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Interaction Notes

Note 433

Alternative Labelling Schemes in Electromagnetic Topology

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ABSTRACT

The aim of this note is to define the problems associated with applying graph theory to interaction sequence diagrams used in electromagnetic topology. Because few applications have been developed, the optimal labelling scheme has yet to emerge. As a result, operations on the graph are difficult to record concisely. This note contains criteria to guide the design of labelling schemes. Some specific schemes are presented and illustrated for a sample topology, and satisfaction of the criteria is discussed.

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INTRODUCTION

Although electromagnetic topology (EMT) is a new concept, its theoretical development has already become overburdened with indices. Baum [1,2,3] has described the subdivision of Euclidean space into components of interest, namely shields and subshields (surfaces), layers and sublayers (volumes), etc., and has suggested one method of labelling them. The same labels apply to the dual graph, in which volumes are represented by points, and surfaces between volumes as lines connecting the points. In this scheme, a given hierarchical level is identified with a volume called a layer. A layer is subscripted once to identify its hierarchical level. Non-intersecting subsets of a layer, called sublayers, receive two indices. The first index is the layer index. All of a layer's sublayers are numbered consecutively to supply the second integer of the pair.

Further decomposition of sublayers into subsets called elementary volumes follows the same pattern with a third subscript. Shields and subshields are tagged similarly, but require two such sets of indices to show enclosed and enclosing volumes. A sample EMT, to sublayer level, is shown in Figure 1 along with its dual graph in Figure 2, labelled according to the scheme described above.

Rather than dealing with these previously used labels, this note defines some characteristics of a well-labelled dual graph and presents a few alternative schemes which possess some of these traits.

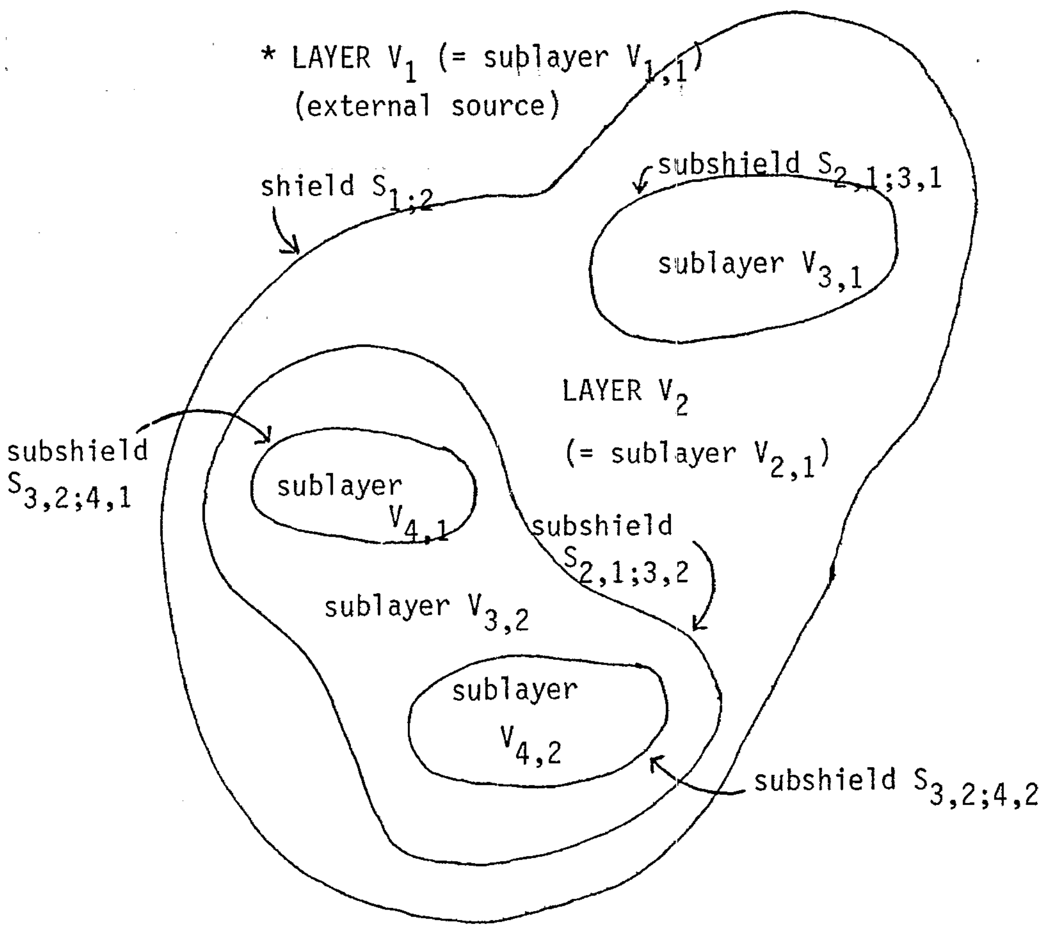


Figure 1. A sample electromagnetic topology to sublayer level.

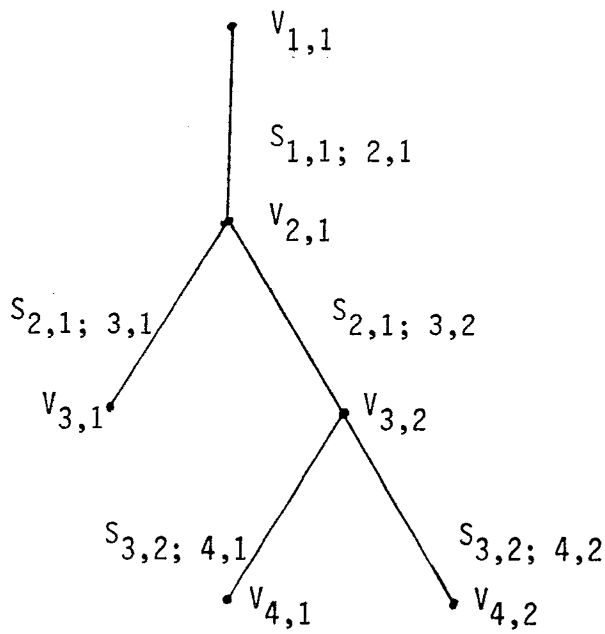


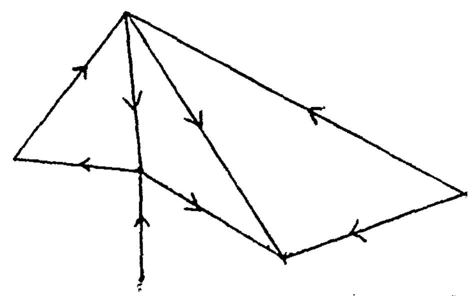
Figure 2. Dual graph of Figure 1.

CHARACTERISTICS OF AN IDEAL LABELLING SCHEME

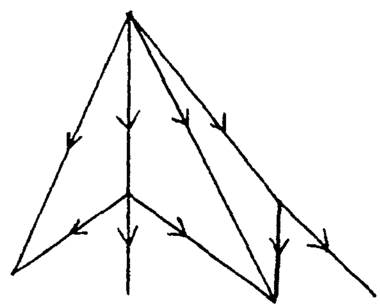
An ideal labelling scheme should meet the following criteria:

1) Labels should be compatible with standard terminology. Representing an EMT by its dual graph invites the introduction of results from graph theory. For ease of application, the EMT graph labels must be clearly related to, if not the same as, standard graph theory notation. This requires identification of each vertex and edge, and adherence to the concepts of adjacency and non-adjacency of edges and vertices. (Two vertices are adjacent if they are the end points of some edge. Two edges are adjacent if some vertex is an end point of both.)

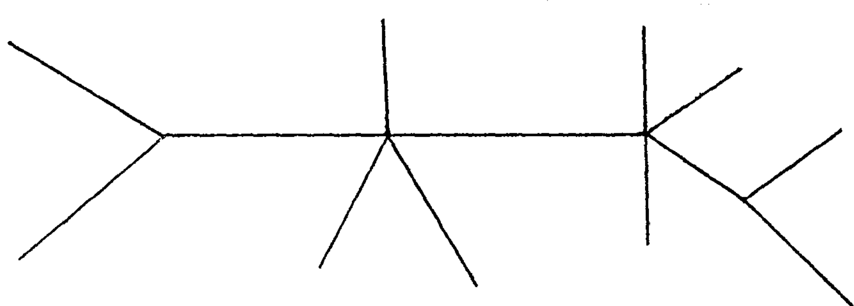
2) The dual graph is said to be a directed graph ("digraph") if each edge is oriented by distinguishing a starting vertex from an ending vertex. A digraph is hierarchical if there exists a partial ordering of the vertices. A tree is a graph containing no closed loops (cycles). Figure 3 illustrates these terms. Labels should indicate direction for digraphs, and ordering for hierarchical graphs, but if no ordering is assumed then the additional structure imposed by this requirement should be removable. That is, the label attached to a point of the graph should indicate which layer(s) enclose and are enclosed by it, as well as which layers are at the same hierarchical level (if the graph is hierarchical). Labels for vertices which neither precede nor follow each other should reflect this fact.



(a) directed graph.



(b) hierarchical graph



(c) tree graph.

Figure 3. A directed graph, a hierarchical graph, and a tree graph.

3) Labels should be flexible enough to absorb changes in the EMT, including

a) insertion of a subshield or sublayer,

b) deletion of a subshield or sublayer,

c) linking of sublayers (by a wire, hole, etc.),

d) frequency-dependent changes,

and e) distinction between "inside" and "outside" of subshields.

Flexibility is the most stringent requirement, because the classification of a graph as a tree, digraph, hierarchical graph, etc., is changed when the topology is altered.

4) Labels contain information about the distance between two vertices. Baum has used the term "relative shielding order" for the number of intervening subshields between two sublayers. Thus labels should generate the distance matrix, $D = (d_{ij})$, where d_{ij} might be defined as the minimum number of edges between vertices i and j, or as some generalization which satisfies the conditions of a metric.

5) Labels should admit an operation representing inversion with respect to a "pivot layer" or "pivotal sublayer". Inversion is a distance-preserving operation which uses reciprocity to consider the pivotal sublayer as the source of electromagnetic excitation. The operation may be visualized as reaching inside the topological diagram and pulling the pivotal sublayer to the outside, turning part of the diagram "inside-out" in the process. The corresponding operation on the dual graph consists of choosing

a new base point, or suspending the graph by the pivotal vertex. If the graph is hierarchical, this raises the pivotal vertex to the top of the hierarchy and partitions the vertices into two subsets, those for which the hierarchy is preserved and those for which the hierarchy is reversed. The latter is an order set, and the new label set can be generated by reversing the order on that set.

6) Labels should suggest how to apply concepts from relevant fields of mathematics, including graph theory, abstract algebra, algebraic topology, matrix theory, differential geometry, and analysis.

This is a lot to ask of a labelling scheme. A reasonable compromise is to allow two or more schemes to collectively satisfy the above requirements. The schemes presented in the following section have been designed to satisfy criteria 2, 4 and 5 for hierarchical graphs, while still observing the guidelines 1 & 6.

ALTERNATIVE LABELLING SCHEMES

The emphasis of this note is on hierarchical tree digraphs, so some of the considerations in criterion 3 are not addressed. The following notation will be used throughout: Unless otherwise specified, X is a partially ordered set which may be represented as a tree graph. $V(X)$, or just V if the set X is understood, is the set of labels representing the vertices of X, called the vertex set of X. Similarly, $E(X)$, or just E, is the edge set of X. A partial ordering $f<g$ is read "f precedes g" or "g follows f". Several schemes attach more than one character to a vertex. One of these should remain fixed during inversion, and is referred to as the identifier of the vertex.

In graph theory, the standard way of labelling a graph is by specifying the vertex set and edge set. If there are p vertices, then there will be at most $q = \binom{p}{2} = p(p-1)/2$ edges. If the graph is a tree, then $q = p-1$. The vertices are arbitrarily labelled from the vertex set

$$V = \{v_1, v_2, \ldots, v_p\},$$

and the edge set is defined by

$$\begin{aligned} E &= \{e_1, e_2, \ldots, e_q\} \\ &= \{v_{i_1 j_1}, v_{i_2 j_2}, \ldots, v_{i_q j_q}\}, \end{aligned}$$

where vertex v_{i_k} is connected by edge e_k to vertex v_{j_k}. Figure 4 shows the standard labels applied to Figure 2, as well as the sets V and E. The order of v_{i_k} and v_{j_k} is important for a digraph and is implicitly assumed to be fixed. There are cases in which this assumption hinders the analysis: Figure 5a shows a partially-directed graph possessing some directed edges and some undirected edges, requiring the introduction of extra edges (see Figure 5b), which create artificial cycles in the graph. The acceptability of this depends on the application of the graph.

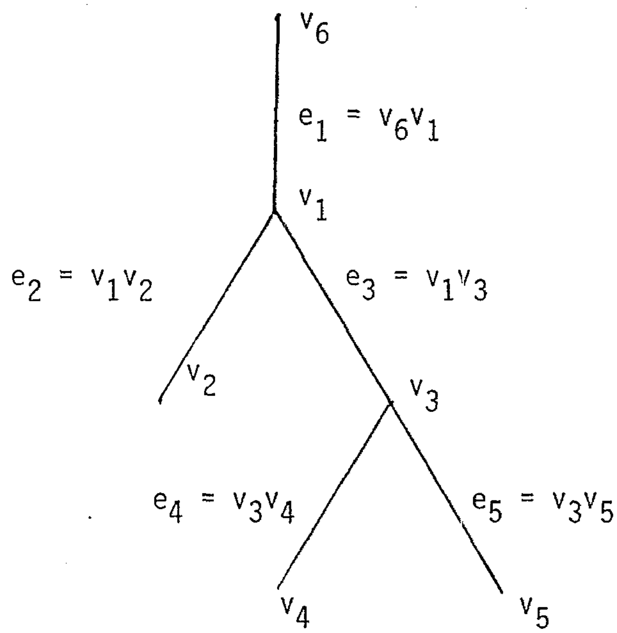
If X is partially ordered (and thus has the property that no vertex v_k appears twice in the second position of the elements of the edge set), then inspection of the elements of E will reveal a unique vertex v_i which appears in the first position of a pair but not in the second position of any other pair. From this fact the tree can be reconstructed and thus admits an inversion operation. This method is sufficiently general to satisfy criteria 1, 2, and 6, but distances cannot be conveniently calculated without first reconstructing the graph.

Another scheme of attaching labels to a graph containing p points is to select elements of a group G generated by p-1 elements, v_1, v_2, ..., v_{p-1}, which satisfy the relations

(1) $v_i^2 = 1$ $(i = 1, \ldots, p-1)$

and

(2) $v_i v_j = v_j v_i$ $(i,j = 1, \ldots, p-1)$

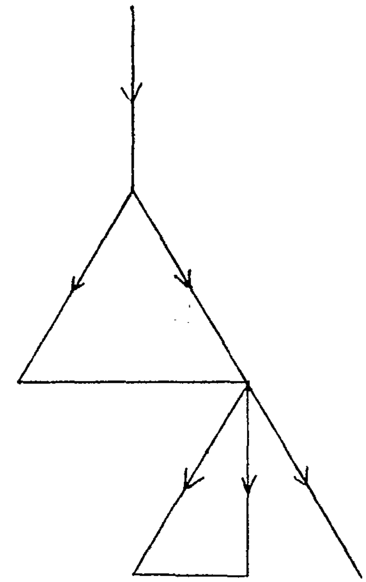


$$V(X) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$$

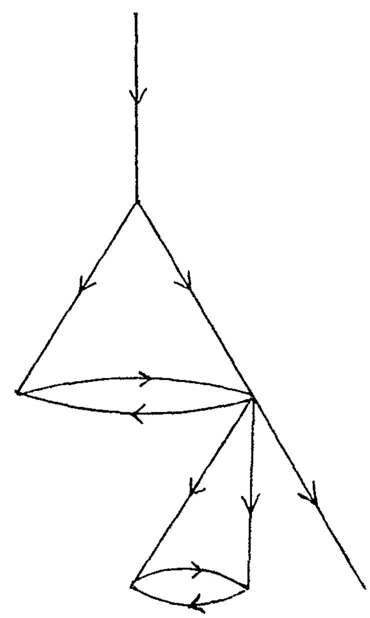
$$E(X) = \{e_1, e_2, e_3, e_4, e_5\}$$

$$= \{v_6v_1, v_1v_2, v_1v_3, v_3v_4, v_3v_5\}$$

Figure 4. Standard labelling scheme, vertex set V(X), and edge set E(X).



(a) A partially-directed graph.



(b) Extra edges introduced.

Figure 5. The problems of partially-directed graphs.

The set (1) identifies G with p copies of the cyclic group Z_2. The second set abelianizes G. This scheme will be named the "pZ_2 scheme".

To understand the concept of groups, generators, and relations, consider a group G generated by two elements, a and b. This is a free group (because it is free from relations) containing infinitely many elements, including

$$1, a, a^{-1}, b, b^{-1}, a^2, ab, ba, aba, ba^{-1}b^3a^{-2}b, \text{etc.}$$

For this example, the relation (1) becomes:

(1a) $a^2 = 1$ or $a = a^{-1}$

and

(1b) $b^2 = 1$ or $b = b^{-1}$.

Including relations (1a) and (1b) reduces the elements of G to the form

$$1, a, b, ab, ba, aba, bab, abab, baba, \text{etc.}$$

The group still has infinitely many elements, however.

Applying the relation (2) to the generators a and b yields:

(2) $ab = ba$.

Now G is a finite group:

$$G = \{ 1, a, b, ab \}.$$

The element abab, for instance, has been reduced to abba by relation (2), then further reduced to 1 by relations (1a) and (1b).

For the EMT in Figure 1, the five generators $v_1, v_2, \ldots, v_5$ yield a group G containing 16 elements:

$$G = \{ 1, v_1, v_2, v_3, v_4, v_5, v_1v_2, v_1v_3, v_1v_4, v_1v_5, v_2v_3, v_2v_4, v_2v_5, v_3v_4, v_3v_5, v_4v_5 \}$$

Figure 6 illustrates the selection of V(X) from G. The identity element of G is assigned to the unbounded region of Figure 1, representing the source of electromagnetic excitation. This region corresponds to the base point at the top of the tree. There is one layer containing all sublayers in the EMT. The corresponding point is labelled by v_1, a generator of G. Two sublayers enclosed by this layer are identified with the group elements v_1v_2 and v_1v_3, showing their relationship to v_1. Finally, elements $v_1v_3v_4$ and $v_1v_3v_5$ are selected from G to represent the two elementary volumes within v_1v_3, again showing the hierarchical dependence. This scheme may be continued by assigning to each vertex a generator of G multiplied by the generator of every preceding vertex.

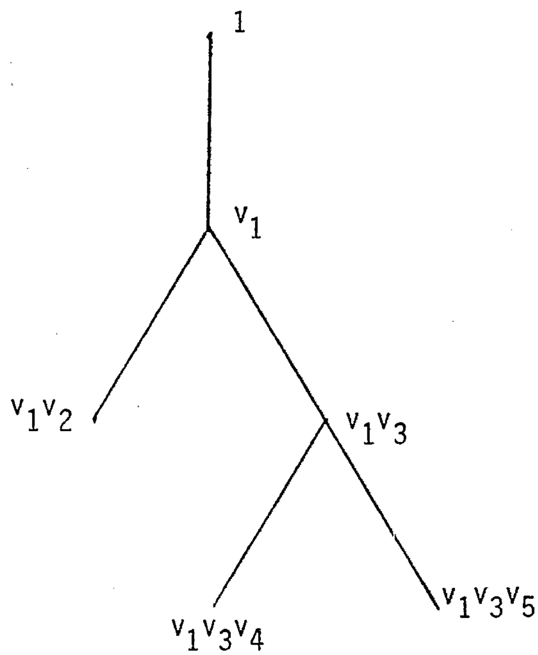


Figure 6. The pZ_2 labelling scheme.

Criterion 1 is not easily satisfied (without first reconstructing the graph from the sets V and E, which takes time) unless the abelianization relations are removed. In that case, X can be identified with the standard vertex set V by keeping only the rightmost multiplicands of each element, and writing v_p for the source element 1. The edge set E may be obtained by keeping the two rightmost multiplicands of each element, remembering to write an edge $v_p v_i$ for each element v_i which consists of only one generator. Thus applications of graph theory may be incorporated into this scheme.

By their selection from the group G, the labels clearly exhibit the hierarchical structure of the EMT: layer f precedes layer g if and only if every generator of f is also present in g (f and g are assumed to be reduced to simplest form by application of the set of relations (1)). The distance between layers f and g is the number of generators in their (reduced) product. This generates the distance matrix D. The multiplication table of the set X and the matrix D are shown in Table I.

Inversion with respect to a pivot layer f is accomplished by multiplying the label of each vertex by f^{-1} (note that $f^{-1} = f$ since G is abelian), then reducing. This operation preserves distances, but the distance matrix is changed by a similarity transformation.

Figure 7 shows the result of inverting with respect to vertex $v_1 v_3$. The original labels are used in Figure 7a, then multiplied by $v_1 v_3$ and reduced to yield Figure 7b. Note that the order of the generators is ambiguous because G is abelian. Table II reveals that the distance

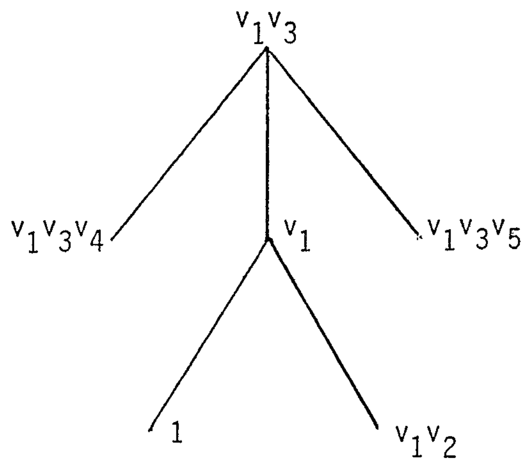
TABLE I

Multiplication Table and Distance Matrix for Figure 6

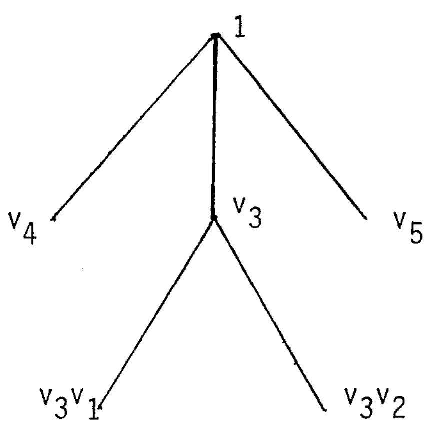
	v_1	v_1v_2	v_1v_3	$v_1v_3v_4$	$v_1v_3v_5$	1
v_1	1	v_2	v_3	v_3v_4	v_3v_5	v_1
v_1v_2	v_2	1	v_2v_3	$v_2v_3v_4$	$v_2v_3v_5$	v_1v_2
v_1v_3	v_3	v_2v_3	1	v_4	v_5	v_1v_3
$v_1v_3v_4$	v_3v_4	$v_2v_3v_4$	v_4	1	v_4v_5	$v_1v_3v_4$
$v_1v_3v_5$	v_3v_5	$v_2v_3v_5$	v_5	v_4v_5	1	$v_1v_3v_5$
1	v_1	v_1v_2	v_1v_3	$v_1v_3v_4$	$v_1v_3v_5$	1

Distance Matrix for Figure 6

$$D = \begin{bmatrix} 0 & 1 & 1 & 2 & 2 & 1 \\ 1 & 0 & 2 & 3 & 3 & 2 \\ 1 & 2 & 0 & 1 & 1 & 2 \\ 2 & 3 & 1 & 0 & 2 & 3 \\ 2 & 3 & 1 & 2 & 0 & 3 \\ 1 & 2 & 2 & 3 & 3 & 0 \end{bmatrix}$$



(a) Before relabelling.



(b) After relabelling by group multiplication.

Figure 7. Inversion of electromagnetic topology of Figure 6 with respect to vertex v_1v_3, using the pZ_2 scheme.

TABLE II

Transformation of Distance Matrix for Figures 6 and 7

$$\text{Before:}\quad D = \begin{bmatrix} 0 & 1 & 1 & 2 & 2 & 1 \\ 1 & 0 & 2 & 3 & 3 & 2 \\ 1 & 2 & 0 & 1 & 1 & 2 \\ 2 & 3 & 1 & 0 & 2 & 3 \\ 2 & 3 & 1 & 2 & 0 & 3 \\ 1 & 2 & 2 & 3 & 3 & 0 \end{bmatrix}$$

After:

$$A^{-1}DA = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 2 & 2 & 1 \\ 1 & 0 & 2 & 3 & 3 & 2 \\ 1 & 2 & 0 & 1 & 1 & 2 \\ 2 & 3 & 1 & 2 & 0 & 3 \\ 2 & 3 & 1 & 2 & 0 & 3 \\ 1 & 2 & 2 & 3 & 3 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 & 1 & 3 & 3 & 2 \\ 2 & 0 & 1 & 3 & 3 & 2 \\ 1 & 1 & 0 & 2 & 2 & 1 \\ 3 & 3 & 2 & 0 & 2 & 1 \\ 3 & 3 & 2 & 2 & 0 & 1 \\ 2 & 2 & 1 & 1 & 1 & 0 \end{bmatrix}$$

matrix for the post-inversion graph is related to the matrix D for the pre-inversion graph by a similarity transformation $D \to A^{-1}DA$, where A is the 6x6 identity matrix I_6 with rows rearranged according to the permutation (316), so A^{-1} is I_6 with columns permuted by (316). (The permutation (316) applied to the sequence [1,2,3,4,5,6] produces the sequence [3,2,6,4,5,1].)

The set X is partitioned into three subsets relative to a pivot element f :

(a) $X_<(f)$ = {elements which precede f }

(b) $X_>(f)$ = {elements which follow f }

(c) $X_0(f)$ = {elements which neither precede nor follow f }

After inverting with respect to f the sets $X_<$ and X_0 vanish, and $X = X_>$. As can be observed from the example, the permutation of I_6 described above reverses the identifiers of elements in $X_<$ while leaving fixed the identifiers in $X_>$ and X_0 . Thus, although this scheme fails criterion 5, there is an algorithm to recover the original identifiers of the vertices.

A variation of the "pZ_2 scheme" removes both sets of relations and redefines inversion with respect to a pivot element f to be $f^{-1}g$. The same remarks apply concerning satisfaction of the criteria, because the two schemes are essentially the same: the pZ_2 scheme defined $f = f^{-1}$ to all multiplication by f instead of f^{-1} in the inversion operation.

The only difference is now G is a free group on $p-1$ generators, a much larger group. This "free group scheme" has the advantage that the ordering of generators in each label is correct after each inversion, whereas multiplication in the pZ_2 scheme changes the order of some labels, so that closer inspection is required to identify predecessors.

The natural correspondence between the group G given in the pZ_2 scheme and the corners of the p-cube defines an embedding of the graph X along certain edges of the cube. A corner is identified by an ordered p-tuple $(b_1,\ldots,b_p)$, where each b_i is either 0 or 1. A label f assigns values to the bi's as follows:

$$b_i = \begin{cases} 1 & \text{if } v_i \text{ is present in the label } f, \\ 0 & \text{if } v_i \text{ is not present.} \end{cases}$$

Here the identity element 1 (base point of the tree) has been changed to v_p. If the cube is suspended by the base point, vertical height corresponds to hierarchical level, and inversion corresponds to suspending the cube by the pivot vertex. Although simple to describe, this embedding is not very useful, because the 3-cube allows graphs with only three vertices, and higher-dimensional cubes are not easy to visualize.

CONCLUSION

Graph theory plays an important role in describing electromagnetic topology. To make full use of the concepts of the theory, a systematic method of labelling graphs must be employed. In this note some criteria were developed to aid in the design of labelling schemes. Using these criteria, various methods of labelling graphs were introduced. Satisfaction of the criteria was discussed. Much work remains to be done, particularly in the area of flexibility of the labels.

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