use of convolution integrals has shown the ability of this method to describe the self and transfer admittances of a lossless line and account for the reflections of pulses in the network.

The differential equation method has satisfactorily described the propagation of a pulse through a 1 km section of coaxial cable. In all probability, the pulse exiting the cable described in this manner is closer to reality than the pulse described by a pure time-domain approach.

This procedure has also been used to predict the secondary breakdown occurring in a buried cable 4 km in length. The time and extent of the predicted nonlinearities and the breakdown described by BLINE are essentially in agreement.

2. RECOMMENDATIONS

The use of constant R, L, G, and C parameters to describe the linear section is most likely inadequate for most applications. However, the use of frequency dependent parameters to describe the admittances is difficult because of the lack of well defined impulse behavior. It is recommended that further study be made of the three media problem to determine the high frequency limits of the self and transfer admittances of a linear transmission line.

The technique of using differential equations to describe convolution integrals can be applied to each spatial cell of a nonlinear time domain code to achieve a significant increase in the predictive capability of such a code. This coupled with the previous recommendation would enable the time-domain code to be as accurate as a frequency-domain code but still enable it to describe nonlinearities.

An easily achieved improvement on the present use of differential equations in this report is the addition of soil conductivity to the procedure. A simple relationship between the total voltage propagated by the linear section and the dielectric voltage needed at the junction to the nonlinear section is

$$(G_d+G_g)\, V_d + (C_d+C_g)\, \frac{\partial V_d}{\partial t} = G_g V + C_g \frac{\partial V}{\partial t}$$

where the d and g subscripts refer to the dielectric and ground respectively and V is the total voltage, V_d+V_g.

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