Electromagnetics, 1983
pp. 145-157

Interaction Notes

Note 417

17 June 1982

Topological Considerations for Low-Frequency Shielding and Grounding

Carl E. Baum
Air Force Weapons Laboratory

Abstract

This note expands on the concepts of electromagnetic volume/surface topology by including some considerations related to reduction of interference from low-frequency magnetic fields; this may be significant in some applications. The topology of a subshield can be chosen as a Baumkugel (tree-sphere) to minimize the induced currents on the subshield conductors. Also, grounding networks can be designed on the basis of sublayer grounding subnetworks in the form of trees which do not have grounding conductors penetrating subshields, but connect (ground) the subshields and other equipment.

CLEARED FOR PUBLIC RELEASE
OASD/PA 15 Mar 83
AFSC 83-202
AFCMD/PA 83-50
SAF/PAS 83-399

I. Introduction

In designing an electromagnetic topology for some practical application, there are various factors to be considered. Basically, one wishes to exclude electromagnetic energy from certain volumes. At "high" frequencies, the magnetic field is excluded by an inductive effect associated with the inductance of the shield volume limited by the resistance around the assumed highly conducting thin shell surrounding this volume [1]. At even "higher" frequencies, electromagnetic field penetration is further attenuated by the skin effect in the thin conducting shell. Actually, in practice, apertures and conductive penetrations are dominant in electromagnetic penetration at sufficiently high frequency. However, the concern of this note is "low" frequencies as they influence the design of electronic systems with respect to "shielding" and "grounding".

At sufficiently low frequencies, the inductance between shield and inner conductors is negligible, so that the currents are determined by resistive division of currents between conductors where such conditions exist. The shielding properties can be considered in a more classical circuit form in such cases.

Also at low frequencies, the shape of the shielding surface can be significant in determining its shielding properties. Specifically, the topology of such a surface strongly influences the surface current density in the presence of an exciting magnetic field.

This note then addresses some topological considerations concerning the design of subshields and low-impedance grounding networks. These questions are considered in light of the general hierarchical topology for the design of complex electronic systems [3,11]. Some of the points discussed here were introduced in a previous presentation [5].

II. Surface Currents on Highly Conducting Subshields in Low-Frequency Magnetic Fields

Subshields are an important part of electromagnetic topology [3,4]. These are the closed surfaces (connected, i.e., not in two or more separate pieces) that form the basis of our electromagnetic shielding. More than one such subshield may be part of our chosen electromagnetic topology. Here attention is given to the topological properties of individual subshields and how these influence the interaction with electromagnetic fields.

There are various measures one might use to estimate shielding effectiveness. For aperture penetration, one is often concerned with the electric and magnetic fields which would be present if the aperture were closed (the short-circuit fields), at least for electrically small apertures [12]. In this note, let us consider the simple case of uniform low-frequency magnetic fields incident on the exterior of individual subshields (assumed to be perfectly conducting). In considering subshields of different topological properties, let us look at the surface current densities on electrically-small subshields in uniform magnetic fields.

Let us compare some canonical examples. If our subshield is a perfectly conducting spherical sheet of radius r_s, we have the well-known result [7,9]

$$
\begin{aligned}
&\frac{J_{s_{max}}}{H_{inc}} = \frac{3}{2} \\
&\vec{H}_{inc} = H_{inc}\vec{1}_h \equiv \text{incident magnetic field} \\
&\vec{1}_h \equiv \text{direction of incident magnetic field} \\
&\vec{J}_s \equiv \text{surface current density} \\
&J_s \equiv |\vec{J}_s|
\end{aligned}
\tag{2.1}
$$

Similarly, for a circular cylinder of radius r_c with the incident magnetic field parallel to the cylinder axis

$$\frac{J_s}{H_{inc}} = 1 \tag{2.2}$$

and for the case of the incident magnetic field perpendicular to the axis of the circular cylinder

$$\frac{J_{s_{max}}}{H_{inc}} = 2 \tag{2.3}$$

Contrast these results with the case of a circular loop (toroid) of major radius a and minor radius b with b << a for which we have an inductance [9]

$$L \simeq \mu_o a \left[\ell n \left(\frac{8a}{b} \right) - 2 \right] \tag{2.4}$$

For an incident magnetic field perpendicular to the plane of the loop, the current in the loop can be estimated from the open-circuit voltage for a slowly changing magnetic field via

$$\begin{aligned} V_{o.c.} &= (\text{area}) \frac{d}{dt} B_{inc} \simeq \mu_o \pi a^2 \frac{d}{dt} H_{inc} \\ &= L \frac{dI}{dt} \end{aligned} \tag{2.5}$$

giving

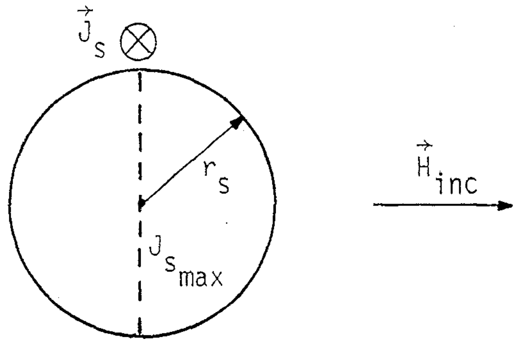
$$\frac{I}{H_{inc}} \simeq \frac{\mu_o \pi a^2}{L} \simeq \pi a \left[\ell n \left(\frac{8a}{b} \right) -2 \right]^{-1} \tag{2.6}$$

The surface current density is given by

$$\begin{aligned} J_s &\simeq \frac{I}{2\pi b} \\ \frac{J_s}{H_{inc}} &\simeq \frac{a}{2b} \left[\ell n \left(\frac{8a}{b} \right) - 2 \right]^{-1} \end{aligned} \tag{2.7}$$

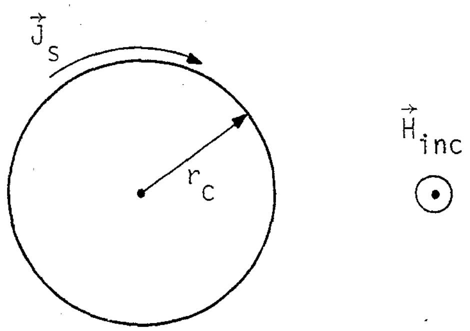
These cases are illustrated in fig. 2.1.

Comparing these examples, we can see some general results. The first three examples (as in figs. 2.1A, 2.1B, and 2.1C) all have the surface current density of the same order as H_{inc}. However, the fourth example (in fig. 2.1D) has a ratio of J_s to H_{inc} which is considerably larger, being proportional to $(a/b)/\ell n(a/b)$ for large a/b. One might then determine what topological property is associated with this large difference.



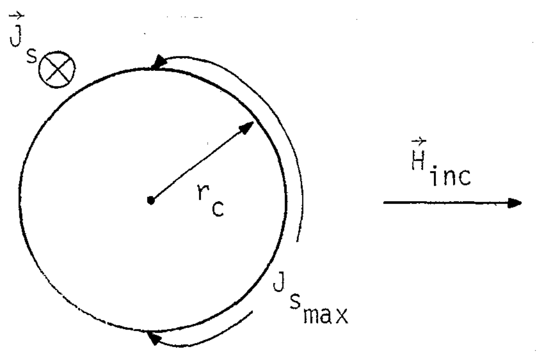
$$\frac{J_{s_{max}}}{H_{inc}} = \frac{3}{2}$$

A. Sphere



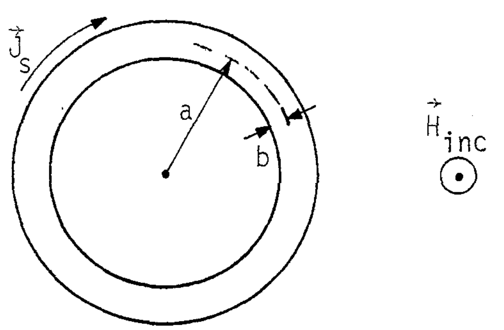
$$\frac{J_s}{H_{inc}} = 1$$

B. Circular cylinder: magnetic field parallel to axis



$$\frac{J_{s_{max}}}{H_{inc}} = 2$$

C. Circular cylinder: magnetic field perpendicular to axis



$$\frac{J_s}{H_{inc}} \simeq \frac{a}{2b}\left[\ell n\left(\frac{8a}{b}\right) - 2\right]^{-1}$$

D. Circular loop: magnetic field perpendicular to plane

Fig. 2.1. Canonical Examples for Surface Current Densities on Perfectly Conducting Objects in Low-Frequency Magnetic Fields

A sphere might be considered a "short, fat" object. A circular cylinder might be considered a "long, slender" object, but they both have about the same J_s/H_{inc} ratio. A sphere is a closed surface. Similarly, if the circular cylinder is truncated with a long length-to-diameter ratio, it can be made a closed surface with about the same J_s/H_{inc} ratio. Alternately, one could consider a long slender prolate spheroid with about the same results. So the "length-to-diameter" ratio going from a sphere to a long, slender cylinder does not seem to make a large difference.

Contrast to this the case of the circular loop for which the surface current density can be much larger. The reason for this is the large area enclosed by the closed loop compared to the area of the loop conductors. A rectangular or elliptical loop has a similar response. This leads to a topological result. Closed loops have large low-frequency magnetic-field responses. Thus, one may wish to avoid loop-like structures in the design of subshields in some cases.

Note that in the circular loop in fig. 2.1D if b/a approaches 1 the above discussion does not apply. The above results are an approximation valid for loop-conductor area small compared to area enclosed by the loop.

III. Topology of Subshields

Now let us formulate the results of the previous section in more formally topological terms. Consider how to define a subshield. There are various topological terms relevant to surfaces in three-dimensional Euclidean space which we use for defining a subshield [8].

Subshield:

a. connected surface (all in one piece)

b. manifold (a connected surface $\ni$ near each point it is homeomorphic to an open disk)

c. closed surface (bounded and has no boundary)

d. two-sided surface (orientable, divides space into inside and outside)

This definition itself uses some terms worth discussing.

Connected:

In a topological context, two sets are connected iff a path can be constructed from any point in one set to any point in the second without leaving the union of the two sets.

Bounded:

A set is bounded iff it can be contained in an open ball (the interior of a sphere, not including the boundary sphere).

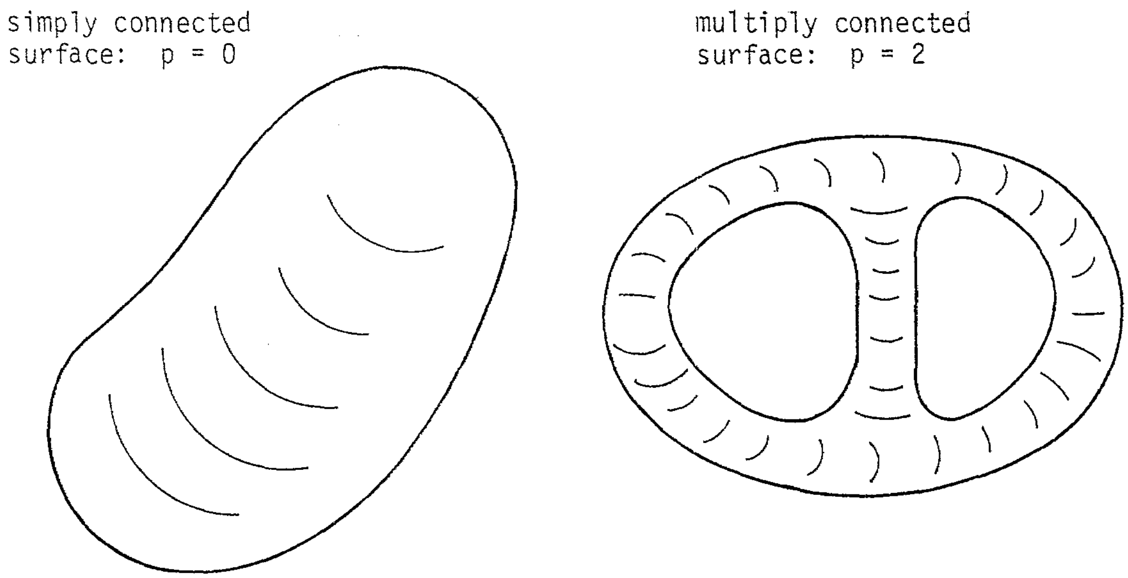
Open:

An open set does not include its boundary. In the case of a surface, each point has a neighborhood (homeomorphic to an open disk) $\ni$ one can construct a path around this point sufficiently close to the point so as to remain in the surface.

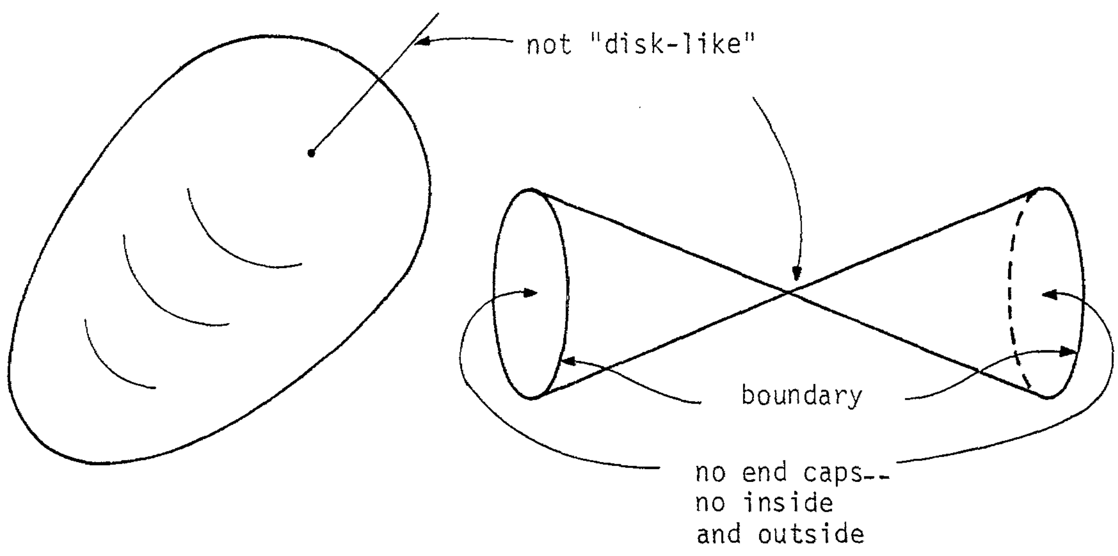
Homeomorphism or topological transformation:

This is a transformation which is continuous and has a continuous inverse transformation. In other words, it is a one-to-one transformation in which points in one set that are sufficiently close to each other correspond to arbitrarily close points in the second set, and the converse.

For a deeper discussion of these points, the reader may consult various books on topology. Figure 3.1 illustrates some of the above points with examples of surfaces. Figure 3.1A exhibits allowable surfaces for subshields; fig. 3.1B exhibits some surfaces that are not subshields.



A. Allowed for subshields



B. Not allowed for subshields

Fig. 3.1. Examples of Surfaces

Now, a basic topological theorem is [8]:

> "Any closed two-sided manifold is topologically equivalent to a sphere with some number of handles."

Such an object is illustrated in fig. 3.2. Cut 2p holes in a sphere and bend p different tubes $\ni$ their ends fit in these holes. Hence we have from our definition of a subshield:

Theorem 3.1: Any subshield is homeomorphic (topologically equivalent) to a sphere with p handles with

$$p = 0,1,2,... \tag{3.1}$$

Of all these possible subshields, the case of $p = 0$ corresponds to no "loops" (or better, no handles) in the subshield in accordance with the discussion in the prevous section. This case of $p = 0$ then has special interest.

The case of no handles can also be considered from the viewpoint of simple connectedness.

Simply connected surface:

> A surface is simply connected iff every simple closed curve in that surface can be deformed into a point while continuously remaining in the surface.

Simple closed curve:

> A simple closed curve is homeomorphic to a circle.

Figure 3.3 shows examples of simply and multiply connected surfaces. In each case, a simple closed curve C is included on S to illutrate the above definition of a simply connected surface. Another test for a simply connected closed manifold is:

> A closed two-sided manifold S is not simply connected (i.e., is multiply connected) iff there exists a simple closed curve C in S which does not divide S into two unconnected paths.

For a subshield we then have

$$\begin{array}{ll} p = 0 & \text{simply connected} \\ p \geq 1 & \text{multiply connected (with multiplicity p)} \end{array} \tag{3.2}$$

The special case of a subshield with no "loops" (or handles) can then be described as a closed two-sided simply connected manifold. Removal of loops

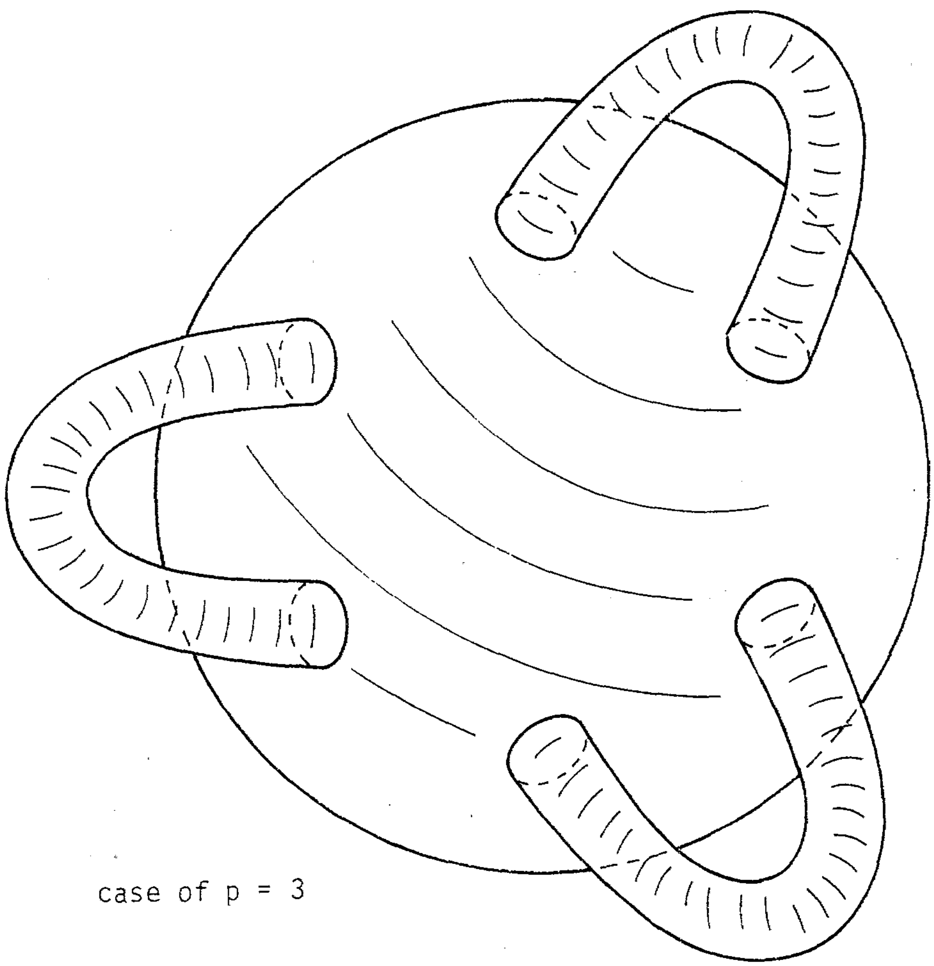
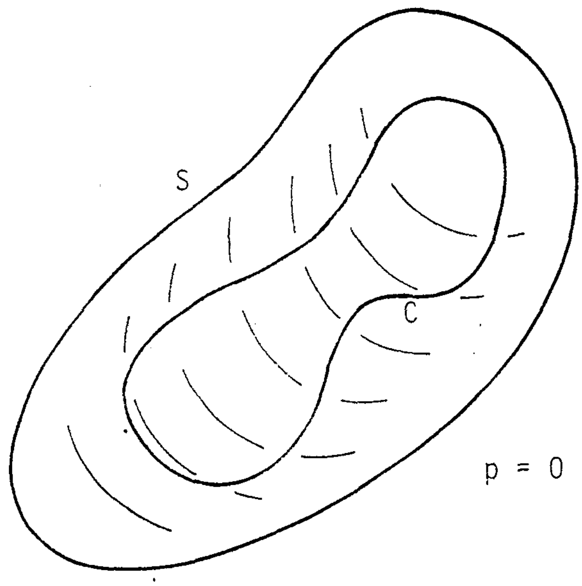
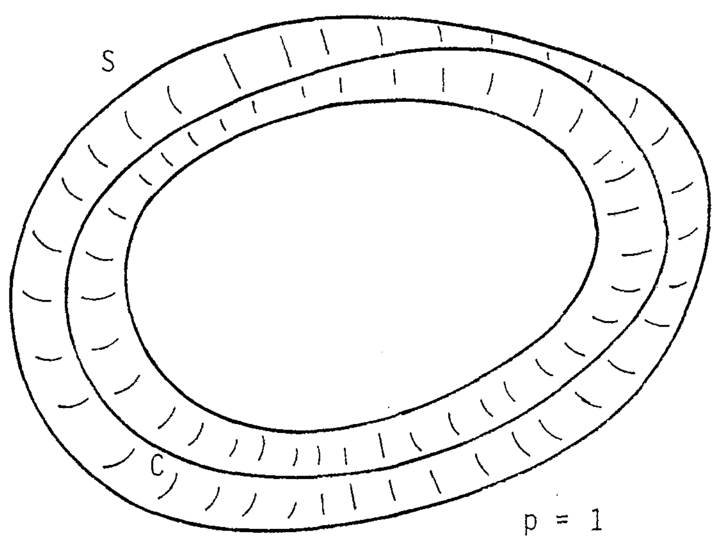


Fig. 3.2. Sphere with p Handles: General Case of Subshield



A. Simply connected



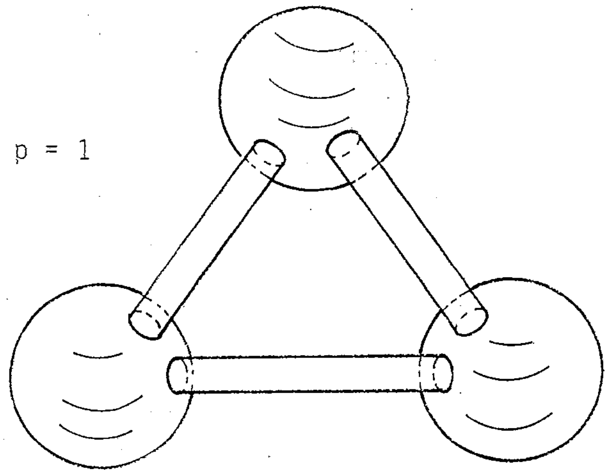
B. Not simply connected (multiply connected)

Fig. 3.3. Examples of Possible Subshields

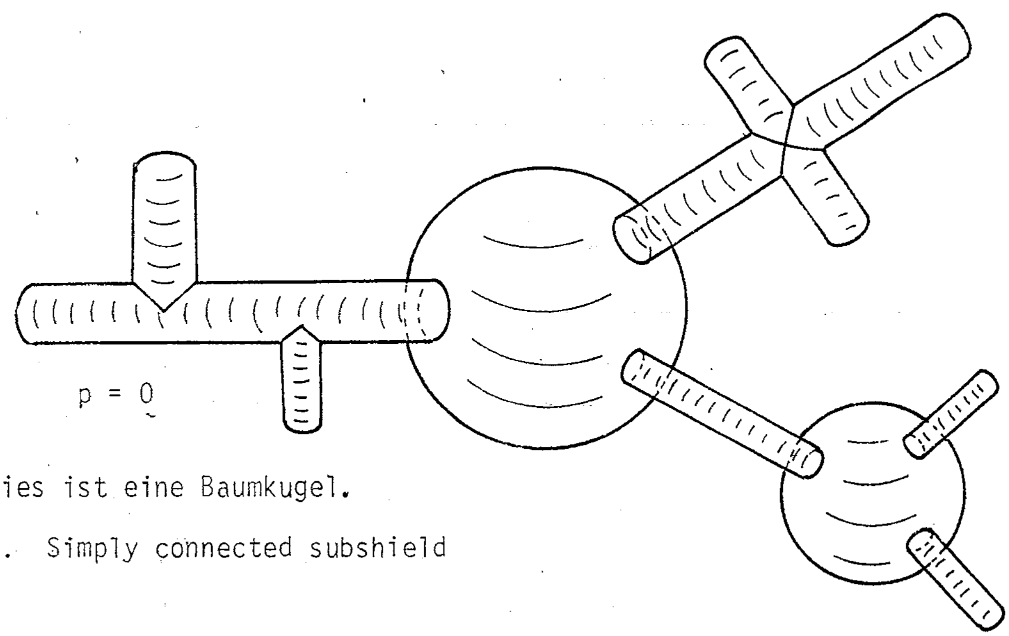
in the network sense from a subshield comprised of "conduit-like" conductors can be obtained by making the subshield "tree like" in the network sense, or better, simply connected or have $p = 0$.

As a practical matter low-frequency shielding problems are sometimes encountered in instrumentation systems which attempt to record slow low-level signals in the presence of significant noise. Such an instrumentation system might have a subshield (typically the outermost shield $S_{1;2}$) as part of its electromagnetic topology. Imagine, if you will, one or more instrumentation vans as part of this subshield together with various conduit-like structures connecting the van shells and some arrays of sensors together for shielding and "ground" reference. Two examples of this are illustrated in fig. 3.4. In fig. 3.4A, note that the "conduits" form a "loop" or handle in connecting the "van shells." Contrast this with the case in fig. 3.4B in which there are no handles and hence no loops in the circuit sense, even for a rather elaborate layout of "conduits" and "van shells" (in a generalized sense). Looking at this last example, note that it illustrates what is both a tree in the circuit or network sense and a sphere (with no handles) in the surface sense. Noting the historical German influence on topology (e.g., well-known connections to Möbius and Euler), let us name such a subshield as a

$$\begin{aligned} \text{Baumkugel} &\equiv \text{tree-sphere} \\ \text{Baum} &\equiv \text{tree} \\ \text{Kugel} &\equiv \text{sphere} \\ \text{Baumkugel} &\equiv \text{subshield with no handles or simply connected subshield} \end{aligned} \tag{3.3}$$



A. Multiply connected subshield



Dies ist eine Baumkugel.

B. Simply connected subshield

Fig. 3.4. Examples of Subshields (for Instrumentation Systems)

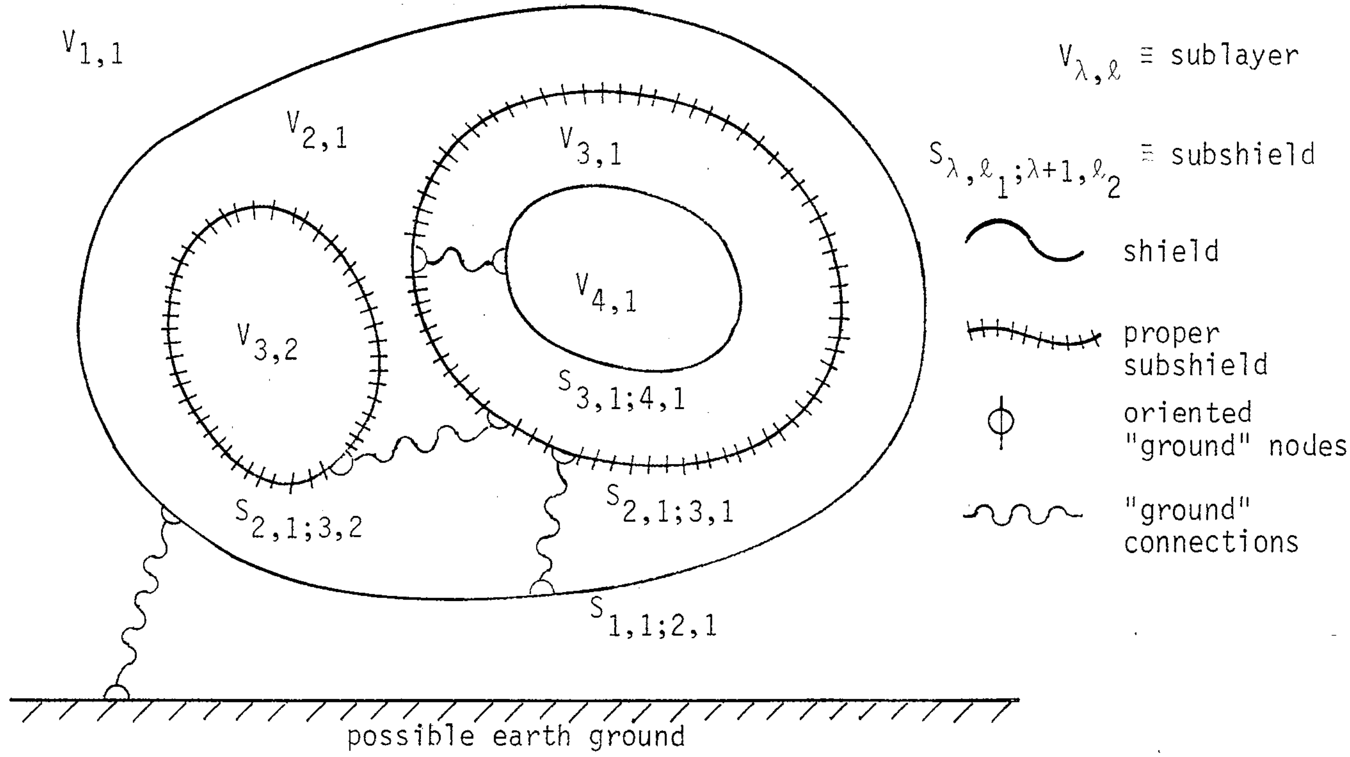
IV. Inclusion of Grounding Networks in Volume/Surface Topology

"Grounding" is a term often misused in electrical engineering practice. It is a term which is sometimes given some absolute status even though some frequencies of concern have wavelengths small compared to the distance from the conductor to be "grounded" to the "ground" for the system, as if by some magic, connecting a conductor between these two distant "points" (or modes) will solve all electromagnetic interference problems. This absurdity results from an inadequate view of the electromagnetic interference problem, particularly at high frequencies. An adequate grounding system must be consistent with other electromagnetic concepts, such as shielding, particularly at high frequencies. One cannot cavalierly run grounding conductors in systems with shielding as though the shield conductors were not significant. It is precisely such an attitude which can result in a destruction of the effectiveness of the intended shielding. Cutting holes in conducting shields and running (insulated) wires through these holes is one of the worst things which one can generally do and which should not be tolerated [2].

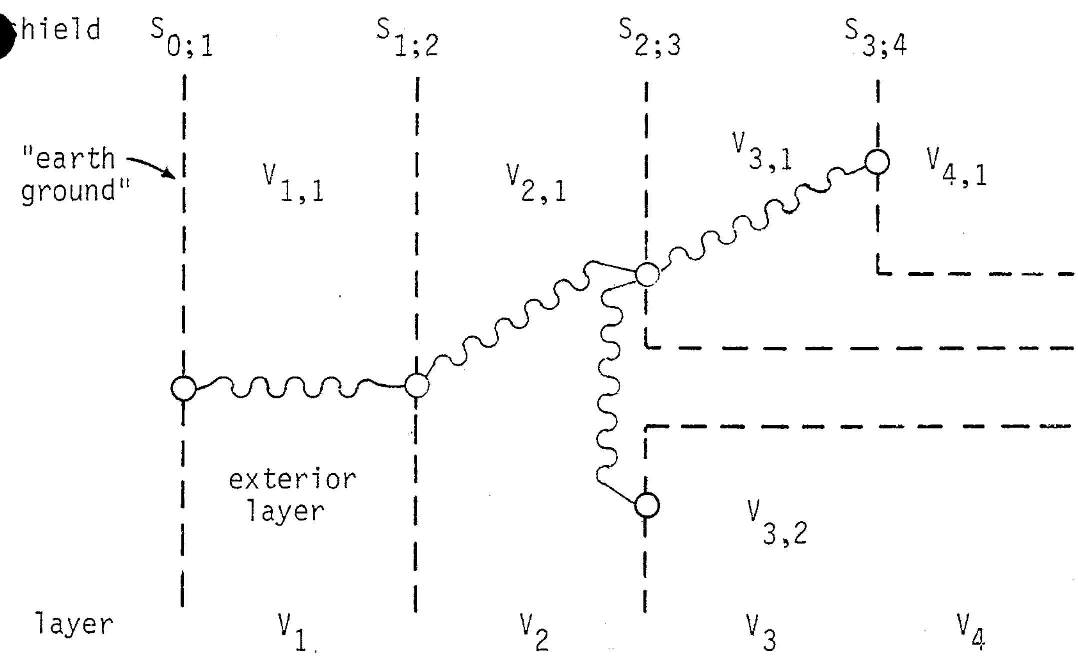
Our problem is to formulate the electromagnetic topology of grounding conductors in such a way that they do not violate the topology of the subshields in our electronic system of concern. Basically, in a topological sense, we work down from the top, beginning with volumes and surfaces in three-dimensional Euclidean space to include branches and nodes in a circuit sense.

For some systems, low-frequency interference is sufficiently troublesome and the subshields have sufficiently small L/R time constant to be ineffective for this part of the interference. Noting that practical subshields are typically mostly highly conducting metal, one needs to consider these conductors as well as others as part of a "grounding" scheme. Given a hierarchical volume/surface topology [3,4] then let us consider the grounding network which interconnects the subshields as well as the electronic equipment.

Figure 4.1 shows some simple considerations for "grounding" subshields. Figure 4.1A shows an example volume/surface hierarchical topology. Not only are the sublayers and subshields indicated, but other topological "grounding" entities are introduced. In particular, we have oriented (or two-sided) ground nodes corresponding to each subshield and ground connections (or branches) connecting these nodes. Note that since ground conductors are not



A. Volume/surface topology with grounding connections



B. Grounding graph

Fig. 4.1. Grounding of Subshields in Hierarchical Volume/Surface Topology

allowed to penetrate subshields it is important to identify on which side (inside or outside) of the subshield the ground connection is made. Each connection is then contained within some one sublayer, but may connect to the boundary subshields of that sublayer only on the side of each boundary corresponding to that same sublayer.

Figure 4.1B shows an equivalent grounding graph which emphasizes the grounding nodes and connections. Note that there is only one oriented node for each subshield. This convention means that no matter where one physically connects to a subshield, it corresponds to the same grounding node. Hence, in our example, the two connections to the exterior of $S_{2,1;3,1}$ are connections to the exterior side of the same node in fig. 4.1B. In this example, the grounding graph is a tree graph and the earth ground corresponds to an artificial reference subshield. Note that the grounding graph is not the same as the interaction sequence diagram or dual bipartite graph for the volume/surface topology. This can be seen by a comparison to this same volume/surface topology as illustrated in fig. 2.1 of a previous note [4]. However, the volume/surface topology does introduce some similar ordering in the two graphs because of the requirement that ground connections do not cross subshields.

Generalizing this discussion, let us define

Grounding network:

graph consisting of

Φ oriented ground nodes (vertices) corresponding to subshields

○ unoriented ground nodes (vertices) within sublayers, and

∿ ground connections (branches or edges)

Now, in such a grounding network, since ground connections are restricted to not cross subshields, one can partition the grounding network by splitting the oriented ground nodes. This gives a set of grounding subnetworks where each subnetwork is contained within a particular sublayer. There are as many grounding subnetworks in general as there are sublayers in the volume/surface topology.

Consider then the design of sublayer grounding subnetworks. An example is given in fig. 4.2. In the sublayer $V_{\lambda,\ell}$ we have

$$G_{\lambda,\ell}^{(n)} \equiv \text{nth ground node}$$

$$C_{\lambda,\ell}^{(n,m)} \equiv \text{connection between nth and mth ground node where such exists} \tag{4.1}$$

In our example we have

$$n = 1,2,\ldots,7$$

$$\text{oriented nodes for } n = 1,2,3 \tag{4.2}$$

$$\text{unoriented nodes for } n = 4,5,6,7$$

Note that there are three subshields bounding $V_{\lambda,\ell}$ in this example. Note also that connections to a given subshield, even if in two (or more) locations, still connect to the same "node" with the same symbol. Furthermore, not all pairs of ground nodes have direct connections between them in this example; there are six connections between the seven nodes in this tree graph. Ideally, the ground connections have zero D.C. impedance.

In the sublayer grounding subnetwork, additional ground nodes (unoriented) have been introduced. These can serve the role of "equipment grounds" or "instrumentation grounds." They serve as local ground reference for whatever one may need and can be arbitrary in number to serve one's D.C. (i.e., sufficiently low frequency) requirements.

In designing a sublayer grounding network for $V_{\lambda,\ell}$, we have numbers of grounding nodes as

$$N_{\lambda,\ell}^{(S)} \equiv \text{number of subshields bounding } V_{\lambda,\ell}$$

$$\equiv \text{number of independent oriented grounding nodes}$$

$$N_{\lambda,\ell}^{(E)} \equiv \text{number of "equipment" ground nodes inside } V_{\lambda,\ell}\text{, not including the boundaries}$$

$$\equiv \text{number of unoriented ground nodes} \tag{4.3}$$

$$N_{\lambda,\ell}^{(G)} \equiv N_{\lambda,\ell}^{(S)} + N_{\lambda,\ell}^{(E)}$$

$$\equiv \text{total number of independent ground nodes in } V_{\lambda,\ell}$$

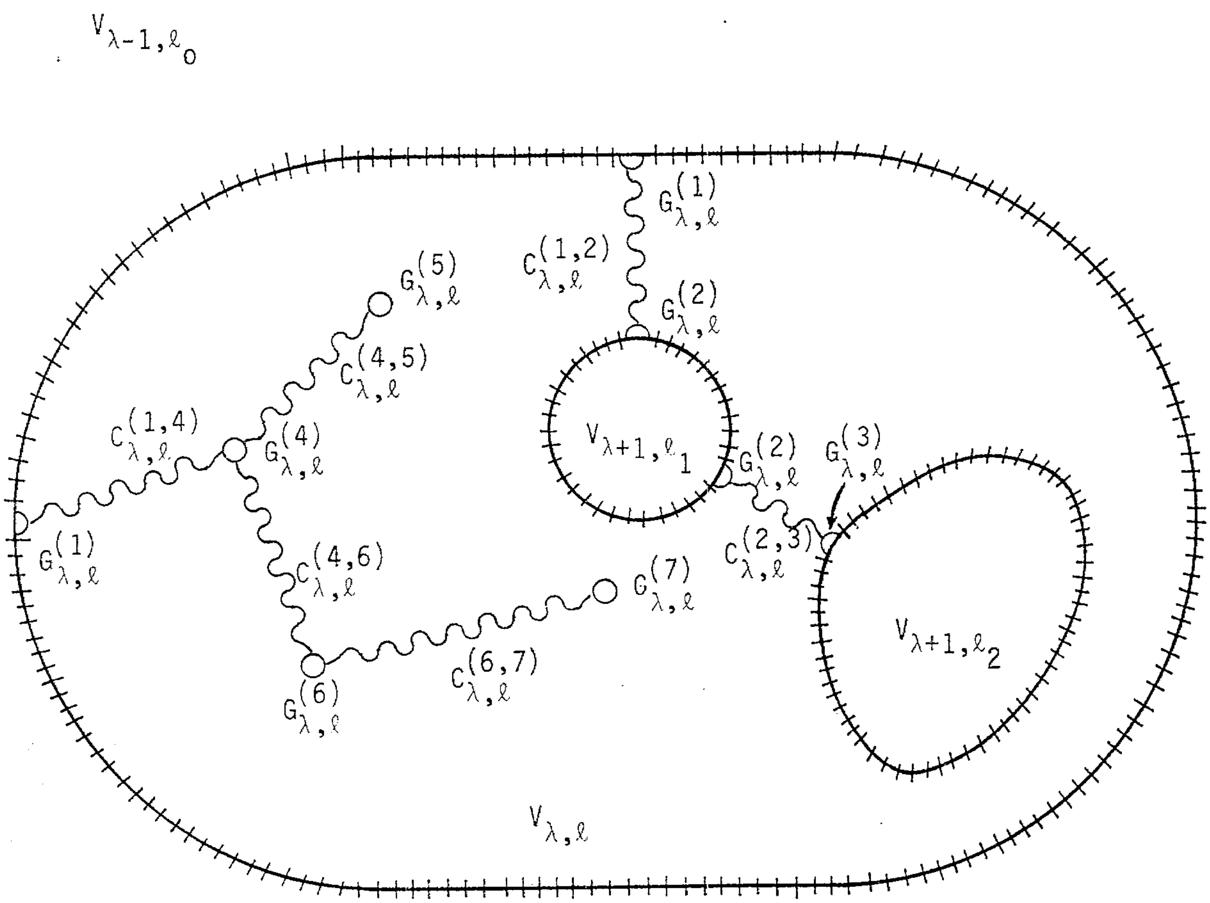


Fig. 4.2. Sublayer Grounding Subnetwork (Subgraph)

Similarly, for ground connections we have

$$N_{\lambda,\ell}^{(C)} \equiv \text{number of ground connections in } V_{\lambda,\ell} \tag{4.4}$$

The number of ground connections is bounded as

$$0 \le N_{\lambda,\ell}^{(C)} \le \frac{1}{2} N_{\lambda,\ell}^{(G)} \left[N_{\lambda,\ell}^{(G)} - 1 \right] \tag{4.5}$$

which merely expresses all possible connections of $N_{\lambda,\ell}^{(G)}$ things taken two at a time and no connections at all as the limiting cases.

Some sets of ground connections may give superior grounding network designs from various low-frequency viewpoints. Section 2 of this note has discussed the undesirability of loops in the presence of a low-frequency interferring magnetic field. For our sublayer grounding subnetwork to have no loops (closed), the number of connections must be no more than the number of nodes minus one. One may also wish that all the grounds be connected via some path in the grounding network for, say, instrumentation and/or safety reasons. This requires that the number of ground connections be at least as great as the number of ground nodes minus one. These points are covered in standard circuit theory texts [10]. Summarizing we have

$$N_{\lambda,\ell}^{(C)} = \begin{cases} \le N_{\lambda,\ell}^{(G)} - 1 & \text{for a constraint of \underline{no loops}} \\ \ge N_{\lambda,\ell}^{(G)} - 1 & \text{for a constraint of \underline{no floating grounds}} \\ = N_{\lambda,\ell}^{(G)} - 1 & \text{for a constraint of a \underline{tree graph} in which both above requirements are met} \end{cases} \tag{4.6}$$

Thus, analogous to a subshield design as a sphere with no handles, or Baumkugel (tree-sphere) in section 3, we have a tree sublayer grounding subnetwork in this section.

V. Summary

In this note, we have explored some of the implications of shielding against low-frequency magnetic-field interference, particularly in the context of a shielding design which is to be effective at higher frequencies as well. This is begun with the more general volume/surface electromagnetic topology consistent with Maxwell's equations and the control of electromagnetic fields in volumes by the control of electromagnetic parameters on closed boundary surfaces. Then low-frequency circuit considerations are added in a manner consistent with this. A special type of sublayer (Baumkugel) and a special sublayer grounding subnetwork (tree) are derived. This is not to say that such low-frequency considerations are always important. However, one must be careful if one has a more general (higher frequencies included) shielding problem to not violate the volume/surface topology by the low-frequency techniques used.

It is interesting to note that by the inclusion of grounding networks in the volume/surface electromagnetic topology we have a topological model that includes volumes, surfaces, lines, and points. These are the four possible types of topological entities in a three dimensional space [6]. This is summarized in the following table. Note that our present discussion has not considered the low-frequency inplications of elementary volumes and elementary surfaces; perhaps some further development would give some further insight.

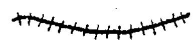
Number of Dimensions	Topological Category	Specific Types	Pictorial Symbol	Algebraic Symbol
3	volume (V)	layer		V_λ
		sublayer		$V_{\lambda,\ell}$
		elementary volume		$V_{\lambda,\ell,\tau}$
2	surface (S)	shield		$S_{\lambda;\lambda+1}$
		subshield	 (proper subshield)	$S_{\lambda,\ell;\lambda+1,\ell'}$
		elementary surface		$S_{\lambda,\ell,\tau;\lambda+1,\ell',\tau'}$
1	line (ground connection C)			$C^{(n,m)}_{\lambda,\ell}$
0	point (ground node G)	oriented	ϕ	$G^{(n)}_{\lambda,\ell}$
		unoriented	○	

Table 5.1. Topological Entities in Three Dimensional Space

References

1. R. W. Latham and K.S.H. Lee, Theory of Inductive Shielding, Interaction Note 12, March 1968.

2. E. F. Vance, On Electromagnetic Interference Control, Interaction Note 380, October 1979.

3. C. E. Baum, Electromagnetic Topology: A Formal Approach to the Analysis and Design of Complex Electronic Systems, Interaction Note 400, September 1980.

4. C. E. Baum, Sublayer Sets and Relative Shielding Order in Electromagnetic Topology, Interaction Note 416, April 1982.

5. C. E. Baum, Multiple Electromagnetic Topologies for Describing Various Aspects of Electronic Systems and Various System States, Proc. 1980 Spring FULMEN Meeting (FULMEN 4), p. 33, March 1980.

6. F. H. Branin, Jr., The Relation Between Network Theory, Vector Calculus, and Theoretical Physics, Proc. International Symposium on Operator Theory of Networks and Systems, vol. 2, August 1977, Texas Tech. U., pp. 97-102.

7. R. V. Langmuir, Electromagnetic Fields and Waves, McGraw-Hill, 1961.

8. B. H. Arnold, Intuitive Concepts in Elementary Topology, Prentice-Hall, 1962.

9. W. R. Smythe, Static and Dynamic Electricity, 3rd ed., McGraw-Hill, 1968.

10. M. E. Van Valkenburg, Network Analysis, 3rd ed., Prentice-Hall, 1974.

11. C. E. Baum, The Role of Scattering Theory in Electromagnetic Interference Problems, in P.L.E. Uslenghi (ed.), Electromagnetic Scattering, Academic Press, 1978.

12. K.S.H. Lee (ed.), EMP Interaction: Principles, Techniques, and Reference Data, EMP Interaction 2-1, December 1980.