3-5 March 1987
pp. 321-329

p. 13, 14, 15, 22, 23

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Carl E. Baum
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I. Introduction

The coupling of incident electromagnetic waves to wires is a subject of long-standing interest. Usually one considers this as a linear electromagnetic scattering problem. However, there are situations in which the resulting fields near the wire(s) are so large that electrical breakdown can occur in the surrounding air. This makes the problem in general nonlinear. In this note, we consider an approximate solution to this problem for the case that there is a wire which can be considered infinite in length, at least for times before signals can reach the observer from the wire ends.

The model is based first on some experimental work concerning the propagation of a pulse on a wire above a ground plane [6,7]. For this purpose this author proposed a model involving an equivalent corona radius discussed in [6,7,10,11]. For this case of nonlinear propagation on a wire in corona a solution was found in general form by Chen [12,13]. This solution has been applied to the lightning leader by Baum [8], and to the lightning return strike by Baum and Baker [9].

This paper applies the equivalent-corona-radius model in a different type of situation. Here the forced solution is considered instead of the previously considered free solution. A uniform plane wave (or two planes waves, including the reflection from a perfectly conducting plane parallel to the wire) is incident on the wire. For an effectively infinite length wire this leads to a closed-form solution for the wire response in the presence of corona. This gives a canonical solution applicable (approximately) to the interaction of pulsed plane waves, such as the high altitude EMP, with wires of interest such as power or communication lines or long-wire antennas.

II. Infinite Wire in Free Space

Consider first the geometry of an infinite straight wire in free space. As illustrated in fig. 2.1 there is a wire (perfectly conducting) of radius Ψ_0 with the z axis aligned along the center of this wire. As will be used later there is an effective corona radius Ψ_c. There is also an effective reference conductor of radius Ψ_∞ which gives a characteristic impedance (without corona) of

$$Z_{c_0} = f_{g_0} Z_0 \text{ (characteristic impedance)}$$

$$f_{g_0} = \frac{1}{2\pi} \ln\left(\frac{\Psi_\infty}{\Psi_0}\right) \tag{2.1}$$

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \text{ (wave impedance of free space)}$$

Associated with this we have the parameters (without corona)

$$L_0' = \mu_0 f_{g_0} \text{ (inductance per unit length)}$$

$$C_0' = \frac{\varepsilon_0}{f_{g_0}} \text{ (capacitance per unit length)}$$

$$Z_{c_0} = \sqrt{\frac{L_0'}{C_0'}} \tag{2.2}$$

$$\mu_0 \equiv \text{permeability of free space}$$

$$\varepsilon_0 \equiv \text{permittivity of free space}$$

$$c \equiv \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \equiv \text{wave speed in free space}$$

Now the incident plane wave takes the form

$$\vec{E}^{(inc)} = E_0 f\left(t - \frac{\vec{1}_1 \cdot \vec{r}}{c}\right) \vec{1}_p$$

$$\vec{r} = (x,y,z) \tag{2.3}$$

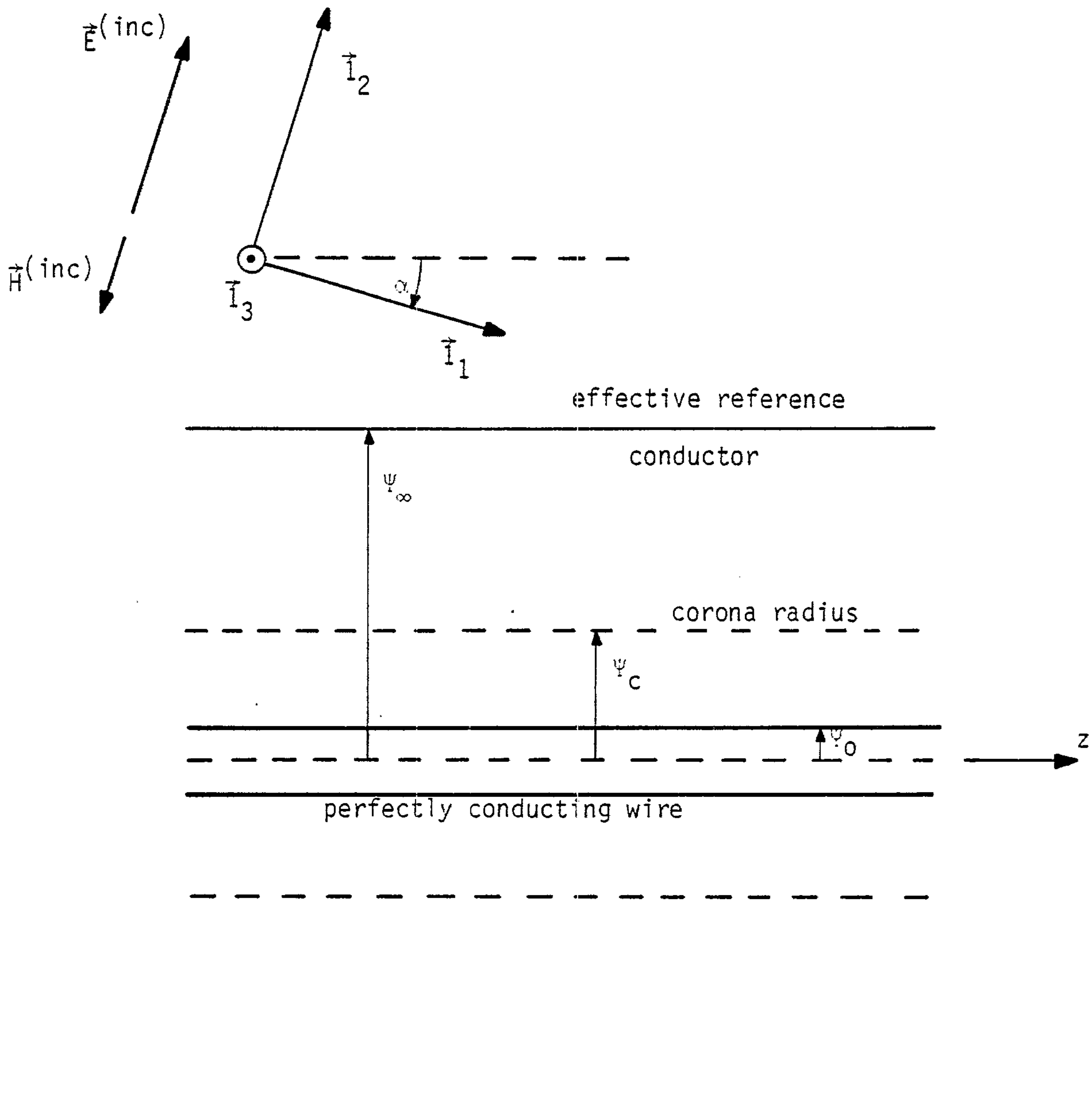


Fig. 2.1. Corona Model of Infinite Wire in Free Space

$\vec{1}_p \equiv$ polarization unit vector

With a right-handed set of unit vectors for the incident wave related as

$$\vec{1}_1 \times \vec{1}_2 = \vec{1}_3 \quad , \quad \vec{1}_2 \times \vec{1}_3 = \vec{1}_1 \quad , \quad \vec{1}_3 \times \vec{1}_1 = \vec{1}_2 \tag{2.4}$$

the incident electric-field vector is taken with a constant polarization (not essential) with unit vector $\vec{1}_p$ as a linear combination of $\vec{1}_2$ and $\vec{1}_3$. Specializing $\vec{r}$ to the z axis and noting

$$\begin{aligned} \vec{1}_1 \cdot \vec{1}_z &= \cos(\alpha) \\ v_p &= \frac{c}{\cos(\alpha)} \end{aligned} \tag{2.5}$$

we have the incident tangential electric field as

$$\begin{aligned} E_t &= \vec{E}^{(inc)} \cdot \vec{1}_z = E_0 f\left(t - \frac{z}{v_p}\right) \vec{1}_p \cdot \vec{1}_z \\ &= E_0 f\left(t - \frac{z}{v_p}\right) \sin(\alpha) \end{aligned} \tag{2.6}$$

For more general cases that $\vec{E}^{(inc)}$ is not parallel to $\vec{1}_2$ the form $\vec{1}_p \cdot \vec{1}_z$ can be used. If one defines δ as the angle of rotation of $\vec{1}_p$ from $\vec{1}_2$ we have an additional factor of $\cos(\delta)$ so that

$$E_t = E_0 f\left(t - \frac{z}{v_p}\right) \cos(\delta) \sin(\alpha) \tag{2.7}$$

For a given direction of incidence this is maximized in magnitude for $\delta = 0, \pi$. It is this tangential component of the incident electric field that we will need for later analysis. Note that any component of the incident field parallel to $\vec{1}_3$ does not contribute to the incident tangential electric field.

For later use we have for this problem

$$\tau \equiv t - \frac{z}{v_p}$$

$$g(\tau) \equiv \int_{-\infty}^{\tau} f(\tau')\, d\tau' \tag{2.8}$$

$$\int_{-\infty}^{\tau} E_t(\tau')\, d\tau' = E_0 g(\tau)\, \vec{1}_p \cdot \vec{1}_z = E_1 g(\tau)$$

$$E_1 \equiv E_0 \vec{1}_p \cdot \vec{1}_z = E_0 \cos(\delta) \sin(\alpha)$$

This integral of the incident tangential electric field is a key parameter in our solution to the transmission-line problem.

For comparison the reader is referred to the exact wave solution to the perfectly conducting wire in free space [5]. Note that in the transmission-line approximation the parameter Ψ_∞ (the radius of the effective reference conductor) and hence Z_{c_0} must be assumed as some approximate number of meters, say based on some time of interest (after the initial wave arrival). Fortunately Ψ_∞ only enters logarithmically as in (2.1).

III. Infinite Wire Above Ground Plane

Now let the perfectly conducting wire be above at height h and parallel to a perfectly conducting ground plane as illustrated in fig. 3.1. In this case we have a characteristic impedance (without corona) of [1]

$$Z_{c_0} = f_{g_0} Z_0 \text{ (characteristic impedance)}$$

$$f_{g_0} = \frac{1}{2\pi} \operatorname{arc\,cosh}\left(\frac{h}{\Psi_0}\right) = \frac{1}{2\pi} \ell n\left[\left(\frac{h}{\Psi_0}\right) + \left(\left(\frac{h}{\Psi_0}\right)^2 - 1\right)^{1/2}\right] \tag{3.1}$$

$$= \frac{1}{2\pi} \ell n\left[\frac{2h}{\Psi_0}\left(1 + O\left(\left(\frac{\Psi_0}{h}\right)^2\right)\right)\right] \quad \text{as } \frac{\Psi_0}{h} \to 0$$

$$= \frac{1}{2\pi} \ell n\left(\frac{2h}{\Psi_0}\right) + O\left(\left(\frac{\Psi_0}{h}\right)^2\right) \quad \text{as } \frac{\Psi_0}{h} \to 0$$

This result can be used to define an equivalent outer radius for a coax by comparison to (2.1) giving

$$\frac{\Psi_\infty}{\Psi_0} = \frac{h}{\Psi_0} + \left(\left(\frac{h}{\Psi_0}\right)^2 - 1\right)^{1/2}$$

$$= \frac{2h}{\Psi_0}\left(1 + O\left(\left(\frac{\Psi_0}{h}\right)^2\right)\right) \quad \text{as } \frac{\Psi_0}{h} \to 0 \tag{3.2}$$

$$\Psi_\infty = 2h\left(1 + O\left(\left(\frac{\Psi_0}{h}\right)^2\right)\right) \quad \text{as } \frac{\Psi_0}{h} \to 0$$

Note that the concept of a characteristic impedance for this configuration is accurate provided h is electrically small.

Let the z axis as before be along the wire axis. As shown in fig. 3.1, let there be an $\vec{r}\,'$ coordinate system with

$$\vec{r}\,' = (x', y', z')$$

$$x' = \Psi' \cos(\phi') = r' \sin(\theta') \cos(\phi') \tag{3.3}$$

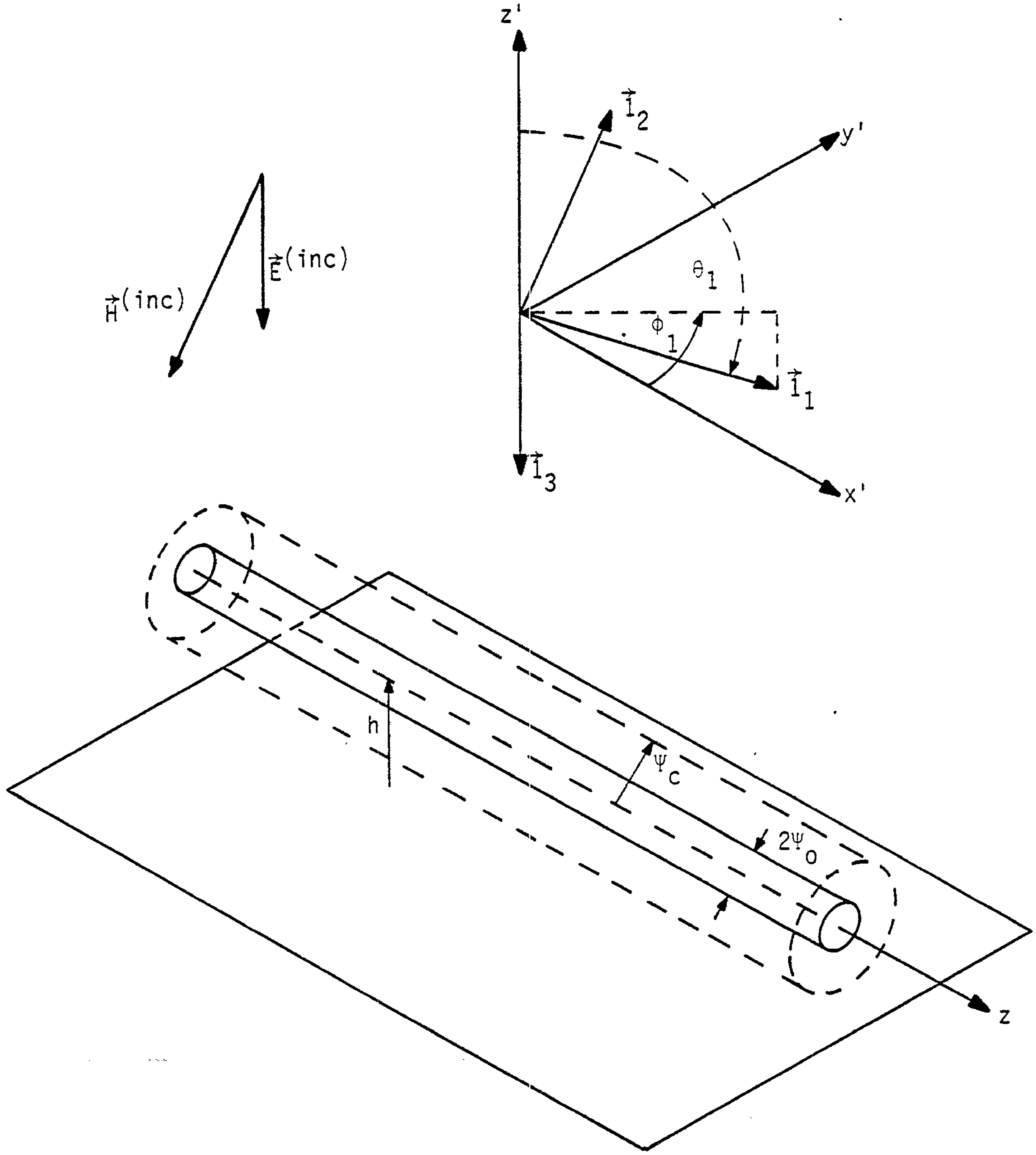


Fig. 3.1. Corona Model of Infinite Wire Parallel to Ground Plane

$$y' = \Psi' \sin(\phi') = r' \sin(\theta') \sin(\phi')$$

$$z' = r' \cos(\theta')$$

with the x' axis parallel to the z axis (i.e., the wire). Again define a set of unit vectors for the incident wave as in (2.4). Here $\vec{1}_1$ is the direction of propagation of the incident wave, and $\vec{1}_2$ and $\vec{1}_3$ are polarization unit vectors. In the $\vec{r}'$ coordinate system we have

$$\begin{aligned}
\vec{1}_1 &= \sin(\theta_1)\ \vec{1}_{\Psi'} + \cos(\theta_1)\ \vec{1}_{z'} \\
&= \sin(\theta_1)\cos(\phi_1)\ \vec{1}_{x'} + \sin(\theta_1)\sin(\phi_1)\ \vec{1}_{y'} + \cos(\theta_1)\ \vec{1}_{z'} \\
\vec{1}_2 &= -\cos(\theta_1)\ \vec{1}_{\Psi'} + \sin(\theta_1)\ \vec{1}_{z'} \\
&= -\cos(\theta_1)\cos(\phi_1)\ \vec{1}_{x'} - \cos(\theta_1)\sin(\phi_1)\ \vec{1}_{y'} + \sin(\theta_1)\ \vec{1}_{z'} \\
\vec{1}_3 &= -\vec{1}_{\phi'} \\
&= \sin(\phi_1)\ \vec{1}_{x'} - \cos(\phi_1)\ \vec{1}_{y'}
\end{aligned} \tag{3.4}$$

Let the incident field consist of the sum of a wave coming down to the ground plane plus one reflected from the ground plane as

$$\vec{E}^{(inc)} = \vec{E}^{(1)} + \vec{E}^{(2)} \tag{3.5}$$

with

$$\begin{aligned}
\vec{E}^{(1)}(\vec{r}',t) &= E_0 f\left(t - \frac{\vec{1}_1 \cdot \vec{r}'}{c}\right) \vec{1}_p \\
\vec{1}_p &= \text{polarization vector} \\
\vec{1}_p \cdot \vec{1}_1 &= 0
\end{aligned} \tag{3.6}$$

Now the polarization vector can be written as a linear combination of $\vec{1}_2$ and $\vec{1}_3$ as

$$\vec{1}_p = \cos(\beta)\,\vec{1}_2 + \sin(\beta)\,\vec{1}_3 \tag{3.7}$$

so that β is the angle of rotation in the $\vec{1}_2,\vec{1}_3$ plane in the direction from $\vec{1}_2$ toward $\vec{1}_3$.

The tangential component of the polarization vector (x' component or z component) is

$$\begin{aligned} \vec{1}_p \cdot \vec{1}_{x'} &= \vec{1}_p \cdot \vec{1}_z \\ &= -\cos(\beta)\cos(\theta_1)\cos(\phi_1) + \sin(\beta)\sin(\phi_1) \end{aligned} \tag{3.8}$$

This can be used for the tangential component of $\vec{E}^{(1)}$ in (3.5). The tangential component of the direction of incidence is

$$\begin{aligned} \vec{1}_1 \cdot \vec{1}_{x'} &= \vec{1}_1 \cdot \vec{1}_z \\ &= \sin(\theta_1)\cos(\phi_1) \end{aligned} \tag{3.9}$$

For convenience now place $\vec{r}' = \vec{0}$ on the perfectly conducting ground plane. The wire is now centered on $(y',z') = (0,h)$. The ground plane can be considered as a plane of symmetry and the resulting electromagnetic fields are antisymmetric with respect to this plane [4]. An antisymmetric electric field has the form

$$\begin{aligned} \vec{E}_{as}(\vec{r}',t) &= \frac{1}{2}\left[\vec{E}(\vec{r}',t) - \vec{E}_m(\vec{r}',t)\right] \\ \vec{E}_m(\vec{r}',t) &= \vec{R} \cdot \vec{E}(\vec{r}'_m,t) \quad \text{(mirror electric field)} \\ \vec{R} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \text{(reflection matrix)} \\ \vec{r}'_m &= \vec{R} \cdot \vec{r}' \quad \text{(mirror position)} \end{aligned} \tag{3.10}$$

Identifying then

$$\vec{E}^{(2)}(\vec{r}\,',t) = -\vec{E}_m(\vec{r}\,',t) \tag{3.11}$$

let us construct

$$\vec{E}^{(inc)}(\vec{r}\,',t) = \vec{E}^{(1)}(\vec{r}\,',t) - \overleftrightarrow{R} \cdot \vec{E}^{(1)}(\vec{r}\,'_m,t) \tag{3.12}$$

as the equivalent antisymmetric incident field. Then

$$\vec{E}^{(2)}(\vec{r}\,',t) = -E_o f\left(t - \frac{\vec{1}_1 \cdot \vec{r}\,'_m}{c}\right) \overleftrightarrow{R} \cdot \vec{1}_p \tag{3.13}$$

The tangential component is found from

$$\begin{aligned}
(\overleftrightarrow{R} \cdot \vec{1}_p) \cdot \vec{1}_{x'} &= (\overleftrightarrow{R} \cdot \vec{1}_p) \cdot \vec{1}_z \\
&= -\cos(\beta)\cos(\theta_1)\cos(\phi_1) + \sin(\beta)\sin(\phi_1) \\
&= \vec{1}_p \cdot \vec{1}_z
\end{aligned}$$

$$\begin{aligned}
\vec{1}_1 \cdot \vec{r}\,'_m &= \vec{1}_1 \cdot \overleftrightarrow{R} \cdot \vec{r}\,' \\
&\equiv \vec{1}_{1_m} \cdot \vec{r}\,'
\end{aligned} \tag{3.14}$$

$$\vec{1}_{1_m} = \vec{1}_1 \cdot \overleftrightarrow{R}$$

$$\begin{aligned}
\vec{1}_{1_m} \cdot \vec{1}_{x'} &= \vec{1}_{1_m} \cdot \vec{1}_z \\
&= \vec{1}_1 \cdot \vec{1}_{x'} \\
&= \sin(\theta_1)\cos(\phi_1)
\end{aligned}$$

So now we have the x' (or z) component of the incident electric field as

$$E_t = E_o \left\{ f\left(t - \frac{\vec{1}_1 \cdot \vec{r}\,'}{c}\right) - f\left(t - \frac{\vec{1}_{1_m} \cdot \vec{r}\,'}{c}\right)\right\} \vec{1}_p \cdot \vec{1}_z \tag{3.15}$$

Setting

$$\vec{r}\,' = (x',0,h) \quad , \quad \vec{r}\,'_m = (x',0,-h) \tag{3.16}$$

on the center of the wire, we have

$$\begin{aligned} \vec{1}_1 \cdot \vec{r}\,' &= \sin(\theta_1)\cos(\phi_1)x' + \cos(\theta_1)\,h \\ \vec{1}_{1_m} \cdot \vec{r}\,' = \vec{1}_1 \cdot \vec{r}\,'_m &= \sin(\theta_1)\cos(\phi_1)x' - \cos(\theta_1)\,h \end{aligned} \tag{3.17}$$

For convenience, and comparison to the results of section 2, let us choose the origin of the $\vec{r}\,'$ coordinates by our choice of $x' = 0$ such that (3.15) has its first term in time domain the same as in (2.6). This makes the early part of the time-domain response the same with the effect of the reflection from the ground plane appearing later. Note that $\pi/2 < \theta_1 < \pi$ (i.e., the wave is incident from above); then set at $z = 0$

$$\vec{1}_1 \cdot \vec{r}\,' = 0 = \sin(\theta_1)\cos(\phi_1)x' + \cos(\theta_1)\,h \tag{3.18}$$

giving

$$\begin{aligned} \vec{1}_{1_m} \cdot \vec{r}\,' &= \sin(\theta_1)\cos(\phi_1)x' - \cos(\theta_1)\,h \\ &= -2\cos(\theta_1)\,h \end{aligned} \tag{3.19}$$

As in section 2 we have a phase velocity

$$v_p = \frac{c}{\vec{1}_1 \cdot \vec{1}_z} = \frac{c}{\vec{1}_1 \cdot \vec{1}_{x'}} = c[\sin(\theta_1)\cos(\phi_1)]^{-1} \tag{3.20}$$

and a time variable

$$\tau \equiv t - \frac{z}{v_p} \tag{3.21}$$

The tangential component of the incident electric field (at the wire center) is then of the form

$$\begin{aligned} E_t &= E_0\left\{f(\tau) - f\left(\tau + 2\cos(\theta_1)\frac{h}{c}\right)\right\}\vec{1}_p \cdot \vec{1}_z \\ &\cos(\theta_1) < 0 \end{aligned} \tag{3.22}$$

For later use we have

$$g(\tau) \equiv \int_{-\infty}^{\tau} \{f(\tau') - [f(\tau' + 2\cos(\theta_1)\,\tfrac{h}{c})]\}\,d\tau'$$

$$\int_{-\infty}^{\tau} E_t(\tau')\,d\tau' = E_0 g(\tau)\,\vec{1}_p \cdot \vec{1}_z = E_1 g(\tau) \tag{3.23}$$

$$E_1 = E_0\,\vec{1}_p \cdot \vec{1}_z$$

Thus the problem of a wire over a ground plane (in this section) has the same form for the tangential electric field as the problem of a wire in free space (in the previous section) provided the incident waveform is appropriately interpreted in the two cases (comparing (2.8) and (3.23)).

IV. Telegrapher Equations in Corona Model

As discussed in [8] the telegrapher equations take the form

$$\frac{\partial V}{\partial z} = - L' \frac{\partial I}{\partial t} + V'_s$$

$$\frac{\partial I}{\partial z} = - \frac{\partial}{\partial t} (C'V) + I'_s \tag{4.1}$$

as in fig. 4.1 where the capacitance per unit length C' is included in the time derivative since it is time varying and of the form

$$
\begin{aligned}
C' &= C'(Q') = \frac{Q'}{V} = \text{capacitance per unit length} \\
Q' &= \text{charge per unit length} \\
L' &= \text{inductance per unit length} \\
V'_s &= \text{longitudinal voltage source per unit length} \\
I'_s &= \text{transverse current source per unit length}
\end{aligned}
\tag{4.2}
$$

These equations are related to a physical model illustrated in fig. 4.1. Here there is a corona of some conductivity (small compared to that of the wire) so that the current is primarily in the wire. The corona conductivity also has to be small enough that the skin depth (or diffusion depth) in the corona is large compared to Ψ_c. In this case L' does not depend on the corona and we have

$$L' = f_{g_0} Z_0 = L'_0 \tag{4.3}$$

with f_{g_0} chosen as in sections 2 or 3 as appropriate.

On the other hand the corona conductivity is assumed to be high enough that G' dominates C'_c and that the voltage between Ψ_0 (the wire) and Ψ_c (the corona boundary) is small compared to the voltage across C'_c. Then the per-unit length equivalent circuit in fig. 4.2A reduces to that in fig. 4.2B. This is the form used for the telegrapher equations (4.1).

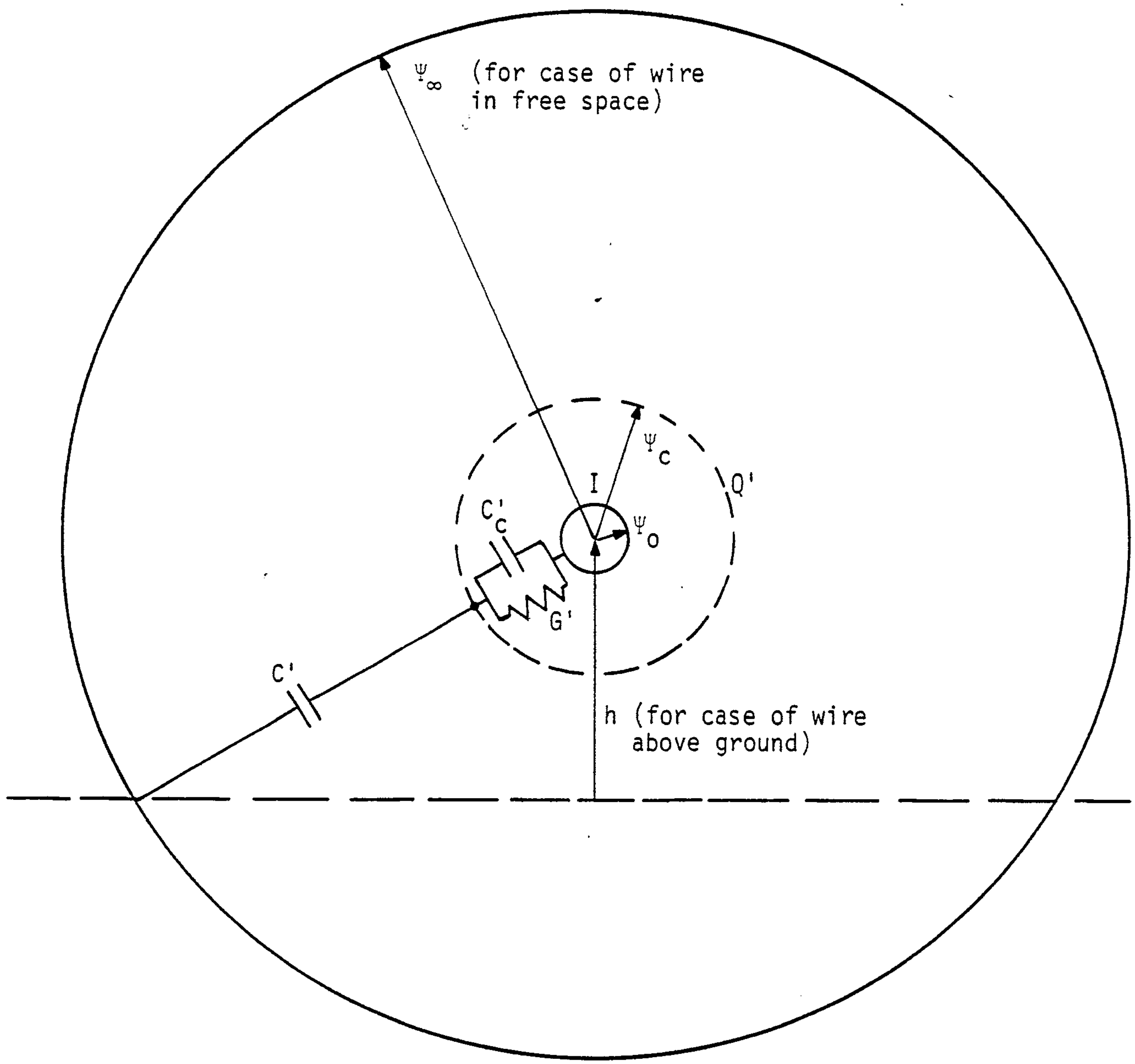
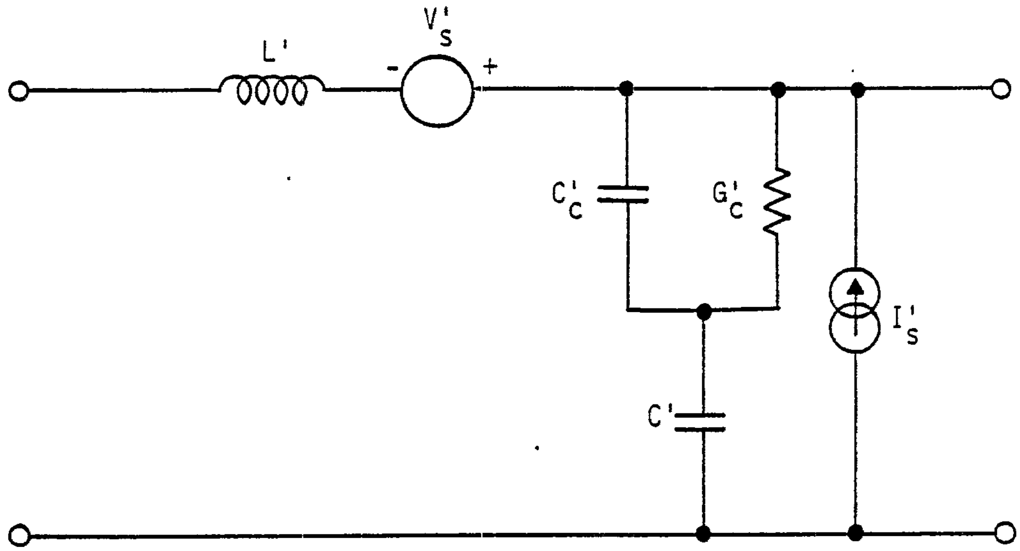
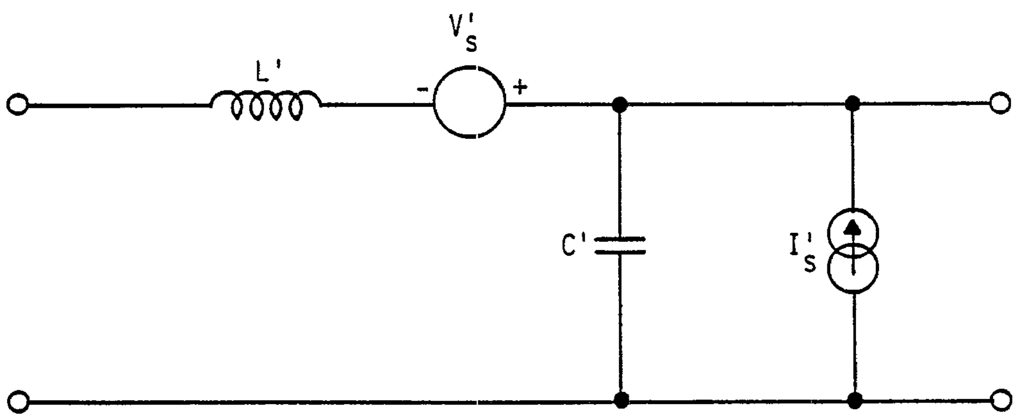


Fig. 4.1. Equivalent-Corona-Radius Model



A. General case with uniform corona



B. Case with sufficiently large corona conductivity

Fig. 4.2. Per-Unit-Length Transmission-Line Equivalent-Circuit Representation

This leads to a corona model of the wire in which the important term is the capacitance per unit length between the corona, which is assumed to be of radius Ψ_c centered on the wire, and the reference conductor, whether the ground plane as in section 3, or some equivalent coaxial cylinder of radius Ψ_∞ as in section 2. In the corona model Q' is assumed to reside on the outer boundary of the corona based on a breakdown electric field as

$$\Psi_c = \frac{|Q'|}{2\pi\varepsilon_0 E_b} = \text{corona radius}$$

$$E_b = \text{effective breakdown electric field} \tag{4.4}$$

provided $\Psi_c > \Psi_0$. For $\Psi_c < \Psi_0$ then Ψ_0 is used for the capacitance per unit length. This gives

$$C' = \varepsilon_0 \left\{\text{smaller of } [f_{g_0}, f_{g_c}]\right\}^{-1} \tag{4.5}$$

where f_{g_c} is the same as f_{g_0} except that Ψ_0 is replaced by Ψ_c, i.e.,

$$f_{g_c} = \begin{cases} \frac{1}{2\pi}\,\ell n\left(\frac{\Psi_\infty}{\Psi_c}\right) & \text{for wire in free space} \\ \frac{1}{2\pi}\,\ell n\left(\frac{2h}{\Psi_c}\right) + O\left(\left(\frac{\Psi_c}{h}\right)^2\right) \quad \text{as } \frac{\Psi_c}{h} \to 0 & \text{for wire above ground plane} \end{cases} \tag{4.6}$$

Identifying $2h \approx \Psi_\infty$ we have

$$f_{g_c} = \frac{1}{2\pi}\,\ell n\left(\frac{\Psi_\infty}{\Psi_c}\right) = \frac{1}{2\pi}\,\ell n\left(\frac{Q'_\infty}{|Q'|}\right)$$

$$Q'_\infty = 2\pi\varepsilon_0 E_b \Psi_c$$

In this form we have the capacitance per unit length in (4.5) as a function of the charge per unit length in (4.7).

The incident tangential electric field E_t has been considered in sections 2 (see (2.7)) and 3 (see (3.23)) for the two problems of interest. This needs to be related to the per-unit-length voltage and current sources V'_s and I'_s.

For the wire in free space I_s' is related to the transverse component of the electric field incident on the wire as in fig. 2.1. However, one does not know how to define the voltage related to this term. Voltage is a line integral [3]. One might take such an integral along a path normal to the wire toward the effective outer reference conductor. However, this path might have any azimuth (ϕ in cylindrical or spherical coordinates based on the z axis). Adding π to the azimuth reverses the sign of the radial electric field (in cylindrical sense).

The wire next to a ground plane is a more appropriate case to consider since the voltage can be defined as a potential difference between the wire and ground plane. Of course, one should be careful how this voltage is defined since voltage is dependent on the contour on which the line integral of the electric field is defined [3]. For present purposes let us define voltage via

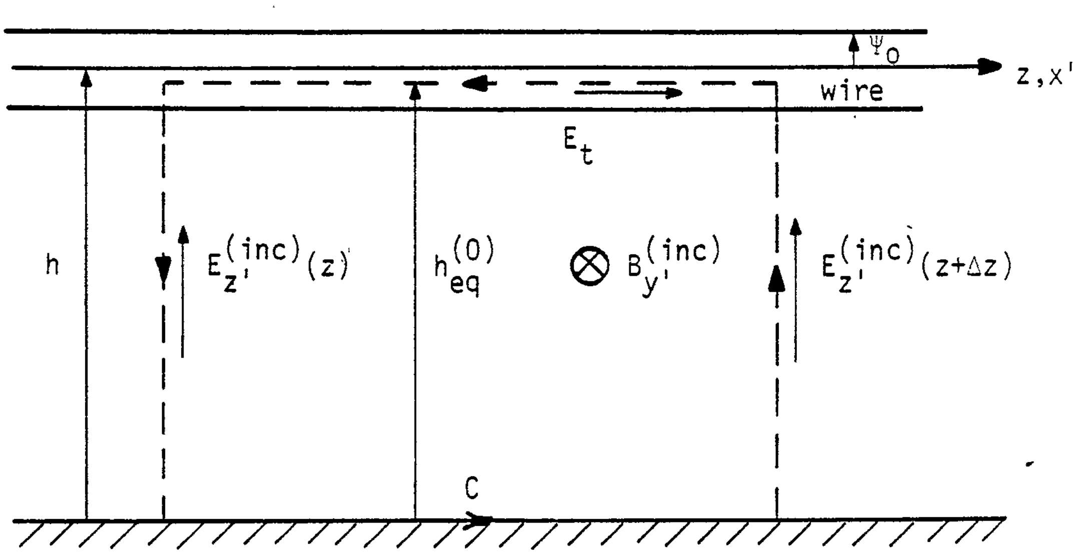
$$V(z) = -\int_0^h E_{z'}\Big|_{\vec{r}'=(x',0,z')} dz' \qquad x' = z \tag{4.8}$$

i.e., the contour is taken (for any z, or equivalently x') as a vertical line between the ground plane and the center of the wire. At low frequencies the incident electric field has a vertical (or z') component which is uniform; this contributes to the voltage in the form $-E_{z'}h$. However, the electric field associated with the charge on the wire or corona is not uniform with respect to z' even at low frequencies.

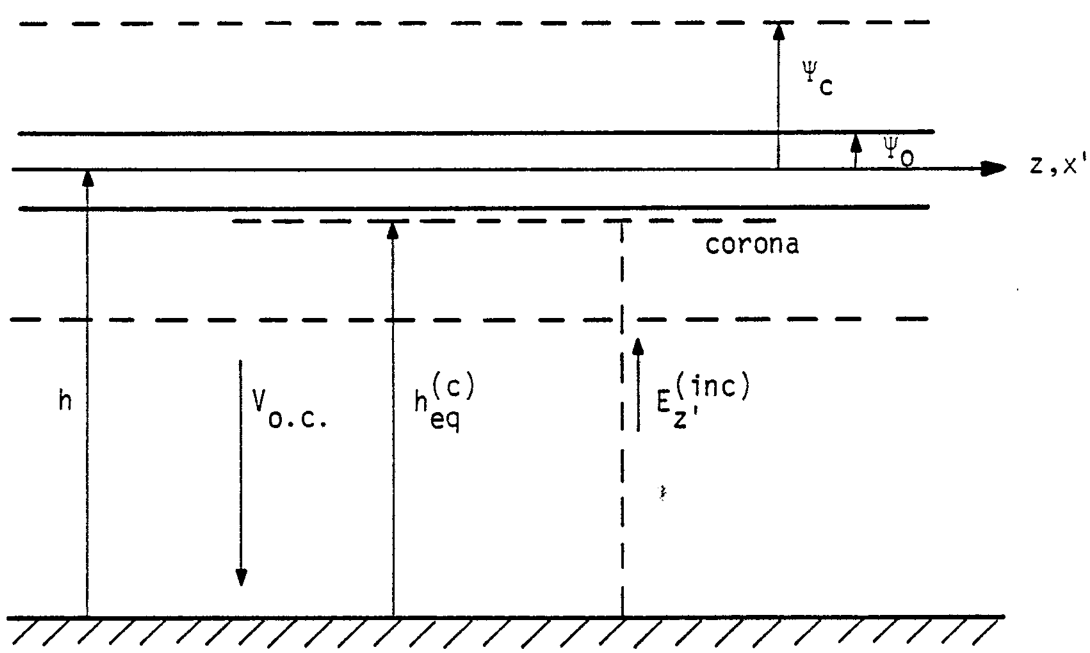
Referring to fig. 4.3 consider some incremental length of the transmission line. The voltage source per unit length is indicated in fig. 4.3A by a contour integral (around the contour C) as

$$V_s'(z) = \lim_{\Delta z \to 0} -\frac{1}{\Delta z}\oint_C \vec{E}^{(inc)} \cdot d\vec{\ell}$$

$$= \lim_{\Delta z \to 0} -\int_S B_{y'}^{(inc)} dS$$



A. Computation of voltage source per unit length



B. Computation of current source per unit length

Fig. 4.3. The Per-Unit-Length Voltage and Current Sources for a Wire Above a Ground Plane

$$= \lim_{\Delta z \to 0} \frac{1}{\Delta z} \{ \int_z^{z+\Delta z} E_t dz - \int_0^{h_{eq}^{(0)}} [E_{z'}^{(inc)}(\vec{r}\,') \Big|_{\vec{r}\,'=(x'+\Delta z,0,z')}$$

$$+ E_{z'}^{(inc)}(\vec{r}\,') \Big|_{\vec{r}\,'=(x',0,z')}] \, dz' \}$$

$$= E_t - \frac{\partial}{\partial z} \int_0^{h_{eq}^{(0)}} E_{z'}^{(inc)}(\vec{r}\,') \Big|_{y'=0} dz' \tag{4.9}$$

Note that the wire diameter Ψ_0 is small for this result. In general if one defines $h_{eq}^{(0)}$ as the equivalent height of the wire for magnetic-flux purposes (in the low-frequency or quasi-static limit) then one has an approximation as

$$V_s' = E_t - \frac{\partial}{\partial z} \int_0^{h_{eq}^{(0)}} E_{z'}^{(inc)}(\vec{r}\,') \Big|_{y'=0.} dz'$$

$$\simeq E_t - h_{eq}^{(0)} \frac{\partial}{\partial z} E_{z'}^{(inc)} \tag{4.10}$$

Considered from a circuit viewpoint what we are doing is considering an incremental length of the transmission line as a two-port network. The per-unit-length voltage source as above is the difference of the voltages at the two open-circuited ports. The per-unit-length current source is the sum of the currents at the two short-circuited ports.

The current source per unit length is illustrated in fig. 4.3B. To express this first consider the open-circuit voltage of an incremental length of line with respect to the ground plane. This is

$$V_{o.c.} = - \int_0^{h_{eq}^{(c)}} E_{z'}^{(inc)}(\vec{r}\,') \Big|_{y'=0} dz' \tag{4.11}$$

where for symmetry the open circuit voltage is defined at the center of our incremental section. Note that $h_{eq}^{(c)}$ is the equivalent height of the corona surrounded wire. This equivalent height accounts for the induced charge distribution around the corona boundary. The capacitance per unit length of this corona surrounded wire is C' giving $V_{o.c.}C'$ as an equivalent charge per unit length on our section of open-circuited line. The short-circuit current source per unit length which provides this charge per unit length is

$$I'_s = \frac{\partial}{\partial t}(V_{o.c.}C')$$

$$= -\frac{\partial}{\partial t}\{C' \int_0^{h_{eq}^{(c)}} E_{z'}^{(inc)}(\vec{r}')\Big|_{y'=0} dz'\} \tag{4.12}$$

Now change variables in the telegrapher equations as

$$V(z) = \frac{Q'}{C'} - \int_0^{h_{eq}} E_{z'}^{(inc)} dz' \tag{4.13}$$

where h_{eq} is approximately h and can be compared to $h_{eq}^{(0)}$ and $h_{eq}^{(c)}$ as desired. Then (4.1) become

$$\frac{\partial}{\partial z}\{\frac{Q'}{C'} - \int_0^{h_{eq}} E_{z'}^{(inc)} dz'\} = -L'\frac{\partial I}{\partial t} + V'_s$$

$$\frac{\partial I}{\partial z} = -\frac{\partial}{\partial t}[Q' - C'\int_0^{h_{eq}} E_{z'}^{(inc)} dz'] - \frac{\partial}{\partial t}\{C'\int_0^{h_{eq}} E_{z'}^{(inc)} dz'\} \tag{4.14}$$

Selecting h_{eq} as $h_{eq}^{(c)}$ to simplify the second of (4.14) gives

$$\frac{\partial}{\partial z}[\frac{Q'}{C'}] = -L'\frac{\partial I}{\partial t} + V'_s + \frac{\partial}{\partial z}\int_0^{h_{eq}^{(c)}} E_{z'}^{(inc)} dz'$$

$$\frac{\partial I}{\partial z} = -\frac{\partial Q'}{\partial t} \tag{4.15}$$

so that the second of the equations reduces to a continuity equation.

The first of (4.15) is simplified by noting that

$$V'_s + \frac{\partial}{\partial z}\int_0^{h_{eq}^{(c)}} E_{z'}^{(inc)} dz'$$

$$= E_t - \frac{\partial}{\partial z}\int_0^{h_{eq}^{(0)}} E_{z'}^{(inc)} dz' + \frac{\partial}{\partial z}\int_0^{h_{eq}^{(c)}} E_{z'}^{(inc)} dz'$$

$$= E_t + \frac{\partial}{\partial z}\int_{h_{eq}^{(0)}}^{h_{eq}^{(c)}} E_{z'}^{(inc)} dz'$$

$$\approx E_t + \left[h_{eq}^{(c)} - h_{eq}^{(0)}\right] \frac{\partial}{\partial z} E_{z'}^{(inc)} \tag{4.16}$$

Let us assume that the corona radius Ψ_c is sufficiently small so that we can set

$$h_{eq}^{(c)} \equiv h_{eq}^{(0)} \tag{4.17}$$

giving

$$V_s' + \frac{\partial}{\partial z} \int_0^{h_{eq}^{(c)}} E_{z'}^{(inc)} \, dz' = E_t \tag{4.18}$$

Note that the use of an equivalent height is most appropriate for the case that h is electrically small. For h electrically large a quasi-static estimate of h_{eq} is inappropriate. However, it does show the difference between the coupling terms associated with magnetic fields (or current) and electric fields (or charge).

With our selection in (4.17) now the telegrapher equations are transformed to

$$\begin{aligned} \frac{\partial}{\partial z} \left[\frac{Q'}{C'}\right] &= -L' \frac{\partial I}{\partial t} + E_t \\ \frac{\partial I}{\partial z} &= - \frac{\partial Q'}{\partial t} \end{aligned} \tag{4.19}$$

Note that E_t has been computed in sections 2 and 3. This form of the telegrapher equations has Q' and I as the response variables instead of V and I. In order to convert Q' to V one can use (4.13) which brings in an extra term, the incident vertical electric field (used either in integral form or multiplied by the equivalent height).

V. Form of Solution for Plane-Wave Excitation

As discussed in sections 2 and 3 the source term E_t is a function of

$$\tau \equiv t - \frac{z}{v_p}$$
$$v_p > c \tag{5.1}$$

For an infinite uniform transmission line we also expect a solution which combines time and space via τ. Let us then change our derivatives as

$$\frac{\partial}{\partial z} = \frac{\partial \tau}{\partial z}\bigg|_{t \text{ const.}} \frac{d}{d\tau}$$
$$= -\frac{1}{v_p}\frac{d}{d\tau}$$
$$\frac{\partial}{\partial t} = \frac{\partial \tau}{\partial t}\bigg|_{z \text{ const.}} \frac{d}{d\tau} \tag{5.2}$$
$$= \frac{d}{d\tau}$$

The telegrapher equations are now

$$-\frac{1}{v_p}\frac{\partial}{\partial \tau}\left(\frac{Q'}{C'}\right) = -L' \frac{\partial I}{\partial \tau} + E_t$$
$$\frac{1}{v_p}\frac{\partial I}{\partial \tau} = \frac{\partial Q'}{\partial \tau} \tag{5.3}$$

and we wish to solve for $I(\tau)$ and $Q'(\tau)$. Assuming zero initial conditions the second of (5.3) can be directly integrated to give

$$\frac{1}{v_p} I = Q' \tag{5.4}$$

This may in turn be substituted in the first of (5.3) to give

$$-\frac{1}{v_p}\frac{\partial}{\partial \tau}\left(\frac{Q'}{C'}\right) + v_p L' \frac{\partial Q'}{\partial \tau} = E_t \tag{5.5}$$

Again assuming zero initial conditions this equation can be integrated to give

$$
-\frac{1}{v_p}\frac{Q'}{C'} + v_p\ L'Q' = \int_{-\infty}^{\tau} E_t(\tau')\ d\tau'
$$

$$
= E_1\ g(\tau) \tag{5.6}
$$

where the results of (2.8) and (3.23) are included. This is quite a simple result. The source term is just the time integral of the component of the incident field parallel to the wire.

VI. Solution with No Corona

If there is no corona then

$$C' = \frac{\varepsilon_0}{f_{g_0}} \quad , \quad L' = \mu_0 f_{g_0} \tag{6.1}$$

and (5.6) takes the form

$$\begin{aligned}
Q'(\tau) &= \{-\frac{1}{v_p}\frac{f_{g_0}}{\varepsilon_0} + v_p\,\mu_0 f_{g_0}\}^{-1} E_1\, g(\tau) \\
&= \{-\frac{c}{v_p}\sqrt{\frac{\mu_0}{\varepsilon_0}} + \frac{v_p}{c}\sqrt{\frac{\mu_0}{\varepsilon_0}}\}^{-1} f_{g_0}^{-1} E_1 g(\tau) \\
&= \{-\frac{c}{v_p} + \frac{v_p}{c}\}^{-1} \frac{1}{Z_0 f_{g_0}} E_1 g(\tau) \\
&= \{-\frac{c}{v_p} + \frac{v_p}{c}\}^{-1} \frac{1}{Z_{c_0}} E_1 g(\tau)
\end{aligned} \tag{6.2}$$

This can be converted to current via

$$I(\tau) = v_p Q'(\tau) = (\frac{v_p}{c})\, cQ'(\tau) \tag{6.3}$$

Note that as $v_p \to c$ then $\vec{1}_1$ is becoming parallel to the wire. In that case we have from (3.20) and (2.5)

$$\frac{v_p}{c} = [\sin(\theta_1)\, \cos(\phi_1)]^{-1} = \cos(\alpha)^{-1} \tag{6.4}$$

Also from (3.8) and (3.23) we have

$$E_1 = [-\cos(\beta)\, \cos(\theta_1)\, \cos(\phi_1) + \sin(\beta)\, \sin(\phi_1)]\, E_0 \tag{6.5}$$

for the case of a wire parallel to a ground plane. If we regard α as the angle of $\vec{1}_1$ with respect to the wire and δ as the angle of $\vec{1}_p$ from the plane of $\vec{1}_1$ and the wire (as in section 2), then from (2.7) and (2.8) we have

$$E_1 = \cos(\delta)\, \sin(\alpha)\, E_0 \tag{6.6}$$

This can also be applied to a wire above a ground plane for which we note that, for fixed α, E_1 is maximized in magnitude for $\delta = 0,\pi$.

Substituting from (6.4) and (6.6) in (6.2) gives

$$Q'(\tau) = \{- \cos(\alpha) + \frac{1}{\cos(\alpha)}\}^{-1} \frac{1}{Z_{c_0}} \cos(\delta) \sin(\alpha) E_0 g(\tau)$$

$$= \cot(\alpha) \cos(\delta) \frac{E_0}{Z_{c_0}} g(\tau) \qquad (6.7)$$

Note that the current behaves as

$$I(\tau) = \frac{1}{\cos(\alpha)} cQ'(\tau) = \frac{\cos(\delta)}{\sin(\alpha)} \frac{c}{Z_{c_0}} E_0 g(\tau)$$

$$= \frac{\cos(\delta)}{\sin(\alpha)} \frac{E_0}{\mu_0 f_{g_0}} g(\tau) = \frac{\cos(\delta)}{\sin(\alpha)} \frac{E_0}{L'} g(\tau) \qquad (6.8)$$

As $\alpha \to 0$, if $g(\tau)$ is not also a function of α, then $Q'(\tau)$ and $I(\tau)$ blow up, as is well known for the case of an infinitely long perfectly conducting wire in free space [5]. As we can see this behavior even applies in the transmission-line approximation. One can also compare the present results to those in [5] to see the degree of approximation present in the transmission-line formalism as applied to a wire in free space. Of course, as $\alpha \to 0$ a finite length wire will have a signal propagate to the observer from one end, arriving a very short time after the "direct" signal and having an effect of reducing the peak signal.

Another case of interest is $\alpha = \pi/2$, or broadside incidence. Then we have

$$Q'(\tau) = 0$$

$$I(\tau) = \cos(\delta) \frac{E_0}{\mu_0 f_{g_0}} g(\tau) = \cos(\delta) \frac{E_0}{L'} g(\tau) \qquad (6.9)$$

$$= \cos(\delta) \frac{E_0 c}{Z_{c_0}} g(\tau)$$

In this case since Q' is zero for all time, then no corona will develop and the solution for the case of no corona will apply for all time (or until the presence of actual wire ends can propagate a signal to the observer).

Turning to the case of a wire above a ground plane (3.23) indicates the dependence of $g(\tau)$ as

$$g(\tau) \equiv \int_{-\infty}^{\tau} \{f(\tau') - f(\tau' + 2\cos(\theta_1)\frac{h}{c})\}\, d\tau'$$

$$\pi/2 < \theta_1 < \pi \qquad (6.10)$$

Note that for $\theta_1 = \pi/2$ then $g(\tau) = 0$. So let us expand the second term in the integrand for small $\cos(\theta_1)$ as

$$f(\tau' = 2\cos(\theta_1)\frac{h}{c}) = f(\tau') + 2\cos(\theta_1)\frac{h}{c}\frac{df(\tau')}{d\tau'}$$

$$+ O((2\cos(\theta_1)\frac{h}{c})^2 \frac{d^2f(\tau')}{d\tau'^2}) \text{ as } \theta_1 \to \frac{\pi}{2} \qquad (6.11)$$

where $f(\tau')$ is assumed to have a first derivative, and a second derivative as well if the order symbol is to apply. Substituting (6.11) into (6.10) with zero initial conditions on $f(\tau')$ and its first and second derivatives gives

$$g(\tau) = -2\cos(\theta_1)\frac{h}{c} f(\tau) + O((2\cos(\theta_1)\frac{h}{c})^2 \frac{df(\tau)}{d\tau}) \qquad (6.12)$$

This result can also be obtained by expanding the time integral of $f(\tau)$ in (6.10) in a manner similar to (6.11).

Noting as in (6.4) that

$$\cos(\alpha) = \sin(\theta_1)\cos(\phi_1) \qquad (6.13)$$

then (6.6) can be written

$$E_1 = \cos(\delta)\, [1 - \sin^2(\theta_1)\cos^2(\phi_1)]^{1/2} E_0 \qquad (6.14)$$

This allows us to consider the response of the wire above a ground plane as a function of δ and either α or θ_1 and ϕ_1.

Now express (6.2) and (6.3) as

$$
\begin{aligned}
Q'(\tau) &= \{-\cos(\alpha) + \frac{1}{\cos(\alpha)}\}^{-1} \frac{1}{Z_{c_0}} \cos(\delta)\sin(\alpha) E_0 g(\tau) \\
&= \cot(\alpha)\cos(\delta)\frac{E_0}{Z_{c_0}} g(\tau) \\
&= \cos(\delta)\frac{E_0}{Z_{c_0}}\{-2\frac{h}{c}\cos(\theta_1)\cot(\alpha) f(\tau) \\
&\qquad + O((2\cos(\theta_1)\frac{h}{c})^2 \cot(\alpha)\frac{df(\tau)}{d\tau})\} \\
&\text{as } \theta_1 \to \frac{\pi}{2}
\end{aligned}
\tag{6.15}
$$

$$
\begin{aligned}
I(\tau) &= \frac{1}{\cos(\alpha)} cQ'(\tau) \\
&= \cos(\delta)\frac{E_0}{Z_{c_0}}\{-\frac{2h}{c}\frac{\cos(\theta_1)}{\sin(\alpha)} f(\tau) \\
&\qquad + O((2\cos(\theta_1)\frac{h}{c})^2 \frac{1}{\sin(\alpha)}\frac{df(\tau)}{d\tau})\} \\
&\text{as } \theta_1 \to \frac{\pi}{2}
\end{aligned}
$$

Since α is a function of θ_1 and ϕ_1 as in (6.13) then making α small requires both θ_1 near $\pi/2$ and θ_1 near zero (or π). If θ_1 is bounded away from 0 and π then α cannot approach zero where singular behavior has been noticed for the case of a wire in free space as in (6.7) and (6.8). Let us then choose a special case of $\phi_1 = 0$ for which

$$\cos(\alpha) = \sin(\theta_1) \ , \ \cos(\theta_1) = -\sin(\alpha) \tag{6.16}$$

The wire-above-ground-plane result in (6.15) then becomes

$$
\begin{aligned}
Q'(\tau) &= \cos(\delta)\frac{E_0}{Z_{c_0}}\{\frac{2h}{c}\cos(\alpha) f(\tau) \\
&\qquad + O(2\cos(\alpha)\sin(\alpha)(\frac{h}{c})^2 \frac{df(\tau)}{d\tau}
\end{aligned}
$$

as $\alpha \to 0$

$$I(\tau) = \frac{1}{\cos(\alpha)} c\dot{Q}'(\tau) \tag{6.17}$$

$$= \cos(\delta) \frac{c}{Z_{c_0}} E_0 \left\{\frac{2h}{c} f(\tau) + 0\left(4 \sin(\alpha)\left(\frac{h}{c}\right)^2 \frac{df(\tau)}{d\tau}\right)\right\}$$

as $\alpha \to 0$

Note that this result is bounded as $\alpha \to 0$ and the response is proportional to $f(\tau)$, not to its time integral. Also note that the wire length is taken to be infinite <u>before</u> the limit $\alpha \to 0$ is taken. In this way the termination impedance at the wire end in the direction toward the source of the incident wave does not enter into the present result.

Thus while the formulation of the two cases (both without corona) is similar, the practical results are somewhat different. The presence of the reflection from the perfectly conducting ground plane significantly reduces and alters the shape of the wire response.

VII. Solution with Corona

If there is corona then

$$C' = \frac{\varepsilon_0}{f_{g_c}} \quad , \quad L' = \mu_0 f_{g_0}$$
$$f_{g_c} = \frac{1}{2\pi} \ln\left(\frac{Q'_\infty}{|Q'|}\right) \tag{7.1}$$

There is a threshold for corona given by

$$Q'_0 = 2\pi\varepsilon_0 E_b \tag{7.2}$$

If $|Q'| > Q'_0$ then (7.1) applies. For $|Q'| < Q'_0$ then (6.1) applies. Our general solution in (5.6) then applies to both cases where

$$C'(Q') = \begin{cases} \dfrac{\varepsilon_0}{f_{g_0}} & \text{for } |Q'| < Q'_0 \\ \dfrac{\varepsilon_0}{f_{g_c}} & \text{for } |Q'| > Q'_0 \end{cases} \tag{7.3}$$

noting that C' is continuous through Q'_0.

Rewriting (5.6) as

$$\left\{- \frac{c}{v_p} \frac{1}{cC'} + \frac{v_p}{c} cL'\right\} Q'(\tau) = E_1 g(\tau) \tag{7.4}$$

we have

$$\left\{- \cos(\alpha)\, f_{g_c} + \frac{1}{\cos(\alpha)} f_{g_0}\right\} Z_0 Q'(\tau) = E_1 g(\tau)$$

$$f_{g_c} = \frac{1}{2\pi} \ln\left(\frac{Q'_\infty}{|Q'|}\right)$$
$$f_{g_0} = \frac{1}{2\pi} \ln\left(\frac{Q'_\infty}{Q'_0}\right) \tag{7.5}$$

Note that

$$|Q'| > Q'_0 \quad \Rightarrow \quad f_{g_c} < f_{g_0} \tag{7.6}$$

This last result shows that in the presence of corona then as $\alpha \to 0$ the coefficient of Q' in (7.5) remains positive away from zero. This prevents the blowup of Q' as $\alpha \to 0$.

Since Q_0' is the corona threshold let us normalize our results to this and rewrite (7.5) as

$$\{-\cos(\alpha) + \frac{1}{\cos(\alpha)} + \cos(\alpha)\left[1 - \frac{f_{g_c}}{f_{g_0}}\right]\} \frac{Q'(\tau)}{Q_0'} = \frac{E_1 g(\tau)}{Z_{c_0} Q_0'}$$

$$E_1 = \cos(\delta)\sin(\alpha) E_0 \tag{7.7}$$

In this form there is a correction term, the third term in the braces, which accounts for the corona. For the case of no corona this correction term is zero. Using the expressions in (7.5) for the geometric factors we have

$$\{-\cos(\alpha) + \frac{1}{\cos(\alpha)} + \cos(\alpha) \frac{\ln\left(\frac{|Q'(\tau)|}{Q_0'}\right)}{\ln\left(\frac{Q_\infty'}{Q_0'}\right)}\} \frac{Q'(\tau)}{Q_0'} = \frac{E_1 g(\tau)}{Z_{c_0} Q_0'}$$

applying for $\frac{|Q'(\tau)|}{Q_0'} > 1$

$$\frac{I(\tau)\cos(\alpha)}{cQ_0'} = \frac{Q'(\tau)}{Q_0'} \tag{7.8}$$

Solving for $Q'(\tau)/Q_0'$ note that one has $I(\tau)$ appropriately normalized as well. One might define then

$$I_0 \equiv \frac{cQ_0'}{\cos(\alpha)} \tag{7.9}$$

giving

$$\frac{I(\tau)}{I_0} = \frac{Q'(\tau)}{Q_0'} \tag{7.10}$$

Consider the special case as before in which $\alpha \to 0$. Now we have

$$\ln\left(\frac{|Q'(\tau)|}{Q_0'}\right) \frac{Q'(\tau)}{Q_0'} = 0$$

$$\frac{|Q'(\tau)|}{Q'_0} = \begin{cases} 0 \text{ if } g(\tau) = 0 \\ 1 \text{ if } g(\tau) \neq 0 \end{cases} \tag{7.11}$$

$$I = cQ'_0$$

Note that this simple result indicates that the charge per unit length (and hence the current as well) does not blow up as $\alpha \to 0$; the corona limits it. This applies irrespective of $g(\tau)$, whether for a wire in free space or a wire above a ground plane. Before the incident wave reaches the observer the solution is zero (by causality). After the wave arrives Q' immediately rises to $\pm Q'_0$. If $g(\tau)$ is unipolar (for constant polarization δ) and we adjust the zero-time reference so that

$$g(\tau) \begin{cases} = 0 \text{ for } \tau < 0 \\ \neq 0 \text{ for } \tau > 0 \end{cases} \tag{7.12}$$

then we have

$$\frac{|Q'(\tau)|}{Q'_0} = u(\tau) \tag{7.13}$$

with polarity depending on the polarity of the incident field.

These results are graphed in figs. 7.1 through 7.7. Each of these figures is for a particular value of f_{g_0} (as in (2.1), (3.1), and (7.5)). This is a geometrical impedance factor and can be related to the characteristic impedance (without corona) in the transmission-line model by multiplication by the free-space wave impedance Z_0 ($\approx 377\ \Omega$).

Each of the figures has two parts. Part A plots $|Q'(\tau)|/Q'_0$ as a function of $E_1 g(\tau)/(Z_{c_0} Q'_0)$ as in (7.8) and (7.11) for the case of corona, and for comparison the normalized solution in (6.2) is plotted as dashed lines. Note that for $|Q'| > Q'_0$ the response is significantly reduced. Various choices of α between 0° and 90° are given. Note that for $\alpha = 90°$ the charge per unit length is zero, but the current is nonzero and can be found from (6.9) since there is no corona in this case.

Part B puts these results in terms of a corona reduction factor. The response under corona conditions is divided by the linear response (no

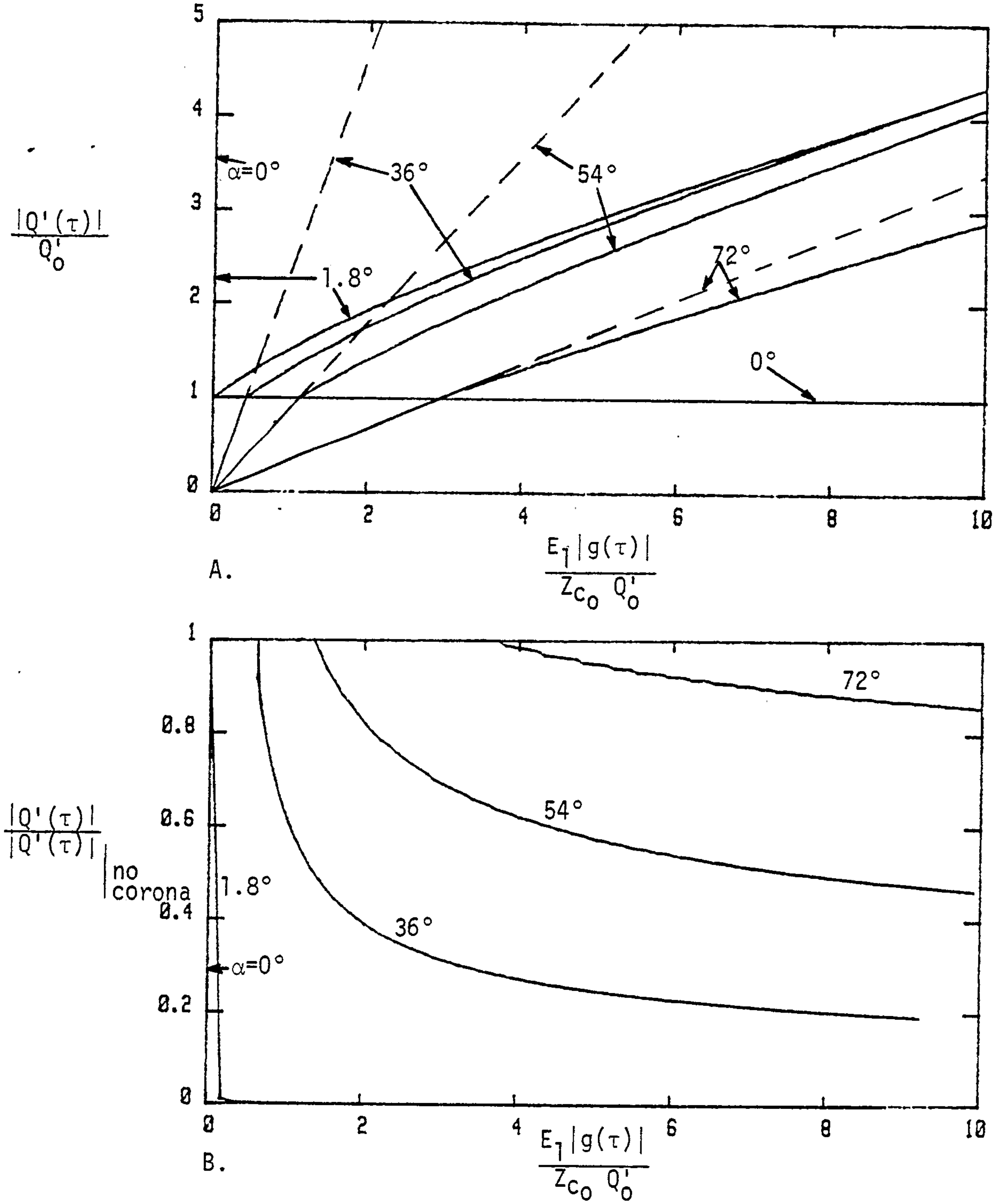


Fig. 7.1. Normalized Response: $f_{g_o} = 0.1$

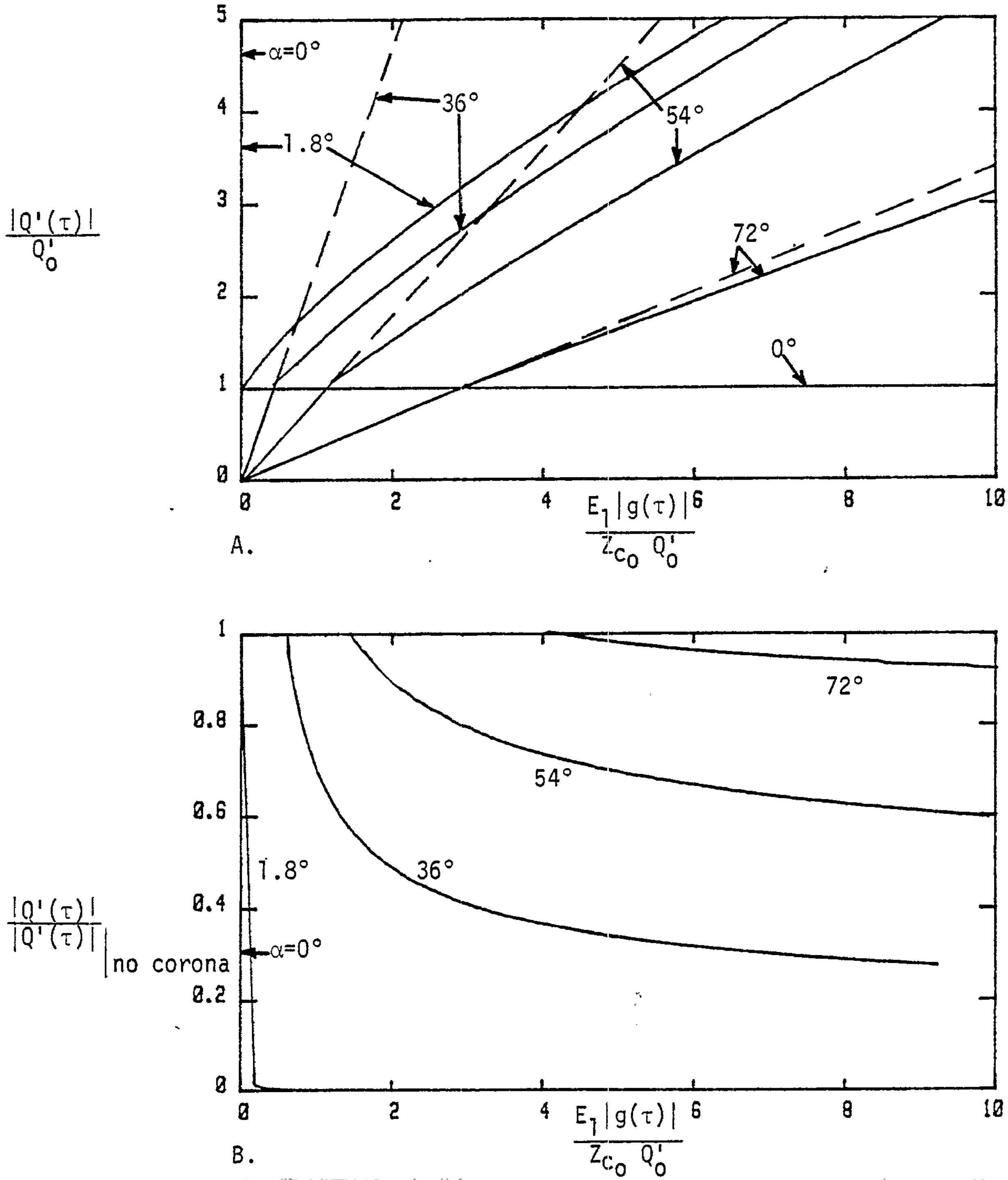


Fig. 7.2. Normalized Response: $f_{g_o} = 0.2$

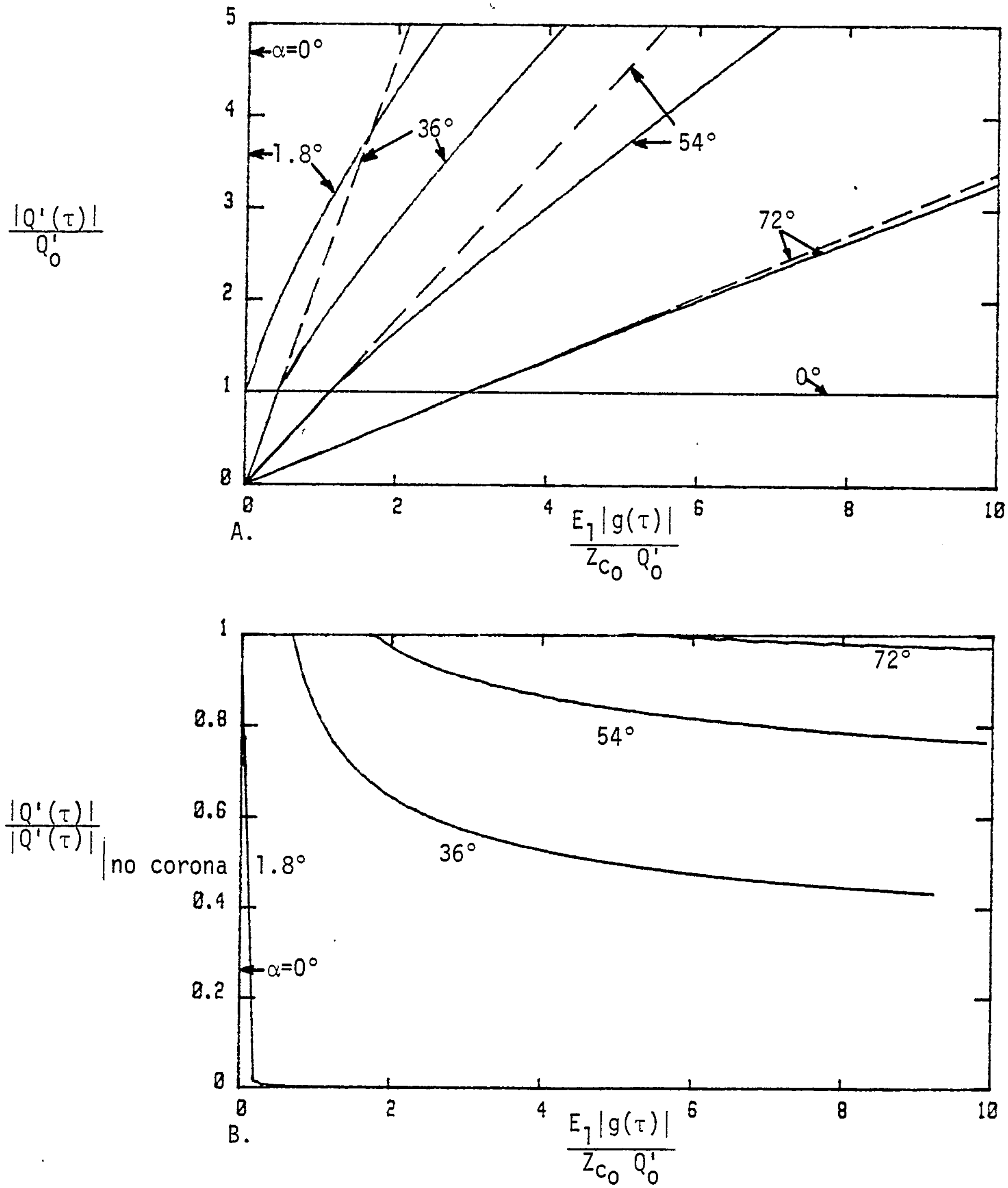


Fig. 7.3. Normalized Response: $f_{g_0} = 0.5$

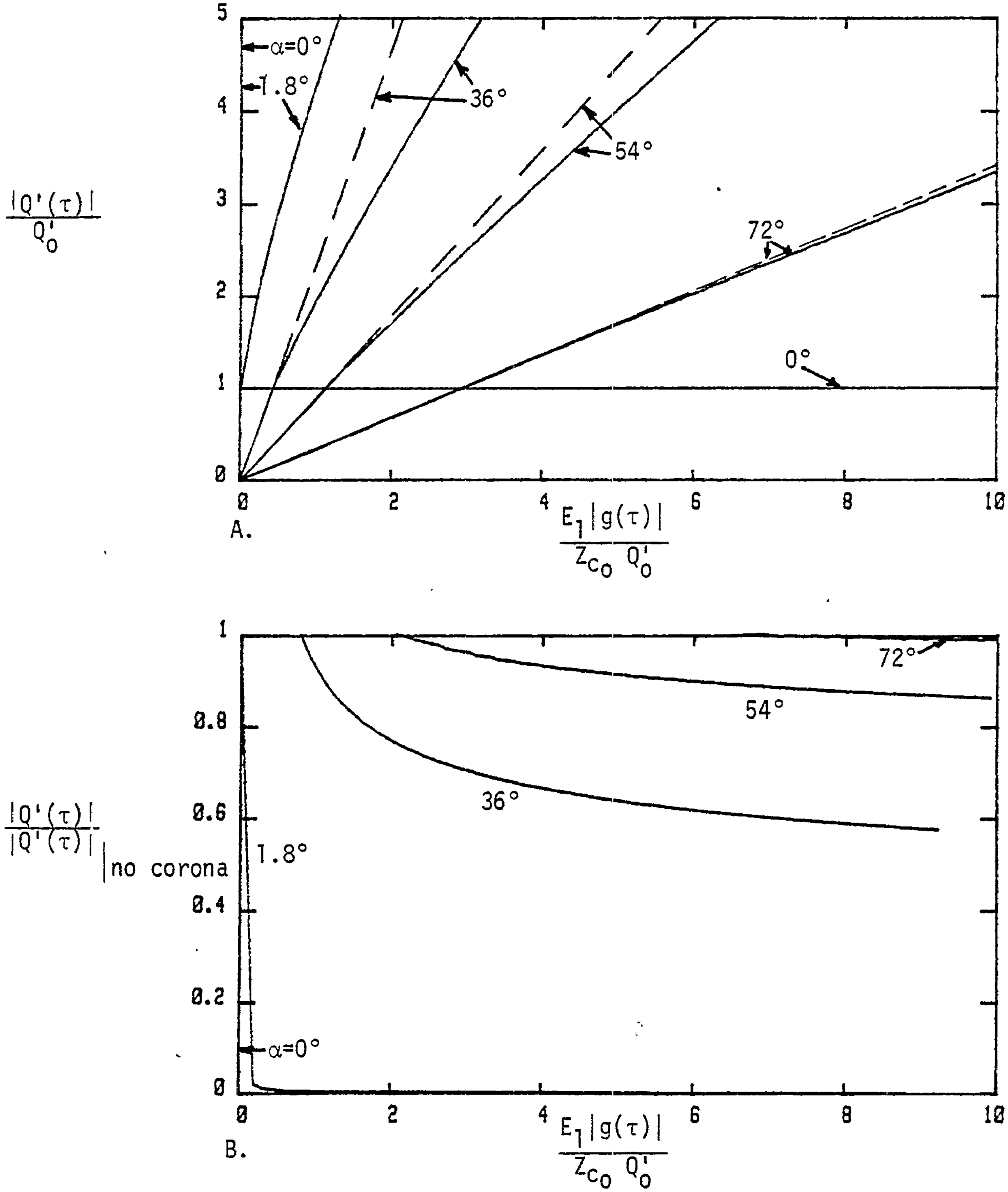


Fig. 7.4. Normalized Response: $f_{g_o} = 1$

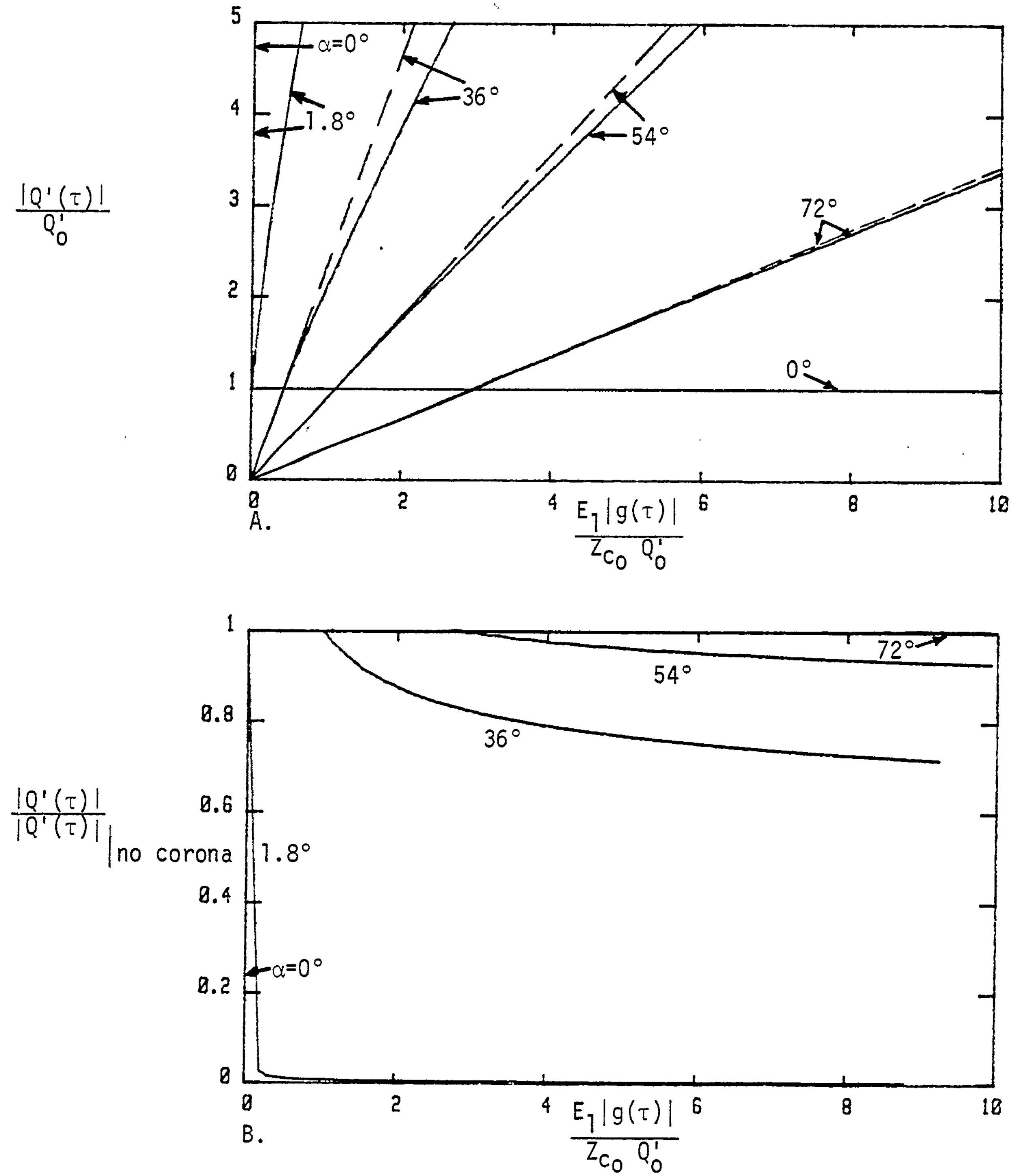


Fig. 7.5. Normalized Response: $f_{g_0} = 2$

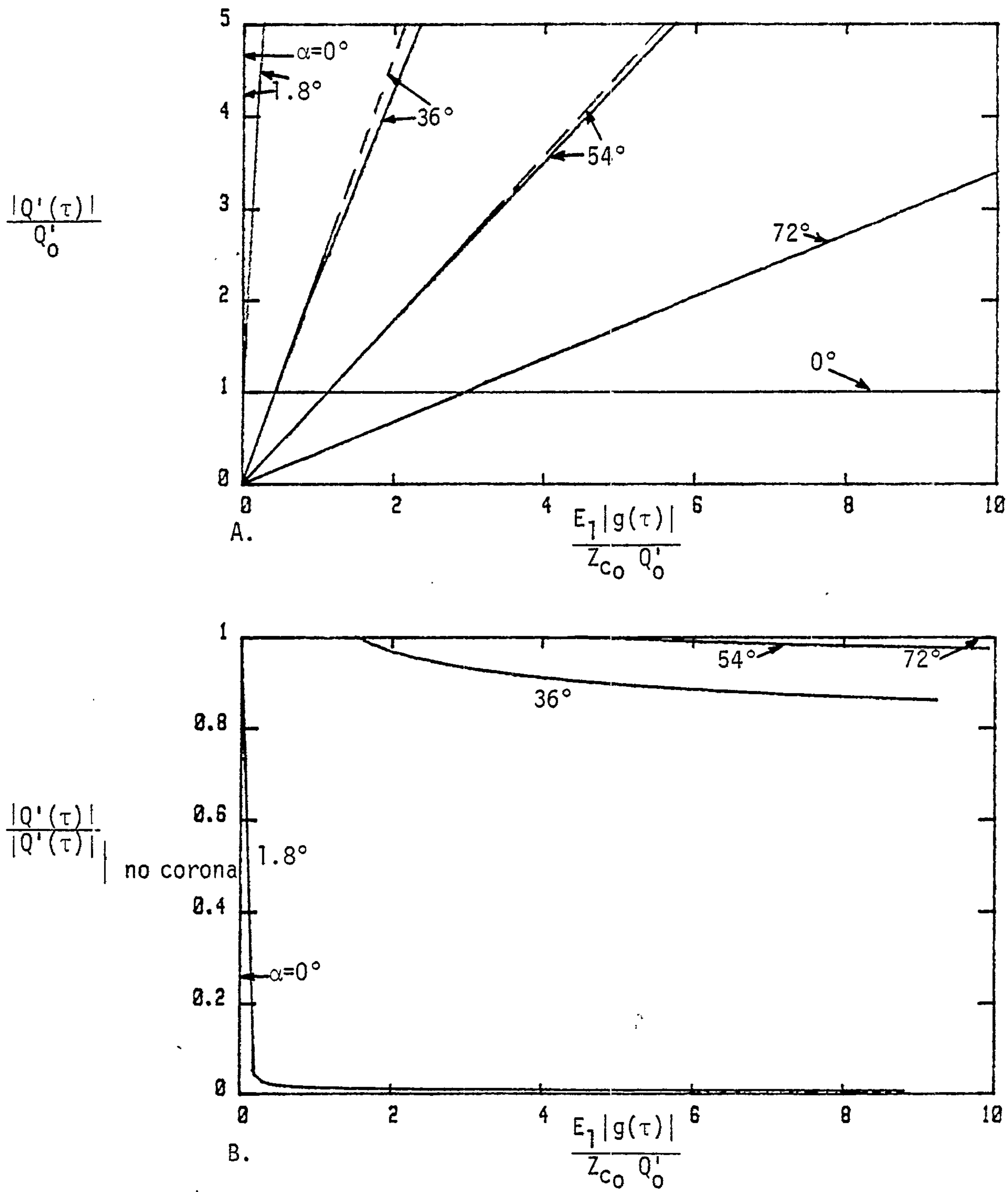


Fig. 7.6. Normalized Response: $f_{g_o} = 5$

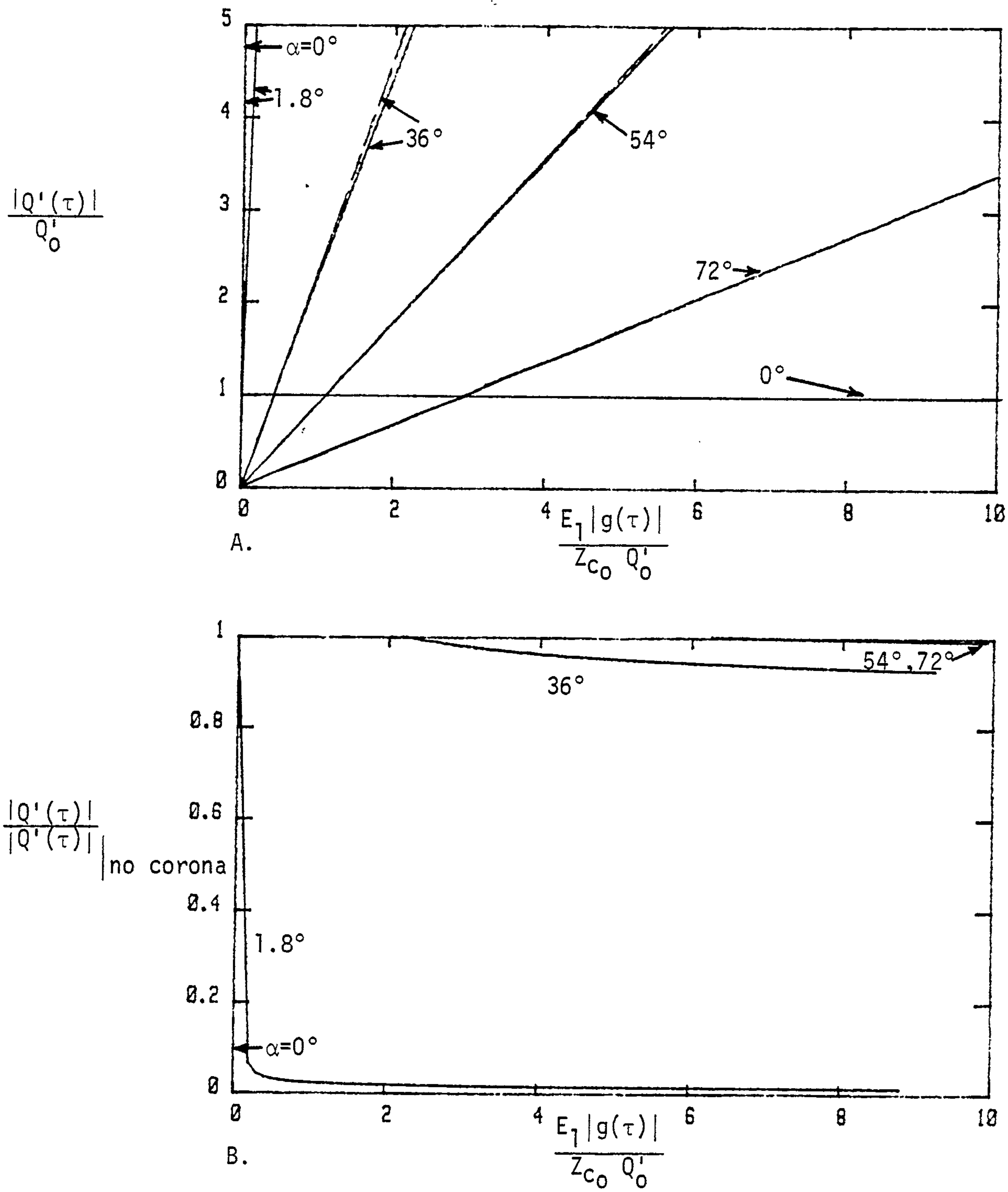


Fig. 7.7. Normalized Response: $f_{g_o} = 10$

corona). Before onset of corona this form of the normalized response is 1.0. After corona onset a significant reduction is noted due to corona.

In these plots various choices of α are taken. At $\alpha = 90°$ there is no corona and the linear result applies. At $\alpha = 0$ the case of no corona gives an infinite response for a nonzero source term $E_1 g(\tau)/(Z_{c_0} Q_0')$. However, as in (7.13) the corona limits the response. For intermediate α there is some reduction of the charge per unit length due to corona. The charge per unit length increases for small α until α is quite near 0°, in which case it decreases again toward its limiting value.

VIII. Experiments

As mentioned before, the corona model used in the telegrapher equations developed from some experimental results in which a high-voltage pulser launched a wave on one end of a wire above a ground plane [6,7]. However, instead of the "free" solution appropriate to that case, this paper considers the "forced" solution appropriate to the case of a plane wave incident on a wire, either in free space or above a ground plane.

This solution comes from the same nonlinear transmission-line equations, but with different sources. As such one might construct one or more experiments based upon such plane-wave illumination which test to what degree the present formulas model reality.

One type of experiment is indicated in fig. 8.1 in which a transient electromagnetic field, an approximate plane wave, is incident on a wire. To obtain large fields which induce air breakdown around a wire, a guided TEM wave, such as propagated between parallel conducting sheets, or in a region of conical conducting sheets of small angle of divergence would be appropriate [2]. The approximately uniform fields between the planar conducting sheets in turn approximate an incident uniform plane wave over a finite region of space.

In this parallel-plate region let us introduce a wire at an angle ν with respect to the direction of incidence $\vec{1}_1$ and in the plane of $\vec{1}_1$ and $\vec{E}^{(inc)}$ as indicated in fig. 8.1. Let us define

$$w \equiv \text{plate spacing}$$

$$\ell \equiv \text{wire length}$$

$$\frac{w}{\ell} = \sin(\nu) \tag{8.1}$$

Let us consider an observation position on the wire (wire axis for simplicity) at a distance w_0 from the "bottom" plate and a corresponding slant distance ℓ_0 along the wire with

$$\frac{w_0}{\ell_0} = \sin(\nu) \tag{8.2}$$

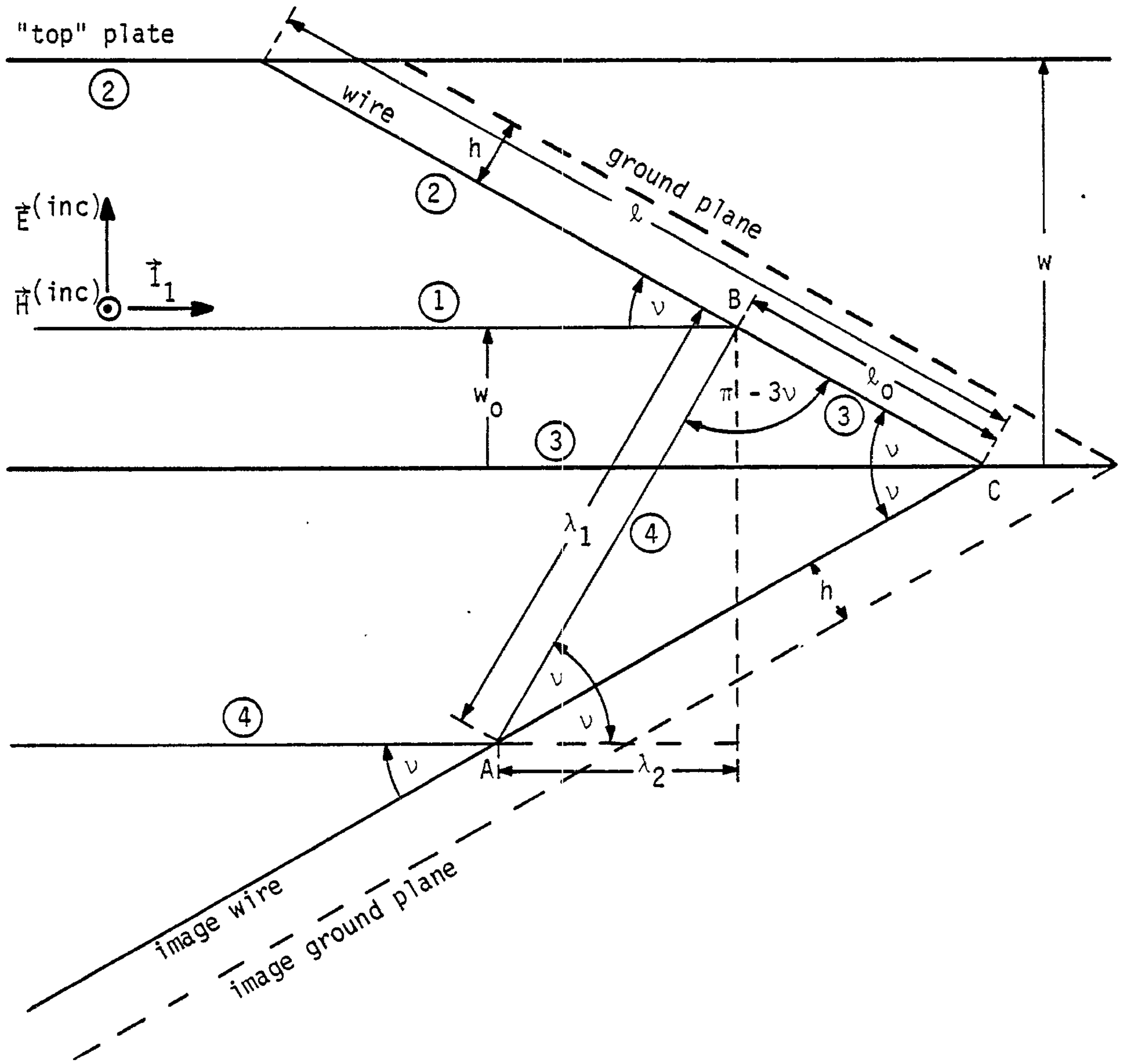


Fig. 8.1. A Wire and Possible Ground Plane Introduced into a Parallel-Plate Region

In designing an experiment like this we need to consider the time for which the results are applicable. Specifically, let us at least initially design the experiment to apply to the case of an infinite length wire. For the case of $\nu = \pi/2$ for which the wire is orthogonal to the parallel plates, then one can consider the wire to include an infinite set of images in both directions making an infinite straight wire. However, in practice the plates have finite width (say 2a). Then the scattered field from the wire will propagate to the plate edge and return to the wire in a time 2a/c if the wire is centered between the plate edges. If the observer is in the middle of the wire ($w_0 = w/2$) the signal from the edge of the plates arrives at $[a + \sqrt{a^2 + w_0^2}]/c$ after the first signal reaches the observer. If there is a ground plane placed a distance h behind the wire (see fig. 8.1) then the signal from the edge of this plate to the observer also needs to be considered. These considerations lead to a concept of clear time for which the experimental results apply.

Note that our reference to "top" and "bottom" plates of the parallel-plate waveguide is arbitrary. The plates could as well as "sides" depending on the orientation of the waveguide in space.

By varying the amplitude of the incident pulse one can in principle drive the wire into corona. However, the case of $\nu = \pi/2$ is not of much interest in this regard since our results (and symmetry) indicate that corona is not produced in this case.

Varying ν over the range $0 < \nu < \pi/2$, corona can in general be generated around the wire. This requires a more careful consideration of the clear time for the experimental result to apply.

Consider first the signals from the wire ends which reach the observer. Several ray paths are indicated in fig. 8.1. The incident field of interest travels on path (1). The field from the top of the wire travels on path (2); it arrives at the observer at a time

$$t_2 = \frac{1}{c}[\ell - \ell_0][1 - \cos(\nu)] = 2\,\frac{\ell - \ell_0}{c}\sin^2\left(\frac{\nu}{2}\right) \tag{8.3}$$

after the field on path (1). The field from the bottom of the wire travels on path (3); it arrives at the observer at a time

$$t_3 = \frac{1}{c} \ell_0[1 + \cos(\nu)]$$

$$= 2 \frac{\ell_0}{c} \cos^2(\frac{\nu}{2}) \tag{8.4}$$

after the field on path 1. Equating these two results gives

$$\ell[1 - \cos(\nu)] = 2\ell_0^{(2,3)}$$

$$\frac{\ell_0^{(2,3)}}{\ell} = \frac{w_0^{(2,3)}}{w} = \frac{1 - \cos(\nu)}{2} = \sin^2 (\frac{\nu}{2}) \tag{8.5}$$

So as $\nu \to 0$ then $w_0^{(2,3)} \to 0$ and the optimum observer position moves toward the bottom plate. Note that we have superscripts to designate which t_n and t_m are used for this choice of w_0 or ℓ_0 as an optimum measurement location. Substituting (8.5) into (8.3) or (8.4) then gives the corresponding clear time

$$t_2 = t_3 = \frac{2\ell}{c} \sin^2 (\frac{\nu}{2}) \cos^2 (\frac{\nu}{2})$$

$$= \frac{1}{2} \frac{\ell}{c} \sin^2(\nu) = \frac{1}{2} \frac{w}{c} \sin(\nu) \tag{8.6}$$

So as $\nu \to 0$ the clear time gets less and less at the optimum measurement location.

However, there are other disturbances to consider. As ν becomes smaller consider the arrival of a scattered field from some other part of the wire reflecting from the ground plane and subsequently reaching the observer. This is conveniently considered by means of an image wire below the lower plate and path 4 for the field scattered to the observer. As indicated in fig. 8.1 this path makes an angle ν for both incident and scattered field with respect to the image wire.

Considering the triangle ABC we have the length λ_1 of the ray AB diffracted from the image wire related to the wire length ℓ_0 of the wire from the lower conducting plate (BC) by the law of sines as

$$\frac{\lambda_1}{\ell_0} = \frac{\sin(2\nu)}{\sin(\nu)} = 2 \cos(\nu) \tag{8.7}$$

Furthermore the projected length λ_2 of the ray (4) through the image wire to a position under the observer is

$$\frac{\lambda_2}{\lambda_1} = \cos(2\nu) = 1 - 2\sin^2(\nu) \tag{8.8}$$

The extra transit time along the ray (4) is then

$$t_4 = \frac{1}{c}[\lambda_1 - \lambda_2] = \frac{\ell_o}{c}\frac{\lambda_1}{\ell_o}\lfloor 1 - \frac{\lambda_2}{\lambda_1}\rfloor = 2\frac{\ell_o}{c}\cos(\nu)\lfloor 1 - \cos(2\nu)\rfloor$$
$$= 4\frac{\ell_o}{c}\sin^2(\nu) = 4\frac{w_o}{c}\sin(\nu) \tag{8.9}$$

Now the experimental design problem for any particular ν is to maximize the minimum clear time from all undesired scattering locations. Let us compare t_4 with t_3. Equating the two gives

$$t_4 = t_e = \frac{1}{c}\ell_o^{(3,4)}[1 + \cos(\nu)] = \frac{4}{c}\ell_o^{(3,4)}\sin^2(\nu)$$

$$1 + \cos(\nu) = 4[1 - \cos^2(\nu)] = 4[1 + \cos(\nu)][1 - \cos(\nu)]$$

$$\cos(\nu) = \frac{3}{4}$$

$$\nu = \text{arc}\cos(\tfrac{3}{4}) \simeq 41.3^0 \simeq .72 \text{ radians} \tag{8.10}$$

$$t_4 = t_3 = \frac{1}{c}\ell_o^{(3,4)}\frac{7}{4} = \frac{1}{c}\frac{w_o^{(3,4)}}{\sin(\nu)}\frac{7}{4}$$
$$= \sqrt{7}\,\frac{w_o^{(3,4)}}{c}$$

This shows a critical angle. If ν is smaller than this angle then t_4 is less than t_3 and then t_4 is the clear time to be considered.

Assuming that ν is smaller than this critical angle then t_4 is less than t_3 and t_4 should be compared instead to t_2. Equating t_4 and t_2 gives

$$t_4 = t_2 = 4\frac{\ell_o^{(2,4)}}{c}\sin^2(\nu) = \frac{\ell - \ell_o^{(2,4)}}{c}[1 - \cos(\nu)]$$

$$4 \frac{\ell_o^{(2,4)}}{\ell} [1 + \cos(\nu)][1 - \cos(\nu)] = \left[1 - \frac{\ell_o^{(2,4)}}{\ell}\right][1 - \cos(\nu)]$$

$$\frac{\ell_o^{(2,4)}}{\ell} (5 + 4 \cos(\nu)] = 1 \tag{8.11}$$

$$\frac{\ell_o^{(2,4)}}{\ell} = \frac{w_o^{(2,4)}}{w} = [5 + 4 \cos(\nu)]^{-1}$$

$$t_4 = t_2 = 4 \frac{w_o^{(2,4)}}{c} \sin(\nu) = 4 \frac{w}{c} \frac{\sin(\nu)}{5 + 4 \cos(\nu)}$$

Note that

$$\frac{w_o^{(2,4)}}{w} \sim \frac{1}{9} \quad \text{as } \nu \to 0$$

$$t_4 = t_2 \sim \frac{4}{9} \frac{w}{c} \nu \quad \text{as } \nu \to 0 \tag{8.12}$$

so that the clear time becomes small for small ν establishing some practical limit to the allowable range of ν in the experiment.

As illustrated in fig. 8.1 we can also have a ground plane placed behind the wire parllel to it and separated from the wire by a distance h. Also indicated is the appropriate image ground plane. In general this ground plane is not of infinite width and so the incident wave can be scattered from the ground-plane edge. This suggests that the ground plane be sufficiently wide that this scattering does not reach the observer before the scattering from the wire ends and image wire discussed previously.

Having selected an appropriate observer position on the wire for selected values of ν then measurements can be made. Measurement of the current should not be too difficult if the wire is really a coax or other kind of shielded cable with sufficient shielding to conduct the signal from the sensor through the "bottom" plate to the recording instruments. By varying the intensity of the incident pulse one should observe the differences between a case of no corona (low amplitude) and of significant corona (high amplitude). Note that the current model is a transmission-line model, which is itself an approximation, particularly for the case of the wire in free space (no ground plane). Such an approximation should be allowed for when considering the effect of the corona.

IX. Summary

We now have a simple result for the effect of corona on the response of an infinite wire (in free space or parallel to a ground plane) to an incident transient plane wave. Essentially the corona reduces the charge per unit length and current once a certain corona threshold Q_0' is exceeded in magnitude. For directions of incidence orthogonal to the wire axis there is no corona. For directions of incidence nearly parallel to the wire the corona can prevent the response from becoming very large (for the wire in free space).

Since the present theory is based on a simplified model of the corona it would be good to compare these results with more detailed calculations involving various air chemistry parameters. In addition, one might compare the present results to the result of appropriate experiments involving the assumed incident-wave and geometric conditions. By this means one could see how well the model works for various regimes of times, amplitudes, etc. In addition, one could determine what choice is best made for the breakdown electric field E_b.

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