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Interaction Notes

Note 392

July 1978

On Electromagnetic Waves in Chiral Media

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ABSTRACT

The propagation of electromagnetic waves through chiral media, i.e., through composite media consisting of macroscopic chiral objects randomly embedded in a dielectric is analyzed. The peculiar effects that such media have on the polarization properties of the waves are placed in evidence. To demonstrate the physical basis of these effects a specific example, chosen for its analytical simplicity, is worked out from first principles.

Acknowledgement

We would like to thank Dr. J. P. Castillo of the Air Force Weapons Laboratory for his interest in this work. This work was supported by the U.S. Air Force Office of Scientific Research under Grant No. AFOSR-77-3451.

CONTENTS

ILLUSTRATIONS

SECTION I

INTRODUCTION

Chirality is quite common. It occurs not only in nature but also in works of art and architecture as well as in manufactured articles (refs. 1 and 2). In nature we find chirality on a molar scale in, for example, snails, flowers, and vines, and on a molecular scale in such substances as grape sugar and fruit sugar. Moreover, chirality is an operational feature of such manufactured articles as screws, springs, and golf clubs.

Since chirality begets handedness and handedness begets optical activity, it is not surprising that the interaction between an electromagnetic wave and a collection of randomly oriented chiral objects can be such as to rotate the plane of polarization of the wave to the right or to the left depending on the handedness of the objects.

The concept of chirality is not new, nor has it been ignored. Since the early part of the nineteenth century, it has played an increasingly important role in chemistry (refs. 3, 4, and 5), optics (refs. 6 and 7), and elementary particle physics (ref. 8). In 1811 D.F. Arago (ref. 9) discovered that crystals of quartz rotate the plane of polarization of plane polarized light and hence are optically active. Shortly thereafter, circa 1815, J.B. Biot (ref. 10) discovered that this optical activity is not restricted to crystalline solids but appears as well in other media such as oil of turpentine and aqueous solutions of tartaric acid. These discoveries led to the fundamental problem of determining the basic cause of optical activity. In 1848 Louis Pasteur (ref. 3) solved the problem by postulating that the optical activity of a medium is caused by the chirality of its molecules. Thus, Pasteur introduced geometry into chemistry and originated the branch of chemistry we now call stereochemistry. More recently, in 1920 and 1922, K.F. Lindman (refs. 11 and 12) devised a macroscopic (molar) model for the phenomenon by using microwaves instead of light, and wire spirals instead of chiral molecules. The validity of the model was verified a few years later by W.H. Pickering (ref. 13).

To obtain a better understanding of chirality and assay its future role in electrical design, we shall examine in the following pages the interaction between electromagnetic waves and chiral objects. In particular, we shall study the case of a composite medium consisting of randomly oriented chiral conductors embedded in a dielectric.

This work constitutes one aspect of the general problem to uncover and exploit the symmetry properties of the electromagnetic field and of the structures with which it interacts (ref. 14). The importance of symmetry to the response of general scattering problems such as encountered in the nuclear electromagnetic pulse (EMP) is becoming clearer. Geometrical symmetry is useful for reducing the size of numerical computations in scattering problems by decomposing the problem into "smaller" parts. The modal expansions for general linear scatterers which have been recently developed, i.e., the singularity expansion method (SEM) and the eigenmode expansion method (EEM) [20], have the modes divided into separate sets based on the symmetry of the modes resulting from the symmetry of the scatterer, e.g., as in the simple case of a symmetry plane [14]. Symmetry then should be regarded as one of the important areas of future research in electromagnetic theory and computation [20]. We would like to thank C.E. Baum for some interesting discussion concerning this general application of symmetry.

SECTION II

TWO CONJECTURES ON CHIRAL OBJECTS

Chirality is a purely geometric notion which refers to the lack of symmetry of an object. By definition, an object is chiral if it cannot be brought into congruence with its mirror image by translation and rotation. An object that is not chiral is said to be achiral. Thus all objects are either chiral or achiral. Some chiral objects occur naturally in two versions related to each other as a chiral object and its mirror image. Objects so related are said to be enantiomorphs of each other.

A chiral object has the property of handedness; it must be either left-handed or right-handed. If a chiral object is left- (right-) handed, its enantiomorph is right- (left-) handed. For example, if the chiral object is a left- (right-) handed helix, its enantiomorph is a right- (left-) handed helix.

The handedness of helices was made clear by Lindman's and Pickering's experimental results which showed that a collection of randomly oriented left-handed helices would rotate the plane of polarization of a linearly polarized microwave one way but that a collection of randomly oriented right-handed helices would rotate the plane of polarization the opposite way.

Assuming that this relation between the handedness of the helices and the sense of rotation of the microwave is not peculiar only to helices but is a property of all chiral objects and their enantiomorphs, we are led to the following conjecture: Any medium composed of randomly oriented equivalent (simple-connected) chiral objects will rotate the plane of polarization one way, say, to the left, while a medium composed of the enantiomorphs of these objects will rotate the plane of polarization the opposite way, i.e., to the right.

In figure 1 we see common examples of chiral objects: a helix, a Möbius strip, an irregular tetrahedron, and a glove. On one side of the figure is the chiral object and on the other is its enantiomorph.

A type of (multiply-connected) chiral object that has recently attracted considerable attention is the wire braid. The theory of braids is a developing branch of topology (refs. 15 and 16) and a study of how an electromagnetic wave interacts with a braid may help in the development of the theory.

Examining the forces that are exerted on certain simple chiral configurations of wire when an electromagnetic wave falls on them, we conjecture that the forces are such as to reduce the chirality of the configurations. This is true for the wire helix, for the three-stranded braid, and appears to be true in general. This tendency of the forces makes the object more nearly symmetrical.

CHIRAL OBJECTS AND
THEIR ENANTIOMORPHS

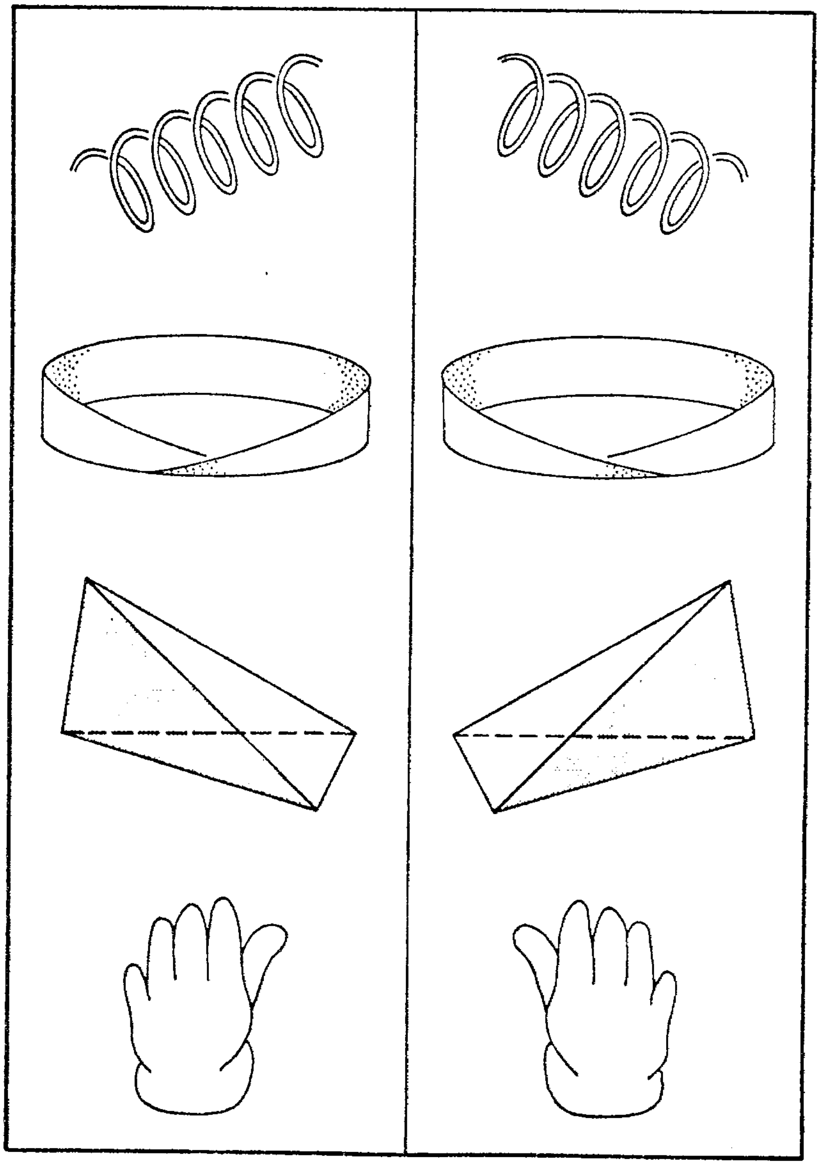

Figure 1. A Sketch Showing Chiral Objects (Left Column) and their Enantiomorphs (Right Column). From Top to Bottom are Shown a Helix, a Möbius Strip, an Irregular Tetrahedron, and a Glove.

SECTION III

THE SHORT HELIX

To demonstrate the plausibility of the above conjectures, we examine the scattering of electromagnetic waves from a metallic chiral object. For computational simplicity, the chiral object is chosen to be an electrically small perfect conductor having the form of a short right- or left-handed helix, as shown in figure 2. The calculation is simplified by referring the incident and scattered waves to the scattering plane defined by the incident and scattered wave vectors $\underline{k}' = k\,\hat{e}_n'$ and $\underline{k} = k\,\hat{e}_n$, respectively (figure 3). The incident plane wave is composed of the electric field

$$\underline{E}' = [a_{||}\,\hat{e}_{||}' + a_{\perp}\,e^{i\delta}\,\hat{e}_{\perp}']e^{ikz} \qquad (1)$$

and the corresponding magnetic field

$$\underline{B}' = \hat{\underline{e}}_n{}' \times \underline{E}'/c \qquad (2)$$

where c is the free-space speed of light, $a_{||}$, $a_{\perp}$ and δ are real numbers with $a_{||}^2 + a_{\perp}^2 = 1$, and z is the distance along $\hat{e}_n{}'(=\hat{e}_{||}' \times \hat{e}_{\perp}')$. The circumflexed quantities are unit vectors, the primes denote quantities associated with the incident wave, and the subscripts identify quantities parallel or perpendicular to the scattering plane. The harmonic time dependence $\exp(-i\omega t)$ (where $\omega = ck$) has been suppressed.

The scattered electric field $\underline{E}_{sc}(\theta)$ depends on the observation angle θ, defined by the relation $\cos\theta = \hat{e}_n{}' \cdot \hat{e}_n$, and on the induced electric

THE SHORT HELIX

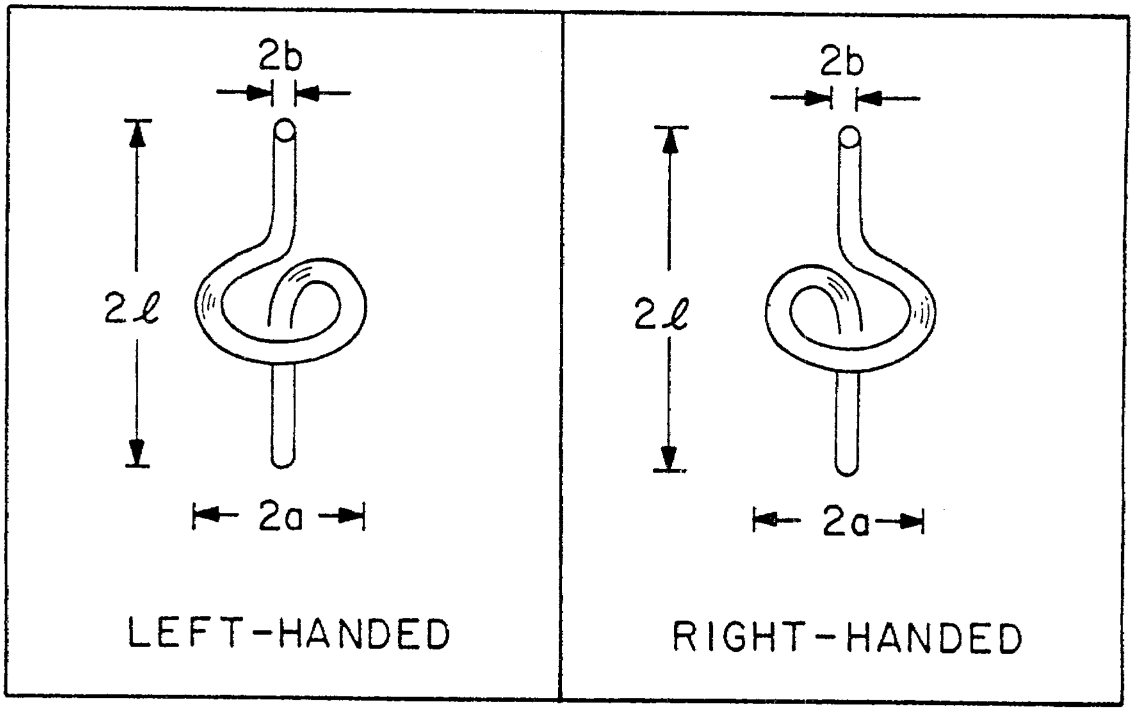

Figure 2. Idealized Short Helices Used in Calculations. The Plane of the Loop is Perpendicular to the Axis of the Straight Portion of the Wire.

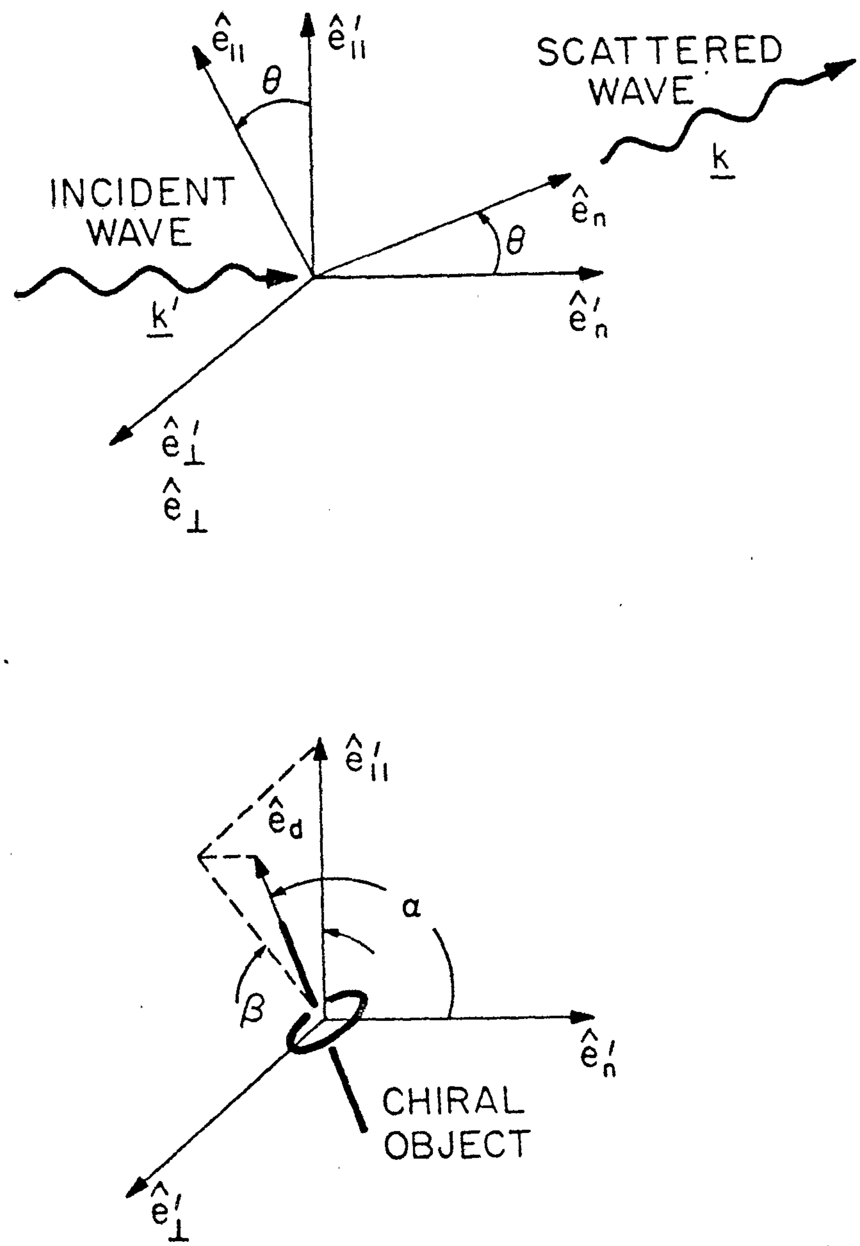

Figure 3. Vectors Indicating the Directions and Angles of Incident and Scattered Waves and the Orientation Angles of the Chiral Object.

and magnetic dipole moments $\underline{p}$ and $\underline{m}$. It is apparent (figure 2) that both dipole moments are directed parallel to the axis of the helix. This axis lies along the unit vector $\hat{e}_d$ whose orientation angles are α and β (figure 3).

The incident electric field induces currents in the straight portion of the chiral object, and by continuity these currents must also flow in the circular portion of the object. The current in the straight portion contributes to the electric dipole moment of the object and the current in the circular portion contributes to its magnetic dipole moment. In a complementary manner, the incident magnetic field induces currents in the circular portion and by continuity in the straight portion. Thus, also the magnetic field contributes to the electric and magnetic dipole moments of the object. In a first-order (Born) approximation we find from the heuristic argument above that the electric and magnetic dipole moments of the object are given by

$$\underline{p} = \varepsilon_0[\chi_e(\hat{e}_d \cdot \underline{E}') \pm i\chi_{em}c\, \hat{e}_d \cdot \underline{B}']\hat{e}_d \tag{3}$$

$$\underline{m} = \eta^{-1}[-\chi_m c(\hat{e}_d \cdot \underline{B}') \mp i\chi_{me}(\hat{e}_d \cdot \underline{E}')]\hat{e}_d \tag{4}$$

Here, as in the remainder of the report, the upper (lower) sign corresponds to the right-handed (left-handed) helix of figure 2. The permittivity, the permeability, and the impedance of free space are denoted by ε_0, μ_0, $\eta(= (\mu_0/\varepsilon_0)^{\frac{1}{2}})$. The electric and magnetic self-susceptibilities, χ_e and χ_m, are the real positive quantities as are the cross-susceptibilities χ_{em} and χ_{me}. Clearly, χ_e and χ_m are the usual electric and magnetic susceptibilities associated with electrically small metallic bodies. The cross-susceptibilities χ_{em} and χ_{me} are, in a certain sense, a measure of chirality or handedness since for archiral bodies $\chi_{em} = \chi_{me} = 0$.

Using known approximations, the self-susceptibilities can be written as

$$\chi_e = (2\ell)^2 \, C/\varepsilon_o \tag{5}$$

$$\chi_m = (\pi a^2)^2 \, \mu_o/L \tag{6}$$

where C and L are respectively the capacitance and the inductance of the body, and 2ℓ and $2a$ represent the length and the width of the short helix (figure 2). It can be shown that the cross-susceptibilities are given by

$$\chi_{em} = \chi_m (2\ell/\pi a^2 k) \tag{7}$$

$$\chi_{me} = \chi_e (\pi a^2 k/2\ell) \tag{8}$$

From physical considerations it appears that χ_{em} and χ_{me} are equal and real, i.e.,

$$\chi_{em} = \chi_{me} = \chi_c \tag{9}$$

where χ_c is their real common value.

It follows from (5) through (9) that the constraint

$$LC = \omega^{-2} \tag{10}$$

is placed upon the inductance and capacitance of the helix and the common value χ_c for the cross-susceptibilities is related to the inductance and capacitance by

$$\chi_c = 2\ell(\pi a^2)\eta/\omega L = 2\ell(\pi a^2)\eta\omega\, C \qquad (11)$$

From the knowledge of $\underline{p}$ and $\underline{m}$ the scattered field can be calculated by the formula

$$\underline{E}_{sc}(\theta) = \frac{k^2 e^{ikr}}{4\pi\varepsilon_o r}\,[(\hat{e}_n \times \underline{p}) \times \hat{e}_n - \hat{e}_n \times \underline{m}/c] \qquad (12)$$

To gain further insight into the problem, it is useful to find the constitutive relations of a medium composed of randomly oriented equivalent chiral objects. These constitutive relations must have the form (ref. 17)

$$\underline{P} = \gamma_e\, \underline{E} + \gamma_{em}\, \underline{B} \qquad (13)$$

$$\underline{M} = \gamma_{me}\, \underline{E} + \gamma_m\, \underline{B} \qquad (14)$$

where $\underline{P}$ and $\underline{M}$ are respectively the polarization and magnetization of the medium.

Energy conservation dictates that for a lossless medium

$$\gamma_{me} = \gamma_{em}^{*} \qquad (15)$$

where the asterisk denotes complex conjugate. If γ_{me} and γ_{em} not only satisfy (15) but also are purely imaginary quantities, then the constitutive relations (13) and (14) are those of an optically active medium (ref. 17).

To find the constitutive parameters for a medium composed of N short helices per unit volume we compare (3) and (4), averaged over orientation angles α and β, with (13) and (14). Thus we obtain

$$\gamma_e = N \varepsilon_o \chi_e / 4 \quad (16)$$

$$\gamma_m = -N \chi_m / 4\mu_o \quad (17)$$

$$\gamma_{em} = \pm i N \chi_{em} / 4\eta \quad (18)$$

$$\gamma_{me} = \mp i N \chi_{me} / 4\eta \quad (19)$$

Since χ_{em} and χ_{me} are real and equal, we see from (18) and (19) that γ_{me} and γ_{em} are purely imaginary and satisfy (15). Hence the medium composed of short helices exhibits optical activity.

SECTION IV

COLLECTION OF SHORT HELICES

To find the scattered field of a collection of randomly oriented identical helices we can use one of two approaches. One approach uses (12) averaged over orientation angles α and β; the other approach uses (13) and (14) directly. These two approaches give the same result. Here we use the former of the two approaches.

Let us suppose that we have a collection of N non-interacting helices per unit volume occupying a small volume ΔV. When the incident wave is circularly polarized, the scattering cross-sections per unit solid angle Ω are found to be

$$\left(\frac{d\sigma(\theta)}{d\Omega}\right)_{RCP\rightarrow RCP} = \frac{(k^2 N\Delta V)^2}{1024\pi^2} |\chi_e - \chi_m \pm 2\chi_c|^2 (1+\cos\theta)^2 \qquad (20)$$

when the incident wave is right circularly polarized (RCP) and only the right circularly polarized part of the scattered field is considered,

$$\left(\frac{d\sigma(\theta)}{d\Omega}\right)_{LCP\rightarrow LCP} = \frac{(k^2 N\Delta V)^2}{1024\pi^2} |\chi_e - \chi_m \pm 2\chi_c|^2 (1+\cos\theta)^2 \qquad (21)$$

when the incident wave is left circularly polarized (LCP) and only the left circularly polarized part of the scattered field is considered, and

$$\left(\frac{d\sigma(\theta)}{d\Omega}\right)_{RCP\rightarrow LCP} = \left(\frac{d\sigma(\theta)}{d\Omega}\right)_{LCP\rightarrow RCP} = \frac{(k^2 N\Delta V)^2}{1024\pi^2} |\chi_e + \chi_m|^2 (1-\cos\theta)^2 \qquad (22)$$

when the incident wave is RCP or LCP and the scattered field is LCP or RCP respectively.

From these expressions we see that for scattering in the forward direction ($\theta=0$) right (left) circular polarization produces a right (left) circularly polarized scattered field whereas in the backward direction ($\theta=\pi$) right (left) circular polarization produces a left (right) circularly polarized scattered field.

Next, we consider a wave normally incident on an electrically thin slab of width d which contains N randomly oriented helices per unit volume. Using again the assumption that the helices are non-interacting, we find from averaging (12) that the transmitted field is given by

$$\underline{E}_{tr} \simeq \Bigg[a_{||}\hat{e}_{||} + a_{\perp} e^{i\delta} \hat{e}_{\perp} + \frac{iNkd}{8}\Big\{ [a_{||}(\chi_e-\chi_m) \mp ia_{\perp} e^{i\delta} 2\chi_c]\hat{e}_{||} + i[\pm a_{||}2\chi_c - ia_{\perp} e^{i\delta} (\chi_e-\chi_m)]\hat{e}_{\perp}\Big\}\Bigg] e^{ikz} \tag{23}$$

and the reflected field by

$$\underline{E}_{ref} \simeq \frac{iNkd}{8}(\chi_e+\chi_m)\Big[a_{||}\hat{e}_{||}+a_{\perp} e^{i\delta} \hat{e}_{\perp}\Big]e^{-ikz} \tag{24}$$

where the above expressions are correct to the first order in $(Nkd\chi)$ (here χ stands for χ_e, χ_m or χ_c).

From (23) it can be shown that the plane of polarization is rotated through the angle ϕ where

$$\phi \simeq \tan\phi = \mp Nkd\chi_c/4 \tag{25}$$

for waves which pass through the chiral medium. Here ϕ is measured from $\hat{e}_{||}$ towards $\hat{e}_{\perp}$. Expression (25) is again correct to the first order in ($Nkd\chi$). This equation expresses a general result which holds for any medium composed of objects characterized by parallel electric and magnetic dipole moments with non-zero cross-susceptibilities. For the short helices pictured in figure 2 we can find a lower bound on the capacitance C by the expression (ref. 18)

$$C \geq \varepsilon_0 (4\pi)^{2/3} (3V_h)^{1/3} \tag{26}$$

where V_h ($= 2\pi\, b^2(\ell + \pi a)$) is the volume occupied by the short wire helix and b is the wire radius. This inequality, with the aid of (11) and (25), yields the following lower bound for the magnitude of the rotation angle:

$$|\phi| \approx |\tan \phi| \geq 4^{-1/3}\pi^{2/3} N(kd)(2\ell)(\pi a^2)(3k^3 V_h)^{1/3} \tag{27}$$

This bound is proportional to the product of the third root of the volume V_h of the wire helix and the cylindrical volume containing the helix ($= 2\ell\pi a^2$).

From (23) the eccentricity of the transmitted polarization ellipse differs from that of the incident polarization ellipse by a factor of order $(Nkd\chi)^2$. However, this transmitted field is correct only to order ($Nkd\chi$). Therefore, the change in the eccentricity of the polarization ellipse cannot be determined exactly from this model. To the first order in ($Nkd\chi$) the eccentricity is unchanged.

The reflected wave (24) to order ($Nkd\chi$) shows zero rotation for the plane of polarization and zero change in the eccentricity for the polarization ellipse. Therefore, for reflected waves, the slab of chiral medium behaves as an ordinary dielectric slab. These polarization characteristics are due to the fact that in the backscatter direction, in the first order, the effects of chirality are not present in the scattered field (equations (20), (21) and (23)).

From Noether's theorem (ref. 19) it can be shown that the angular momentum of the electromagnetic field is conserved for a medium described by equation (13) through (19). This implies that no torque is exerted on a slab of chiral medium. It is not surprising that there is no torque since the electrical properties of the slab are invariant under rotations of the slab about $\hat{e}_n'$. With a knowledge of the state of polarization of the incident wave, conservation of field angular momentum further implies that the state of polarization of the reflected wave can be determined from the state of polarization of the transmitted wave and vice versa. Some experimental results indicate that there is a change of eccentricity between the incident and scattered fields due to the chiral medium (refs. 11 and 12).

From the above considerations, the conjecture that a collection of chiral objects will rotate the plane of polarization becomes plausible.

SECTION V

REDUCTION OF CHIRALITY

Assuming that the helix in figure 4 is made of flexible wire, we can see that the currents that are induced tend to deform the helix. The current along the circular portion tends to open up the circle and make a planar figure out of the original helix. Moreover, interaction with the current along the straight portion of the helix tends to elongate the planar figure into a straight line. Since planar figures are achiral, we thus see that the helix evolves into a planar figure and that the chirality of the configuration is reduced.

Suppose now that we have a flexible helix of many turns, figure 5. In this case the induced current forces adjacent turns together and at the same time makes each turn expand into a turn of larger radius. Thus the original helix becomes a shortened helix of larger radius. Since the shortened helix is less chiral than the original helix, we see that here again the induced currents tend to reduce the chirality of the configuration.

Another type of chiral object is a braid of non-intersecting wires. Following Artin's theory of braids (ref. 15) we may describe a braid by projecting it on a plane and expressing the projected pattern as the product of terms, each of which is σ_i or σ_i^{-1}. Here σ_i denotes that the strand in position i crosses in front of the strand in position $i+1$ and σ_i^{-1} denotes that the latter crosses in front of the former.

Let us consider the three-stranded braid $\sigma_2\sigma_1^{-1}\sigma_2$ shown in figure 6. The bus bars L_1 and L_2 are connected by three flexible wires at the freely

REDUCTION OF CHIRALITY (SHORT HELIX)

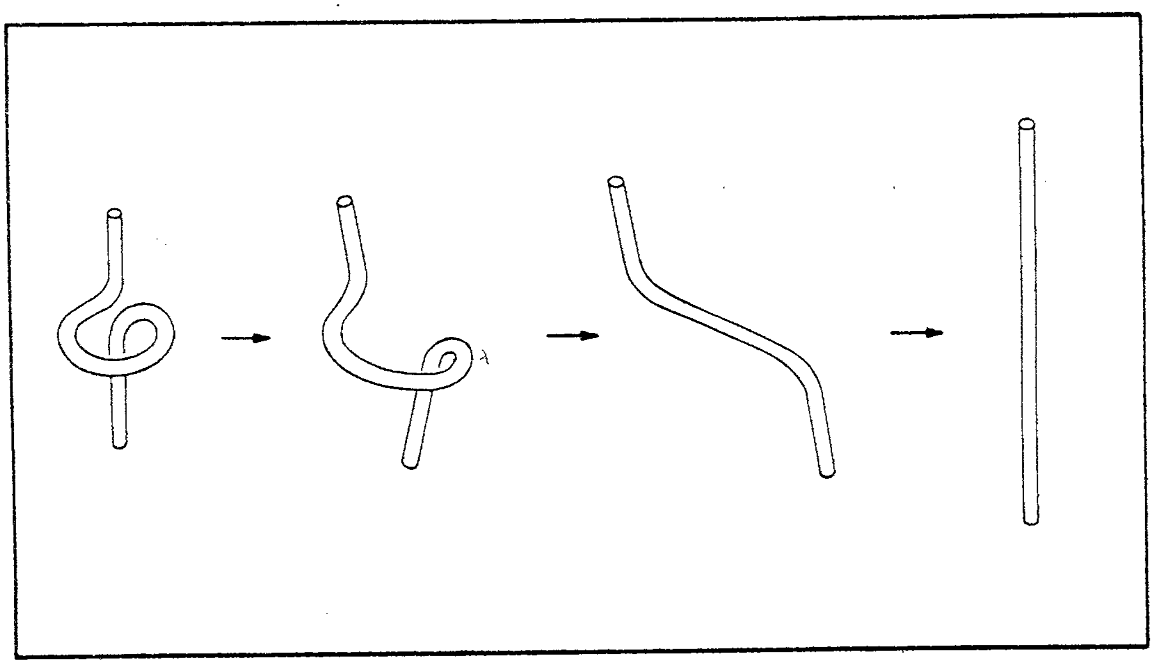

Figure 4. A Schematic of a Short Helix (Chiral) Evolving into a Straight Wire (Achiral) under the Influence of Forces Produced by Induced Currents.

REDUCTION OF CHIRALITY (LONG HELIX)

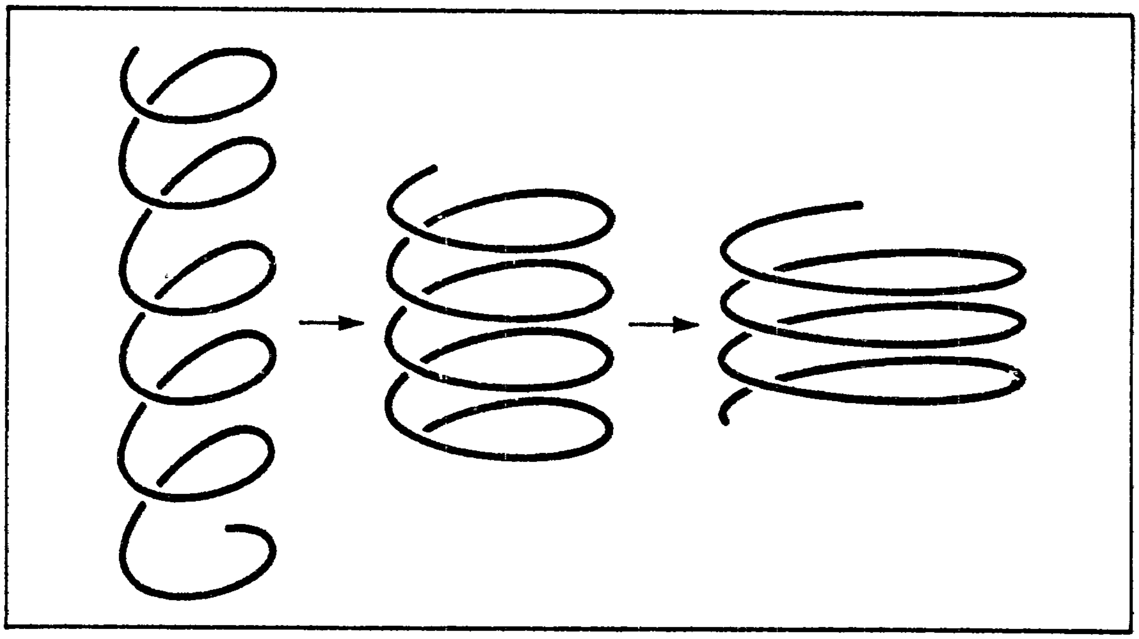

Figure 5. Under the Action of Forces Produced by the Induced Currents a Long Helix (Shown at Left) is Gradually Shortened into More Closely Spaced Loops of Increased Radius (Shown at Right).

WIRE BRAID

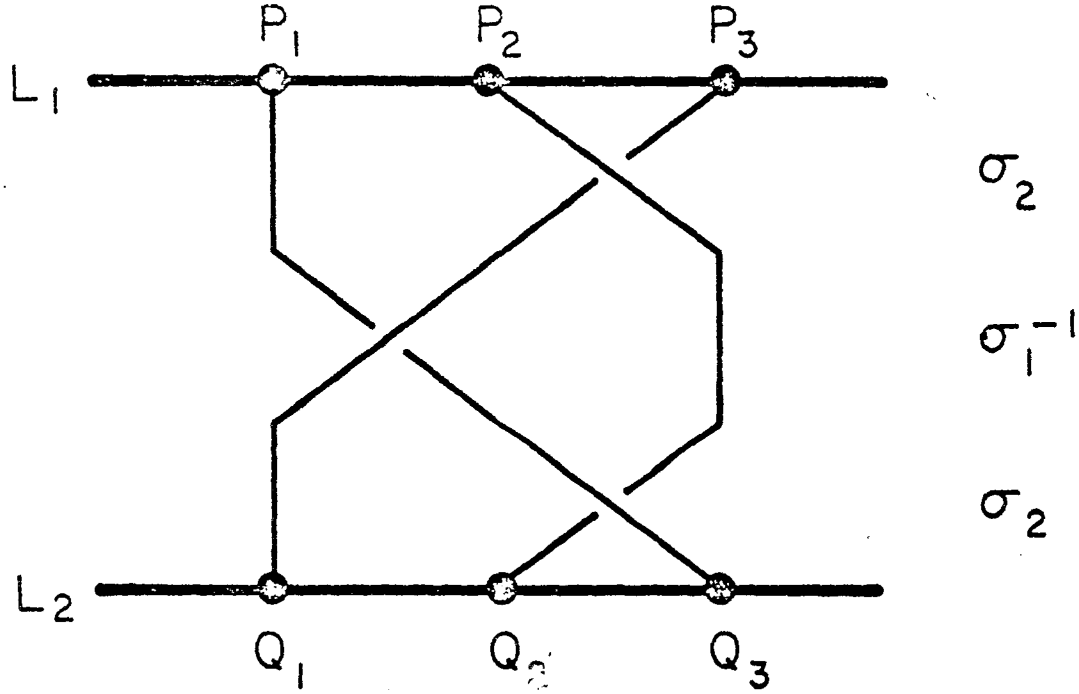

Figure 6. Bus Bars L_1 and L_2 Connected by Three Strands of the Braid Defined by $\sigma_2\sigma_1^{-1}\sigma_2$.

movable but ordered terminals P_1, P_2, P_3 and Q_1, Q_2, Q_3. Clearly, the braid and bus bars form a chiral object. An incident wave will induce current in the braid and bus bars and these currents will deform the configuration, viz., will make it more nearly planar, and thus reduce its chirality.

SECTION VI

CONCLUSIONS

We direct attention to the interaction of electromagnetic fields with macroscopic chiral objects. By examining the wire helix and the wire braid as chiral objects, we obtain results which conform to the conjecture that composite media composed of macroscopic chiral objects are optically active, and to the conjecture that electrodynamic forces tend to reduce chirality.

These considerations are expected to play a role in the development of diagnostic tools for remote sensing, in the design of electromagnetic shields and in the prediction of structural deformations.

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