Interaction Notes

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ELECTROMAGNETIC TOPOLOGY: A FORMAL APPROACH TO THE ANALYSIS AND DESIGN OF COMPLEX ELECTRONIC SYSTEMS

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Abstract

This paper summarizes several of the concepts included in the new and expanding subject of electromagnetic topology. Beginning with the elementary concepts of volumes and boundary surfaces, these entities are specialized to layers and shields, and sublayers and subshields, and also associated with equivalent graphs. Next one defines a set of indices associated with the various levels of topological decomposition. The general supermatrix equation describing electromagnetic propagation is partitioned according to these indices. The partitioned equation leads to useful approximate solutions which can be used to bound signals, relate the performance of the system to its parts, and allocate shielding to various subshields so that a system of interest can be protected against various electromagnetic environments.

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This paper summarizes several of the concepts included in the new and expanding subject of electromagnetic topology. Beginning with the elementary concepts of volumes and boundary surfaces, these entities are specialized to layers and shields, and sublayers and subshields, and also associated with equivalent graphs. Next one defines a set of indices associated with the various levels of topological decomposition. The general supermatrix equation describing electromagnetic propagation is partitioned according to these indices. The partitioned equation leads to useful approximate solutions which can be used to bound signals, relate the performance of the system to its parts, and allocate shielding to various subshields so that a system of interest can be protected against various electromagnetic environments.

I. Introduction

In designing or analyzing the response of an electronic system to some kind of electromagnetic interference such as the nuclear electromagnetic pulse (EMP), one is often overwhelmed by the complexity of the problem. There are too many individual components with an enormous number of interconnections. The corresponding numbers of variables and equations make it very difficult to get simple insights into how to control the system performance in an electromagnetic environment.

To deal with this complexity one needs ways to identify and deal with a set of important variables which, if controlled, control the system performance. An approach to this problem has been developed [9] which can be referred to as electromagnetic topology. This concept involves the definition of principal surfaces and principal volumes which divide up the space occupied by the system. These surfaces and volumes are further divided corresponding to various features of the system design or analysis. One can show that electromagnetic topology is related to and is a generalization of graph theory used in electrical network analysis.

Having defined the electromagnetic topology and the related interaction sequence diagram (graph) one can write a general matrix equation (a form of the BLT equation) which is partitioned according to the topology which has been defined. The resulting supermatrix equation admits an approximate solution which directly shows the dependence of the interior signals on the shielding properties of the principal surfaces. Certain approximate bounds for the interior signals result from norm concepts. Other aspects of the electromagnetic topology can be used to control low-frequency grounding and undesirable signal transport within the system.

Electromagnetic topology can then be used as a synthesis technique. Given some requirements for signal attenuation (isolation) one can choose an appropriate electromagnetic topology and allocate performance specifications for the design of each important part of the system.

II. Volume/Surface Topology

Let us divide three-dimensional space into a set of volumes $\{V_\delta\}$ as indicated in fig. 2.1A. The common boundaries between two volumes are

$$S_{\delta;\delta'} = V_\delta^+ \bigcap V_{\delta'}^+ \tag{2.1}$$

with the superscript + indicating that the volumes are augmented by their boundary surfaces. This begins the elementary basis for electromagnetic topology for describing signal propagation through complex systems [9].

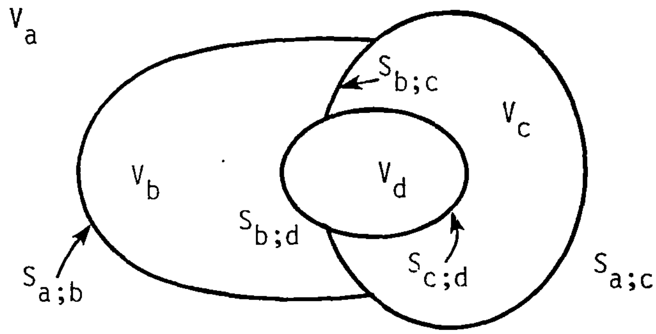
Related to the volume/surface topology it can be shown [2] that there is an equivalent bipartite graph as in fig. 2.1B with vertices ⊚ to represent the volumes and a second set of vertices o to represent the connecting surfaces. This graph can be referred to as the interaction sequence diagram [2,4,9]. The edges represent the electromagnetic signal transport through the system.

One of the uses of this topological description involves the identification of selected closed surfaces as "subshields" which attenuate unwanted signals passing through these surfaces. As indicated in fig. 2.2A such subshields are closed surfaces separating an "inside" from and "outside." "Shields" are the unions of subshields including all subshields which do not surround one another. Shields are nested in the sense that every pair of shields has the property that one surrounds the other.

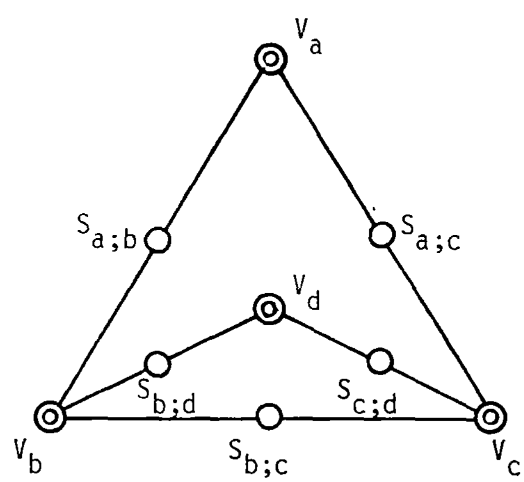
Define some indices

$$\begin{aligned} &\lambda = 1,2,\ldots,\lambda_{max} \quad \text{(layers)} \\ &\mu = 1,2,3 \quad \text{(layer parts)} \\ &\ell = 1,2,\ldots,\ell_{max}(\lambda) \quad \text{(sublayers)} \\ &\tau = 1,2,\ldots,\tau_{max}(\lambda,\ell) \quad \text{(elementary volumes)} \\ &\sigma = 1,2 \quad \text{(dual-wave index)} \end{aligned} \tag{2.2}$$

An elementary volume would be designated $V_{\lambda,\ell,\tau}$ while in fig. 2.2A the decomposition is carried to sublayer $V_{\lambda,\ell}$ level. Further decomposition of a sublayer into elementary volumes significantly increases the complexity of the interaction

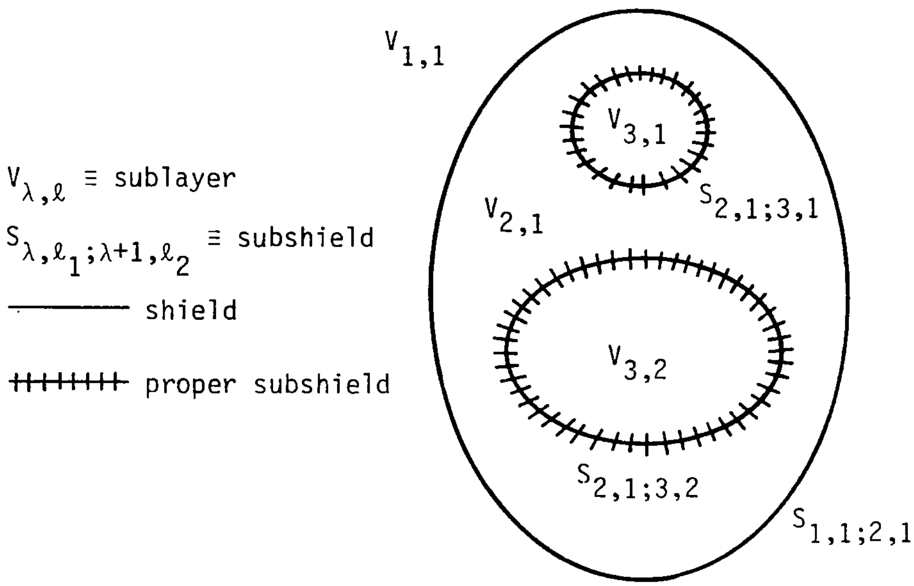


A. Volume/surface topology

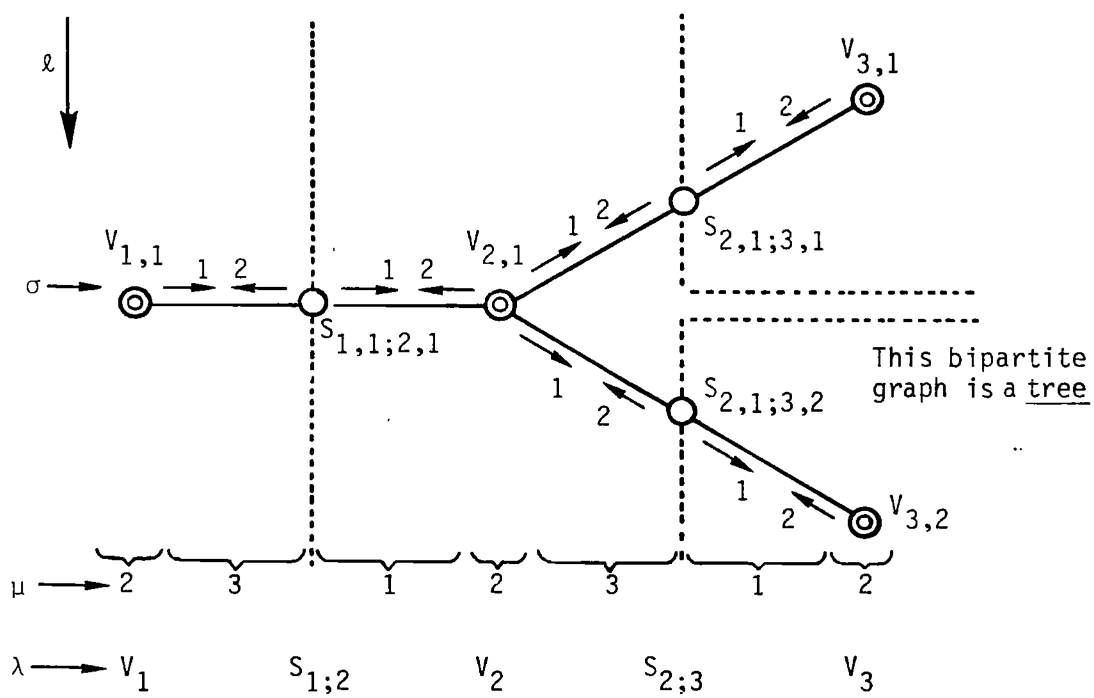


B. Related bipartite graph (interaction sequence diagram)

Fig. 2.1. Scatterer Topology



A. Volume/surface topology



B. Interaction sequence diagram

Fig. 2.2. Sublayers and Subshields in Hierarchical Topology

sequence diagram from that in fig. 2.2B [2]. Note that the indices μ and σ are not used with the volumes but correspond to the waves connecting between volumes and surfaces on the edges of the bipartite graph as in fig. 2.2B. Within a layer three sets of such connections are identified, $\mu = 2$ corresponding to connections between elementary volumes and surfaces within a sublayer. The dual-wave index σ indicates the two possible wave directions on an edge. Considering δ in (2.1) as an index set (such as (λ,ℓ,τ)) the various types of surfaces can be generated.

The various types of volumes are related by

$$V_\lambda^+ = \bigcup_{\ell=1}^{\ell_{max}(\lambda)} V_{\lambda,\ell}^+ \qquad \text{(layers from sublayers)}$$

$$V_{\lambda,\ell}^+ = \bigcup_{\tau=1}^{\tau_{max}(\lambda,\ell)} V_{\lambda,\ell,\tau}^+ \qquad \text{(sublayers from elementary volumes)} \tag{2.3}$$

The corresponding surfaces are found from (2.1). Some interesting relations are

$$S_{\lambda;\lambda+1} = V_\lambda^+ \bigcap V_{\lambda+1}^+ \qquad \text{(shields from layers)}$$

$$S_{\lambda,\ell_1;\lambda+1,\ell_2} = V_{\lambda,\ell_1}^+ \bigcap V_{\lambda+1,\ell_2}^+ \qquad \text{(subshields from sublayers)} \tag{2.4}$$

$$S_{\lambda;\lambda+1} = \bigcup_{\ell_1=1}^{\ell_{max}(\lambda)} \bigcup_{\ell_2=1}^{\ell_{max}(\lambda+1)} S_{\lambda,\ell_1;\lambda+1,\ell_2} \qquad \text{(shields from subshields)}$$

From the viewpoint of protecting electrical equipment the shields and subshields are the important entities in this hierarchical electromagnetic topology.

III. Supermatrix BLT Equation for Scatterers

Partition an N-component vector (x_n) into a set of vectors $(x_n)_\lambda$ where $(x_n)_1$ is the first N_1 components of (x_n), $(x_n)_2$ is the second N_2 components, etc. Write this as a divector $((x_n)_\lambda)$. Continue this process by partitioning each $(x_n)_\lambda$ as a set of $(x_n)_{\mu;\lambda}$ giving a trivector $(((x_n)_\mu)_\lambda)$. This process can be continued indefinitely and the general types of vectors so defined are all referred to as supervectors [1,2,5,6]. Using the indices in section 2 one can have

$$((((((x_n)_\sigma)_\tau)_\ell)_\mu)_\lambda) \equiv \text{hexavector}$$

$$x_{n;\sigma;\tau;\ell;\mu;\lambda} \equiv \text{individual vector components} \tag{3.1}$$

The elementary vectors $(x_n)_{\sigma;\tau;\ell;\mu;\lambda}$ have an index n corresponding to the individual variables (such as voltage, current, etc.) at that level of topological decomposition, as on an edge in the interaction sequence diagram. Supermatrices are similarly defined from matrices as $(A_{n,m})$ is partitioned as [1,2,5,6]

$$((((((A_{n,m})_{\sigma,\sigma'})_{\tau,\tau'})_{\ell,\ell'})_{\mu,\mu'})_{\lambda,\lambda'}) \equiv \text{hexamatrix}$$

$$A_{n,m;\sigma,\sigma';\tau,\tau';\ell,\ell';\mu,\mu';\lambda,\lambda'} \equiv \text{individual matrix components} \tag{3.2}$$

Now it is important that the partitioning be done in such a way that the usual addition and dot multiplication operations carry through in generalized form. Some combination of supervectors and supermatrices combined by such operations are said to be of <u>compatible order</u> with respect to such operations. In particular for supermatrices it is necessary that partitioning according to rows be done uniformly across the columns at every level of partition, i.e., every column is partitioned, or assigned, an index set such as $n;\sigma;\tau;\ell;\mu;\lambda$ in exactly the same way. Similarly the partitioning according to columns is done uniformly across the rows.

While these concepts can be applied to rectangular matrices our concern is with square matrices which are partitioned in exactly the same manner for both rows and columns. Such <u>symmetric</u> partitioning of square matrices leaves the diagonal blocks square at every level of partition. With all supermatrices under consideration of the same size and square with identical symmetric partitions we refer to these as being of symmetric compatible order; together with this we require the supervectors to have the same size and partition for compatibility. Then the generalized addition (commutative) carries through as illustrated on dimatrices

$$\begin{aligned}
&((A_{n,m})_{u,v}) + ((B_{n,m})_{u,v}) = ((C_{n,m})_{u,v}) \\
&(C_{n,m})_{u,v} = (A_{n,m})_{u,v} + (B_{n,m})_{u,v} \\
&C_{n,m;u,v} = A_{n,m;u,v} + B_{n,m;u,v}
\end{aligned} \tag{3.3}$$

and similarly for generalized dot multiplication (non-commutative in general)

$$\begin{aligned}
&((A_{n,m})_{u,v}) \odot ((B_{n,m})_{u,v}) = ((D_{n,m})_{u,v}) \\
&(D_{n,m})_{u,v} = \sum_{u'} (A_{n,m})_{u,u'} \cdot (B_{n,m})_{u',v} \\
&D_{n,m;u,v} = \sum_{u'} \sum_{n'} A_{n,n';u,u'} B_{n',m;u',v}
\end{aligned} \tag{3.4}$$

which is readily generalized to any order of partition. Note that addition (or subtraction) is immediately carried to the smallest partition, while generalized dot multiplication corresponds to successive dot multiplication at each level of partition. The above ((3.3) and (3.4)) also apply to supervectors by the deletion of appropriate indices.

These supervectors and supermatrices are used in the BLT equation which was originally developed to describe the response of transmission-line networks [1]. By shrinking the tubes (multiconductor transmission lines) to zero length the BLT equation takes the form [2,5,6]

$$[((1_{n,m})_{u,v}) - ((\tilde{\Sigma}_{n,m}(s))_{u,v})] \odot ((\tilde{V}_n(s))_u) = ((\tilde{\Sigma}_{n,m}(s))_{u,v}) \odot ((\tilde{V}_{s_n}(s))_u)$$

$((1_{n,m})_{u,v}) \equiv$ identity supermatrix

$$((\tilde{\Sigma}_{n,m}(s))_{u,v}) \equiv \text{scattering supermatrix} \tag{3.5}$$

$((\tilde{V}_n(s))_u) \equiv$ combined voltage supervector (response)

$((\tilde{V}_{s_n}(s))_u) \equiv$ combined voltage source supervector

~ above a variable ≡ Laplace transform (two-sided)

$s \equiv \Omega + j\omega \equiv$ complex frequency

where u,v can be further partitioned as in (3.1) and (3.2) to correspond to the levels of the hierarchical topology. The index u (or corresponding index set) corresponds to the waves on the interaction sequence diagram which take on two directions on each edge. Accordingly for use with the scattering matrices corresponding to the vertices, the supervectors are wave variables combining voltages and currents which we write as

$$(\tilde{V}_n(s))_u \equiv (\tilde{V}_n^{(+)}(s))_u + (\tilde{Z}_{c_{n,m}}(s))_u) \cdot (\tilde{I}_n^{(+)}(s))_u$$

$$(\tilde{V}_{s_n}(s))_u \equiv (\tilde{V}_{s_n}^{(+)}(s))_u + (\tilde{Z}_{c_{n,m}}(s))_u \cdot (\tilde{I}_{c_n}^{(+)}(s))_u \tag{3.6}$$

$(\tilde{Z}_{c_{n,m}}(s))_u \equiv$ chosen normalizing impedance (matrix)

where the quantities superscripted with a + are true voltages and currents with the + indicating that current is positive in the direction of the wave indicated by u (or specifically one of its component indices σ). The source terms are voltages in series with the wire(s) and currents from local ground to the wire(s) in a transmission-line interpretation. Voltages and currents are appropriate for "conductive" penetrations where wires or other conductors pass through metallic sheets (bulkheads, etc.). In the case of apertures other interpretations need to be given to these variables.

While this form of the BLT equation can be used for detailed calculations of signal transport through complex systems, a perhaps more important application is for bounding signals in complex systems. Consider the form of the scatterer BLT equation (3.5) as

$$((\tilde{I}_{n,m}(s))_{u,v}) \odot ((\tilde{V}_n(s))_u) = ((\tilde{E}_n(s))_u)$$

$$((\tilde{I}_{n,m}(s))_{u,v}) \equiv ((1_{n,m})_{u,v}) - ((\tilde{\Sigma}_{n,m}(s))_{u,v}) \equiv \text{interaction supermatrix} \tag{3.7}$$

$$((\tilde{E}_n(s))_u) \equiv ((\tilde{\Sigma}_{n,m}(s))_{u,v}) \odot ((\tilde{V}_{s_n}(s))_u) \equiv \text{source supervector}$$

This places (3.5) in the form of a standard vector/matrix equation which is readily solved for the response voltage supervector. The interaction supermatrix can also be diagonalized in terms of eigenvectors and can be expanded in terms of its singularities in the complex s plane (SEM) [1].

The interaction supermatrix has some important properties [2,5,6] based on the structure of the identity and scattering supermatrices. Writing u as an index set as in (3.2) we have

$$(((((I_{n,m}(s))_{\sigma,\sigma'})_{\tau,\tau'})_{\ell,\ell'})_{\mu,\mu'})_{\lambda,\lambda'} \equiv (((((0_{n,m})_{\sigma,\sigma'})_{\tau,\tau'})_{\ell,\ell'})_{\mu,\mu'})_{\lambda,\lambda'} \quad \text{for } |\lambda - \lambda'| \geq 2 \tag{3.8}$$

i.e., the interaction supermatrix is block tridiagonal at the layer level of partition.

Starting with this block-tridiagonal property let us next assume that the off-diagonal blocks are small (in the norm sense) compared to the diagonal blocks. Then one has an approximate solution of the scatterer BLT equation ((3.5) or (3.7)) which can be called the good shielding approximation [2,5,6] in

$$\begin{aligned} (((((\tilde{V}_n(s))_{\sigma})_{\tau})_{\ell})_{\mu})_{\lambda} \simeq (-1)^{\lambda+1} \Bigg\{ \bigodot_{\lambda'=0}^{\lambda-2} \Big[(((((\tilde{I}_{n,m}(s))_{\sigma,\sigma'})_{\tau,\tau'})_{\ell,\ell'})_{\mu,\mu'})^{-1}_{\lambda-\lambda',\lambda-\lambda'} \\ \odot (((((\tilde{I}_{n,m}(s))_{\sigma,\sigma'})_{\tau,\tau'})_{\ell,\ell'})_{\mu,\mu'})_{\lambda-\lambda',\lambda-\lambda'-1} \Big] \Bigg\} \\ \odot (((((\tilde{I}_{n,m}(s))_{\sigma,\sigma'})_{\tau,\tau'})_{\ell,\ell'})_{\mu,\mu'})^{-1}_{1,1} \odot (((((\tilde{V}_{s_n}(s))_{\sigma})_{\tau})_{\ell})_{\mu})_{1} \\ \text{for } \lambda \geq 2 \end{aligned} \tag{3.9}$$

$$((((\tilde{V}_n(s)_\sigma)_\tau)_\ell)_\mu)_1 \simeq (((((\tilde{I}_{n,m}(s))_{\sigma,\sigma'})_{\tau,\tau'})_{\ell,\ell'})_{\mu,\mu'})^{-1}_{1,1}$$

$$\odot\ (((((\tilde{\Sigma}_{n,m}(s))_{\sigma,\sigma'})_{\tau,\tau'})_{\ell,\ell'})_{\mu,\mu'})_{1,1} \odot ((((\tilde{V}_{s_n}(s)_\sigma)_\tau)_\ell)_\mu)_1$$

where the generalized dot product $\odot$ is now also used in a continued product form (as in $\prod$) with order of multiplication having ascending λ' from left to right. For this result only external sources (in V_1) are assumed present.

Having the scatterer response in terms of a product of the blocks one can now consider the influence of the individual blocks (corresponding to specific layers and shields) on the response. In particular, though, one can use this result to obtain an approximate bound for the signals in a layer by the use of norm concepts [3,8] in

$$\|((((\tilde{V}_n(s))_\sigma)_\tau)_\ell)_\mu)_\lambda\| \lesssim \left\{ \prod_{\lambda'=0}^{\lambda-2} \|(((((\tilde{I}_{n,m}(s))_{\sigma,\sigma'})_{\tau,\tau'})_{\ell,\ell'})_{\mu,\mu'})^{-1}_{\lambda-\lambda',\lambda-\lambda'}\| \right.$$

$$\left. \|(((((\tilde{I}_{n,m}(s))_{\sigma,\sigma'})_{\tau,\tau'})_{\ell,\ell'})_{\mu,\mu'})_{\lambda-\lambda',\lambda-\lambda'-1}\| \right\}$$

$$\|(((((\tilde{I}_{n,m}(s))_{\sigma,\sigma'})_{\tau,\tau'})_{\ell,\ell'})_{\mu,\mu'})^{-1}_{1,1}\| \ \ \|(((((\tilde{V}_s(s))_\sigma)_\tau)_\ell)_\mu)_1\|$$

$$\text{for } \lambda \geq 2 \tag{3.10}$$

The norms ($\|\ \|$) give positive scalars for non-zero vectors and matrices to characterize the response and layer and shield transfer properties. Various vector (and associated matrix) norms are possible, such as maximum element magnitude and euclidean (length).

This approximate result is carried further by noting the decoupling between sublayers in the same layer. For example, in fig. 2.2B one can define a path from $V_{1,1}$ to $V_{3,1}$ which does not pass through the vertices $S_{2,1;3,2}$ and $V_{3,2}$. This is accomplished by associating a particular value of ℓ with each λ along the path P as $\ell(\lambda,P)$. Then (3.9) is simplified by only considering the response of particular sublayers on the path P in

$$((((\tilde{V}_n(s))_\sigma)_{\tau;\ell})_\mu)_\lambda \simeq$$

$$(-1)^{\lambda+1}\left\{ \bigodot_{\lambda'=0}^{\lambda-2} \left[(((((\tilde{I}_{n,m}(s))_{\sigma,\sigma'})_{\tau,\tau';\ell(\lambda-\lambda',P),\ell(\lambda-\lambda',P)})_{\mu,\mu'})_{\lambda-\lambda',\lambda-\lambda'} \right.\right.$$

$$\left.\left. \odot\ (((((\tilde{I}_{n,m}(s))_{\sigma,\sigma'})_{\tau,\tau';\ell(\lambda-\lambda',P),\ell(\lambda-\lambda'-1,P)})_{\lambda,\lambda'})_{\lambda-\lambda',\lambda-\lambda'-1}\right]\right\}$$

$$\odot\ (((((\tilde{I}_{n,m}(s))_{\sigma,\sigma'})_{\tau,\tau';\ell(1,P),\ell(1,P)})_{\mu,\mu'})_{1,1}$$

$$\odot\ ((((\tilde{V}_{s_n}(s))_\sigma)_{\tau;1})_\mu)_1 \qquad \text{for } \lambda \geq 2 \tag{3.11}$$

Constraining $\ell(\lambda,P)$ has effectively removed one level of matrix/vector partition by specialization to one path in the interaction sequence diagram at sublayer level. Just as in (3.10) this result (3.11) can be used to give an approximate bound on the signals in the $V_{\lambda,\ell}$ sublayer as a product of norms corresponding to the sublayer and subshield characteristics along path P.

This concept can be carried a step further by extending the path P to be any path in the sublayer interaction sequence diagram. Instead of considering signals from exterior sources to an interior sublayer, one can place the sources $((((\tilde{V}_{s_n}(s))_\sigma)_{\tau;\ell})_\mu)_\lambda$ at some other sublayer $V_{\lambda,\ell}$ and consider the response at yet another sublayer. In the example of fig. 2.2B this might be a path from say $V_{3,1}$ to the exterior $V_{1,1}$. By reciprocity this should have the same attenuation as the signal transport from exterior to interior. Another type of path begins at one sublayer, goes toward the exterior, but then returns toward another interior sublayer; in fig. 2.2B this is illustrated by the path from $V_{3,1}$ to $V_{3,2}$ (or conversely). By a topological transformation (inversion) any beginning sublayer can be transformed to the exterior so that (3.11) applies to this path. In this topological transformation various indices are changed to correspond to the transformed topology. Viewed in a simpler way one looks at the norms of the blocks of the scattering matrices in (3.11) corresponding to the sublayers and subshields along P; it is the product of these norms which is important, and these can be identified by inspection of the appropriate interaction sequence diagram.

IV. Electromagnetic Design and Specification

Having characterized the signal transport through the electromagnetic topology of a system in terms of norms of sublayer and subshield scattering matrices, the process can be turned around for design purposes [7].

1. Consider some elementary topology defined to at least sublayer level (e.g., fig. 2.2).

2. Identify the sources in each sublayer corresponding to electromagnetic environments of interest.

3. Identify the allowed maximum signal levels in each sublayer of concern associated with each electromagnetic environment.

4. Identify the paths P_n associated with each pair of source sublayer (2) and response sublayer (3) of concern.

5. Allocate "shielding" along each path P_n such that the sources (2) produce no more response than allowed in sublayers (3).

6. Partition the shielding along each P_n among the corresponding subshields encountered on P_n. This gives maximum allowable values to the norms of each corresponding subshield scattering matrix.

7. For each subshield consider all paths P_n that pass through it. Choose the scattering matrix norm to be the least of those chosen for all P_n as in 6.

This procedure recognizes the dominant role played by subshields in reducing unwanted electromagnetic signal transport through systems. However, the sublayers as indicated by the diagonal blocks in (3.11) also have a role to play in the signal attenuation. If the norms of any such diagonal blocks are greater than one, then one will have to allow for somewhat greater attenuations (smaller norms) for the off-diagonal blocks corresponding to the subshields.

With the required subshield norms specified one next needs design and test procedures to ensure that these norms are as small as specified. Appropriate scattering-matrix measurements for conductive penetrations, small apertures, small antennas, etc. need to be standardized for this purpose. Having all the measured scattering-matrix parameters for a given subshield, these can be combined to give a computed norm of the desired type. This procedure can in principle give a system design procedure in which the meeting of subsystem (subshield) electromagnetic specifications implies the successful passing of system environmental specifications.

The signal bounding here has been cast in complex frequency domain as in (3.9) and (3.11) which of course implies linearity. While this is an important simplification, not all is lost if one remains in time domain. The matrices in the good-shielding approximation become convolution operators in time domain. In a more general case they may even be nonlinear time-domain operators. Norms of such time-domain operators then also need to be considered.

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