Interaction Notes

Note 429

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Black Box Bounds

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Abstract

This note applies time-domain norm concepts to bound the failure of a black box to multiport excitation in terms of the failure responses to single port excitation. Appropriate assumptions concerning the nature of the black box response are made and discussed.

I. Introduction

In characterizing the interaction of electromagnetic fields with complex systems one can make the problem more tractable if, instead of trying to obtain the actual signals at various positions in the system, one settles for something less detailed, in particular, bounds on these signals. Recently several notes have addressed this approach [4,5,6,7], from the points of view of both transmission-line network theory and the scattering equations encountered in (quantitative) electromagnetic topology.

It is becoming clear that the concept of norms plays a central role in bounding the electromagnetic response of complex systems [2,8]. The general interaction equations [3,4] are conveniently cast in forms involving supermatrices which appropriate norms can reduce to scalars.

The electromagnetic signals of concern propagate "down" to the circuit level where various undesirable effects can occur. These effects are usually divided into two sets designated upset and (permanent) damage. In this note we take some set of such circuits which are physically grouped together into what are often termed "black boxes" which are in turn typically interconnected by signal transmission lines (wires, waveguides, etc.).

Characterizing such a black box as an N-port network the N signals (considered independent) are cast in the form of voltages and currents, or equivalent voltages and currents for cases that the signals are in the form of more general electromagnetic waves (modes). In this form black-box terminals are put in a form compatible with the equivalent voltages and currents presented to them by the rest of the system in the format of transmission-line network theory or electromagnetic topology.

II. Black-Box Characteristics

For our purposes the common "black box" is considered to be a network with N input ports as indicated in figure 2.1. There are also M internal "failure ports" [1]. These are indexed as

$$\begin{aligned} n &= 1,2,\ldots,N \qquad \text{(input ports)} \\ m &= 1,2,\ldots,M \qquad \text{(failure ports)} \end{aligned} \tag{2.1}$$

The N input ports are assumed to be known a priori. However, the number of failure ports (M) may be a priori unknown as may be the location of any or all of the individual failure ports.

A failure port is defined as any port (with two terminals) inside the black box where some signal at this port can cause failure. This is interpreted in the sense of any change in the black box function or capability to function resulting from some signal there. This includes any transient upset (change of logic state) as well as permanent damage attributable to the signal driving the failure port.

Let the input signals be

$$F_n(t) = a_n f_n(t) \tag{2.2}$$

where $f_n(t)$ is some appropriately normalized waveform and a_n is an arbitrary (real) amplitude. Here the $F_n(t)$ can be interpreted as voltages, currents, or some linear combination of the two (such as combined voltages [3]). Let the response at the mth failure port be given by

$$G_m(t) = \sum_{n=1}^{N} a_n g_{m,n}(t) \tag{2.3}$$

where $g_{m,n}(t)$ is the response due to $f_n(t)$ from each nth input port. Of course this type of response assumes linearity, at least for times of interest. Stated in vector/matrix form the input is $(F_n(t))$ giving a response

$$(G_m(t)) = (g_{m,n}(t)) \cdot (a_n) \tag{2.4}$$

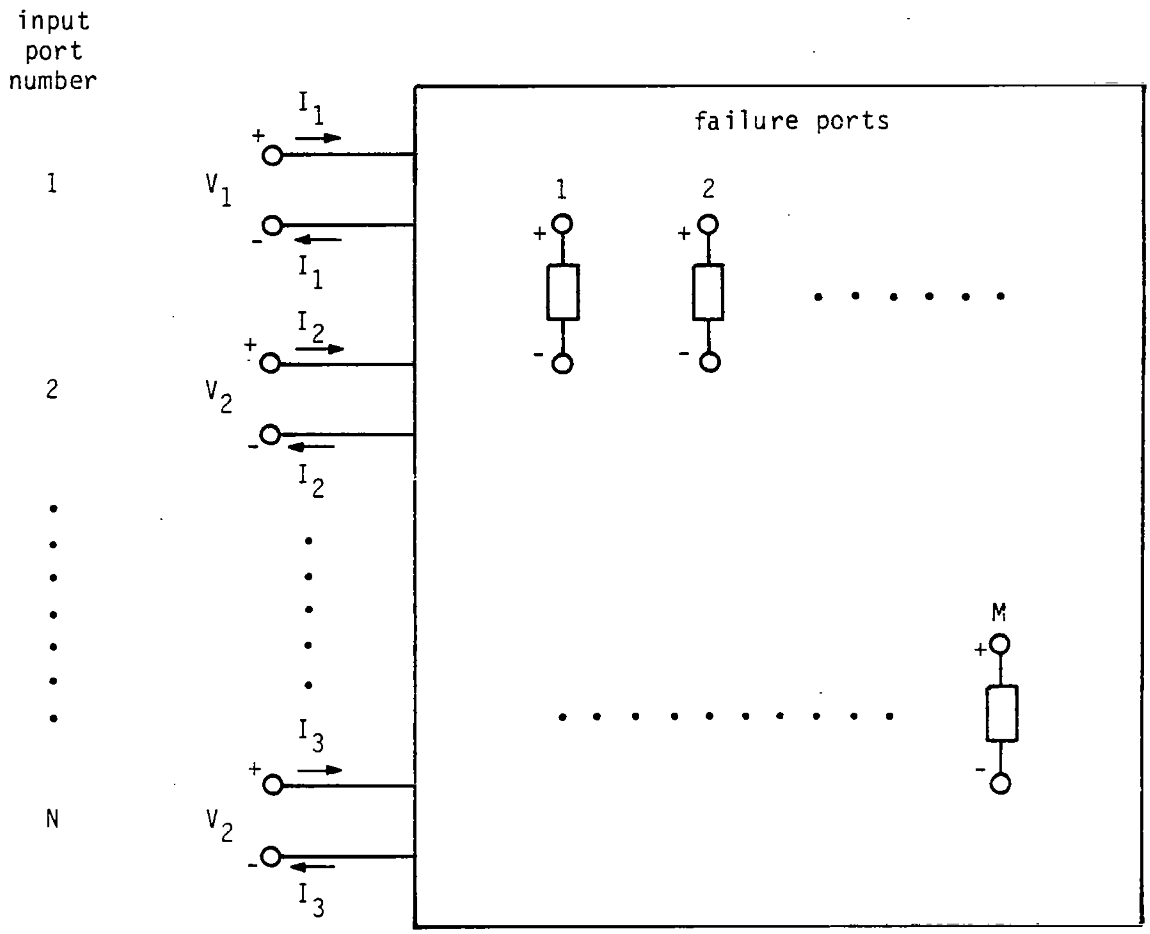

Fig. 2.1. Black box representation

Now $f_n(t)$ might be any kind of waveform, including a δ function such as

$$f_n(t) = \delta(t - t_n) \tag{2.5}$$

If t_n is allowed to vary then $g_{m,n}(t)$ may vary as a function of t_n in a complex way if the system is not time invariant. If it is time invariant then $f_n(t - t_n)$ produces $g_{m,n}(t - t_n)$ and the two are related by a convolution operator. Perhaps we might better assume a certain set of piecewise time-invariant states so that $f_n(t - t_n)$ produces $g_{m,n}^{(\tau)}(t - t_n)$ during system state τ. Waveforms are not "allowed" to cross state boundaries (particular times). This can be stated by (for causal functions)

$$\begin{aligned} g_{m,n}^{(\tau)}(t) &= T_{m,n}^{(\tau)}(t) \circ f_n(t) \\ (G_m^{(\tau)}(t)) &= (g_{m,n}^{(\tau)}(t)) \cdot (a_n) \\ &= (T_{m,n}^{(\tau)}(t) \circ f_n(t)) \cdot (a_n) \\ &= (T_{m,n}^{(\tau)}(t) \circ [a_n f_n(t)]) \end{aligned} \tag{2.6}$$

$\circ \equiv$ convolution with respect to time

$$\begin{aligned} x(t) \circ y(t) &\equiv \int_{-\infty}^{t} x(t - t' y(t')dt' \\ &= y(t) \circ x(t) \end{aligned}$$

Thus far we have not specified whether $f_n(t)$ and $g_{m,n}(t)$ represent voltages or currents or some linear combination of the two. For present purposes this is unnecessary. However, for the $f_n(t)$ one will eventually have to choose some form to perform the experiments involving sequential single-port excitation. The remaining input ports will then have to be properly terminated as indicated in table 2.1.

Table 2.1. Terminations at input ports for different kinds of single-waveform excitation

Input-Port Waveform $f_n(t)$	Input-Port Termination for $n' \neq n$
voltage V	short circuit
current I	open circuit
combined voltage V + ZI (incoming wave) (measure both V and I at nth input port)	impedance Z taken as a frequency-independent resistance (assures only outgoing waves)

III. Failure Norms

Now we need some measure of $G_m(t)$ to determine if failure occurs at the mth failure port. Remember $G_m(t)$ could be a voltage, current, or combined voltage; whichever it is may not be important for present purposes.

Let us define some failure measure as a norm $\ni$

$$\|G_m(t)\| \begin{cases} \geq \Gamma_m \Rightarrow \text{failure at mth failure port} \\ < \Gamma_m \Rightarrow \text{non-failure at mth failure port} \end{cases} \tag{3.1}$$

Let us take system failure as

$$\begin{aligned} &\text{system failure} \Longleftrightarrow \text{at least one failure-port failure} \\ &\text{system non-failure} \Longleftrightarrow \text{no failures at any failure port} \end{aligned} \tag{3.2}$$

It will be further assumed that such a system failure, whether upset or permanent damage, will be observable, even if the particular failure port or ports which fail are not observable. This observation might take the form of a check of system logic states and/or functional performance after the test of interest.

Some questions are:

What is an appropriate norm?
Do all failure ports have the same norm?

Fortunately, if there are such a norm or norms, these answers are not essential since all norms (vector norms) have certain properties [8]

$$\begin{aligned} &\|\alpha G(t)\| = |\alpha| \, \|G(t)\| \\ &\qquad \alpha \equiv \text{scalar} \\ &\|G(t)\| \begin{cases} = 0 \text{ iff } G(t) \equiv 0 \text{ or has zero "measure" per the particular norm} \\ \geq 0 \text{ otherwise} \end{cases} \end{aligned} \tag{3.3}$$

Examples of norms might be

$$\|G(t)\|_1 \equiv \int_{-\infty}^{\infty} |G(t)|dt$$

$$\|G(t)\|_2 \equiv \left\{\int_{-\infty}^{\infty} G^2(t)dt\right\}^{\frac{1}{2}} \tag{3.4}$$

$$\|G(t)\|_\infty \equiv \max_t |G(t)|$$

More generally the p norm is

$$\|G(t)\|_p = \left\{\int_{-\infty}^{\infty} |G(t)|^p dt\right\}^{1/p} \tag{3.5}$$

Here integration is actually over times for which $G(t)$ is significant. In particular integration needs to be limited to times in the τth time-invariant state of the black box. Note that only real $G(t)$ are considered since we are dealing with physical time-domain signals.

If energy is the failure mechanism then the 2 norm might be appropriate. However, suppose the failure mechanism is peak voltage. Then the failure mechanism may not be bipolar,

e.g., failure requires +1 volt or -10 volts

This difficulty can be overcome by defining the experiment so that both $G_m(t)$ and $-G_m(t)$ are produced (different tests) from $(F_n(t))$ and $-(F_n(t))$ with failure in either polarity defining failure-port failure. Such norms then apply to bipolar experiments. Of course, if $G_m(t)$ has equal positive and negative peaks only one test is needed. This might be the case if the $F_n(t)$ were sinusoids (of a common frequency) making the $G_m(t)$ sinusoids. Practically this would require slowly and smoothly turning the exciting sinusoids on and off. Damped sinusoids are more problematical.

Let us define a special kind of norm as a <u>time-invariant norm</u> iff

$$\|G(t - t_\ell)\| \neq \text{function of } t_\ell \tag{3.6}$$

Of course this is only meant to apply within a time-invariant state of the system. Note that the above p norms are all time invariant.

The p norms in (3.4) and (3.5) have the property that if the integration is truncated one obtains a lesser value for the norm since the integrands are positive semi-definite. Stated another way, we can define

$$\| G(t) \|_{p,t_f} \equiv \left\{ \int_{-\infty}^{t_f} |G(t)|^p dt \right\}^{1/p}$$

$$= \text{monotone non-decreasing function of } t_f \tag{3.7}$$

which also applies to (3.4) by restricting $-\infty < t \leq t_f$. If we have a failure at the mth failure port we can define a time of failure by

$$\| G_m(t) \|_{p,t_{f_m}} = \Gamma_m \tag{3.8}$$

since

$$\| G_m(t) \|_p \geq \Gamma_m \tag{3.9}$$

if failure occurs (from (3.1)). Thus t_{f_m} can be interpreted as a failure time for the mth failure port.

In section 2 transfer convolution functions were defined relating $G_m(t)$ to the $f_n(t)$. This involves fundamentally the assumption of linearity. For our failure norms to apply it is only necessary for a failure port (and the signal transport to it) to be linear for times up to t_{f_m}. After this time the failure port will have failed, which by assumption is detected as a box failure. Stated another way $t > t_{f_m}$ is irrelevant and linearity for such times is not needed to insure the result. Even more generally define

$$t_f = \min_{1 \leq m \leq M} t_{f_m} \tag{3.10}$$

giving the first failure at any failure port, which of course gives black box failure. Times greater than this t_f are unimportant and linearity is not required for such times.

Thus we do not need a completely linear system for our results to hold. Define this lesser required kind of linearity as <u>linearity to failure</u>.

IV. Single-Port Tests

Suppose now that we apply $F_n(t)$ at the nth input with all other inputs zero and terminated per table 2.1. Noting that

$$F_n(t) = a_n f_n(t)$$
$$F_{n'}(t) \equiv 0 \quad \text{for } n' \neq n \tag{4.1}$$

vary a_n (real) until a failure occurs at some (perhaps unknown) mth failure port. Determine the maximum (positive) a_n and minimum (negative) a_n for failure. Define

$$A_n \equiv \min\left[a_{n_{max}}, -a_{n_{min}}\right] \tag{4.2}$$

$$> 0 \text{ so no failure occurs without an input}$$

so that

$$A_n = \text{minimum } |a_n| \text{ causing failure anywhere within the black box} \tag{4.3}$$

Note then that for all $G_m(t)$ under the above condition

$$\|G_m(t)\| = \|a_n g_{m,n}(t)\|$$
$$= |a_n| \, \|g_{m,n}(t)\| \leq \Gamma_m \text{ for all m with equality for at least one failure port (giving the black-box failure)} \tag{4.4}$$

with failure norm (and Γ_m per (3.1)) being that appropriate for each mth failure port.

The point is that for all a_n individually with

$$0 \leq |a_n| < A_n \quad , \quad n = 1,2,\ldots,N \tag{4.5}$$

there is no black-box failure. Furthermore each case of

$$|a_n| < A_n \text{ with } a_n' = 0 \text{ for all } n' \neq n \tag{4.6}$$

gives

$$\|G_m(t)\| < \Gamma_m \text{ for all } m = 1,2,\ldots,M$$

which is the requirement for no system failure.

V. Bounds on Failure under Multi-Port Excitation

Now let there be signals on all N input ports. The failure port signals are

$$(G_m(t)) = (g_{m,n}(t)) \cdot (a_n) \tag{5.1}$$

with failure norms

$$\begin{aligned} \|G_m(t)\| &= \left\| \sum_{n=1}^{N} g_{m,n}(t) a_n \right\| \\ &\leq \sum_{n=1}^{N} |a_n| \, \|g_{m,n}(t)\| \end{aligned} \tag{5.2}$$

This last result is interpretable as the 1 norm of a vector whose elements are failure norms of the signals from each nth input port, i.e.,

$$\|G_m(t)\| \leq \|(|a_n| \, \|g_{m,n}(t)\|)\|_1 \tag{5.3}$$

with

$$(F_n(t)) = (a_n f_n(t)) \tag{5.4}$$

Our task here is to find conditions under which the black box will <u>not</u> experience a failure. This is based on

$$\|G_m(t)\| \begin{cases} \geq \Gamma_m \text{ for any } m = 1,2,\ldots,M & \Rightarrow \text{box failure} \\ < \Gamma_m \text{ for all } m = 1,2,\ldots,M & \Rightarrow \text{box non-failure} \end{cases} \tag{5.5}$$

Non-failure of the box is then assured if

$$\|(|a_n| \, \|g_{m,n}(t)\|)\|_1 < \Gamma_m \quad \text{for all } m = 1,2,\ldots,M \tag{5.6}$$

From the single-input-port tests (section 4) one has

$$\begin{aligned} &A_n \|g_{m,n}(t)\| \leq \Gamma_m \quad \text{for all } m = 1,2,\ldots,M \text{ with equality for at least one } m \text{ and for all } n = 1,2,\ldots,N \\ &A_n > 0 \quad \text{for all } n = 1,2,\ldots,N \end{aligned} \tag{5.7}$$

or

$$\|g_{m,n}(t)\| \le \frac{\Gamma_m}{A_n} \qquad \text{for all } \begin{array}{l} m = 1,2,\ldots,M \\ n = 1,2,\ldots,N \end{array} \tag{5.8}$$

Replacing $\|g_{m,n}(t)\|$ by Γ_m/A_n only increases the sum (1 norm) in (5.6) so that requiring

$$\|(\frac{|a_n|}{A_n}\Gamma_m)\|_1 < \Gamma_m \tag{5.9}$$

or

$$\|(\frac{|a_n|}{A_n})\|_1 < 1 \tag{5.10}$$

also assures non-failure. Consider the N component vector $(\frac{a_n}{A_n})$. Then

$$\|(\frac{a_n}{A_n})\|_1 < 1 \implies \text{box non-failure} \tag{5.11}$$

This is then a bound on multi-port excitation (the set $\{a_n\}$ of input-port excitations) to assure non-failure in terms of the results of single-port excitations (the levels $|a_n| = A_n$ for black-box failure due to single port excitation). Note that the index m does not appear in (5.11) so that the location (m) of a failure in the black box is not needed in determining this bound. Note that $\|(\frac{a_n}{A_n})\|$ for all $A_n > 0$ is a valid norm for arbitrary (a_n). One might call this norm a weighted 1 norm. (Similarly one could define a weighted p norm.)

Note that the above norm is a tight one since for

$$\text{all } a_n = 0 \text{ except for } n = n' \tag{5.12}$$

this norm in (5.11) reduces to

$$\frac{|a_{n'}|}{A_{n'}} < 1 \tag{5.13}$$

which is exactly the result from an experiment concerning non-failure for single-port excitation at the n'th input port.

A looser but simpler bound can also be obtained. Write

$$\left(\frac{a_n}{A_n}\right) = \left(\frac{1}{A_n} 1_{n,m}\right) \cdot (a_n) \tag{5.14}$$

so that

$$(a_n) = (A_n 1_{n,m}) \cdot \left(\frac{a_n}{A_n}\right) \tag{5.15}$$

$$\|(a_n)\| \leq \|(A_n 1_{n,m})\| \, \|\left(\frac{a_n}{A_n}\right)\| \tag{5.16}$$

Now for diagonal matrices for any associated matrix norm (see [6])

$$\|(A_n 1_{n,m})\| = \max_n |A_n| = \max_n A_n \tag{5.17}$$

Also we have

$$\|\left(\frac{a_n}{A_n}\right)\| \leq \|\left(\frac{1}{A_n} 1_{n,m}\right)\| \, \|(a_n)\| \tag{5.18}$$

with

$$\|\left(\frac{1}{A_n} 1_{n,m}\right)\| = \max_n \frac{1}{A_n} = [\min_n A_n]^{-1} \tag{5.19}$$

Combining these results

$$[\min_n A_n] \, \|\left(\frac{a_n}{A_n}\right)\| \leq \|(a_n)\| \leq [\max_n A_n] \, \|\left(\frac{a_n}{A_n}\right)\| \tag{5.20}$$

This result holds for all norms and all possible sets $\{a_n\}$.

Replacing $\|\left(\frac{a_n}{A_n}\right)\|_1$ in (5.11) by the larger quantity $[\min_n A_n]^{-1} \|(a_n)\|$ gives the looser condition

$$\|(a_n)\|_1 < \min_n A_n \implies \text{box non-failure} \tag{5.21}$$

This is then a sufficient condition for box non-failure. This bound is tight only in the sense that for at least one $n = n'$ (given by $\min_n A_n$) $|a_{n'}| \geq A_n$ (with other $a_n = 0$) gives a box failure condition.

Note that if

$$\max_t |f_n(t)| = 1 \quad \text{(normalized waveform)} \tag{5.22}$$

then

$$\|(a_n)\|_1 = \sum_{n=1}^{N} \text{peak signal magnitudes} \tag{5.23}$$

VI. Summary

By defining appropriate norms (failure norms) of time-domain signals we have obtained bounds on black-box failure for multi-port drive in terms of the failure results for single-port drive. This shows the potential importance of time-domain norms for electromagnetic interaction problems.

Linearity is a basic consideration in this bounding process. However, in this type of time-domain problem complete linearity is not required. This can be replaced by a concept of linear to failure, whether failure includes upset and/or permanent damage.

The kinds of applicable exciting waveforms are quite arbitrary as indicated by the $f_n(t)$. These could be damped sinusoids or any other type of interesting canonical waveforms. It is not even necessary that all the $f_n(t)$ be of the same type for these results to hold. Furthermore, the $f_n(t)$ need not all begin at the same time, the delay between them still falling within the bounding procedure of the time-domain norms defined in time-invariant form.

The bounding results for box non-failure are expressed in a relatively tight form in (5.11) and in a somewhat looser form in (5.21). These are expressed in terms of a 1 norm of the exciting input-port waveform amplitudes, either normalized to the various single-port failure levels (as in (5.11)) or to the smallest single-port failure level (as in (5.21)).

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