INTERACTION NOTES
NOTE 482

Bounding Calculations in RF Coupling*

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ABSTRACT

Various analytical formulas for bounding the absorption cross section in RF coupling are derived and discussed. Some are derived rigorously from first principles, while others are based on heuristic arguments. What needs to be done in gaining more quantitative and qualitative understanding of system coupling is suggested.

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INTRODUCTION

When an electromagnetic wave impinges on a system, a sequence of penetration and propagation starts to take place from the system outer surface into its interior and, ultimately, RF energy will appear at the system electronics (Fig. 1). The energy arrival can be by way of either front-door or back-door paths. A front-door path is an intended path for RF transmission and reception, an example of which is an antenna connected to a coaxial cable terminated at an electronic box. A back-door path is an inadvertent point of entry (POE) for energy penetration, such as windows, doors, cracks, seams, connectors, cable shields or non-electrical lines.

A convenient quantity to describe the above coupling to electronics is the absorption cross section (σ) which gives, when multiplied by the incident power density, the total power delivered to the terminals of the electronic component.

Recently, an integral bound on σ has been derived giving [1]

$$\int_0^\infty \sigma \, d\lambda \leq \pi^2 V(P_{11} + M_{22}) \qquad (1)$$

where V is the volume enclosing the system, and P_{11} and M_{22} are, respectively, the normalized electric and magnetic polarizabilities of the system in the directions of the incident electric and magnetic fields. For example, the right hand side of (1) is equal to $2\pi\,(4\pi a^3/3)$ for a circular hole with radius a, and equal to $\pi^3 L^3/(12\,\ell n\,(4L/w))$ for a narrow slot with length L and width w. Eq. (1) is useful in two ways. It bounds the behavior of σ at low and high frequencies. It can also be utilized to obtain some upperbound energy or power that an incident electromagnetic wave can penetrate into the interior of a system, given the geometries and the distribution of its POEs.

Since (1) is of such a fundamental nature, it is worthwhile to derive it in more than one way. A derivation slightly different from the one given in [1] can be found in Appendix A. Reference [2] offers another look at (1) from a complex frequency and time domain consideration.

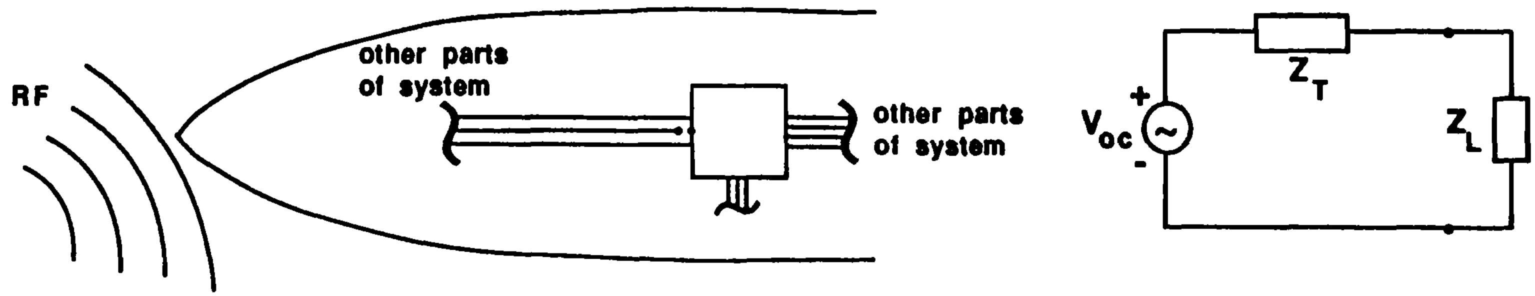

Figure 1. Example of RF coupling to electronic pin with an impedance loading represented by Z_L. V_{OC} and Z_T are the Thevenin parameters representing the induced RF stress.

Warne and Chen have skillfully applied (1) to bound EMP coupling problems [3]. Noticing that the left-hand side of (1) is proportional to the total absorbed energy for a step-function indicent plane wave, they have shown that

$$2\int_0^\infty \sigma S_{inc}\, df \le \frac{1}{2}\mathbf{E}_o \cdot \mathbf{p} + \frac{1}{2}\mathbf{H}_o \cdot \mathbf{m} \tag{2}$$

The left-hand side is the energy absorption, and $S_{inc} \equiv \tilde{E}_{inc}\tilde{H}^*_{inc}$, with $\tilde{E}_{inc}$ being the Fourier transform of the incident field $E_{inc} = E_o u(t)$, $u(t)$ the unit step function. The right-hand side is the total energy stored in the induced electric and magnetic dipoles **p** and **m**. It is interesting to note that the energy absorption is bounded by the sum of the static electric and magnetic stored energy, whereas if one starts with the frequency-domain Poynting vector theorem one will end up with the difference of the two energies[4].

The bound given by (1), although useful, often yields too loose a bound such as the case in EMP interaction problems. It would be most desirable to have a bound tighter than what (1) offers. Since the publication of [1-3], several other forms of bound on σ have been discovered. The purpose of this report is to discuss these bounds, some derived from first principles while others from practical assumptions.

FIELD-THEORETIC CONSIDERATION

In this section the Lorentz reciprocity theorem is invoked to derive rigorously a concise expression for the absorption (or coupling) cross section σ, which will be valid all the way to the pin level within a system. The only assumption invoked is that the intervening medium between the pin's terminals of interest and the RF source outside the system is reciprocal, be it lossy, inhomogeneous, or anisotropic.

Let $(\mathbf{E}_r, \mathbf{H}_r)$ and $(\mathbf{E}_t, \mathbf{H}_t)$ be two electromagnetic fields oscillating at the same angular frequency ω and having no singularities within the volume V bounded by the surface S_{AB} and S_∞ (Fig. 2). The subscript r or t on a quantity denotes that that quantity is associated with the reception or the transmission problem. In the reception problem one is interested in the power received by the load Z_L attached to the terminals (A,B), whereas in the transmission problem one applies a voltage V_t across (A,B) and asks for the power gain function G and the impedance Z_T looking out from (A,B). From Maxwell's equations one has the so-called Lorentz reciprocity theorem,

$$\nabla \cdot (\mathbf{E}_r \times \mathbf{H}_t - \mathbf{E}_t \times \mathbf{H}_r) = 0 \tag{3}$$

within V, which gives, by means of the Gauss theorem,

$$\int_{S_\infty + S_{AB}} (\mathbf{E}_r \times \mathbf{H}_t - \mathbf{E}_t \times \mathbf{H}_r) \cdot \mathbf{n} \, dS = 0 \tag{4}$$

The surface integral on S_∞ can be evaluated by the method of stationary phase and the result is

$$\int_{S_{AB}} (\mathbf{E}_r \times \mathbf{H}_t - \mathbf{E}_t \times \mathbf{H}_r) \cdot \mathbf{n} \, dS = -\frac{\lambda^2}{\pi Z_o} \mathbf{E}_{inc}(\theta_o, \phi_o) \cdot \mathbf{F}(\theta_o, \phi_o) \tag{5}$$

where $\mathbf{E}_{inc}$ is the electric field of the incident plane wave and $\mathbf{F}$ is related to the far field of the transmission problem by

$$\mathbf{E}_t \sim i \mathbf{F} \frac{e^{ikr}}{kr} \qquad r \to \infty \tag{6}$$

and (θ_o, ϕ_o) is the direction of incidence. The surface integral over S_{AB} enclosing the terminals

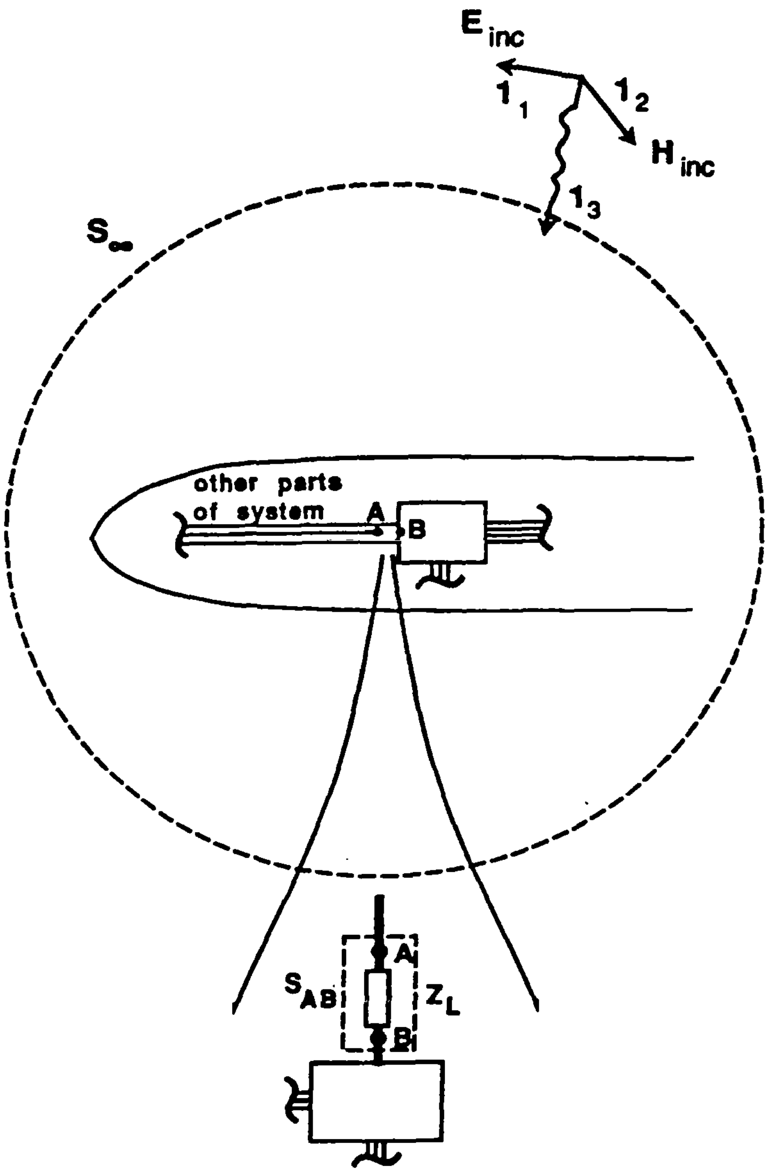

Figure 2. Lorentz reciprocity theorem applied to the volume bounded by the surfaces S_∞ and S_{AB}.

(A,B) can be expressed in terms of the voltages and currents across (A,B). With the usual definitions of V and I in terms of E and H and the assumption that S_{AB} be electrically small, one can obtain from (5)

$$V_t I_r + V_r I_t = -\frac{\lambda^2}{\pi Z_o} \mathbf{E}_{inc} \cdot \mathbf{F} \tag{7}$$

The quantity of interest is the real power (P_r) received by the load in the reception problem, and is given by

$$P_r = \frac{1}{2} R_L I_r I_r^* \tag{8}$$

where R_L is the real part of Z_L. Since $V_t = Z_T I_t$ and $V_r = Z_L I_r$, one gets from (7)

$$I_r = -\frac{\lambda^2}{\pi Z_o} \mathbf{E}_{inc} \cdot \mathbf{F} \frac{1}{I_t (Z_T + Z_L)} \tag{9}$$

and hence

$$P_r = \frac{R_T R_L}{\left|Z_T + Z_L\right|^2} \frac{\lambda^4}{2\pi^2 Z_o^2} \left|\mathbf{E}_{inc} \cdot \mathbf{F}\right|^2 \frac{1}{R_T I_t I_t^*} \tag{10}$$

If one introduces the gain function G defined by

$$G = \frac{4\pi r^2 E_t H_t^*}{R_T I_t I_t^*} = \frac{\lambda^2}{\pi Z_o} \frac{|\mathbf{F}|^2}{R_T I_t I_t^*} \tag{11}$$

into (10), one obtains

$$P_r = \frac{\lambda^2}{4\pi} G(\theta_o,\phi_o) p(\theta_o,\phi_o) q S_{inc} \tag{12}$$

where

$$p = \left|\mathbf{1}_{inc} \cdot \mathbf{1}_t\right|^2, \qquad q = \frac{4R_T R_L}{\left|Z_T + Z_L\right|^2}, \qquad S_{inc} = \frac{1}{2} E_{inc} H^*_{inc} \tag{13}$$

with $\mathbf{1}_{inc}$ and $\mathbf{1}_t$ being the unit vectors in the directions of $\mathbf{E}_{inc}$ and $\mathbf{F}$, respectively.

Recalling that the definition of the coupling or absorption cross section σ is P_r divided by S_{inc}, one finally has the following concise expression:

$$\sigma = \frac{\lambda^2}{4\pi} G\, p\, q \tag{14}$$

which has also been derived by Tai from a circuit consideration [5].

BOUNDS ON ABSORPTION CROSS-SECTION

It is obvious from the definitions of p and q that they are real and never exceed unity. Therefore, the first bound on σ is

$$\sigma \le \frac{\lambda^2}{4\pi} G \tag{15}$$

If (14) is averaged over all angles of incidence one gets

$$\overline{\sigma} \le \frac{\lambda^2}{8\pi} q \le \frac{\lambda^2}{8\pi} \tag{16}$$

since p is averaged to 1/2 and G to less than unity. If losses between the pin's terminals and the outside surface of the system were ignored, G would have been averaged to unity; otherwise the angular average of G is less than unity by the amount of such losses.

One may notice that in the expression for P_r given by (10), R_T cancels out except for the part of R_T contained in Z_T. This means that one may introduce quantities other than G for the representation of P_r. One such quantity is the directivity function D defined by

$$D = \frac{4\pi r^2 E_t H_t^*}{R_{rad}|I_t|^2} = \frac{\lambda^2}{\pi Z_o} \frac{|F|^2}{R_{rad}|I_t|^2} \tag{17}$$

where R_{rad} is the radiation resistance, a measure of real power leaking out of the system when it is driven by a source across the terminals (A, B). With (17) one obtains the following alternative form for σ:

$$\sigma = \frac{\lambda^2}{4\pi} D\, p \frac{4R_{rad}R_L}{|Z_T + Z_L|^2} \tag{18}$$

A comparison of (14) and (18) shows that

$$\frac{G}{D} \equiv \frac{R_{rad}}{R_T}$$

which, for a real system, should be a small quantity.

The most interesting feature of (18) is that D can be interpreted as the directivity function of all the points of entry (POEs) in the outer surface of a system and has nothing to do with the losses (R_{loss}) between the pin's terminals and the POEs.

Since

$$
\begin{aligned}
|Z_T + Z_L|^2 &= (R_T + R_L)^2 + (X_T + X_L)^2 \\
&= (R_{loss} + R_{rad} + R_L)^2 + (X_T + X_L)^2 \\
&\geq (R_{loss} + R_L)^2
\end{aligned}
$$

one gets for (18)

$$\sigma \leq \frac{\lambda^2}{4\pi} D\, p\, \frac{4R_{rad}R_L}{(R_{loss} + R_L)^2} \tag{19a}$$

$$\leq \frac{\lambda^2}{4\pi} D\, p \frac{R_{rad}}{R_L} \tag{19b}$$

The last inequality is obtained when R_L is set equal to R_{loss}.

The advantage of having (19b) over (15) is that the gain function G has been separated into the product of two parts, one part dealing with the radiation pattern (the D function) and the other with the efficiency factor (meaning the losses or, more precisely, the ratio of radiation loss R_{rad} to load loss R_L or to internal coupling loss R_{loss}). Although one may estimate the D and p functions for a given situation, it is rather difficult to have any feel for what R_{rad} should be. To be sure, R_{rad} is a very small quantity in a real system and is given by

$$R_{rad} = R_T - R_{loss} \tag{20}$$

If R_{rad} is to be measured, one can first measure the real part of the input (or driving point) impedance Z_T and measures it again with all the POEs closed, as suggested by Warne [6]. The difference between the two measurements will give R_{rad}. As of now no known procedures exist to estimate R_{rad}.

DIRECT DECOMPOSITION OF ABSORPTION CROSS-SECTION

In the preceding section the coupling or absorption cross-section is written as a product of other functions, such as G, p and q as shown in (14). In this sense a decomposition of the absorption cross-section has been achieved. As one may recall, such a decomposition comes naturally in the process of deriving the cross-section and it does not come directly from the sequence of interaction that is actually taking place. The sequence of interaction from outside to inside is, first, penetration through POE(s), then coupling to cavity, then through cable shield(s), etc., and finally to connector pin(s). There may be more or less intermediate steps than those just enumerated. For simplicity, consider an interaction sequence with POE(s) to cavity to a wire pin connected to the load impedance Z_L. Then one would be tempted to write for the absorption cross-section at the pin the following expression,

$$\sigma_{pin} \doteq \left(\sum_i \sigma(POE_i) \right) \cdot (\sigma_c)^{-1} \cdot \sigma_w \tag{21}$$

where σ_c represents the cavity effect and should have the dimension of area, σ_w is related to the absorption cross section of the wire (with Z_L) exposed to an angular spectrum of plane wave, and $\Sigma_i \sigma(POE_i)$ represents the sum of transmission cross-sections of all POEs. In the following, two heuristic approaches are suggested to derive an explicit representation for σ_c. The first is borrowed from the ideas developed for mode-stirred chambers [7].

Let P_T be the total net power that penetrates into the system through its POEs in the system outer surface, and W be the corresponding total energy. Assume equilibrium is reached so that P_T and W are related as follows:

$$P_T = \frac{\omega W}{Q} \tag{22}$$

where ω = angular frequency and Q = quality factor of the cavity including all losses. Now a crucial assumption is made that the average power density S be related to the average energy density W/V (V = volume of the cavity) through

$$S = cW/V \tag{23}$$

which is exact for a plane-wave field. Combining (22) and (23) one has

$$S = \frac{\lambda Q}{2\pi V} P_T \equiv P_T / \sigma_c \tag{24}$$

Once the average power density S is known, one may think of the wire being exposed to an angular spectrum of plane waves that make up S. The average power absorbed by the wire is then given by $S\sigma_w$, where σ_w has the form

$$\sigma_w = \frac{\lambda^2}{8\pi} q_w = \frac{\lambda^2}{8\pi} \frac{4R_L R_T^{'}}{|Z_L + Z_T^{'}|^2} \leq \frac{\lambda^2}{8\pi} \tag{25}$$

with $Z_T^{'}$ being the input impedance of the wire in free space.

Recalling that $P_T = \left(\sum \sigma(POE_i)\right) S_{inc}$ one obtains from (21), (24), and (25)

$$\sigma_{pin} = P_r / S_{inc} \doteq \left(\sum \sigma(POE_i)\right) \cdot \frac{\lambda Q}{2\pi V} \cdot \frac{\lambda^2}{8\pi} q_w \tag{26}$$

Comparing (18) with (26), one should have

$$R_{rad} \sim \left(\frac{\lambda}{2\pi}\right)^3 \frac{\pi Q}{V} \cdot \left|\frac{Z_T + Z_L}{Z_T' + Z_L}\right|^2 \cdot R_T' \tag{27}$$

It is worthwhile in the future to investigate if (27) holds true.

An alternative way to derive (26) heuristically is to start with the time-domain Poynting vector theorem, viz.,

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right) + \mathbf{J}\cdot\mathbf{E} = -\nabla\cdot(\mathbf{E}\times\mathbf{H}) \tag{28}$$

which gives, upon integration over the volume V of the unfilled part of the cavity,

$$\frac{d}{dt}W = P_T - P_{loss} \tag{29}$$

where W is the total energy stored inside V, P_T is net flow of power into V through all the POEs, and P_{loss} accounts for all the losses inside V. Eq. (29) is nothing more than a statement of power conservation. When steady state is reached, and if the quality factor Q is introduced to relate P_{loss} to the total energy content W, one obtains (22). If P_{loss} is split into one part due to absorption (P_{ab}) by the wire's load and the other by the cavity, then

$$P_{loss} = \frac{\omega W}{Q_w} + \frac{\omega W}{Q_c} \tag{30}$$

where Q_w = quality factor of the loaded wire, and Q_c = quality factor of the cavity. If there are many loaded wires inside the cavity, one simply adds all the individual Q's in parallel to obtain the total Q to be used in (26). From (30) the received power P_r of the wire is

$$P_r = \frac{Q_c}{Q_c + Q_w} P_{loss} = \frac{Q_c}{Q_c + Q_w} P_T \tag{31}$$

If one makes the assumption that the loss due to re-radiation through all the POEs into the system's exterior is small compared to all other losses, one may say that $P_T \doteq \left[\sum_i \sigma(POE_i)\right] S_{inc}$. In this case, one should have, on comparing (26) and (31),

$$Q_w q_w \doteq 16\pi^2 V / \lambda^3 \qquad (32)$$

The left-hand side contains parameters characteristic of the wire itself, in contradistinction to the right-hand side, which is a function of the cavity volume V and wavelength λ. Perhaps, the roles that various types of losses come into play are more subtle than what has been assumed. This should be investigated further in the future.

CONCLUDING REMARKS

It has been well recognized that accurate prediction of RF coupling to real systems is out of reach. The most that one can hope for is to calculate its bound and/or to estimate its trend. In this report various bounds on the absorption (or coupling) cross section have been discussed. Consistency among some of the bounds has not been established. It is believed that most difficulties that have been encountered here can be overcome if one canonical problem is solved "rigorously." One problem that offers such an opportunity is a cylindrical cavity with an electrically small rectangular slot in its outer surface and a wire monopole terminated at a 50 Ω located within the cavity. This "simple" structure is also amenable to exact measurement and, if one wishes, can be complicated by filling the cavity with various lossy material. The goal of attacking such a problem is not to determine its "exact" solution, but rather to find out the roles that various losses(e.g., R_{rad}, R_L, R_T, Q_c, Q_w, etc.) play and how well the solution can be bounded by the various expressions "derived" in this report.

ACKNOWLEDGMENTS

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APPENDIX A
DERIVATION OF EQUATION (1)

Let the far field scattered off from a target be

$$\mathbf{E} = \mathbf{A}\,\frac{e^{ikr}}{4\pi r}, \qquad r \to \infty \tag{A·1}$$

The total cross section (σ_t), that is, the sum of absorption (σ) and scattering cross section, is related to A via the optical theorem

$$\sigma_t = \frac{1}{k}\,\mathrm{Im}(\mathbf{1}_1 \cdot \mathbf{A}) \tag{A·2}$$

where Im denotes the imaginery part of the quantity following it, $\mathbf{1}_1$ is the unit vector in the direction of the incident electric field, and Im $(\mathbf{1}_1 \cdot \mathbf{A})$ is evaluated along the direction of forward scattering, namely, $\mathbf{1}_3$ in Fig. 2. Let

$$A_1 \equiv A_1' + iA_1'' \equiv \mathrm{Re}(\mathbf{1}_1 \cdot \mathbf{A}) + i\,\mathrm{Im}(\mathbf{1}_1 \cdot \mathbf{A}) \tag{A.3}$$

Then, since the scattering process must satisfy the causality principle, A_1' and A_1'' form a Hilbert pair:

$$\begin{aligned} A_1'(\omega) &= \frac{2}{\pi}\int_0^{\infty} \frac{x}{x^2-\omega^2}\,A_1''(x)\,dx \\ &= \frac{2}{\pi c}\int_0^{\infty} \frac{x^2}{x^2-\omega^2}\,\sigma_t(x)\,dx \end{aligned} \tag{A.4}$$

where the integral is understood to be the Cauchy principal value. Differentiating (A.4) twice with respect to ω and evaluating the resulting equation at ω=0, one obtains

$$\int_o^{\infty}\sigma_t(\lambda)d\lambda = \frac{\pi^2c^2}{2}\frac{d^2}{d\omega^2}A_1'(\omega), \text{ at } \omega = 0 \tag{A·5}$$

Recall that at the low frequency limit the scattered far field along the direction $\mathbf{1}_3$ is given by

$$\mathbf{E} = -\mu_o\omega^2\left[\mathbf{1}_3\times(\mathbf{1}_3\times\mathbf{p})+\frac{1}{c}\mathbf{1}_3\times\mathbf{m}\right]\frac{e^{ikr}}{4\pi r} \quad . \tag{A.6}$$

Hence,

$$\mathbf{1}_1\cdot\mathbf{A} = \mu_o\omega^2\mathbf{1}_1\cdot\mathbf{p} + \omega^2\frac{\mu_o}{c}\mathbf{1}_2\cdot\mathbf{m} \tag{A.7}$$

and

$$\begin{aligned}\frac{\pi^2c^2}{2}\frac{d^2}{d\omega^2}A_1'(\omega) &= \pi^2(\frac{1}{\varepsilon_o}\mathbf{1}_1\cdot\mathbf{p} + Z_o\mathbf{1}_2\cdot\mathbf{m}) \\ &= \pi^2V(P_{11}+M_{22})\end{aligned} \tag{A·8}$$

where the electric and magnetic polarizability tensors have been introduced, via the following definitions:

$$\mathbf{p} = \varepsilon_o V\,\mathbf{P}\cdot\mathbf{E}_{inc}, \qquad \mathbf{m} = V\,\mathbf{M}\cdot\mathbf{H}_{inc}/Z_o \tag{A·9}$$

and $|\mathbf{E}_{inc}|$ has been assumed to be unity.

Since $\sigma \le \sigma_t$, one has from (A·5) and (A·8),

$$\int_o^{\infty}\sigma\, d\lambda \le \pi^2V(P_{11}+M_{22}) \tag{A·10}$$