Interaction Notes

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Natural Frequencies and Natural Modes of an Aircraft Using a Six Stick-Model with Different Radii

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Abstract

This report extends earlier work by Bedrosian [1] dealing with a stick-model characterization of the natural frequencies and natural modes of an aircraft. In this reference, the stick model parameter $\Omega = 2\ell n$ [2(stick length)/(stick radii)] is assumed to be the same for all sticks. A logical extension of Bedrosian's work is then to allow for individual sticks to have different Ω's and compare the accuracies. In this note, we have reformulated the problem allowing for different Ω's. Another improvement over the past work is the use of a frequency dependent junction condition [2] on change densities on various sticks. Numerical results for the B-1, E-4 and EC-135 aircrafts are also presented and compared with the earlier results.

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I. Introduction

As was pointed out in the abstract, Bedrosian's work [1] employs a constant stick-parameter $\Omega = 2\ell n$ [2(stick length)/(stick radius)] for all the six sticks used in modeling an aircraft. This results in the use of realistic lengths for individual sticks, but somewhat adjusted radii in order that the parameter Ω is the same for all sticks. The process of determining the natural frequencies also requires the satisfaction of the Kirchoff's current condition at all junctions, the end conditions and a condition on the charge densities at all junctions. Reference [1] uses the condition that the charge per unit length Q_i (not the surface charge density η_i) be continuous at the junction. In the present work, a frequency dependent junction condition derived by Wu and King [2] has been employed. This condition is that the quantity $[\psi_i \; Q_i]$ is continuous at the junction with $\psi_i \simeq 2\,[\ell n\{2/(ka_i)\} - \gamma]$. Unlike the continuity of the linear (Q_i) or the surface (η_i) density of charge, the condition used here is frequency dependent and is expected to yield more accurate results. Related published work dealing with the determination of natural frequencies consist of a model of perpendicular crossed wires over perfect [3] and imperfect [4] ground media. After finding the natural frequencies, the natural modes can be determined and classified on the basis of symmetry considerations discussed in [5].

The natural frequencies and natural modes are presented here for three aircrafts: the B-1 with wings swept forward, the E-4 and the EC-135. The present results of the modified stick-model are compared with the earlier stick-model characterization [1].

II. The Modified Six-Length Stick Model

The modification of earlier work of Bedrosian [1] consists in letting each stick have its own radius and consequently, different Ω_i parameters. In view of this, the induced current on individual stick by an incident plane wave is approximated by

$$I_{ind_i n}(k, x, \Omega_i, \theta) = \frac{4\pi j E_o}{k Z_o \Omega_n \sin\theta} \exp(jkx\cos\theta) \quad (1)$$

where (see figure 1),

Ω_n = 2 ℓn [2(nth stick length)/nth stick radius)]

k ≡ wave number of the incident field

x ≡ position along the stick

Z_o ≡ impedance of free space

E_o ≡ electric field strength of the incident wave

θ ≡ angle between $\vec{k}$ and $-\vec{1}_x$.

As in reference [1], the phase of the incident field is set to zero at $x = 0$ and $\vec{H}$ is polarized perpendicular to the stick. Note that the induced current of equation (1) is devoid of the end condition and that the lengths and radii of the eight sticks modeling a general aircraft are specified in figure 1.

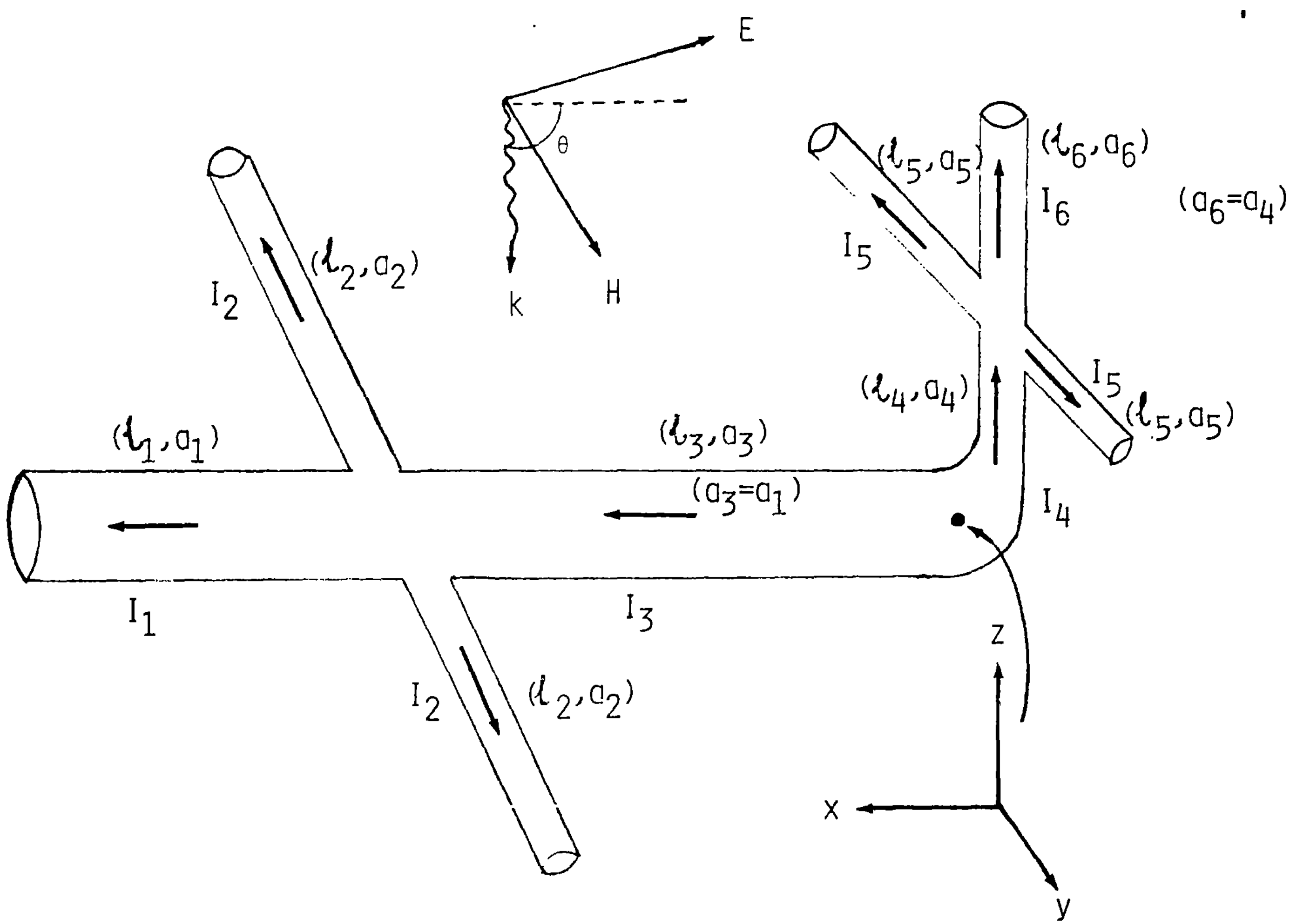

Figure 1. The six stick model with different radii.

To a first order in $(1/\Omega_n)$, the current on each stick can be written as a directly induced component plus a sine and a cosine component, as given by

$$I_n(\xi) = I_{ind,n}(k, \Omega_n, \xi, \theta) + S_n \sin[k(\xi - \ell_n)] + C_n \cos[k(\xi - \ell_n)] \tag{2}$$

for $n = 1$ to 6

The constants S_n and C_n must be chosen to satisfy the free end conditions $I_n(\xi_{end}) = 0$ and the juction conditions of current and charge. The free end conditions lead to the following

$$I_1(x) = I_{ind,1}(k, \Omega, x, \theta) + S, \sin[k(x - \ell_1 - \ell_3)]$$

$$- I_{ind,1}(k, \Omega, \ell_1 + \ell_3, \theta) \cos[k(x - \ell_1 - \ell_3)]$$

$$I_2(y) = S_2 \sin[k(y - \ell_2)] \qquad (y > 0)$$

$$I_3(x) = I_{ind,3}(k, \Omega_3, x, \theta) + S_3 \sin[k(x - \ell_3)] + C_3 \cos[k(x - \ell_3)]$$

$$I_4(z) = -I_{ind,4}(k, \ell_4, z, \pi/2 - \theta) + S_4 \sin[k(z - \ell_4)] + C_4 \cos[k(z - \ell_4)]$$

$$I_5(y) = S_5 \sin[k(y - \ell_5)] \qquad (y > 0)$$

$$I_6(z) = -I_{ind.6}(k, \Omega_6, z, \pi/2 - \theta) + S_6 \sin[k(z - \ell_4 - \ell_6)]$$

$$+ I_{ind,6}(k, \Omega_6, \ell_4 + \ell_6, \pi/2 - \theta) \cos[k(z - \ell_4 - \ell_6)]$$

(3)

Equation (3) satisfies the following 4 end conditions

$$\left.\begin{array}{ll} I_1(\ell_1 + \ell_3) = 0 & ; \quad x = (\ell_1 + \ell_3) \\ I_2(\ell_2) = 0 & ; \quad y = \ell_2 \\ I_5(\ell_5) = 0 & ; \quad y = \ell_5 \\ I_6(\ell_4 + \ell_6) = 0 & ; \quad z = (\ell_4 + \ell_6) \end{array}\right\} \quad (4)$$

Remaining 8 unknowns in equation (3) are

S_1, S_2 , S_3, S_4, S_5, S_6, C_3, and C_4

needing 8 more equations; they are:

1, 2, 3) equations Kirchoff's law at $x = \ell_3$, $x = \ell_3$, $x = z = 0$ and $z = \ell_4$

4) $Q_1 \psi_1 = Q_3 \psi_3$ & 5) $Q_2 \psi_2 = Q_3 \psi_3$ at $x = \ell_3$, $y = 0$

6) $Q_3 \psi_3 = Q_4 \psi_4$ at $x = z = 0$

7) $Q_4 \psi_4 = Q_5 \psi_5$ & 8) $Q_4 \psi_4 = Q_6 \psi_6$ at $z = \ell_4$, $y = 0$

where the charge per unit length Q_n and the expansion parameter ψ_n are given by

$$Q_n(\xi) = -\frac{1}{j\omega}\frac{dI_n(\xi)}{d\xi}, \quad \text{for } n = 1 \text{ to } 6 \tag{5}$$

$$\psi_n = 2\left[\ell n\left(\frac{2}{ka_n}\right) - \gamma\right]; \quad \text{for } n = 1 \text{ to } 6 \tag{6}$$

with

$k = \frac{\omega}{c}$; $\omega = 2\pi f$; f = frequency in Hz and

c = free space speed of light

a_n = radius of the nth wire

γ = 0.577 Euler's constant

8 Boundary conditions needed in solving for the 8 unknowns

(i) $I_1\ (x = \ell_3) + 2\ I_2\ (x = \ell_3) = I_3\ (x = \ell_3)$ (7)

(ii) $I_3\ (x = 0) + I_4\ (z = 0) = 0$ (8)

(iii) $I_6\ (z = \ell_4) + 2\ I_5\ (y = 0) = I_4\ (z = \ell_4)$ (9)

(iv) $\left.\frac{\partial I_1(x)}{\partial x}\right|_{x = \ell_3} \psi_1 = \left.\frac{\partial I_3(x)}{\partial x}\right|_{x = \ell_3} \psi_3$ (10)

(v) $\left.\frac{\partial I_2(y)}{\partial y}\right|_{y = 0} \psi_2 = \left.\frac{\partial I_3(x)}{\partial x}\right|_{x = \ell_3} \psi_3$ (11)

(vi) $\left.\frac{\partial I_3(x)}{\partial x}\right|_{z = \ell_4} \psi_3 = \left.\frac{\partial I_4(z)}{\partial z}\right|_{z = 0} \psi_4$ (12)

(vii) $\left.\frac{\partial I_4(z)}{\partial z}\right|_{z = \ell_4} \psi_4 = \left.\frac{\partial I_5(y)}{\partial y}\right|_{y = 0} \psi_5$ (13)

(viii) $\left.\frac{\partial I_4(z)}{\partial z}\right|_{z = \ell_4} \psi_4 = \left.\frac{\partial I_6(z)}{\partial z}\right|_{z = \ell_4} \psi_6$ (14)

After applying the above conditions, one gets the following matrix equation for the unknown quantities.

TABLE 1. Matrix Equation (15)

$$
\begin{bmatrix}
-\sin k\ell_1 & -2\sin k\ell_2 & 0 & -1 & 0 & 0 & 0 & 0 \\
\cos k\ell_1 & 0 & -\frac{\psi_3}{\psi_1} & 0 & 0 & 0 & 0 & 0 \\
0 & \cos k\ell_2 & -\frac{\psi_3}{\psi_2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\sin k\ell_3 & \cos k\ell_3 & \cos k\ell_4 & -\sin k\ell_4 & 0 & 0 \\
0 & 0 & \cos k\ell_3 & \sin k\ell_3 & -\frac{\psi_4}{\psi_3}\sin k\ell_4 & -\cos k\ell_4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{\psi_4}{\psi_3} & \cos k\ell_5 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{\psi_4}{\psi_6} & 0 & \cos k\ell_6 \\
0 & 0 & 0 & 0 & -1 & 0 & -2\sin k\ell_6 & -\sin k\ell_6
\end{bmatrix}
\begin{bmatrix}
S_1 \\ S_2 \\ S_3 \\ C_3 \\ C_4 \\ S_4 \\ S_5 \\ S_6
\end{bmatrix}
= \frac{4\pi j E_o}{k Z_o}
\begin{bmatrix}
\csc\theta\, e^{jk\ell_3\cos\theta}\left(\frac{1}{\Omega_3} - \frac{1}{\Omega_1}\right) + \frac{\cos k\ell_1}{\Omega_1}\csc\theta\, e^{jk(\ell_1 + \ell_3)\cos\theta} \\
e^{jk\ell_3\cos\theta}\, j(\cot\theta)\left(\frac{\psi_3}{\psi_1}\frac{1}{\Omega_3} - \frac{1}{\Omega_1}\right) + \frac{(\sin k\ell_1)(\csc\theta)}{\Omega_1} e^{jk(\ell_1 + \ell_3)\cos\theta} \\
+ j\frac{\cot\theta}{\Omega_3} e^{jk\ell_3\cos\theta} \\
\left(\frac{\sec\theta}{\Omega_4} - \frac{\csc\theta}{\Omega_3}\right) \\
-j\left(\frac{\psi_4}{\psi_3}\frac{1}{\Omega_4}\tan\theta + \frac{1}{\Omega_3}\cot\theta\right) \\
-j\frac{\tan\theta}{\Omega_4}\left(\frac{\psi_4}{\psi_5}\right) e^{jk\ell_4\sin\theta} \\
-j\tan\theta\, e^{jk\ell_4\sin\theta}\left(\frac{1}{\Omega_4}\frac{\psi_4}{\psi_6} - \frac{1}{\Omega_6}\right) - \sin k\ell_6 \frac{\sec\theta}{\Omega_6} e^{jk(\ell_4 + \ell_6)\sin\theta} \\
\sec\theta\, e^{jk\ell_4\sin\theta}\left(\frac{1}{\Omega_6} - \frac{1}{\Omega_4}\right) - (\cos k\ell_6)\left(\frac{\sec\theta}{\Omega_6}\right) e^{jk(\ell_4 + \ell_6)\sin\theta}
\end{bmatrix}
$$

It is observed that the present matrix equation (15) of Table 1 reduces identically to the matrix equation (4) of reference [1] in the case of $\psi_n = 1$ and $\Omega_n = \Omega$ for $n = 1$ to 6.

III. Determination of Natural Frequencies and Natural Modes

The natural frequencies and natural modes are found by setting $E_o = o$ and investigating equation (15) for non-trivial solutions. Setting the determinant equal to zero, one obtains

$$
\begin{aligned}
0 = {} & 2\, c_1\, c_4\, c_5\, c_6\, s_2\, \psi_{32} - 2\, c_1\, c_3\, c_5\, s_2\, s_4\, s_6\, \psi_{32}\, \psi_{43}\, \psi_{46} \\
& - 4\, c_1\, c_3\, c_6\, s_2\, s_4\, s_5\, \psi_{32}\, \psi_{43}\, \psi_{45} - 4\, c_1\, c_4\, c_6\, s_2\, s_3\, s_5\, \psi_{32}\, \psi_{45} \\
& - 2\, c_1\, c_5\, c_6\, s_2\, s_3\, s_4\, \psi_{32} - 2\, c_1\, c_4\, c_5\, s_2\, s_3\, s_6\, \psi_{32}\, \psi_{46} \\
& + c_2\, c_3\, c_4\, c_5\, c_6\, s_1\, \psi_{31} - c_2\, c_3\, c_5\, s_1\, s_4\, s_6\, \psi_{31}\, \psi_{43}\, \psi_{46} \\
& - 2\, c_2\, c_3\, c_6\, s_1\, s_4\, s_5\, \psi_{31}\, \psi_{43}\, \psi_{45} - 2\, c_2\, c_4\, c_6\, s_1\, s_3\, s_5\, \psi_{31}\, \psi_{45} \\
& - c_2\, c_5\, c_6\, s_1\, s_3\, s_4\, \psi_{31} - c_2\, c_4\, c_5\, s_1\, s_3\, s_6\, \psi_{31}\, \psi_{46} \\
& + c_1\, c_2\, c_3\, c_4\, c_5\, s_6\, \psi_{46} + 2\, c_1\, c_2\, c_3\, c_4\, s_5\, \psi_{45}\, c_6 \\
& + c_1\, c_2\, c_3\, c_5\, c_6\, s_4 + c_1\, c_2\, c_4\, c_5\, s_3\, c_6 \\
& - c_1\, c_2\, c_5\, s_3\, s_4\, s_6\, \psi_{43}\, \psi_{46} - 2\, c_1\, c_2\, c_6\, s_3\, s_4\, s_5\, \psi_{43}\, \psi_{45}
\end{aligned}
\qquad (16)
$$

where the following abbreviated notation is employed

$$c_i = \cos(k\ell_i)\ ; \quad S_i = \sin(k\ell_i)$$

and (17)

$$\psi_{ij} = \psi_i/\psi_j$$

It has also been verified that the above equation (16) reduces to equation (5) of reference [1] in the limit of $\psi_n = \Omega$ for $n = 1$ to 6. However, while in this limiting case of reference [1], the solution for the six constants in terms of one of the arbitrary constants can be written in closed form, such elegance is not possible in the present general case. The matrix equation has been solved numerically. The natural modes are then determined by substituting the matrix solution into equation (3).

The final task is to find the imaginary components of the natural frequencies. The procedure followed is same as in reference [1] and the temporal damping constant α_m is given by

$$\alpha_m = \frac{P_m}{4W_m} = \frac{ck_m P'_m}{D_m} \tag{18}$$

where P_m = average radiated power and W_m = stored magnetic energy per cycle. The denominator D_m is given by

$$D_m = \sum_{n=1}^{6} \int I_n^2 \; (\xi_n) \quad d\xi_n \tag{19}$$

and the numerator factor P'_m is given by

$$P'_m = \int_{\text{sticks}'} \int_{\text{sticks}''} \left[P\,(' \,,\, '')\; I(\xi')\; I(\xi'') - \frac{1}{K_m^2} \left(\frac{\partial}{\partial \xi'} I(\xi') \right) \left(\frac{\partial}{\partial \xi''} I(\xi'') \right) \right] \frac{\sin\,[k_m\, |\vec{r}\,' - \vec{r}\,''|]}{|\vec{r}\,' - \vec{r}\,''|} \; d\xi' \; d\xi'' \tag{20}$$

and,

$$P(',\,'') = \begin{cases} 0 \text{ if stick' is perpendicular to stick''} \\ 1 \text{ if stick' is parallel to stick''} \end{cases}$$

IV. Results and Conclusions

The six length modified stick model has been used to calculate the first several natural frequencies and natural modes for the B-1, E-4 and EC-135.

A I R C R A F T : B-1

Segment Lengths (ℓ, meters) and Radii (r, meters)

Segment		[Set 1]	[Set 2]
1	ℓ	24.5	24.4
	r	1.48	1.60
2	ℓ	21.6	20.8
	r*	1.31	1.48
3	ℓ	18.5	14.1
	r	1.12	1.28
4	ℓ	2.0	1.9
	r*	.12	2.55
5	ℓ	8.0	9.6
	r*	.48	.75
6	ℓ	6.5	3.8
	r*	.39	1.24
7	ℓ		5.8
	r		.64

Natural Resonances (f, MHz) and Damping Constants (α, microsec)

Number		[Res 1]	[Res 2]	[Res 3]	[Res 4]	[Res 5]
1	f	2.2	2.19	2.25	2.34	2.42
	α	.73	.72	.61	.64	.57
2	f	3.2	3.19	3.20	3.25	3.25
	α	.65	.57	.56	.61	.62
3	f	6.0	6.00	6.12	6.46	6.55
	α	.40	.39	.34	.24	.25
4	f	8.2	8.22	8.39	8.42	8.81
	α	.27	.28	.24	.28	.31
5	f	9.6	9.60	9.63	9.68	9.73
	α	.13	.12	.12	.12	.11

Remarks

[Set 1]	Data taken from AFWL Interaction Note 326 or reference [1]
[Set 2]	Date from "Observer's Book of Aircraft" (1976). Some radii (r*) computed to preserve surface areas
[Res 1]	AFWL Interaction Note 326 or reference [1]
[Res 2]	1 and r from [Set 1], using new code: omega=7
[Res 3]	1 from [Set 1], r from [Set 2], using new code: psi
[Res 4]	1 and r from [Set 2], using new code: no psi
[Res 5]	1 and f from [Set 2], using new code: psi

A I R C R A F T : E-4

Segment Lengths (ℓ, meters) and Radii (r, meters)

Segment		[Set 1]	[Set 2]
1	ℓ	25	29.6
	r	2.25	3.31
2	ℓ	36	34.7
	r*	3.24	2.07
3	ℓ	36	35.7
	r	3.24	3.19
5	ℓ	12	11.2
	r*	1.08	1.87
6	ℓ	16	13.3
	r*	1.44	2.31

Natural Resonances (f, MHz) and Damping Constants (α, microsec)

Number		[Res 1]	[Res 2]	[Res 3]	[Res 4]	[Res 5]
1	f	1.3	1.32	1.49	1.30	1.48
	α	1.27	1.35	6.82	1.28	5.37
2	f	2.6	2.59	2.51	2.40	2.37
	α	.57	.52	.55	.59	.59
3	f	3.9	3.77	3.58	3.62	3.32
	α	.50	.31	.24	.34	.29
4	f	5.0	4.87	4.72	4.84	4.70
	α	.37	.28	.38	.28	.41
5	f	5.3	5.24	5.30	5.08	5.09
	α	.23	.21	.21	.22	.24
6	f		7.40	7.32	7.14	7.05
	α		.25	.40	.16	.16
7	f		8.45	8.39	7.86	7.74
	α		.33	.30	.35	.32
8	f		9.68	9.81	9.42	9.52
	α		.12	.15	.26	.52

Remarks

[Set 1]	Data (for E-4) taken from AFWL Interaction Note 326
[Set 2]	Data (for B747-200B) from "Observer's Book of Aircraft" (1976). Some radii (r*) computed to preserve surface areas
[Res 1]	AFWL Interaction Note 326
[Res 2]	1 and r from [Set 1], using new code: omega=6.2
[Res 3]	1 from [Set 1], r from [Set 2], using new code: psi
[Res 4]	1 and r from [Set 2], using new code: no psi
[Res 5]	1 and f from [Set 2], using new code: psi

A I R C R A F T : EC-135

Segment Lengths (ℓ, meters) and Radii (r, meters)

Segment		[Set 1]	[Set 2]
1	ℓ	13	20.6
	r	1.01	1.00
2	ℓ	20	25.7
	r*	1.55	1.70
3	ℓ	23	22.8
	r	1.79	1.00
5	ℓ	7	7.3
	r*	.54	1.16
6	ℓ	11	8.8
	r*	.85	1.46

Natural Resonances (f, MHz) and Damping Constants (α, microsec)

Number		[Res 1]	[Res 2]	[Res 3]	[Res 4]	[Res 5]
1	f	2.1	2.17	2.35	2.05	2.12
	α	.78	.83	-.80	.98	-3.36
2	f	4.8	4.73	4.63	3.37	3.39
	α	.38	.32	.40	.43	.44
3	f	6.4	6.36	6.25	5.33	4.76
	α	.26	.20	.16	.23	.23
4	f	7.8	7.78	7.70	7.43	7.15
	α	.20	.15	.17	.24	.32
5	f	9.0	8.82	8.28	9.05	9.11
	α	.17	.14	.12	.10	.10

Remarks

[Set 1]	Data (for EC-135) taken from AFWL Interaction Note 326
[Set 2]	Data (for B707-320C) from "Observer's Book of Aircraft" (1976). Some radii (r*) computed to preserve surface areas
[Res 1]	AFWL Interaction Note 326
[Res 2]	1 and r from [Set 1], using new code: omega=6.5
[Res 3]	1 from [Set 1], r from [Set 2], using new code: psi
[Res 4]	1 and r from [Set 2], using new code: no psi
[Res 5]	1 and f from [Set 2], using new code: psi

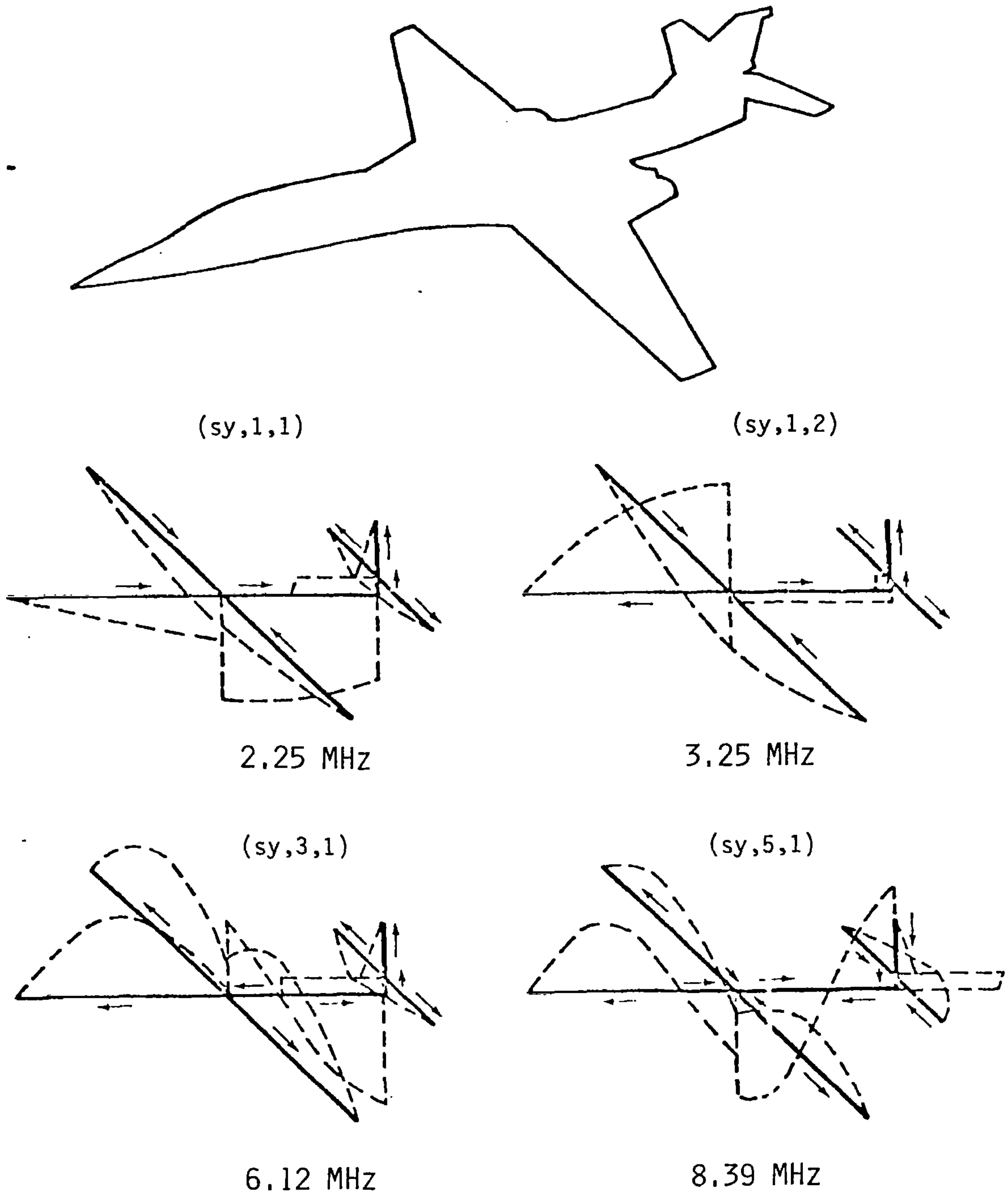

Figure 2. B-1 natural modes. The dashed lines represent the current distribution on the aircraft segments at resonance, while the arrows indicate direction.

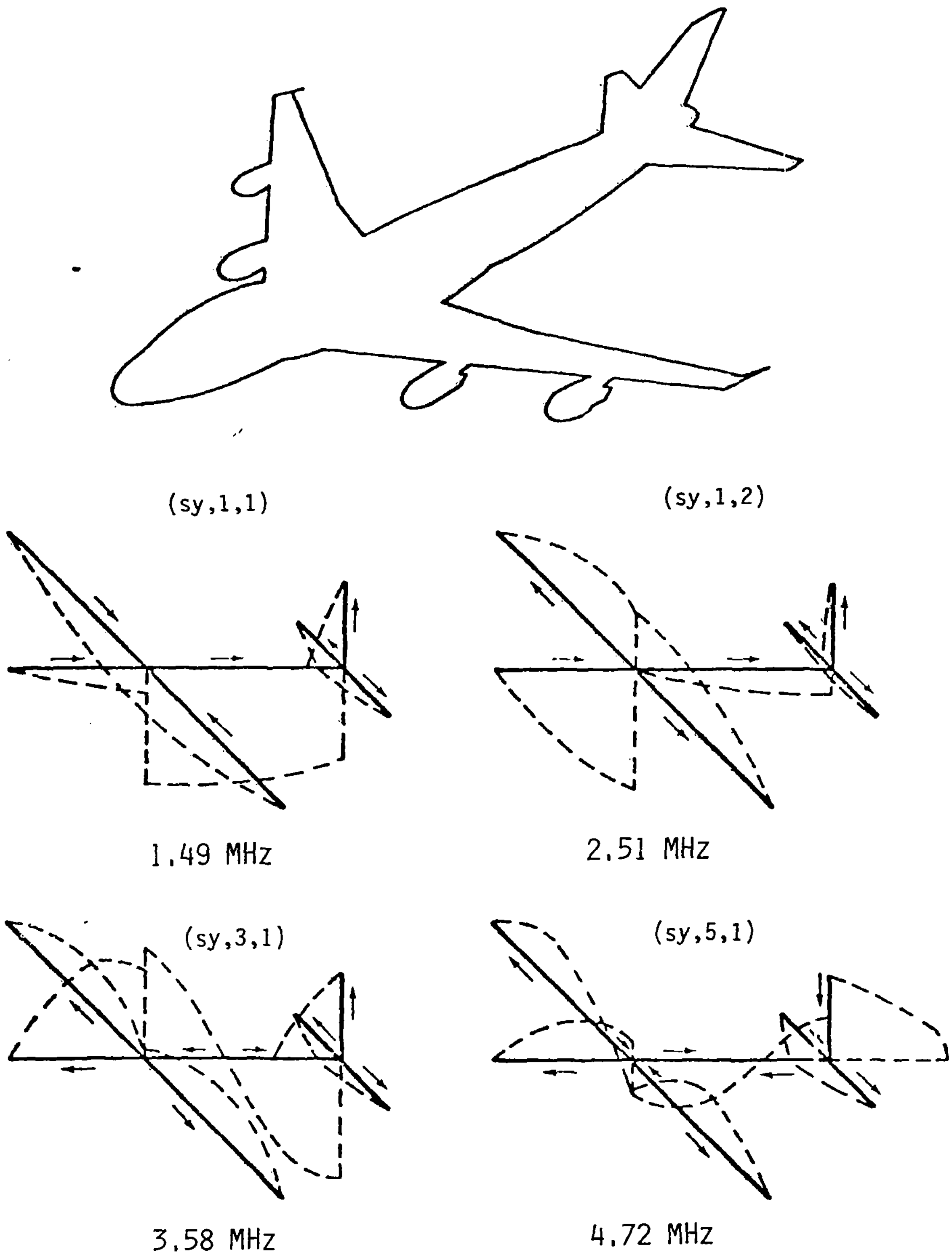

Figure 3. E-4 natural modes. The dashed lines represent the current distribution on the aircraft segments at resonance, while the arrows indicate direction.

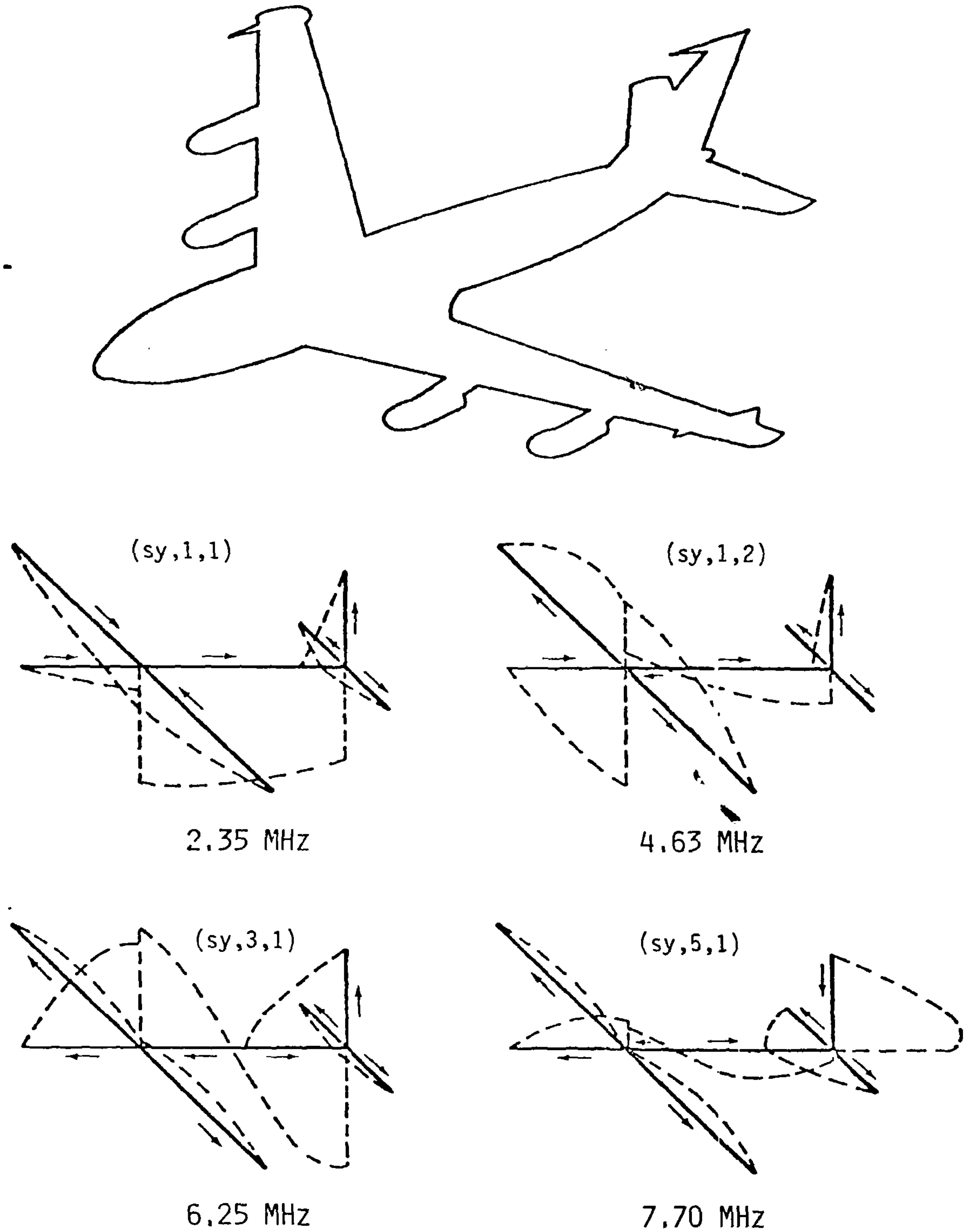

Figure 4. EC-135 natural modes. The dashed lines represent the current distribution on the aircraft segments at resonance, while the arrows indicate direction.

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