Interaction Notes

Note 456

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Norm Limiters Combined with Filters

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Abstract

In the general formalism of electromagnetic topology the propagation through and reflection from subshield penetrations of signals is important. This note considers the characteristics of linear filters and nonlinear devices idealized as norm limiters which can be used at such penetrations. Both frequency-domain and time-domain concepts are employed to limit undesirable signal penetration and reflection.

I. Introduction

In the general development of electromagnetic topology in quantitative form, the volumes (layers, sublayers) and surfaces (shields, subshields) are assigned scattering matrices to transport the signal vectors (N-waves) through these entities. Under suitable assumptions concerning the effective attenuation of signals through the subshields and effective termination of the signals incident on the subshield penetrations, one obtains the good-shielding approximation in which the transport of signals through the system is written as a product of matrix blocks (in frequency domain) [2,8].

Norms have been introduced to scalarize the matrices and vectors in a bounding sense. These have been used in frequency domain [2,3,4,6,7,8] in the context of the BLT equation for transmission line networks, the good-shielding approximation, and general linear-system bounds. The use of norms has been extended to time domain for both functions (waveforms) and convolution operators [5,9].

Mostly norms have been associated with linear problems, but with the use of norms of time-domain entities one would like to extend their use to situations involving nonlinearities. In one paper this is begun by introducing the concept of linearity to failure [5]. In another the restriction of a nonlinear element to being passive allowed one to obtain bounds on the system response in a 2-norm sense in both frequency and time domains [7].

It is common practice to design certain kinds of nonlinear elements, such as diodes, spark gaps, etc., into systems to limit the passage of transient signals. Considering norms in a linear-system context applies to such things as filters to attenuate signals in frequency and time domains. One would like to incorporate such nonlinear protection devices into the norm context in some way so that both linear and nonlinear devices could be evaluated on some common basis, and perhaps combined in useful ways.

II. Concept of a Norm Limiter

Let us now introduce the concept of a special kind of nonlinear device that will be called a norm limiter. In essence this device will limit one or more norms of a waveform transmitted past this device. This can be written as

$$||V_c(t)|| \leq \Lambda \tag{2.1}$$

where the subscript c reminds us that the waveform $V_c(t)$ is a combined voltage of the form

$$V_c(t) = V(t) \pm R\ I(t)$$
$$R > 0 \tag{2.2}$$

with the sign chosen depending on the direction of positive current flow (+ meaning propagation in the direction of positive I on conductor (terminal) with positive convention). As discussed in [6,7,8], R is chosen as a constant (frequency-independent) resistance to obtain desirable properties for the 2-norm of certain scattering matrices (bounded by 1.0).

Note that the particular norm to be used has not yet been specified. The common p-norm for time domain waveforms is

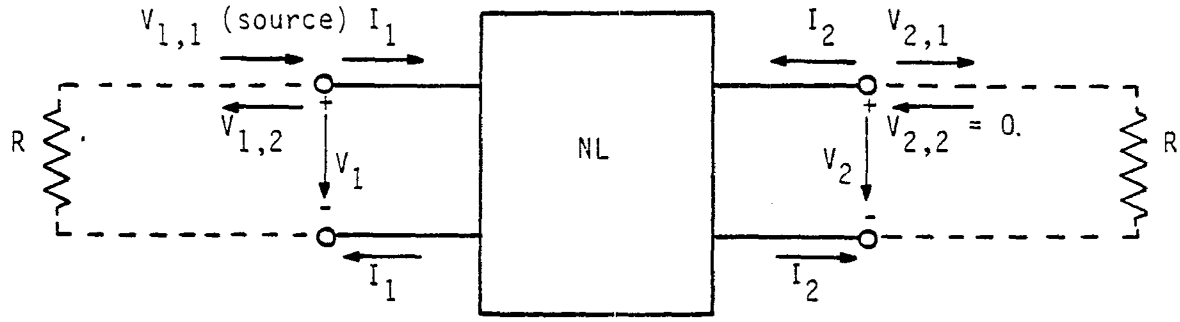
$$||V_c(t)||_p \equiv \{\int_{-\infty}^{\infty} |V_c(t)|^p\ dt\}^{\frac{1}{p}} \tag{2.3}$$

with the special case of the ∞-norm as

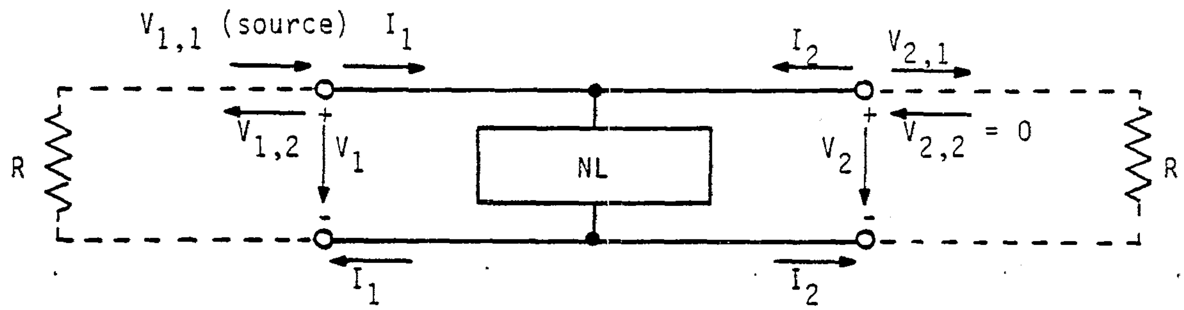
$$||V_c(t)||_p = \sup_{-\infty<t<\infty} |V_c(t)| \tag{2.4}$$

where isolated values of $V_c(t)$ are excluded from consideration [9]. In (2.1), since more than one type of norm may be under consideration, the subscript on the norm symbol $||\ \ ||$ can also be applied to the norm limit Λ.

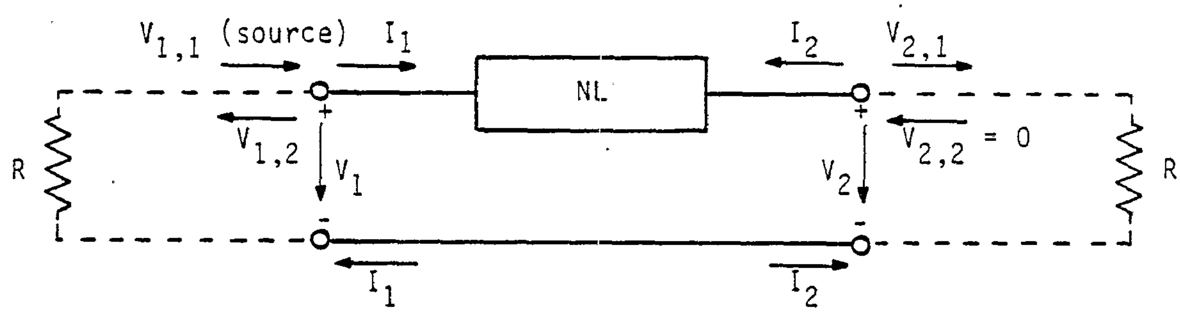
Conceptually let us represent our norm limiter (symbol NL) as a two-port network as in fig. 2.1A. With voltage and current conventions as indicated the waves are constructed as



A. Two-Port Network



B. Parallel Norm Limiter



C. Series Norm Limiter

Fig. 2.1. Types of norm limiters

$$
\begin{aligned}
V_{1,1}(t) &= V_1(t) + R\ I_1(t) \\
V_{1,2}(t) &= V_1(t) - R\ I_1(t) \\
V_{2,1}(t) &= V_2(t) - R\ I_2(t) \\
V_{2,2}(t) &= V_2(t) + R\ I_2(t)
\end{aligned}
\tag{2.5}
$$

Consistent with previous papers the positive wave direction is in the direction of assumed signal propagation into the system (to the right in fig. 2.1).

Note the symbolic inclusion of termination resistors of value R on both sides of the network. This is so that no reflections from the norm limiter on either side can reflect again back to the norm limiter. If this were not the case then the nonlinear character of the norm limiter could make matters much more complicated. So for present purposes let us assume that effective impedances, at least approximately R, are present on both sides of the norm limiter and used to define its properties. Now we have

$$V_{2,2} = 0 \tag{2.6}$$

but we assume for the (orginal) wave incident on the norm limiter

$$V_{1,1} \neq 0 \tag{2.7}$$

which can be modeled by an equivalent source (not included in fig. 2.1) if desired.

Applying (2.1) to the situation and conventions in fig. 2.1A, then the norm limiter is described by

$$||V_{2,1}(t)|| \leq \Lambda \tag{2.8}$$

If the norm limiter is passive (supplies no energy) then we have

$$||V_{2,1}(t)||_2 \leq ||V_{1,1}(t)||_2 \tag{2.9}$$

An interesting question is under what circumstances equality in (2.9) applies for the 2-norm, or for any other norm for that matter. One can envision that the NL might have the property that for "small" signals (in norm sense) the NL might completely pass the signal with no reflection ($V_{2,1}(t)$) and no attenuation, i.e., with

$$V_{2,1}(t) = V_{1,1}(t) \tag{2.10}$$

In addition the NL might have the property that it gives no change to $V_{1,1}(t)$ unless

$$||V_{1,1}(t)|| > \Lambda \tag{2.11}$$

in which case

$$||V_{2,1}(t)|| = \Lambda \tag{2.12}$$

Such a special norm limiter might be referred to as an ideal NL, with of course the particular norm specified.

If we look at the reflected wave $V_{1,2}(t)$, first note that for small signals an ideal NL has

$$\begin{aligned} V_{1,2}(t) &\equiv 0 \\ ||V_{1,2}(t)|| &\equiv 0 \end{aligned} \tag{2.13}$$

However, for larger signals, for which the NL becomes operative and the transmitted wave is limited as in (2.12), there is a reflection with

$$\begin{aligned} V_{1,2}(t) &\not\equiv 0 \\ ||V_{1,2}(t)|| &= \Theta > 0 \end{aligned} \tag{2.14}$$

If the NL is passive then we have

$$||V_{1,2}(t)||_2 \leq ||V_{1,1}(t)||_2 \tag{2.15}$$

and more generally, including the energy of both transmitted and reflected waves,

$$\begin{aligned} ||V_{2,1}(t)||_2^2 + ||V_{1,2}(t)||_2^2 &\leq ||V_{1,1}(t)||_2^2 \\ \Theta_2^2 + \Lambda_2^2 &\leq ||V_{1,1}(t)||_2^2 \end{aligned} \tag{2.16}$$

Now an NL can take various forms. Figure 2.1B shows a parallel NL which might limit the voltage ($V_1 = V_2$) across it. Similarly fig. 2.1C shows a series NL which might limit the current ($I_1 = I_2$) through it. However, this is only a small sample of the possibilities. Note that at least in these two forms the NL is bilateral, i.e., signals incident from the right are also transmitted to the left with a norm limit Λ; this kind of NL is symmetrical with respect to "forward" and "reverse" directions.

III. Some Kinds of Norm Limiters

Considering now the physical realizability of norm limiters, there are various simple idealized networks that one might consider.

A. Voltage Limiter

Corresponding to the parallel norm limiter in fig. 2.1B one might design the NL to limit the voltage (V_2 and hence V_1). This can be accomplished in various ways. One simple way is a voltage clamp as illustrated in fig. 3.1A. In this case

$$\begin{aligned} V_1 &= V_2 \\ |V_2| &\leq V_0 \end{aligned} \tag{3.1}$$

assuming ideal diodes and voltage sources. Note the symmetry of the network so that the voltage is limited to $\pm V_0$. This network need not be symmetrical: all that is required is that V_2 is limited both + and - with the largest magnitude defining the limit.

Now it is not V_2 exactly which is the transmitted wave, but

$$V_{2,1}(t) = V_2(t) - RI_2(t) = 2V_2(t) \tag{3.2}$$

so that our limiter is characterized by

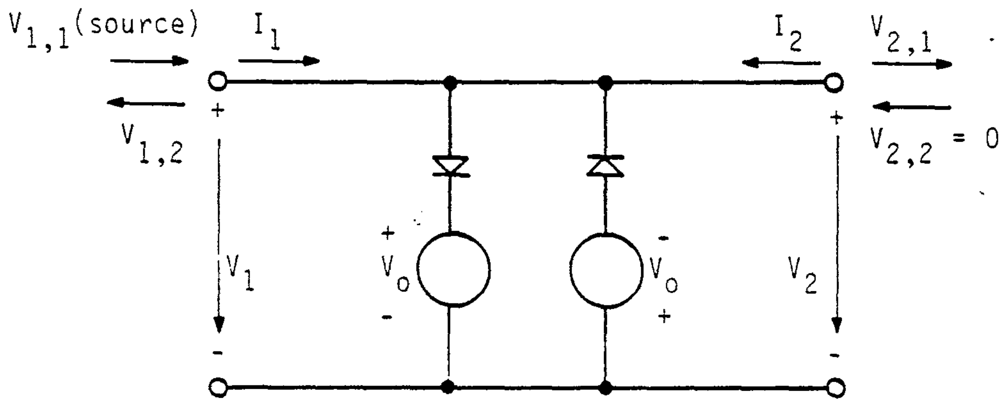
$$||V_{2,1}(t)||_\infty \leq 2V_0 = \Lambda \tag{3.3}$$

Then $2V_0$ is the proper number to use.

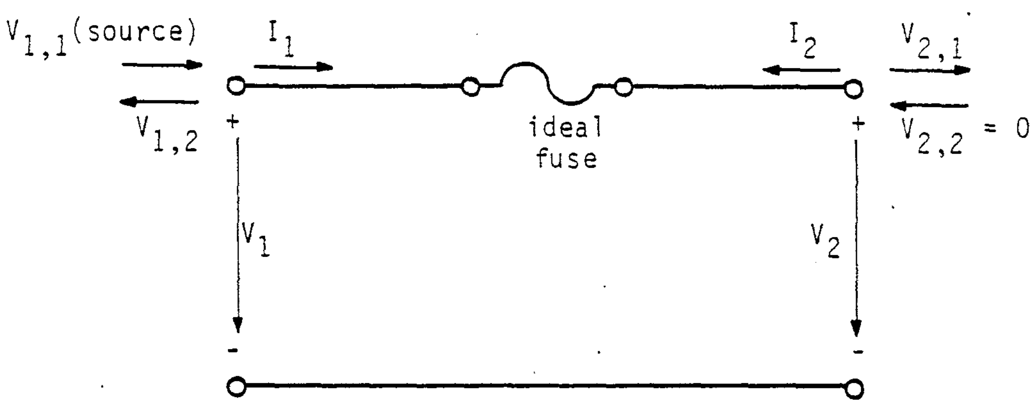
In more general terms this ideal voltage limiter is characterized by

$$V_{2,1}(t) \begin{cases} = V_{1,1}(t) \text{ for } ||V_{1,1}(t)||_\infty \leq \Lambda \\ = \Lambda \text{ for } ||V_{1,1}(t)||_\infty \geq \Lambda \end{cases}$$

$$V_{1,2}(t) \begin{cases} = 0 \text{ for } ||V_{1,1}(t)||_\infty \leq \Lambda \\ = \pm|V_{1,1}(t) - \Lambda| \text{ for } ||V_{1,1}(t)||_\infty \geq \Lambda \end{cases}$$



A. Parallel ∞-Norm Limiter



B. Series 2-Norm Limiter

Fig. 3.1. Some simple norm limiters

$$||V_{2,1}(t)||_\infty \begin{cases} = ||V_{1,1}(t)||_\infty \text{ for } ||V_{1,1}(t)||_\infty < \Lambda \\ = \Lambda \text{ for } ||V_{1,1}(t)||_\infty > \Lambda \end{cases} \tag{3.4}$$

$$||V_{1,2}(t)||_\infty \begin{cases} = 0 \text{ for } ||V_{1,1}(t)||_\infty < \Lambda \\ = ||V_{1,1}(t)||_\infty - \Lambda \text{ for } ||V_{1,1}(t)||_\infty > \Lambda \end{cases}$$

Thus, while the transmitted wave has ∞-norm limited by Λ, the reflected wave has ∞-norm limited by $||V_{1,1}(t)||_\infty - \Lambda$ if this is positive.

The scheme in fig. 3.1A is not the only way to achieve such a voltage limiter. A spark gap firing at $\pm V_0$ achieves the same ∞-norm. Of course the spark gap should be ideal in the sense that any delay in firing once $\pm V_0$ is reached is negligible, and the arc resistance is negligible. While a spark gap can achieve an ∞-norm, it does not do the same thing to the waveform as a clamp. After the spark gap fires, then $|V_2|$ is reduced to $<< V_0$ until the incident wave ($V_{1,1}(t)$) is small enough that the arc is quenched and the gap recovers.

There are other norms of $V_{2,1}(t)$ that one might wish to achieve via a parallel norm limiter. However, such an NL may be more complex than the above.

B. Current Limiter

Corresponding to the series norm limiter in fig. 2.1C one might design the NL to limit the current (I_2 and hence I_1) in some norm sense. As in fig. 3.1B this might be an ideal fuse for which

$$I_1 = -I_2$$
$$||I_2(t)||_2 < \tau \tag{3.5}$$

Here τ has units $As^{1/2}$. The square of this 2-norm times a resistance (say of the fuse) is an energy, and may represent the energy deposited in the fuse (for a sufficiently short pulse). (Under other conditions the fuse might be designed for the ∞-norm of the current.)

Now it is not I_2 which is the transmitted wave, but

$$V_{2,1}(t) = V_2(t) - RI_2(t) = 2RI_2(t) \tag{3.6}$$

so that the NL is characterized by

$$||V_{2,1}(t)||_2 = 2R||I_2(t)||_2 \leq 2R\tau = \Lambda \tag{3.7}$$

Then $2R\tau$ is the proper number to use. Then this ideal fuse is characterized by

$$V_{2,1}(t) = V_{1,1}(t) \text{ for } ||V_{1,1}(t)||_2 \leq \Lambda$$

$$V_{1,2}(t) = 0 \text{ for } ||V_{1,1}(t)||_2 \leq \Lambda$$

$$||V_{2,1}(t)||_2 = \begin{cases} ||V_{1,1}(t)||_2 \text{ for } ||V_{1,1}(t)||_2 \leq \Lambda \\ \\ \Lambda \text{ for } ||V_{1,1}(t)||_2 \geq \Lambda \end{cases} \tag{3.8}$$

$$||V_{1,2}(t)||_2 = \begin{cases} 0 \text{ for } ||V_{1,1}(t)||_2 \leq \Lambda \\ \\ [||V_{1,1}(t)||_2^2 - \Lambda^2]^{1/2} \text{ for } ||V_{1,1}(t)||_2 \geq \Lambda \end{cases}$$

Thus, while the transmitted wave has 2-norm limited by Λ, the reflected wave has 2-norm limited by $[||V_{2,1}(t)||_2^2 - \Lambda^2]^{1/2}$ if this is positive.

The above example is only illustrative. There may be various NL's realizable in the series sense.

IV. Combining Norm Limiters with Filters in Electromagnetic Topology

Let us now look at the possible role of norm limiters in the context of EM topology. As indicated in fig. 4.1 let us consider a simple form of EM topology only divided down to sublayers and subshields [2,8]. The equivalent graph (interaction sequence diagram) is a graph (bipartite) which is a very simple form of graph known as a tree.

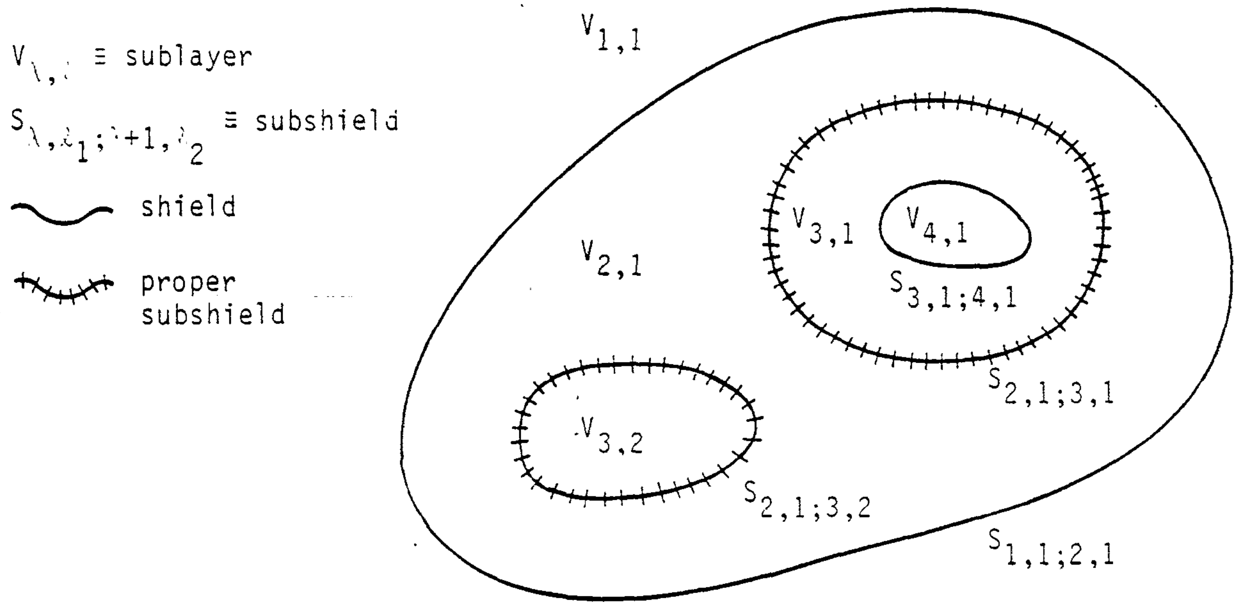
Now consider what happens at a subshield. The interaction sequence diagram in fig. 4.1B shows all the signals incident on and passing through a subshield by two edges, one on each side at each subshield vertex in the graph. This allows for numerous signals passing through various kinds of penetrations (conductor penetrations, aperture penetrations, and diffusion penetration). Another paper [4] discusses measurement techniques for the case of conductor and aperture penetrations.

Let us then concentrate on the canonical problem of a single conductor penetrating a subshield as indicated in fig. 4.2. Note that the norm limiter and filters (on both sides) are considered to be at and part of the subshield. The geometry of this penetration (such as coaxial) is significant, but not part of the present considerations.

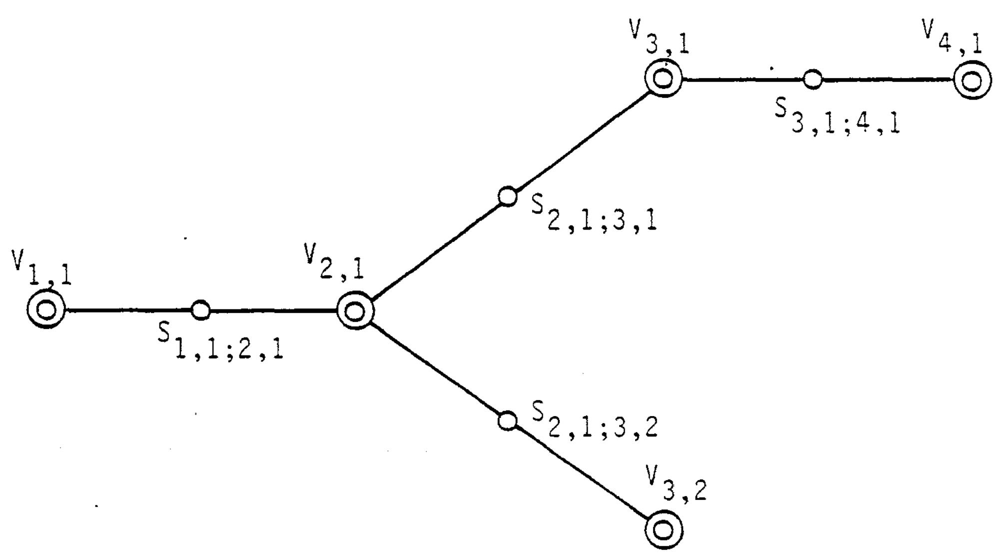
As discussed in section 2, to characterize the NL there is assumed to be a resistance R as the impedance "seen" by the NL on both sides. This will be an important consideration in the design of the filters, to be discussed later.

Since for small signals the NL is assumed to have no effect, then the filters are associated with any signal attenuation on passing through the subshield. Such attenuation may be minimal for signals in the pass band of the filter, while strongly rejecting other frequencies. If, in time domain, a signal passes through a filter with a norm exceeding that allowed by the NL, then the NL reduces the norm to the allowed value. Of course, the waveform and associated frequency content may be considerably altered in this case, but that is an inherent characteristic of such nonlinear devices.

Considering only the filters, then one can compute the transfer function (perhaps in a bound or norm sense) through the system subshields as a function of frequency. Alternatively one can compute the norms of filters for



A. Volume/Surface Topology



B. Interaction Sequence Diagram

Fig. 4.1. Sublayers and subshields in hierarchical topology

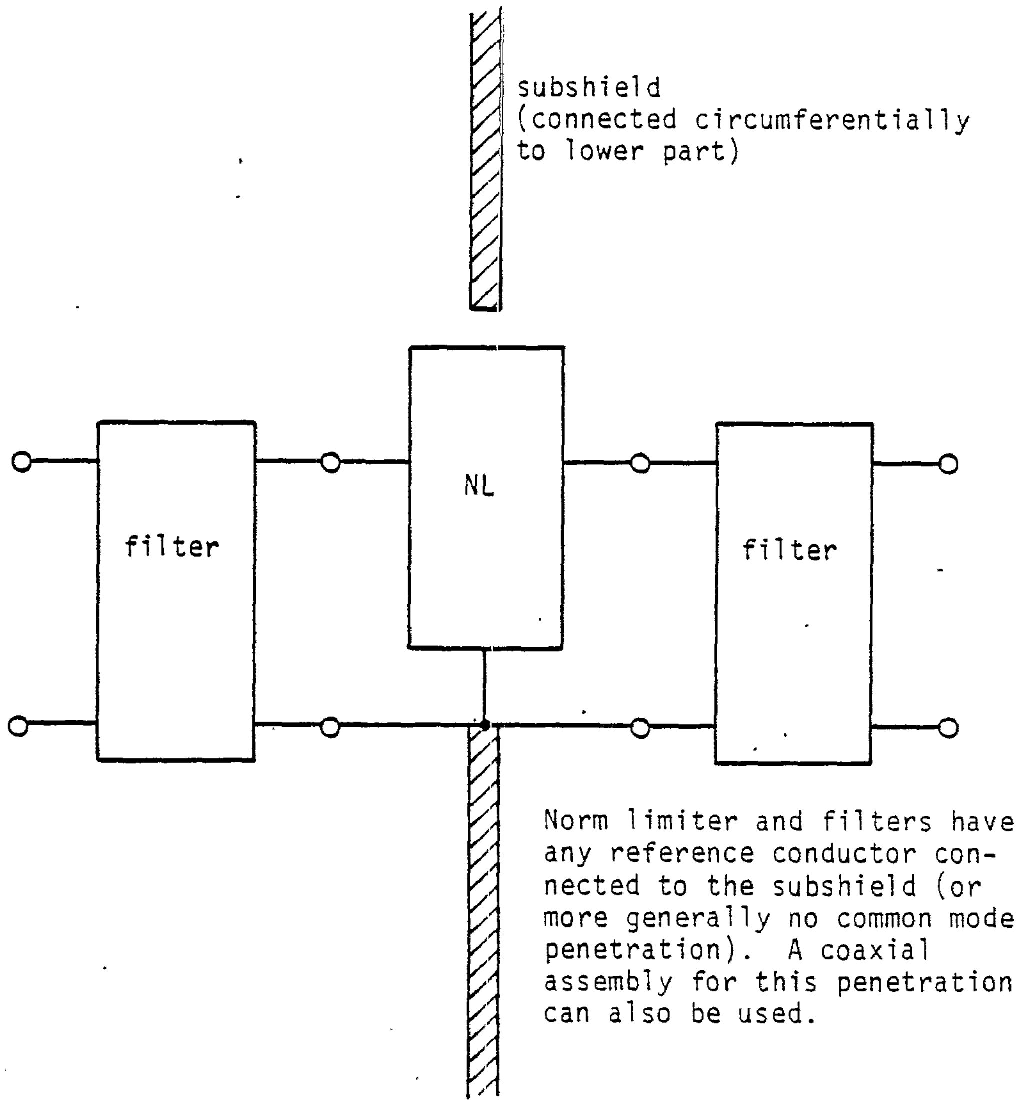


Fig. 4.2. Combined norm limiter and filters

transmitting time-domain waveforms. Unfortunately, in this sense the filters are rather good transmitters (as will be seen later). However, the NLs can compensate for this time-domain problem.

The idea, then, is this. Consider a cascaded set of filters with one or more NLs as indicated in fig. 4.3. Begin with some signal designated as a source S (in time-domain-norm sense) either outside the system, or incident on some subshield within the system. On going through a filter the bound is "reduced" to ST_1. On propagating through the NL the signal bound is either ST_1 or Λ (the NL characteristic), whichever is <u>less.</u> On passing another filter the bound is multiplied by the filter norm T_2, etc.

This is a somewhat simplified view involving a single signal propagation path. In actuality all the signals passing through a subshield are to be considered and bounded on the transmitted side.

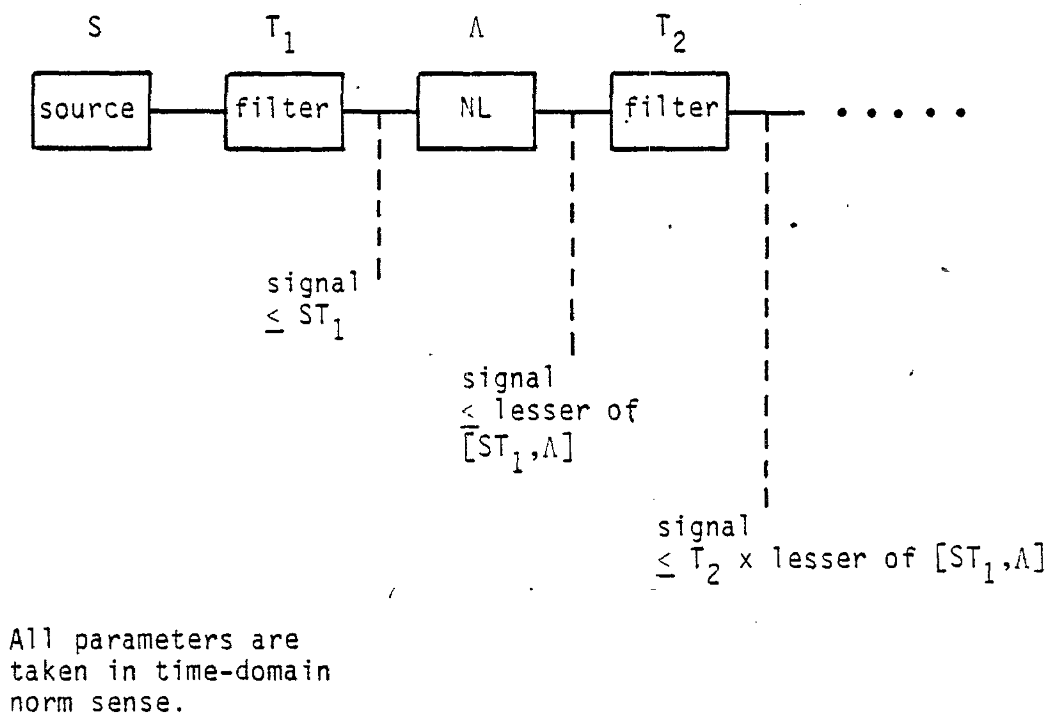


Fig. 4.3. Cascaded filters with an NL

V. Constant-Resistance Filters

For our special type of filters the impedance looking in should be R for all frequencies. The input impedance in general depends on the load impedance on the other side of the filter. Since a constant resistance R appears in our scattering variables let us assume that the load impedance is R. Since we would like the filter to terminate signals in the volume in a resistance R for purposes of the good-shielding approximation [8], let us then consider what is known as a constant resistance filter which has input impedance R from each side when the other side is terminated in R [10].

Since our filter will also be reciprocal ($\tilde{Z}_{1,2} = \tilde{Z}_{2,1}$) which is a kind of electrical symmetry, let us consider a network with physical symmetry as well [10]. As illustrated in fig. 5.1 we begin with a physically and electrically symmetrical network with conventions as in fig. 5.1A. Here we have

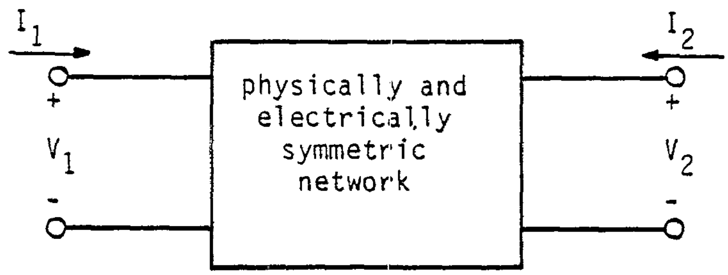
$$\begin{aligned}
(\tilde{V}_n) &= (\tilde{Z}_{n,m}) \quad (\tilde{I}_n) \\
(\tilde{I}_n) &= (\tilde{Y}_{n,m}) \quad (\tilde{V}_n) \\
(\tilde{Y}_{n,m}) &= (\tilde{Z}_{n,m})^{-1} \\
\tilde{Z}_{1,1} &= \tilde{Z}_{2,2} \text{ (input impedances, symmetry)} \\
\tilde{Z}_{1,2} &= \tilde{Z}_{2,1} \text{ (transfer impedances, reciprocity)} \\
\tilde{Y}_{1,1} &= \tilde{Y}_{2,2} \text{ (input admittances, symmetry)} \\
\tilde{Y}_{1,2} &= \tilde{Y}_{2,1} \text{ (transfer admittances, reciprocity)}
\end{aligned} \tag{5.1}$$

where a tilde ~ indicates the Laplace transform (two sided), making these variables functions of the complex frequency s.

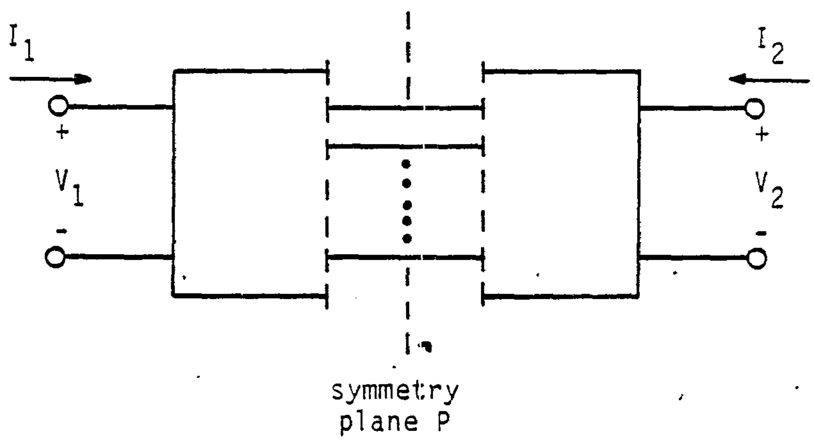
Such a symmetrical network is bisected along a symmetry plane P as indicated in fig. 5.1B by what is known as Bartlett's bisection theorem [10]. In symmetric excitation defined by

$$V_1 \equiv V_2 \quad , \quad I_1 \equiv I_2 \tag{5.2}$$

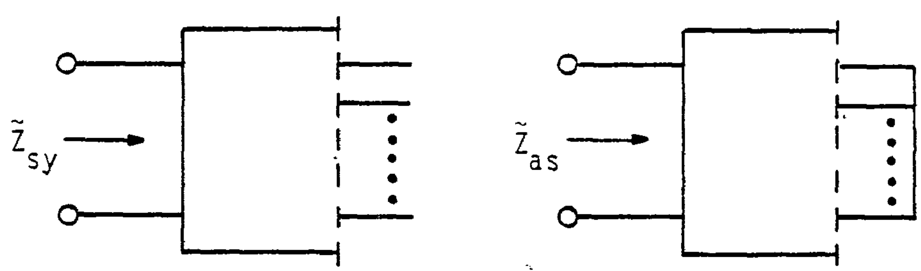
there are no currents in branches crossing P. Equivalently such branches crossing P are cut there. This gives symmetric input impedance and admittance as



A. Original Symmetrical Network



B. Bisected Network



C. Equivalent Networks for Symmetric and Antinsymmetric Parts

Fig. 5.1. Decomposition of symmetrical network

$$\tilde{Z}_{sy} \equiv \frac{\tilde{V}_1}{\tilde{I}_1} = \frac{\tilde{V}_2}{\tilde{I}_2} = \tilde{Z}_{1,1} + \tilde{Z}_{1,2}$$

$$\tilde{Y}_{sy} \equiv \frac{\tilde{I}_1}{\tilde{V}_1} = \frac{\tilde{I}_2}{\tilde{V}_2} = \tilde{Y}_{1,1} + \tilde{Y}_{1,2} = \tilde{Z}_{sy}^{-1} \tag{5.3}$$

In antisymmetric excitation defined by

$$\tilde{V}_1 \equiv -\tilde{V}_2 \quad , \tilde{I}_1 \equiv -\tilde{I}_2 \tag{5.4}$$

there are no voltages between branches crossing P (or better between nodes placed in these brackets on symmetry plane P). Equivalently, P is made a shorting plane on which all branches are connected together. This gives antisymmetric input impedance and admittance as

$$\tilde{Z}_{as} \equiv \frac{\tilde{V}_1}{\tilde{I}_1} = \frac{\tilde{V}_2}{\tilde{I}_2} = \tilde{Z}_{1,1} - \tilde{Z}_{1,2}$$

$$\tilde{Y}_{as} \equiv \frac{\tilde{I}_1}{\tilde{V}_1} = \frac{\tilde{I}_2}{\tilde{V}_2} = \tilde{Y}_{1,1} - \tilde{Y}_{1,2} = \tilde{Z}_{as}^{-1} \tag{5.5}$$

Schematically these are indicated in fig. 5.1C. Note that this kind of network symmetry can be considered as a special case of an electromagnetic symmetry plane discussed in [1], in which case all the electromagnetic parameters split into symmetric and antisymmetric parts.

Reconstructing the impedance and admittance matrix elements from the symmetric and antisymmetric terms gives

$$\tilde{Z}_{1,1} = \tilde{Z}_{2,2} = \frac{1}{2}[\tilde{Z}_{sy} + \tilde{Z}_{as}]$$

$$\tilde{Z}_{1,2} = \tilde{Z}_{2,1} = \frac{1}{2}[\tilde{Z}_{sy} - \tilde{Z}_{as}]$$

$$\tilde{Y}_{1,1} = \tilde{Y}_{2,2} = \frac{1}{2}[\tilde{Y}_{sy} + \tilde{Y}_{as}] \tag{5.6}$$

$$\tilde{Y}_{1,2} = \tilde{Y}_{2,1} = \frac{1}{2}[\tilde{Y}_{sy} - \tilde{Y}_{as}]$$

Now apply the above to a constant resistance symmetrical filter shown in fig. 5.2A. For the general results (5.1) and the constant resistance condition give

$$\tilde{V}_2 = \tilde{Z}_{1,2}\tilde{I}_1 + \tilde{Z}_{1,1}\tilde{I}_2 = -R\tilde{I}_2$$

$$\tilde{T} \equiv \frac{\tilde{V}_2}{\tilde{V}_1} = -\frac{\tilde{I}_2}{\tilde{I}_1} = \tilde{Z}_{1,2}[R + \tilde{Z}_{1,1}]^{-1} \tag{5.7}$$

where $\tilde{T}$ is now the transfer function of the network. Also from (5.1) and the constant resistance condition we have

$$\tilde{V}_1 = \tilde{Z}_{1,1}\tilde{I}_1 + \tilde{Z}_{1,2}\tilde{I}_2 = R\tilde{I}_1$$

$$= \tilde{Z}_{1,1}\tilde{I}_1 - \tilde{Z}_{1,2}\tilde{T}\tilde{I}_1$$

$$R = \tilde{Z}_{1,1} - \tilde{Z}_{1,2}\tilde{T} = \tilde{Z}_{1,1} - \tilde{Z}^2_{1,2}[R + \tilde{Z}_{1,1}]^{-1} \tag{5.8}$$

$$R^2 = \tilde{Z}^2_{1,1} - \tilde{Z}^2_{1,2} = [\tilde{Z}_{1,1} + \tilde{Z}_{1,2}][\tilde{Z}_{1,1} - \tilde{Z}_{1,2}]$$

$$= \tilde{Z}_{sy}\tilde{Z}_{as}$$

This last condition is put in symmetrical form as

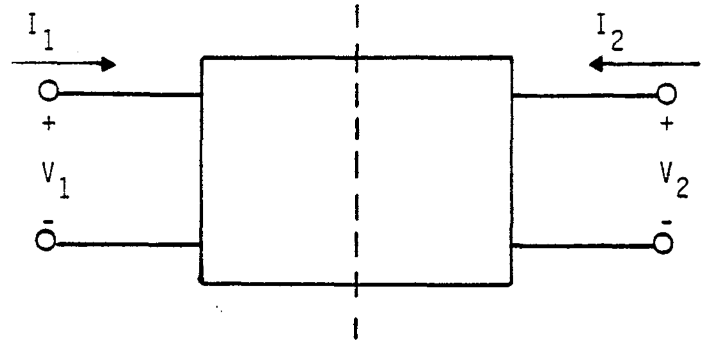
$$\frac{\tilde{Z}_{sy}}{R}\frac{\tilde{Z}_{as}}{R} = 1 \tag{5.9}$$

showing the reciprocal relation of these impedances when normalized by R. Returning to the transfer function we have

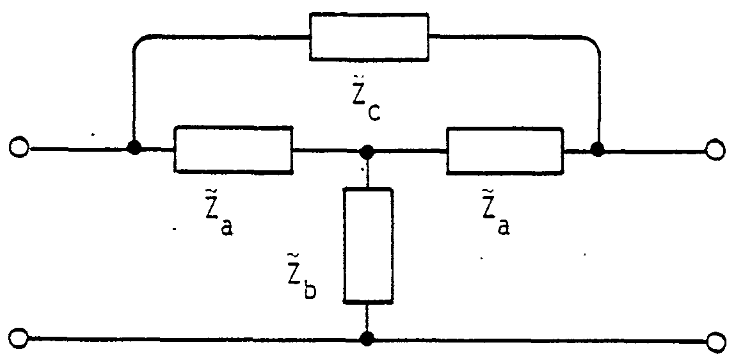
$$\tilde{T} = [\tilde{Z}_{sy} - \tilde{Z}_{as}][\tilde{Z}_{sy} + 2R + \tilde{Z}_{as}]^{-1}$$

$$= [\frac{\tilde{Z}_{sy}}{\tilde{Z}_{as}} - 1][\frac{\tilde{Z}_{sy}}{\tilde{Z}_{as}} + 2\frac{R}{\tilde{Z}_{as}} + 1]^{-1}$$

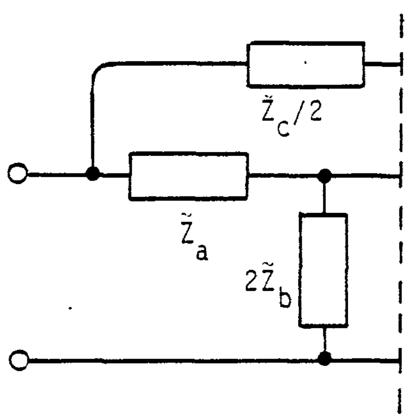
$$= [(\frac{\tilde{Z}_{sy}}{R})^2 - 1][(\frac{\tilde{Z}_{sy}}{R})^2 + 2\frac{\tilde{Z}_{sy}}{R} + 1]^{-1} \tag{5.10}$$



A. Constant-Resistance Symmetrical Network



B. Symmetrical Bridged T



C. Half Section of Symmetrical Bridged T

Fig. 5.2. Constant-resistance bridged-T filter

Removing a common factor gives

$$\tilde{\Gamma} = [\frac{\tilde{Z}_{sy}}{R} - 1][\frac{\tilde{Z}_{sy}}{R} + 1]^{-1}$$

$$= [1 - \frac{\tilde{Z}_{as}}{R}][1 + \frac{\tilde{Z}_{as}}{R}]^{-1} \qquad (5.11)$$

These results can just as easily be expressed in terms of the corresponding admittances.

Now apply these results to a symmetrical bridged-T network in fig. 5.2B, a common realization of the symmetrical network discussed above [10]. Fig. 5.2C shows the decomposition of this network according to its symmetry plane. This gives

$$\tilde{Z}_{sy} = \tilde{Z}_a + 2\tilde{Z}_b$$

$$\tilde{Z}_{as} = \tilde{Z}_a // \frac{\tilde{Z}_c}{2} = \frac{\tilde{Z}_a\tilde{Z}_c}{2\tilde{Z}_a + \tilde{Z}_c} \qquad (5.12)$$

Imposing (5.9) for a constant-resistance filter gives

$$\frac{\tilde{Z}_{sy}}{R}\frac{\tilde{Z}_{as}}{R} = 1$$

$$= \frac{\tilde{Z}_a\tilde{Z}_c}{R^2}\frac{\tilde{Z}_a + 2\tilde{Z}_b}{2\tilde{Z}_a + \tilde{Z}_c}$$

$$= \frac{\tilde{Z}_b\tilde{Z}_c}{R^2}\frac{2 + \frac{\tilde{Z}_a}{\tilde{Z}_b}}{2 + \frac{\tilde{Z}_c}{\tilde{Z}_a}} \qquad (5.13)$$

A common solution of this has [10]

$$\frac{\tilde{Z}_a}{R} = 1$$

$$\frac{\tilde{Z}_b}{R}\,\frac{\tilde{Z}_c}{R} = 1 \tag{5.14}$$

For this solution the transfer function is

$$\tilde{T} = [1 + \frac{R}{\tilde{Z}_b}]^{-1} = [1 + \frac{\tilde{Z}_c}{R}]^{-1} \tag{5.15}$$

With $\tilde{Z}_b$ and $\tilde{Z}_c$ as p.r. (positive real) functions, i.e.,

$$\begin{aligned} \mathrm{Re}[\tilde{Z}_b(s)] &> 0 \qquad \text{for } \mathrm{Re}[s] > 0 \\ \mathrm{Re}[\tilde{Z}_c(s)] &> 0 \qquad \text{for } \mathrm{Re}[s] > 0 \end{aligned} \tag{5.16}$$

then the transfer function is a b.r. (bounded real) function, i.e.,

$$|\mathrm{Re}[\tilde{T}(s)]| < 1 \qquad \text{for } \mathrm{Re}[s] < 0 \tag{5.17}$$

and more generally in this case

$$|\tilde{T}(s)| < 1 \qquad \text{for } \mathrm{Re}[s] < 0 \tag{5.18}$$

VI. Time-Domain Norms for Filters

A recent paper addressed norms of convolution operators [9]. In an electrical-engineering sense a filter can be considered as a convolution operator in time domain. One can define the properties of a linear, time-invariant filter in complex-frequency domain by a transfer function as

$$\tilde{V}_{out}(s) = \tilde{T}(s)\tilde{V}_{in}(s)$$

$$V_{in} \equiv \text{wave incident on filter (input)} \tag{6.1}$$

$$V_{out} \equiv \text{wave transmitted through filter (output)}$$

In the present context the waves are normalized by a resistance R as in the previous sections, and the filters will be of the constant-resistance type so that there is no wave reflected from the input.

In time domain (6.1) is

$$V_{out}(t) = T(t) \circ V_{in}(t)$$

$$= \int_0^t T(t - t')V(t')dt'.$$

$$\circ \equiv \text{convolution} \tag{6.2}$$

The filter is now characterized by a transfer operator $T(t)\circ$. As indicated above, this is not a function of time but an integral operator with integration over t' involving T(t-t'). Note in (6.2) that integration is taken to begin at t'=0 which is a way of stating that $T(t)\circ$ is assumed to be a causal operator (no V_{out} before V_{in}). In our later examples $T(t)\circ$ is also taken to be passive (provides no energy to V_{out}, but may absorb energy from V_{in}).

Using norm concepts (6.2) provides the bound

$$||V_{out}(t)|| \leq ||T(t)\circ|| \; ||V_{in}(t)|| \tag{6.3}$$

where $||\;||$ indicates any of various possible norms. For the p-norm [9] gives the general result

$$||T(t)\circ||_p \leq ||T(t)||_1 \tag{6.4}$$

showing how the p-norm of a convolution operator can be bounded by the 1-norm of the corresponding function.

Special cases of the p-norm of such a convolution operator are [9]

$$||T(t)\circ||_1 = ||T(t)||_1$$

$$||T(t)\circ||_2 = |\tilde{T}(j\omega)|_{max} \leq ||T(t)||_1 \tag{6.5}$$

$$||T(t)\circ||_\infty = ||T(t)||_1$$

which gives equalities instead of bounds. Note that if our filter is passive, then if V_{in} is a CW signal then for all $s = j\omega$ (ω real)

$$0 \leq |\tilde{T}(j\omega)| \leq 1 \tag{6.6}$$

since the output and input waveforms are normalized by the same resistance R. Applying this to the 2-norm in time domain we have

$$0 \leq ||T(t)\circ||_2 \leq 1 \tag{6.7}$$

For the 2-norm in (6.3) this gives

$$||V_{out}(t)||_2 \leq ||V_{in}(t)||_2 \tag{6.8}$$

which is a statement of filter passivity in time domain.

VII. Frequency-Independent Attenuators

A special case of a constant-resistance filter is one with a frequency-independent transfer function, i.e.,

$$\tilde{T}(s) = T \qquad 0 \leq T \leq 1 \tag{7.1}$$

so that

$$T(t) \circ = T\delta(t) \circ \qquad \delta(t) \equiv \text{delta function} \qquad \int_{-\varepsilon}^{\varepsilon} \delta(t)dt = 1 \quad , \quad \varepsilon > 0 \tag{7.2}$$

Now for any operator norm, the norm of a delta-function operator is

$$||\delta(t)\circ|| \equiv \sup_{f(t)\equiv 0} \frac{||\delta(t) \circ f(t)||}{||f(t)||} = \sup_{f(t)\equiv 0} \frac{||f(t)||}{||f(t)||} = 1 \tag{7.3}$$

with appropriate smoothness requirements on $f(t) \ni$

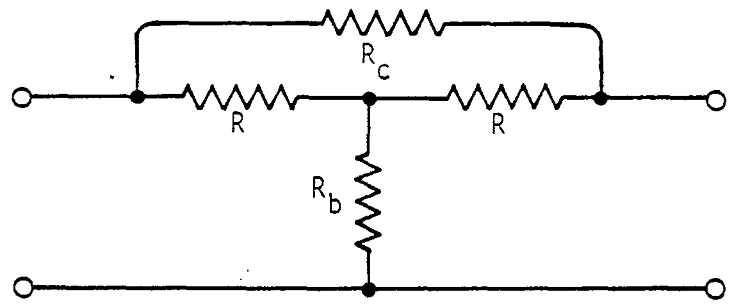
$$\delta(t) \circ f(t) = f(t) \tag{7.4}$$

Then for frequency-independent constant-resistance filter we have

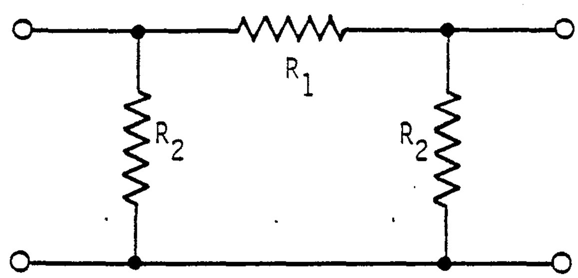
$$||T(t)\circ|| = ||T\delta(t)\circ|| = |T| \; ||\delta(t)\circ|| = |T| \tag{7.5}$$

This applies to the p-norm as well as others.

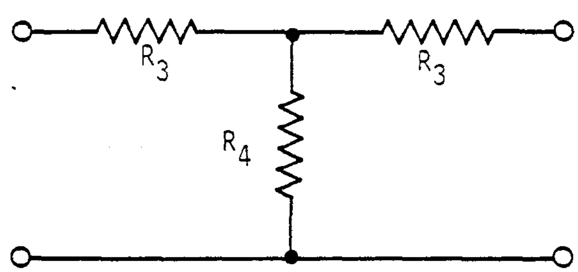
There are various realizations of symmetrical, constant-resistance, frequency-independent attenuators. As indicated in fig. 7.1A, there is a symmetrical bridged-T form as in section 5 with



A. Bridged T



B. Pi



C. Tee

Fig. 7.1. Symmetrical, constant-resistance, frequency-independent attenuators

$$\frac{R_b}{R}\,\frac{R_c}{R} = 1 \quad , \quad R_b > 0 \, , \quad R_c > 0$$

$$T = [1 + \frac{R}{R_b}]^{-1} = [1 + \frac{R_c}{R}]^{-1} \tag{7.6}$$

Another common form is the symmetrical pi network in fig. 7.1B. Terminating one side in R and requiring the impedance at the other side to be R gives for symmetric and antisymmetric parts

$$\begin{aligned} \tilde{Z}_{sy} &= R_2 \\ \tilde{Z}_{as} &= R_2 // (\frac{R_1}{2}) = [\frac{1}{R_2} + \frac{2}{R_1}]^{-1} \end{aligned} \tag{7.7}$$

From (5.9) we have

$$\begin{gathered} 1 = \frac{\tilde{Z}_{sy}}{R}\,\frac{\tilde{Z}_{as}}{R} = \frac{R_2}{2}\,[\frac{1}{R_2} + \frac{2R}{R_1}]^{-1} = [\frac{R^2}{R_2^2} + \frac{2R^2}{R_1 R_2}]^{-1} \\ \frac{1}{R^2} = \frac{1}{R_1^2} + \frac{2}{R_1 R_2} \end{gathered} \tag{7.8}$$

relating R_1 and R_2. From (5.11) we have

$$R = \frac{\frac{R_2}{R} - 1}{\frac{R_2}{R} + 1} \tag{7.9}$$

Yet another common form is the symmetrical tee network in fig. 7.1C. Terminating one side in R and requiring the impedance at the other side to be R gives for symmetric and antisymmetric parts

$$\begin{aligned} \tilde{Z}_{sy} &= R_3 + 2R_4 \\ \tilde{Z}_{as} &= R_3 \end{aligned} \tag{7.10}$$

From (5.9) we have

$$1 = \frac{\tilde{Z}_{sy}}{R} \frac{\tilde{Z}_{as}}{R} = \frac{R_3}{R} \frac{R_3 + 2R_4}{R} \tag{7.11}$$

$$R^2 = R_3^2 + 2R_3R_4$$

relating R_3 and R_4. From (5.11) we have

$$R = \frac{1 - \frac{R_3}{R}}{1 + \frac{R_3}{R}} \tag{7.12}$$

VIII. Canonical Constant-Resistance Bridged-T Filters

A. Low-pass Filter

Consider the simple low-pass filter shown in fig. 8.1A. The constant-resistance constraint in (5.14) gives

$$\frac{L_c}{R} = RC_b \tag{8.1}$$

The transfer function is

$$\Upsilon = [1 + sC_bR]^{-1} = [1 + \frac{sL_c}{R}]^{-1} \tag{8.2}$$

which is quite simple in form.

In time domain we have

$$T(t) = \frac{e^{-\frac{t}{RC_b}}}{RC_b} u(t)$$

$$||T(t)||_1 = \int_0^\infty |T(t)|dt = 1 \tag{8.3}$$

So for the p-norm we have

$$||T(t)\circ||_p < 1 \tag{8.4}$$

For special cases we have

$$||T(t)\circ||_1 = ||T(t)\circ||_\infty = 1$$

$$||T(t)\circ||_2 = \max_\omega |\Upsilon(j\omega)| = 1 \tag{8.5}$$

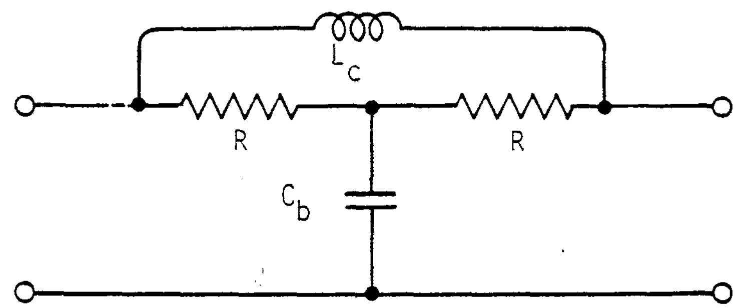
B. High-pass Filter

As in fig. 8.1B we have a high-pass filter. The constant-resistance constraint in (5.14) gives

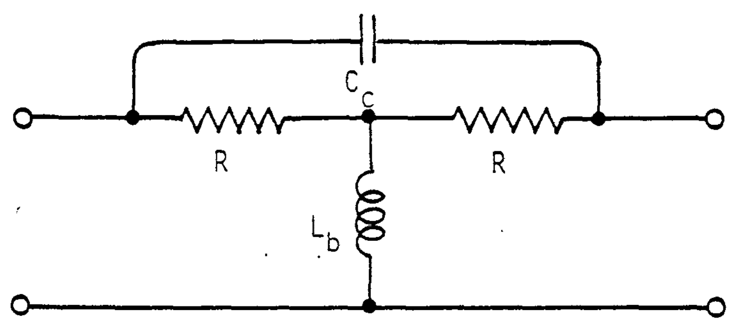
$$\frac{L_b}{R} = RC_c \tag{8.6}$$

The transfer function is

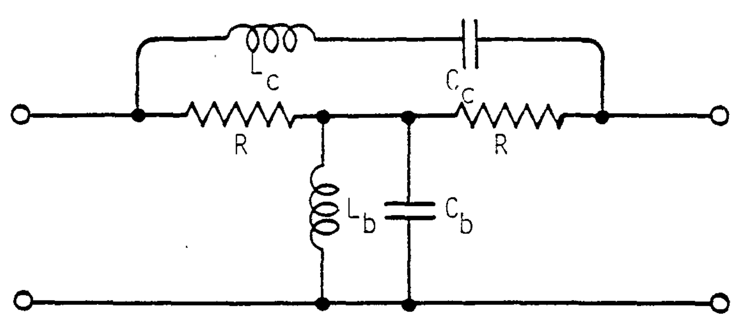
$$\Upsilon = [1 + \frac{1}{sC_cR}]^{-1} = [1 + \frac{R}{sL_b}]^{-1} \tag{8.7}$$



A. Low-pass Filter



B. High-pass Filter



C. Band-pass Filter

Fig. 8.1. Canonical constant-resistance bridged-T filter

In time domain we have

$$T(t) = \delta(t) - \frac{e^{-\frac{t}{RC_c}}}{RC_c} u(t) \tag{8.8}$$

$$||T(t)||_1 = \int_{0-}^{\infty} |T(t)|dt = \int_{0-}^{0+} \delta(t)dt + \int_{0+}^{\infty} \frac{e^{-\frac{t}{RC_c}}}{RC_c} dt$$

$$= 2$$

So for the p-norm we have

$$||T(t)\circ||_p \leq 2 \tag{8.9}$$

For special cases we have

$$||T(t)\circ||_1 = ||T(t)\circ||_\infty = 2$$

$$||T(t)\circ||_2 = \max_\omega |\Upsilon(j\omega)| = 1 \tag{8.10}$$

C. Band-pass Filter

Fig. 8.1C gives an example of a band-pass filter. The constant-resistance constraint in (5.14) gives

$$\frac{Z_b}{R} = [sRC_b + \frac{R}{SL_b}]^{-1}$$

$$\frac{Z_c}{R} = \frac{sL_c}{R} + \frac{1}{sRC_c} \tag{8.11}$$

$$\frac{L_c}{R} = RC_b \quad , \quad \frac{L_b}{R} = RC_c$$

The transfer function is

$$\Upsilon = [1 + \frac{R}{Z_b}]^{-1} = [1 + sRC_b + \frac{R}{sL_b}]^{-1}$$

$$= [1 + \frac{Z_c}{R}]^{-1} = [1 + \frac{sL_c}{R} + \frac{1}{sRC_c}]^{-1} \tag{8.12}$$

By defining

$$t_1 \equiv \frac{L_c}{R} = RC_b$$
$$t_2 \equiv RC_c = \frac{L_b}{R} \tag{8.13}$$

then

$$\Upsilon = \left[1 + st_1 + \frac{1}{st_2}\right]^{-1} = \frac{1}{t_1} \frac{s}{s^2 + \frac{s}{t_1} + \frac{1}{t_1 t_2}}$$
$$= \frac{1}{t_1} \frac{s}{(s - s_1)(s - s_2)} \tag{8.14}$$

where

$$s_{1,2} = -\frac{1}{2t_1} + \sqrt{\frac{1}{4t_1^2} - \frac{1}{t_1 t_2}} \tag{8.15}$$

Our case of interest is that of a narrowband filter for which s_1 and s_2 are near the $j\omega$ axis as

$$s_{1,2} = -\frac{1}{2t_1} \pm j\sqrt{\frac{1}{t_1 t_2} - \frac{1}{4t_1^2}} = \Omega_0 \pm j\omega_0$$
$$s_2 = s_1^*$$
$$\Omega_0 = -\frac{1}{2t_1} = -\frac{R}{2L_c} = -\frac{1}{2RC_b} \equiv \text{damping constant} \tag{8.16}$$
$$\omega_0 = \sqrt{\frac{1}{t_1 t_2} - \frac{1}{4t_1^2}} = \sqrt{\frac{1}{L_c C_c} - \frac{1}{4}\left(\frac{R}{L_c}\right)^2}$$
$$= \sqrt{\frac{1}{L_b C_b} - \frac{1}{4(RC_b)^2}} \equiv \text{center frequency (radian)}$$

For a narrow pass band one needs

$$\frac{t_2}{t_1} = \frac{R^2 C_c}{L_c} = \frac{L_b}{R^2 C_b} \ll 1 \tag{8.17}$$

or

$$\sqrt{\frac{L_c}{C_c}} \gg R \gg \sqrt{\frac{L_b}{C_b}} \tag{8.18}$$

which is effectively an impedance requirement on the reactive elements. Note that

$$\frac{\Omega_o}{\omega_o} = -\left[\frac{4t_1}{t_2} - 1\right]^{-1/2} \approx -\frac{1}{2}\left(\frac{t_2}{t_1}\right)^{1/2} = -\frac{R}{2}\sqrt{\frac{C_c}{L_c}} = -\frac{1}{2R}\sqrt{\frac{L_b}{C_b}}$$

$$\omega_o = \frac{1}{\sqrt{t_1 t_2}}\left[1 - \frac{t_2}{4t_1}\right]^{1/2} \approx \frac{1}{\sqrt{L_c C_c}} = \frac{1}{\sqrt{L_b C_c}} \tag{8.19}$$

We also have

$$|s_{1,2}| = [\Omega_o^2 + \omega_o^2]^{1/2} = \frac{1}{\sqrt{t_1 t_2}} = \frac{1}{\sqrt{L_c C_c}} = \frac{1}{\sqrt{L_b C_b}} \tag{8.20}$$

In time domain we have

$$T(t) = \frac{1}{t_1}\,\frac{s_1 e^{s_1 t} - s_2 e^{s_2 t}}{s_1 - s_2}\, u(t)$$

$$||T(t)||_1 < \frac{1}{|s_1 - s_2|\, t_1}\left\{|s_1| \int_0^\infty e^{\mathrm{Re}[s_1]t}\, dt + |s_2| \int_0^\infty e^{\mathrm{Re}[s_2]t}\, dt\right\} \tag{8.21}$$

$$= \frac{2}{|s_1 - s_2|t_1}\,\frac{|s_1|}{-\mathrm{Re}[s_1]} = \frac{\frac{1}{\sqrt{t_1 t_2}}}{\sqrt{\frac{1}{t_1 t_2} - \frac{1}{4t_1^2}}\;\frac{1}{2}}$$

$$= 2\left[1 - \frac{t_2}{4t_2}\right]^{1/2}$$

Invoking (8.17) gives

$$||T(t)||_1 \lessapprox 2 \tag{8.22}$$

For special cases we have

$$||T(t)\circ||_1 = ||T(t)\circ||_\infty \lessapprox 2$$

$$||T(t)\circ||_2 = \max_\omega |\tilde{T}(j\omega)| = 1 \tag{8.23}$$

IX. Including Loss in Pass Band of Bridged-T Filters

A. General Case

As illustrated in fig. 9.1A we have decomposed $\tilde{Z}_b$ and $\tilde{Z}_c$ as

$$\begin{aligned}
\tilde{Z}_b &= R_b // \tilde{Z}_b' \\
\tilde{Z}_c &= R_c + \tilde{Z}_c' \qquad (9.1) \\
\tilde{Z}_b' \,,\ \tilde{Z}_c' &= \text{reactive impedances}
\end{aligned}$$

Then (5.14) gives an impedance constant

$$\begin{aligned}
&[R_b // \tilde{Z}_b'][R_c + \tilde{Z}_c'] = R^2 \\
&\frac{R_c}{R} + \frac{\tilde{Z}_c'}{R} = \frac{R}{R_b} + \frac{R}{\tilde{Z}_b'}
\end{aligned} \qquad (9.2)$$

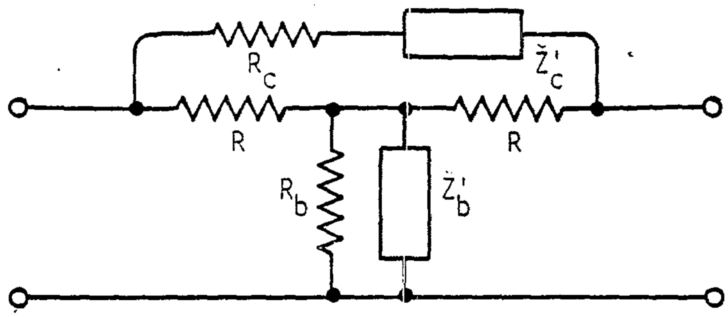
Identifying resistive and reactive parts gives

$$\begin{aligned}
&\frac{R_b}{R}\frac{R_c}{R} = 1 \\
&\frac{\tilde{Z}_b'}{R}\frac{\tilde{Z}_c'}{R} = 1
\end{aligned} \qquad (9.3)$$

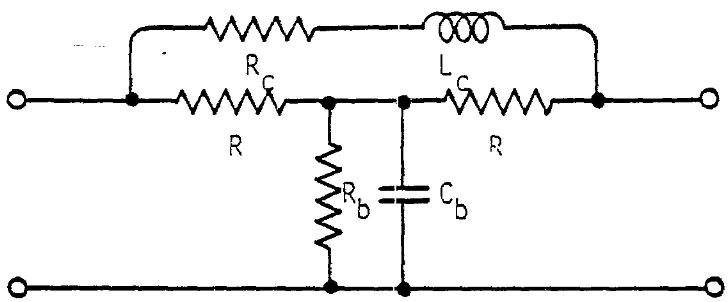
So now $\tilde{Z}_b'$ and $\tilde{Z}_c'$ are related as $\tilde{Z}_b$ and $\tilde{Z}_c$ in (5.14).

The transfer function now becomes

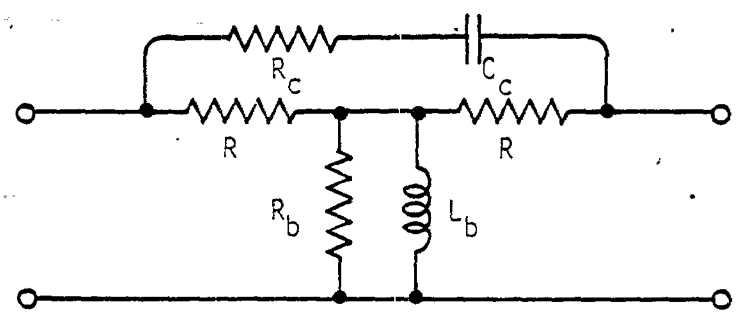
$$\begin{aligned}
\tilde{T} &= [1 + \frac{\tilde{Z}_c}{R}]^{-1} = [1 + \frac{R_c}{R} + \frac{\tilde{Z}_c'}{R}]^{-1} \\
&= T_0\ [1 + \frac{\tilde{Z}_c'}{R + R_c}\ ]^{-1} \qquad (9.4) \\
T_0 &\equiv [1 + \frac{R_c}{R}]^{-1} = \max_{\omega}\ |\tilde{T}(j\omega)|
\end{aligned}$$



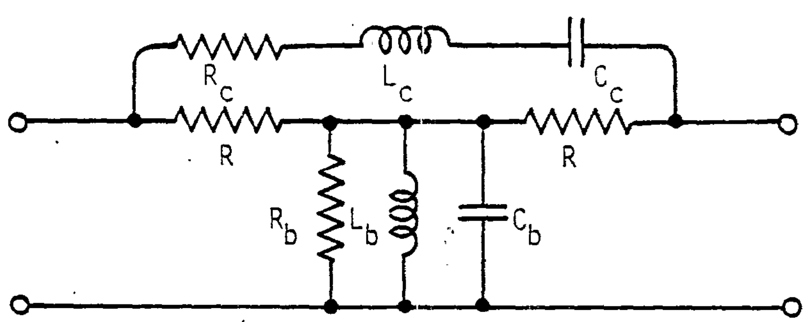
A. General Case



B. Low-pass Filter



C. High-pass Filter



D. Band-pass Filter

Fig. 9.1. Bridged-T filters with loss in pass band

With the factor T_0 pulled out the result is like that in (5.15), except that R is replaced by $R + R_c$ in normalizing $\tilde{Z}'_c$. Summarizing, the transfer function peak is reduced to T_0 and the filter time constants are changed by changing $R \rightarrow R + R_c$ in normalizing $\tilde{Z}'_c$. (Similarly, $R \rightarrow R + R_b$ in normalizing $\tilde{Z}'_b$ and T_0 can be expressed as $[1 + R/R_b]^{-1}$.

B. Low-pass Filter

As in fig. 9.1B the low-pass filter with loss has

$$\frac{L_c}{R} = RC_b \tag{9.5}$$

The transfer function is

$$\begin{aligned} \tilde{T} &= T_0 \left[1 + \frac{sL_c}{R + R_c}\right]^{-1} \\ T_0 &= \left[1 + \frac{R_c}{R}\right]^{-1} \end{aligned} \tag{9.6}$$

Comparing this to (8.2) note the shift in the time constant as well as the amplitude.

In time domain we have

$$\begin{aligned} T(t) &= T_0 \frac{R + R_c}{L_c} e^{-\frac{R+R_c}{L_c}t} u(t) \\ ||T(t)||_1 &= \int_0^\infty |T(t)| \, dt = T_0 \end{aligned} \tag{9.7}$$

So for the p-norm we have

$$||T(t)\circ||_p \leq T_0 \tag{9.8}$$

For special cases we have

$$\begin{aligned} ||T(t)\circ||_1 &= ||T(t)\circ||_\infty = T_0 \\ ||T(t)\circ||_2 &= \max_\omega |\tilde{T}(j\omega)| = T_0 \end{aligned} \tag{9.9}$$

C. High-pass Filter

As in fig. 9.1C the high-pass filter with loss has

$$\frac{L_b}{R} = RC_c \tag{9.10}$$

The transfer function is

$$\begin{aligned} \Upsilon &= T_o \left[1 + \frac{1}{sC_c[R + R_c]}\right]^{-1} \\ T_o &= \left[1 + \frac{R_c}{R}\right]^{-1} \end{aligned} \tag{9.11}$$

Compared to (8.7) similar shifts have been made as before.

In time domain we have

$$\begin{aligned} T(t) &= T_o \left\{ \delta(t) - \frac{e^{-\frac{t}{[R+R_c]C_c}}}{[R + R_c]C_c} \right\} u(t) \\ ||T(t)||_1 &= \int_0^\infty |T(t)|\, dt = 2\, T_o \end{aligned} \tag{9.12}$$

So for the p-norm we have

$$||T(t)\circ||_p < 2\, T_o \tag{9.13}$$

For special cases we have

$$\begin{aligned} ||T(t)\circ||_1 &= ||T(t)\circ||_\infty = 2\, T_o \\ ||T(t)\circ||_2 &= \max_\omega |\Upsilon(j\omega)| = T_o \end{aligned} \tag{9.14}$$

D. Bandpass Filter

As in fig. 9.1D the bandpass has

$$\frac{\tilde{Z}'_b}{R} = [sRC_b + \frac{R}{sL_b}]^{-1}$$

$$\frac{\tilde{Z}'_c}{R} = \frac{sL_c}{R} + \frac{1}{sRC_c} \quad (9.15)$$

$$\frac{L_c}{R} = RC_b \ , \ \frac{L_b}{R} = RC_c$$

The transfer function is

$$\tilde{T} = T_0 \ [1 + \frac{\tilde{Z}'_c}{R + R_c}]^{-1} = T_0 \ [1 + \frac{sL_c}{R + R_c} + \frac{1}{s[R + R_c]C_c}]^{-1}$$

$$T_0 = [1 + \frac{R_c}{R}]^{-1} \quad (9.16)$$

By defining

$$t'_1 \equiv \frac{L_c}{R + R_c}$$

$$t'_2 \equiv [R + R_c]C_c \quad (9.17)$$

then the results are the same as in section 8C with $t_1 \to t'_1$, $t_2 \to t'_2$, and the factor of T_0 in the transfer function.

For a narrow pass band for the filter one now needs

$$\frac{t'_2}{t'_1} = [R + R_c]^2 \frac{C_c}{L_c} \ll 1 \quad (9.18)$$

or

$$\sqrt{\frac{L_c}{C_c}} \gg R + R_c \quad (9.19)$$

showing the shift $R \to R + R_c$.

In time domain the results of section 8C carry over directly as

$$||T(t)||_1 \lessapprox 2 \ T_0 (9.20)$$

For special cases we have

$$\begin{aligned} ||T(t)\circ||_1 &= ||T(t)\circ||_\infty \lessapprox 2\ T_0 \\ ||T(t)\circ||_2 &= \max_\omega |\tilde{T}(j\omega)| = T_0 \end{aligned} \tag{9.21}$$

E. Some Comments

This special form of including R_b and R_c in the bridged-T constant-resistance filter then basically reduces the peak of the transfer function to $T_0 < 1$. By appropriately scaling the element values, the filter transfer function can be kept the same except for a multiplication by T_0.

This simple scaling also manifests itself in the reduction of the time-domain-operator norms to 2 T_0 or T_0 as appropriate. By choice of T_0 then one can reduce the time-domain norms for the filter to desired values. Note, however, that this is done at a price; the transfer function in the pass band (say for quasi CW signals) is also reduced to T_0.

X. Mismatched Constant-Resistance Filters

In previous sections the constant-resistance filter has been considered under the assumption that it is terminated in a resistance R at one port when driven from the other port. With a constant-resistance filter attached to a norm limiter one would like R to be the impedance connected to the NL. In general, however, one may not have a resistance R connected to the NL, but rather some other impedance associated with the complex electronic system into which NLs and constant-R filters are installed. One may wish to know what kind of errors such a mismatch introduces into the analysis.

A. Frequency-Domain Considerations

Consider the symmetrical bridged-T filters as in figs. 8.1 and 5.2. For frequencies in the stop band of the filter we have

$$\tilde{Z}_b \simeq 0 \ , \ \tilde{Z}_c \simeq \infty$$

$$\tilde{Z}_{in} \simeq \tilde{Z}_a = R \tag{10.1}$$

$$\tilde{T} \simeq 0$$

irrespective of the load on the other side of the filter. Then in the stop band of the filter a mismatched load has no significant effect. It presents an input impedance of R in both directions for all frequencies, except possibly those in the pass band.

In the pass band, however, things are different. As illustrated in fig. 10.1 let us consider this mismatch in a scattering-matrix formalism. In passing through the filter a wave $\tilde{V}_{in}$ is transformed to $\tilde{T}\ \tilde{V}_{in}$. This wave is scattered off some impedance designated as $\tilde{Z}_t$ which we assume to be passive. The reflection coefficient is

$$\tilde{\alpha} = \frac{\tilde{Z}_t - R}{\tilde{Z}_t + R} \tag{10.2}$$

The passivity of $\tilde{Z}_t$ assures for all $s = j\omega$

$$|\tilde{\alpha}| \leq 1 \tag{10.3}$$

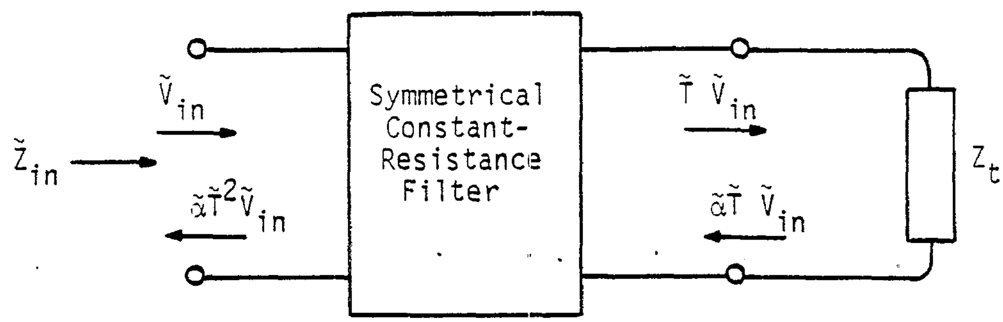


Fig. 10.1. Transmission and reflection through symmetrical constant-resistance filter

from which we can define

$$\alpha_0 \equiv \max_{\omega} |\tilde{\alpha}| < 1 \quad (10.4)$$

The wave reflected back into the filter is then $\tilde{\alpha}\,\tilde{T}\,\tilde{V}_{in}$. On passing back through the (symmetric) filter the wave is $\tilde{\alpha}\,\tilde{T}^2\,\tilde{V}_{in}$, or

$$\frac{\tilde{V}_{refl}}{\tilde{V}_{in}} = \tilde{\alpha}\,\tilde{T}^2 \tag{10.5}$$

$$\left|\frac{\tilde{V}_{refl}}{\tilde{V}_{in}}\right| = |\tilde{\alpha}|\,|\tilde{T}|^2 < 1$$

Noting that

$$|\tilde{T}| < T_0$$
$$|\tilde{\alpha}| < \alpha_0 \tag{10.6}$$

then

$$\left|\frac{\tilde{V}_{refl}}{\tilde{V}_{in}}\right| < \alpha_0 T_0^2 < 1 \tag{10.7}$$

To the extent that $\alpha_0 T_0^2$ is small compared to 1, then $\tilde{V}_{refl}$ has negligible significance and $\tilde{Z}_{in}$ at the filter can be approximated as R. In section 8 the maximum transmission T_0 of the filter is 1, so that it is required that $\alpha_0 \ll 1$ or $\tilde{Z}_t \approx R$ for $\tilde{Z}_{in}$ to be approximated as R for all ω. If as in section 9 the maximum transmission T_0 has $T_0 < 1$, then $\alpha_0 T_0^2 < 1$ and then $\tilde{Z}_{in}$ can also be approximated as R for all ω, and the size of passive $\tilde{Z}_t$ is not significant. However, if it is only known that $\tilde{Z}_t$ is passive, then

$$|\tilde{\alpha}| < \alpha_0 < 1 \tag{10.8}$$

and is it necessary that T_0 be constrained to be small enough that T_0^2 can be neglected compared to 1. Note that

$$\tilde{Z}_{in} = R\,\frac{1 + \tilde{\alpha}\tilde{T}^2}{1 - \tilde{\alpha}\tilde{T}^2} \qquad (10.9)$$

from which a desired degree of closeness of $\tilde{Z}_{in}$ to R can be interpreted as a corresponding constraint on T_0.

B. Time-Domain Considerations

In time domain let us consider the norms of the waveforms passing through the filter. The incident wave is characterized by $||V_{in}(t)||$ or $||V_{1,1}(t)||$ (using wave conventions as in section 2) as

$$||V_{2,1}(t)|| = ||T(t)\circ V_{1,1}(t)|| < ||T(t)\circ||\ ||V_{1,1}(t)|| \qquad (10.10)$$

on passing through the filter.

At the load there is a reflection. If the load is described by an impedance (linear by definition) with a frequency-domain reflection coefficient $\tilde{\alpha}$, the transient reflected wave is in general rather complicated, but some limits on its norm can be established. In section 9 various forms of transfer functions are considered with time-domain norms limited to like $2\alpha_0$ for 1-norm and ∞-norm, and α_0 for 2-norm. This leads to a reflection norm of order α_0.

It is also possible to consider this load as nonlinear (as, for example, an NL), and ipso facto not characterizable by an impedance. If the nonlinear load is passive, then the reflected wave is still limited in 2-norm as

$$||V_{2,2}(t)||_2 < ||V_{2,1}(t)||_2 \qquad (10.11)$$

Considering now α as a nonlinear reflection operator with

$$V_{2,2}(t) = \alpha(V_{2,1}(t)) \qquad (10.12)$$

then for a passive α we have

$$||V_{2,2}(t)||_2 < ||\alpha||_2\ ||V_{2,1}(t)||_2 \qquad (10.13)$$

with by definition of the 2-norm in this special case

$$0 \leq ||\alpha||_2 \leq 1 \qquad (10.14)$$

This, as one should see, is a generalization of the concept of norm in 2-norm sense, being defined by the above inequality. Note that merely passivity is needed for (10.14). In particular cases the upper bound could be even less.

On passing back through the filter the wave is reduced again to $||V_{1,2}(t)||$ or $||V_{refl}(t)||$ as

$$||V_{1,2}(t)|| = ||T(t) \circ V_{2,2}(t)|| \leq ||T(t)\circ|| \; ||V_{2,2}(t)|| \qquad (10.15)$$

Combining these results we have

$$V_{refl}(t) = T(t)\circ\{\alpha(T(t)\circ V_{in}(t))\} \qquad (10.16)$$

which in norm sense gives

$$0 \leq \frac{||V_{refl}(t)||}{||V_{in}(t)||} \leq ||\alpha|| \; ||T(t)\circ||^2 \qquad (10.17)$$

where $||\alpha||$ has to be defined for the particular norm used since $\alpha(\ )$ is possibly a nonlinear operator. In 2-norm sense we have

$$\begin{aligned} ||T(t)\circ||_2 = T_0 &= \max_{\omega} |\tilde{T}(j\omega)| \\ 0 \leq ||\alpha||_2 &\leq 1 \quad \text{(for passive } \alpha(\ )) \end{aligned} \qquad (10.18)$$

giving

$$0 \leq \frac{||V_{refl}(t)||_2}{||V_{in}(t)||_2} \leq ||\alpha||_2 \, T_0^2 \leq T_0^2 \qquad (10.19)$$

C. Some Comments

Whether in frequency domain (assuming a linear Z_t) as in (10.7), or in time domain (including the possibility of a nonlinear reflection operator $\alpha(\ )$) as in (10.17) and (10.19), it is possible to make V_{refl} small compared to V_{in}, at least in norm sense. If $||\alpha||$ is about 1 then it is necessary that $||T(t)\circ||^2$ be small compared to 1. The examples in section 9 show that $||T(t)\circ||$ is like T_0 for the 2-norm, but as much as 2 T_0 for 1-norm and ∞-norm. In any event, the filter can be designed with T_0 small enough so that the input to the filter looks almost like a resistor R.

There are various ways to introduce loss into the filter to achieve a desired T_0 with $0 < T_0 < 1$. One way to include loss in the filter is to simply cascade a simple constant-resistance bridged-T filter (section 8) with a constant-resistance frequency-independent attenuator (section 7). Alternatively, one can include loss in the constant-resistance bridged-T filter (section 9) to achieve the same result.

XI. Summary

So now we have the basic outlines of the combination of nonlinear norm limiters (NLs) with linear constant-R filters at penetrations through subshields. The filter stops frequencies out of the intended pass band from penetrating, and also presents a resistance R on both sides in the stop band.

The NL can be characterized with load resistances R on each side, provided that constant-R filters are used to isolate the NL from other loads, and that in the pass band there is some loss indicated by not-too-large T_0.

The constant-R filters can also be used to terminate the signals incident on the subshields and provide some isolation from the NLs. This is particularly the case again for frequencies in the stop band of the filter for which the filter looks like a resistance R. In time-domain sense one can include a nonlinear device (such as an NL, perhaps connected to yet another filter on its other side). In this case, a passive NL can be characterized by a nonlinear operator $\alpha(\ )$ with $||\alpha||_2 < 1$. By limiting T_0 of the constant-R filter to values sufficiently small compared to 1 the reflection off the filter input can be made small enough (in time-domain norm sense) compared to 1.

In this paper only a single wire penetrating a subshield has been considered. Perhaps various types of multiwire penetrations (and associated filters and NLs) should also be considered.

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