Interaction Notes

Note 488

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ELECTROMAGNETIC COUPLING ON COMPLEX SYSTEMS: TOPOLOGICAL APPROACH

by

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ABSTRACT

The paper presents the general principles of Electromagnetic Topology, method developed in the seventies in the USA by C. E. Baum.

Using this method, one can break down a complex electromagnetic problem into several small problems that are easier to solve. An example explains how this theory can be applied to a practical case.

A formalism is then described that can be used to deal with all these problems from an electromagnetic point of view. It is based on the equation derived by C. E. Baum, T. K. Liu and F. Tesche [6] (BLT equation) dealing with multiconductor transmission line network theory. This theory is generalized to topological networks. An equation is derived that expresses the propagation and scattering of disturbances into and through different volumes.

Keywords (NASA thesaurus): Electromagnetic topology — Electromagnetic noise — Wave interaction — BLT equation — Multiconductor transmission line.

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I. – INTRODUCTION

Current aircraft use electronic equipment operating at a very low energy level to perform vital functions (fly-by-wire controls). The vulnerability of such equipment to external electromagnetic interference (lightning, radiofrequency transmitters, EMP) or internal sources (EMC) leads to the requirement for protecting the on-board equipment. A protection method based only on "common sense" leads to oversizing, detrimental to the performance of the aircraft.

Correlatively, the massive use of composites for their mechanical properties and low weight does not lead to an improvement in the equipment protection against electromagnetic interference. In effect, these materials, dielectric or poor conductors, have shielding properties inferior to those of metals. In addition, their interfaces (between one another or with the metal parts surrounding them) are often major sources of interference. These new technologies risk leading to such an increase in the shielding that the resulting weight increment exceeds the benefit expected from the use of composites.

Engineers are therefore confronted with the problem of optimizing the shieldings.

A predictive computation of the electromagnetic coupling on a complete complex structure would allow a reduction in cost and a better quality of the end product compared with the method consisting of making successive improvements to a system whose electrical and mechanical architectures are for the most part already finalized and therefore difficult to modify a posteriori.

The question of feasibility of computer-aided design in the area of electromagnetic compatibility can therefore be raised.

The problem to be solved is that of the response of a complex system (complete wiring of an aircraft, electronic equipment, etc.) to an electromagnetic aggression. Strictly speaking, to solve this problem, it is necessary to solve the Maxwell equations for the entire volume of the system, considering the presence of all the conductors and their shieldings. The size and computation speed of today's computers make numerical analysis of this problem impossible with three-dimensional codes. In addition, it would often be unsuited, since the interference energy for a given aggression generally follows privileged paths which do not involve the entire structure.

However, the choice of these privileged paths cannot be made arbitrarily, since it represents an approximation which it must be possible to evaluate. A quantitative method for solving this problem was developed by **C. E. Baum** [1] to [4] in recent years in the USA under the name "electrogmagnetic topology".

To our knowledge, this method is the only one that exists. It has the advantage of formally separating the respective influence of the interference sources, the scattering of signals between the various conductors and the propagation problems. It therefore seems very well suited to predictive computations of the interference received by the equipment subjected to several sources. In addition, it has the advantage of being able to test different paths followed by the energy through the structure.

After **C. E. Baum** [5, 7, 8, 9], many authors worked on various problems raised by application of the method. As far as we know, no compilation of the principles and methods used by the topology has yet been published nor has it been applied to practical examples.

In this paper, we summarize the principles published by various authors, stressing what is specific and interesting in the topological method.

II. – THE TOPOLOGICAL METHOD

II,1. – GENERAL DESCRIPTION OF THE METHOD

In practice, the topological approach to a problem includes two stages:

– the first consists of making a geometric description of the problem. It uses the concepts of "volume" and "surfaces" (these concepts are defined accurately below) and leads to establishing a "topological diagram" of the problem. This breakdown of space into regions privileges a path of the interference energy. It therefore requires making a number of assumptions to break the global problem down into one or more partial problems adapted to each situation. It should be noted that the results obtained in the framework of this breakdown can be compared with (or possibly overlaid on) those obtained in the framework of other breakdowns.

This breakdown into "volumes" allows an interaction graph to be established. The graph theory can then be used to model a problem with N independent sources as a superimposition of N elementary problems (principle of state superimposition).

The second part is an electromagnetic description of propagation of the interference. The path follows a network whose geometry is dictated by the interaction graph. The formalism used to describe all the elements and signals is drawn from that of "multiconductor transmission line networks" [6].

This leads us to writing a general equation (BLT equation in the terminology of C. E. Baum) relating the source quantities to the unknowns (these quantities can be electromagnetic fields, voltages, currents, etc.) while taking into account the propagation and scattering effects on the network.

II,2. – THEORETICAL MODEL

II,2.1. – *Topological Diagram*

The basis of the topological method consists of privileging certain axes of penetration of the electromagnetic energy so as not to have to solve the general problem. In a complex system including partitions, recesses, openings, cables, the system must be broken down into elementary subproblems.

The initial concept is that of "volume". A volume can be defined as a region of space in which the fields and currents are created by the same source(s) of interference. Typically, a "source" is any quantity of the electromagnetic field or voltage or current generator type susceptible to coupling with a system. We then define volumes included within the preceding volumes. They are subjected to the same initial source as the outer volume but the effect is modified (generally attenuated) when entering the subvolume. A subvolume can therefore be considered a volume that is excited by an "equivalent" source, expressed as a function of the initial source.

Now let us consider a system subjected to a single aggression which is localized in a volume, called outer volume. We can then structure the path of the interference by establishing a diagram where the various volumes are included in one another, as is shown in the example of Figure 1. V_{ij} is the notation used for the different volumes. "i" indicates the hierarchical order of the volume, "j" indicates a number in the volume defined by i. For instance, volume 3 actually breaks down into two elementary volumes, $V_{3,1}$ and $V_{3,2}$, both of which are included in $V_{2,1}$.

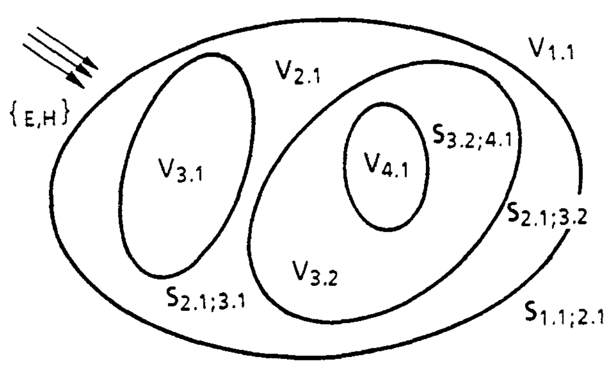


Fig. 1. – Topological diagram.

With this concept of volume is associated the concept of surface $S_{ij,\,kl}$ separating volumes V_{ij} and V_{kl}.

For instance, to reach volume $V_{4,1}$, the interference must cross surfaces $S_{1,1:2,1}$, $S_{2,1:3,1}$ and finally $S_{3,2:4,1}$.

It should be noted that the breakdown into volumes is a difficult step, essential for correctly representing the problem [4, 7].

II,2.2. – *Interaction Graph*

A topological diagram allows us to establish another graphic representation of the interactions between volumes: the graph corresponding to the diagram of Figure 1 is given in Figure 2. This graph involves nodes connected by edges used to model the path of the electromagnetic interference:

– The volume nodes (black circles in Figure 2) correspond to points in space located inside the volumes.

Several volume nodes can be included in a given volume so as to display certain points of space to be analyzed (electrical equipment, composite walls, etc.).

– Surface nodes (white circles in Figure 2) are nodes in space located on topological surfaces.

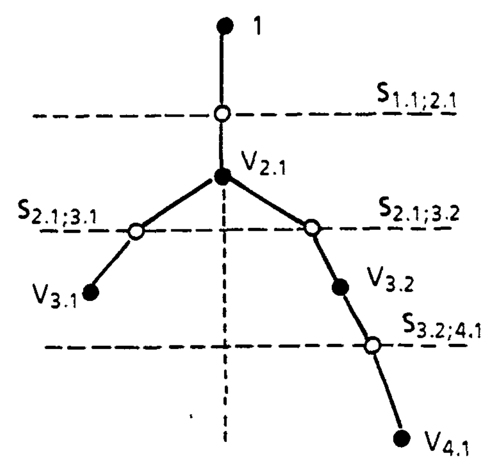


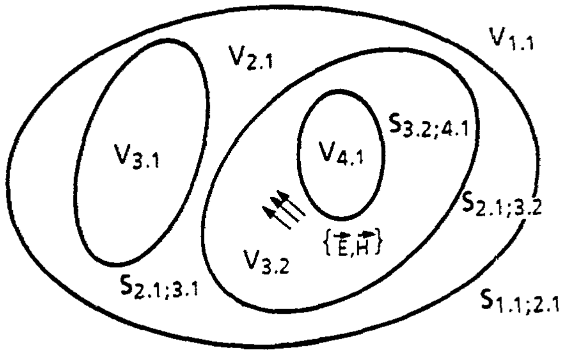
Fig. 2. – Interaction graph.

Several surfaces nodes can be included on a given surface to display certain lines of penetration.

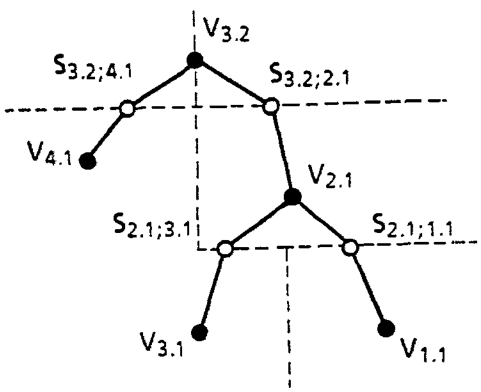
The best way of making a synopsis of the topological diagram consists of grouping the different energy penetration and prcpagation channels and representing each volume by a volume node and each surface by a surface node. The advantage of such a representation is that it lends itself well to the superimposition theorem. In effect, let us consider N sources, each located in a volume. The problem can be treated as a superimposition of N problems with a single source.

For each of these problems, it is sufficient to reorganize the topological graph so as to place the node corresponding to the source volume at the top of the tree. This operation is called "topological inversion" and easily allows the corresponding topological diagram to be reconstructed.

In Figures 3 *a*, *b*, *c* we show how the problem is approached when an electromagnetic source is located in volume $V_{3,2}$.



(a)



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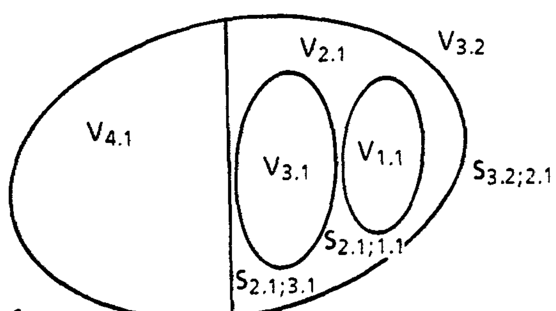


(c)

Fig. 3. – (*a*) Source located in $V_{3.2}$; (*b*) inversion of the graph; (*c*) corresponding diagram.

II,3. – TOPOLOGICAL APPROACH IN A REAL CASE

II,3.1. – *Statement of the Problem*

In order to illustrate the concepts of topological diagram and graph, we show how it is possible to approach a real case concretely. The example we chose is that corresponding to the diagram of Figure 4.

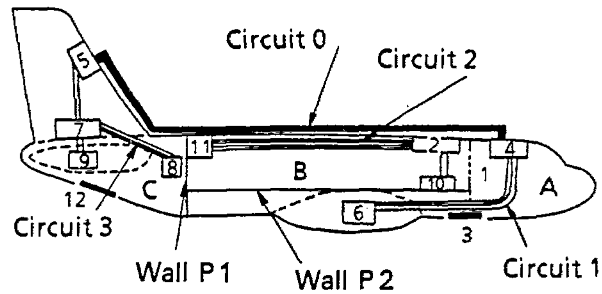


Fig. 4. – Breakdown of an aircraft into volumes.

Figure 4 shows an aircraft on which we determined three regions of space, *A*, *B*, *C*, allowing us to have a better view of our breakdown into volumes.

We included imaginary multiconductor and shielded cables (the shielded cables were connected to the metal structure of the aircraft) that we called circuits 0, 1, 2, 3.

Circuit 0 is assumed to be on the outside of the aircraft (meaning that the cable shielding is directly in contact with the outside). Circuits 1, 2, 3 are located in physical volumes *A*, *B*, *C* respectively. It is assumed that all the walls are metal.

These circuits interconnect electrical equipment shown in Figure 4 as rectangular boxes.

– Circuit 0 includes the cables interconnecting units 5 and 4;

– Circuit 1, units 4 and 6;

– Circuit 2, units 2, 10 and 11;

– Circuit 3, units 5, 7, 8 and 9.

The units are in certain cases cable terminals such as 6, 8, 9 and 10; others correspond to cable branches, such as 2, 4, 5 and 7.

In addition, we showed the paths of penetration from one physical volume to another, symbolized by apertures located in 1, 3 and 12.

It should be mentioned that we voluntarily numbered the units and apertures from 1 to 12 without distinction since both units and apertures are considered below as points of penetration of the energy into the structure.

In Sec. II,2, we showed that a good topological breakdown assumed a breakdown of space into independent volumes included in one another and was valid at any level of the analysis. We will now show that in practical cases, this breakdown can be called into question for particular processing. This concerns in particular the analysis of cable circuits which, because of their multiple branches, can extend into areas of space that can be considered independent when it is attempted, for instance, to determine the current distribution on the shielding. However, when it is attempted to analyze the electromagnetic interactions on the cables located within the shieldings, the same regions of space become interdependent.

II,3.2. – *First Stage: Breakdown into Physical Volumes*

1. *Topological Diagram*

The construction of this diagram consists of transforming the above physical breakdown into a breakdown into topological volumes. It should be noted that this breakdown depends on the choice made by the user according to the equipment he wants to privilege. In addition, it can also depend on the frequency range of the aggressions considered: for instance the phenomena of diffusion through the walls and diffraction of electromagnetic fields by the apertures are involved in different spectra for which the volume breakdown is not the same.

In the case of Figure 4, we assume that the aircraft is subjected to an external aggression of the electromagnetic field type. We assume that there is no diffusion phenomenon through the walls.

To establish the topological diagram corresponding to this textbook case, we will attempt to follow the path of the energy through the aircraft structure. The energy is first coupled on an external metal surface, in particular on the shielding of circuit 0. It then penetrates the structure through apertures 3 and 12 located in volumes *A* and *C* respectively. In each of these volumes, it is coupled to the shieldings of circuits 1 and 3. From volume *A* in particular, it penetrates volume *B* through aperture 1 then couples to the shielding of circuit 2.

We can then establish the topological diagram of the problem, illustrated in Figure 5, showing the path followed by the energy from an external volume $V_{1,1}$ up to the cable shieldings.

The three following topological volumes are distinguished first:

– $V_{2,1}$ corresponds to volumes *A* and *B*. Its surface $S_{1,1:2,1}$ consists of the aircraft fuselage in the location of volumes *A* and *B* and by the separation wall *P* 1 (*see* Fig. 4).

– $V_{2,2}$ corresponds to volume *C*. Its surface $S_{1,1.2,2}$ consists of the aircraft fuselage surrounding *C* as well as wall *P* 1.

– $V_{2,3}$ corresponds to circuit 0; surface $S_{1,1:2,3}$ identifies with the circuit shielding.

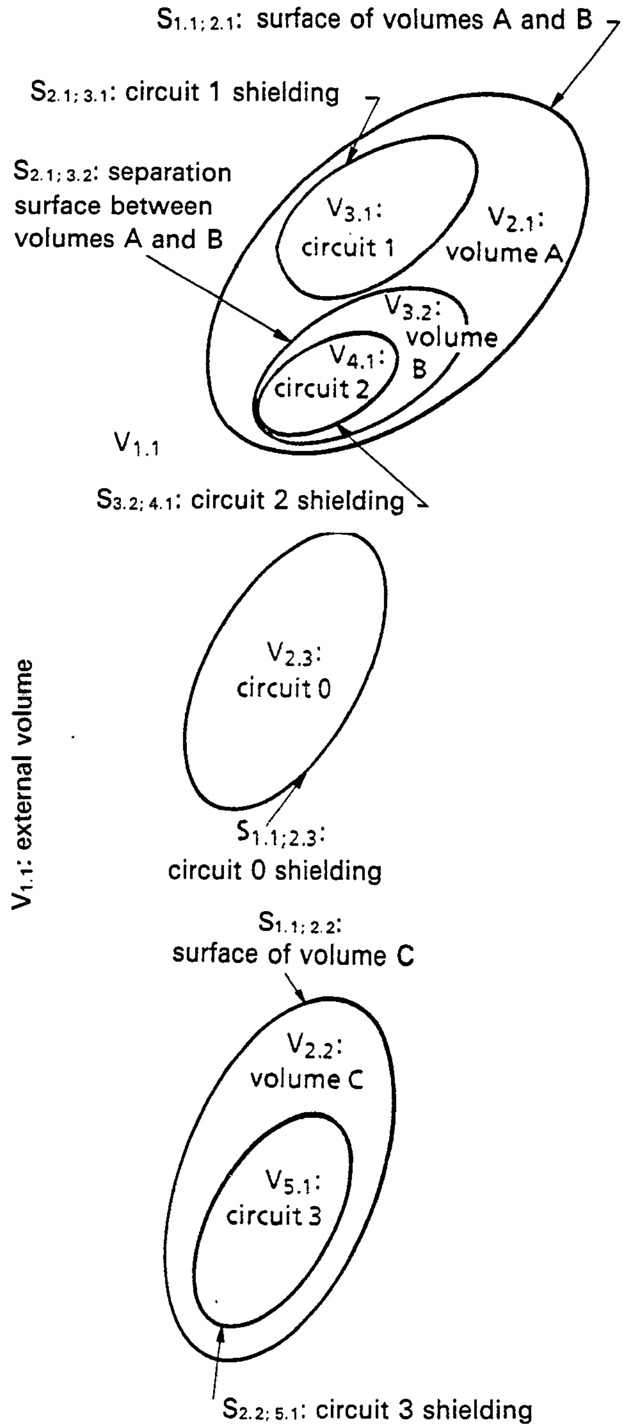


Fig. 5. – Topological diagram corresponding to the breakdown of Figure 4.

In volume $V_{2,2}$, we can distinguish a subvolume $V_{5,1}$ consisting of circuit 3 whose topological surface $S_{2,2:5,1}$ is the cable shielding.

Volume $V_{2,1}$ separates into two volumes, $V_{3,1}$ and $V_{3,2}$.

– $V_{3,1}$ corresponds to circuit 1; the topological surface $S_{2,1.3,1}$ consists of the cable shielding.

– $V_{3,2}$ corresponds to volume *B*: its separation surface $S_{2,1.3,2}$ consists of the aircraft fuselage opposite B as well as wall $P2$ (containing aperture 1). This volume itself contains a subvolume $V_{4,1}$ whose separation surface $S_{3,2;4,1}$ corresponds to the shielded sheath of circuit 2.

It can then be seen that it is not possible to continue the breakdown to the cables located inside the shielding in volumes $V_{3,1}$, $V_{4,1}$, $V_{2,3}$ and $V_{5,1}$ as this would call into question the topological breakdown already defined (*see* Sec. II, 3.3).

2. *Interaction Graph*

The interaction graph corresponding to the diagram of Figure 5 is shown in Figure 6, superimposed on the volume breakdown defined earlier so as to clearly show the concept of volume nodes and surface nodes (black circles and white circles respectively).

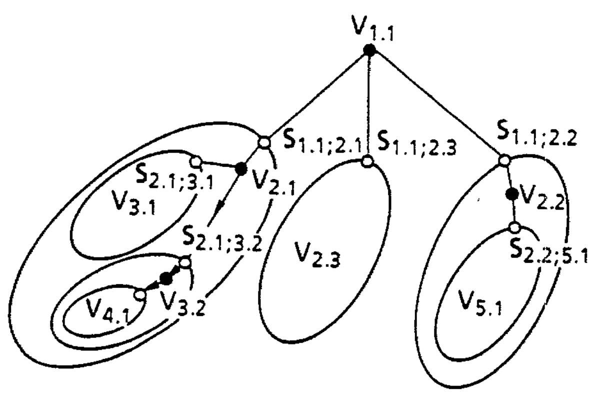


Fig. 6. — Superimposition of the topological graph on the topological diagram of Figure 5.

Here, the volume nodes are each associated with a topological volume in which an equivalent source, determined from the source of the outer volume, must be computed. In the textbook case studied, this amounts to calculating the electromagnetic fields in each of volumes A, B, C (volumes nodes $V_{2,1}$; $V_{3,2}$ and $V_{2,2}$).

The surface nodes are divided into two categories:

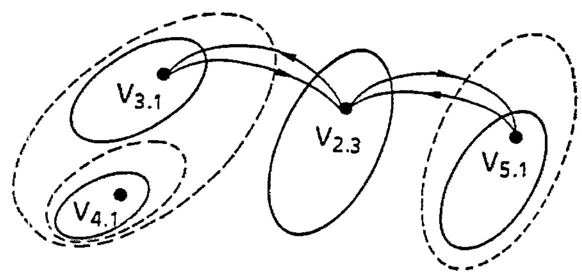
– those corresponding to penetration of the energy through an aperture (aperture 3 for $S_{1,1;2,1}$; aperture 12 for $S_{1,1;2,2}$; aperture 1 for $S_{2,1;3,2}$;

– those symbolizing shielding of the various circuits ($S_{2,1;3,1}$ for circuit 1; $S_{3,2;4,1}$ for circuit 2; $S_{1,1;2,3}$ for circuit 0; $S_{2,2;5,1}$ for circuit 3).

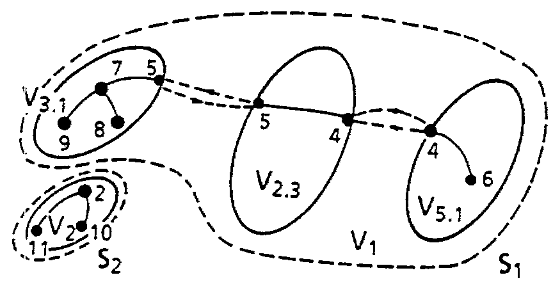
They correspond to surfaces on which is calculated the distribution of the electric currents due to the coupling of the electromagnetic fields of the higher volumes.

II,3.3. – *Second Step: Processing of the Cables*

After solving the problem broken down as shown in Figures 5 and 6, we can now determine the current distribution on all the surfaces of the topological volumes, more particularly on the cable shieldings.



(*a*)



(*b*)

Fig. 7. — (*a*) Display of the interactions between the volumes internal to the cables. (*b*) Breakdown of the volume nodes in each volume internal to the cables.

Figure 7 *a* shows the interactions between volumes corresponding to the inside of the cable shieldings, $V_{3,1}$, $V_{2,3}$, $V_{5,1}$ and $V_{4,1}$. We clearly see that at this level of problem solving, we must call into question the volume breakdown previously established.

This is why we grouped volumes $V_{3,1}$, $V_{2,3}$ and $V_{5,1}$ into a single volume V_1, distributing the currents on its surface S_1 according to their initial distribution on $S_{2,1;3,1}$, $S_{1,1;2,3}$, $S_{2,2;5,1}$. As for volume $V_{4,1}$, it is completely isolated from V_1. We will call it V_2 and we will call its surface S_2.

Figure 7 *b* shows these two volumes, V_1 and V_2, and the breakdown of the volume nodes of the interaction graph into other nodes, corresponding to the points of penetration defined in Figure 4. For instance, node $V_{3,1}$ is broken down into 5, 7, 8 and 9; node $V_{2,3}$ into 4 and 5; node $V_{5,1}$ into 4 and 6; node $V_{4,1}$ into 2, 10 and 11.

We thus obtain two new graphs reflecting the interactions on the cables, represented in Figures 8 *a* and *b*. The cable problem is broken down into two independent problems limited to topological volumes

V_1 and V_2 on which we redistributed the electromagnetic sources previously determined by the volume breakdown of Figure 5.

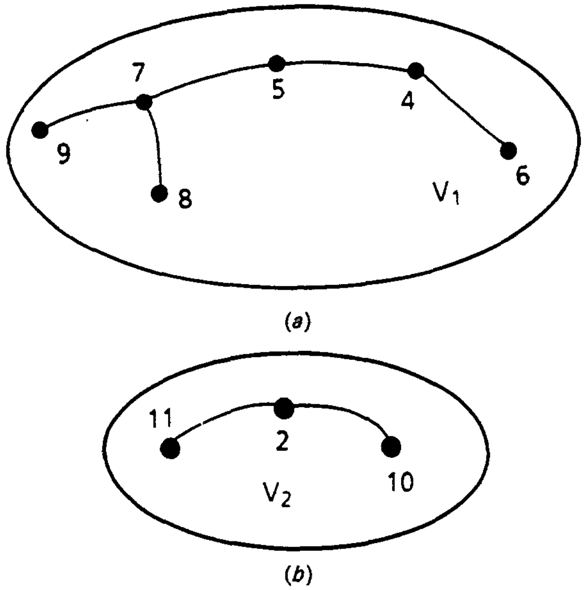


Fig. 8. – (a) Interaction graph for the cables of topological volume V_1. (b) Interaction graph for the cables of topological volume V_2.

III. – PROCESSING OF ELECTROMAGNETIC INTERACTIONS

III,1. – THE ELECTROMAGNETIC NETWORK: PROPAGATION SUPPORT FOR THE INTERFERENCE

III,1.1. – *Constituents*

The interaction graph described above is used to represent the interactions between volumes. We now need a model to physically represent the propagation channels of the electromagnetic signals and their scattering in the different topological volumes. For this purpose, we define a topological network inspired from the interaction graph, on which propagates an electromagnetic signal called "wave".

The signal propagation channels are materialized by tubes associated with the nodes of the graph. It should be noted that the term "propagation" must be understood as movement of a signal along the tube in a single direction, from an origin 0 taken at the end of the tube to the other end L, where L is the length of the tube (z can represent a curvilinear abscissa). This general propagation term obviously includes the propagation of signals whose phase φ can be represented by $\varphi = \varphi(z - ct)$, where t is the time; c is the speed of propagation; z is the abscissa in the direction of propagation.

The signal scattering is modeled by junctions located in the nodes of the topological graph. A distinction is therefore made between volume junctions associated with the volume nodes and surface junctions associated with the surface nodes.

The network then consists of a set of junctions connected by tubes. Figure 9 *a* shows the network associated with the graph of Figure 8 *a*. It should be noted that this network involves only volume junctions.

III,1.2. – *Electromagnetic Quantities Propagating in the Network: Waves*

The "wave" type quantities are denoted $W(z)$, where z is the position abscissa on the tube. It is also a vector quantity whose components are expressed as a function of the various voltage and current quantities on lines or the propagation modes in guides, etc. It should also be noted that this quantity depends on the frequency, even if this does not appear explicitly in the notation of the wave.

In order to identify the waves on the different parts of the network, waves propagating in the opposite direction on each tube are arbitrarily assigned. The direction of propagation is modeled by an arrow leaving each junction, as is shown by the numbers of the waves of Figure 9 *a* shown in Figure 9 *b*.

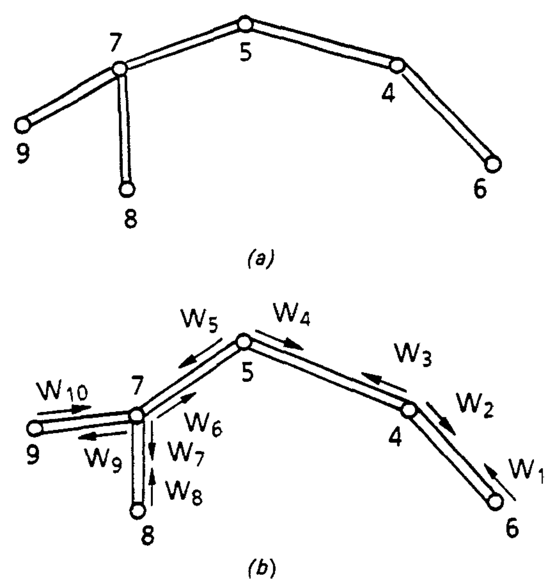


Fig. 9. – (a) Topological network corresponding to the graph of Figure 8 a. (b) Numbering of the waves on the network of Figure a.

It should be noted that arbitrarily assigning a direction of propagation to each wave and using two waves in opposite directions agrees with the conventional forward and backward wave propagation concept.

III,1.3. — *The Equations of the Network*

III,1.3.1. — *The Main Unknowns of the Network*

The main unknowns are the waves to and from the junctions. In effect, the volume junction can be considered as a point in space physically scattering a signal, but also as a point where the interference level is to be evaluated. This is the case, for instance, of an electrical unit located at the end of a cable harness.

The surface junction corresponds to the channel of penetration of a wave from one topological volume into another. It is therefore used to reduce equivalent sources calculated in a higher volume to the level of penetration in a lower volume.

The "outgoing" waves are those leaving a junction. It is observed that they are identified with $W_i(0)$. The "incoming" waves are those entering the junctions: they are therefore identified with $W_i(L_i)$ where L_i is the length of the tube on which propagates wave $W_i(z_i)$.

III,1.3.2. — *Grouping of the Incoming and Outgoing Waves on the Entire Network: Concept of Supervector*

We now propose to group all the waves.

Since waves W_i are already vector quantities, the resulting vector becomes a vector whose components are also vectors. It is a supervector denoted between brackets $[W(z)]$.

Accordingly, if we define the order of a network as the number of waves propagating in the network and where N is this number, the supervector has the following structure:

$$[W(z)]=\begin{bmatrix} W_1(z) \\ W_2(z) \\ \vdots \\ W_N(z) \end{bmatrix}$$

We can therefore define:

— the incoming wave supervector, denoted $[W(L)]$ where $z=L$ means that the waves are taken at the end of the tubes in which they propagate.

— the outgoing wave supervector, denoted $[W(0)]$ where $z=0$ means that the waves are taken at the origin of the tube.

III,1.3.3. — *Equations Relating the Incoming and Outgoing Wave Supervectors*

1. Concept of Supermatrix

The wave supervectors defined above remain vectors broken down by blocks. Accordingly, the matrices defining linear operations on these vectors can also be broken down into blocks. Let us consider a supermatrix $[A]$ multiplied by a supervector $[V]$. The result is the supervector denoted $[Y]$.

Processing of the supervectors by blocks clearly shows blocks in supermatrix $[A]$.

In effect, we have:

$$[Y]=[A].[X] \qquad (1)$$

let us assume that $[Y]=\begin{bmatrix} Y_1 \\ \vdots \\ Y_N \end{bmatrix}$ and whatever $1 \leqq i \leqq N$, $Y_i=\begin{pmatrix} y_1^i \\ \vdots \\ y_{n_i}^i \end{pmatrix}$, where n_i is the dimension of vector Y_i and y_k^i, is the k-th component of Y_i and, similarly, $[X]=\begin{bmatrix} X_1 \\ \vdots \\ X_m \end{bmatrix}$ and, whatever $1 \leqq j \leqq M$, $X_j=\begin{pmatrix} x_1^i \\ \vdots \\ x_{m_j}^i \end{pmatrix}$, where m_j is the dimension of vector X_j and x_l^i, $1 \leqq l \leqq m_j$ is the l-th component of X_j.

Equation (1) can be developed as follows:

$$\begin{bmatrix} Y_1 \\ \vdots \\ Y_i \\ \vdots \\ Y_N \end{bmatrix} = \begin{bmatrix} & & \\ & A_{i,j} & \\ & & \end{bmatrix} \times \begin{bmatrix} X_1 \\ \vdots \\ X_j \\ \vdots \\ X_M \end{bmatrix} \qquad (2)$$

where:

$[A]$ = supermatrix with dimension $N \times M$ (N rows, M columns);

$A_{i,j}$ = matrix with dimension $n_i \times m_j$.

Equation (2) can be broken down still further by using coordinates y_k^i and x_e^i

$$\begin{bmatrix} \begin{pmatrix} y_1^1 \\ \vdots \\ y_{n_1}^1 \end{pmatrix} \\ \vdots \\ \begin{pmatrix} y_1^i \\ \vdots \\ y_{n_i}^i \end{pmatrix} \\ \vdots \\ \begin{pmatrix} y_1^N \\ \vdots \\ y_{n_N}^N \end{pmatrix} \end{bmatrix} = \begin{bmatrix} & & \\ & A_{i,j} & \\ & & \end{bmatrix} \begin{bmatrix} \begin{pmatrix} x_1^1 \\ \vdots \\ x_{m_1}^1 \end{pmatrix} \\ \vdots \\ \begin{pmatrix} x_1^j \\ \vdots \\ x_{m_i}^j \end{pmatrix} \\ \vdots \\ \begin{pmatrix} x_1^M \\ \vdots \\ x_{m_M}^M \end{pmatrix} \end{bmatrix} \qquad (3)$$

where $A_{i,j}=\begin{pmatrix} a_{1,1}^{i,j} \ldots a_{1,m_j}^{i,j} \\ \vdots \\ a_{n_i,1}^{i,j} \ldots a_{n_i,m_j}^{i,j} \end{pmatrix}$ and $a_{k,l}^{i,j}$ corresponds to the coefficient of matrix $A_{i,j}$ (k-th row and l-th column).

Expression (3) shows that the supervectors can be processed as vectors (they are then placed between parentheses like vectors although they represent the same quantity as the supervectors). For instance, for [Y] and [X]:

(Y) has dimension $N_Y = \sum_{i=1}^{n_N} n_i$;

(X) has dimension $M_X = \sum_{j=1}^{m_M} m_j$.

Similarly, the supermatrices can also be considered as conventional matrices (denoted between parentheses), *i.e.* for [A]:

(A) has dimension $N_Y \times M_X$.

2. Propagation Equation

The purpose of this equation is to relate the incoming wave $[W(L)]$ and outgoing wave $[w(0)]$ supervectors using a so-called "propagation" supermatrix $[\Gamma]$ with dimension $N \times N$ (N: order of the network).

Actually, this matrix expresses the movements of the waves at the ends of each tube.

We therefore set an equation of the type:

$$[W(L)] = [\Gamma][W(0)] + [W^{(s)}] \qquad (4)$$

The $[W^{(s)}]$ term is the source wave supervector expressing coupling of the external sources on the tubes. It can be seen that components $W_i^{(s)}$ are actually source vectors that can be put on the junction where $W_i(L)$ enters. Supermatrix $[\Gamma]$ is a physical characteristic of each tube of the network and does not depend on the waves propagating in it.

3. Scattering Equation

This equation expresses the scattering properties of each junction. It thus relates the outgoing waves $[W(0)]$ with the incoming waves $[W(L)]$ by a supermatrix $[S]$ with dimension $(N \times N)$ (N: number of waves associated with the network), which can be considered as a transfer function. The equation is written:

$$[W(0)] = [S][W(L)] \qquad (5)$$

It should also be specified that $[S]$ is a physical characteristic of all the junctions of the network. Moreover, it assumes that there is no propagation effect in the junction. In effect, the junction always has a characteristic dimension which is less than the wavelength studied.

III,1.3.4. – *General Description of Interactions in the Entire Network: the Generalized BLT Equation*

Equations (4) and (5) allow us to group the propagation and scattering properties of the network in a single equation.

This yields:

$$([1] - [S][\Gamma][W(0)] = [S][W^{(S)}] \qquad (6)$$

in which the only unknown is that of the outgoing waves $[W(0)]$ and [1] represents the unit supermatrix with dimension $N \times N$, where N is the order of the network.

We have called this equation "generalized BLT equation" since its structure is similar to the equation established by **C. E. Baum, T. K. Liu** and **F. Tesche** [3] in the specific case of networks of multiconductor propagation lines.

III,2. – EXPRESSION OF QUANTITIES [S], [Γ], [W]

III,2.1. – *General Case*

As was seen above, quantities $[S]$ and $[\Gamma]$ are characteristic of the electromagnetic properties of the network. Quantity $[W^{(S)}]$ expresses the coupling of the electromagnetic sources on the network. Quantity $[W(z)]$ corresponds to the signal propagating in the network for a given coupling with the outside.

There is no rule for choosing the expression of waves $W(z)$. They must however contain the electromagnetic information best suited to the medium. Thus, for electric lines, the waves are expressed as a function of the voltages and currents; for electromagnetic fields, the waves are a combination of the electric and magnetic fields.

Actually, their expression can be obtained by writing the propagation equations for the medium (as will be seen for multiconductor lines). In any case, the choice of the wave quantities unequivocally conditions quantities $[S]$, $[\Gamma]$ and $[W^{(S)}]$. In particular, the wave quantities determine the units of the supervectors and the amplitude and phase of the components of $[\Gamma]$ and $[S]$ (which are with arbitrary units).

III,2.2. – *Particular Case of Multiconductor Transmission Line Networks*

III,2.2.1. – Advantage and Description of the Model

The model whose results we will describe has the advantage of being able to unambiguously process a genuine propagation on each tube, thereby giving an expression of the propagation matrix $[\Gamma]$. It is actually the model which must be approached to represent all forms of coupling between wire structures.

The electrical model of a tube is that shown in Figure 10. It shows a group of N lines over a length dz. On each of the lines can be defined voltages $V_n(z, s)$ with respect to a common voltage reference and currents $I_n(z, s)$.

To express the electrical couplings between wires, we used impedance matrices $(Z_{n,m}(s))$ and admittance

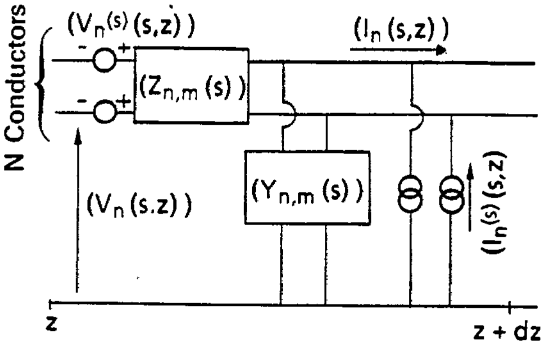


Fig. 10. — Model of an elementary cell for a multiconductor line.

matrices $(Y_{n,m}(s))$, both assumed in independent of the abscissa z and defined per unit length dz.

To express the external coupling, we used serial voltage generators $V_n^{(s)}(z, s)$ and parallel current generators $I_n^{(s)}(z, s)$, defined per unit length dz.

We can write two differential matrix equations:

$$\frac{d}{dz}(V_n(z, s)) = -(Z_{n,m}(s)).(I_n(z, s)) + (V_n^{(s)}(z, s)) \quad (7)$$

$$\frac{d}{dz}(I_n(z, s)) = -(Y_{n,m}(s)).(V_n(z, s)) + (I_n^{(s)}(z, s)) \quad (8)$$

By setting

$$(\gamma_{cn,m}(s)) = [(Z_{n,m}(s)).(Y_{n,m}(s))]^{1/2}, \quad (9)$$

propagation coefficient matrix (actually we take the main value of the matrix product, *see* Appendix 1), and

$$(Z_{cn,m}(s)) = (\gamma_{cn,m}(s))(Y_{n,m}(s))^{-1}, \quad (10)$$

characteristic impedance matrix, making the following changes of variables:

$$(V_n(z, s))_+ = (V_n(z, s)) + (Z_{cn,m}(s))(I_n(z, s)) \quad (11)$$

$$(V_n(z, s))_- = (V_n(z, s)) - (Z_{cn,m}(s))(I_n(z, s)) \quad (12)$$

$$(V_n^{(s)}(z, s))_+ = (V_n^{(s)}(z, s)) + (Z_{cn,m}(s))(I_n^{(s)}(z, s)) \quad (13)$$

$$(V_n^{(s)}(z, s))_- = (V_n^{(s)}(z, s)) - (Z_{cn,m}(s))(I_n^{(s)}(z, s)) \quad (14)$$

differential equations (7) and (8) are written:

$$\left[(1_{n,m})\frac{d}{dz} + (\gamma_{cn,m}(s))\right] \times (V_n(z, s))_+ = (V_n^{(s)}(z, s))_+ \quad (15)$$

$$\left[(1_{n,m})\frac{d}{dz} - (\gamma_{cn,m}(s))\right] \times (V_n(z, s))_- = (V_n^{(s)}(z, s))_- \quad (16)$$

where $(1_{n,m})$ represents the unit matrix.

To relate these results to the topological network formalism, we set:

$$W_n(z) = (V_n(z, s))_+, \quad (17)$$

which solution corresponds effectively to a wave propagating in the direction of increasing z on the tube (forward waves), as is shown by equation (15).

It should be noted that to recover the quantities with a physical meaning, $(V_n(z, s))$ and $(I_n(z, s))$, we need quantities $(V_n(z, s))$ corresponding to backward waves. In effect, according to (11) and (12) we have

$$(V_n(z, s)) = \frac{1}{2}[(V_n(z, s))_+ + (V_n(z, s))_-] \quad (18)$$

and

$$(Z_{cn,m}(s)).(I_n(z, s)) = \frac{1}{2}[(V_n(z, s)_+ - (V_n(z, s))_-] \quad (19)$$

The backward wave concept is preserved in the case of topological networks, since we arbitrarily assigned two forward waves propagating in opposite directions to each tube. Figure 11 shows the correspondence between these quantities.

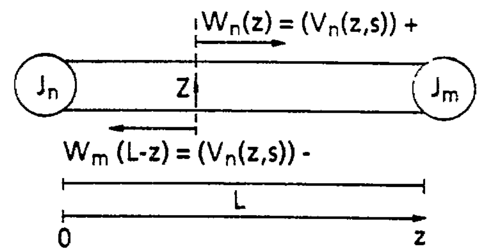


Fig. 11. — Concept of backward wave on topological networks.

Generally, if two waves $W_n(z)$ and $W_m(z)$ propagate in opposite directions on a tube, we can write that since

$$(V_n(z, s))_+ = W_n(z)$$

hence (20)

$$(V_n(z, s))_- = W_m(L-z)$$

III,2.2.2. — Solution of the Propagation Equations

The solution of equation (15) gives:

$$(V_n(z, s))_+ = \{\exp-(\gamma_{cn,m}(s))[z-z_0]\}(V_n(z_0, s))_+ + \int_{z_0}^{z}\{\exp-(\gamma_{cn,m}(s))[z-z']\} \times (V_n^{(s)}(z', s))_+ dz' \quad (*) \quad (21)$$

(*) See Appendix 2: "Exponential of a Matrix" and Appendix 3: "Integral of a Matrix".

where z_0 is an arbitrary origin on the tube. It should be noted that this expression allows the signal to be determined in any point of the tube.

If, in equation (21), we take $z_0=0$ and $z=L$, we obtain:

$$(V_n(L, s))_+ = \{\exp-(\gamma_{cn,m}(s)).L\}(V_n(0, s))_+ + \int_0^L \{\exp-(\gamma_{cn,m}(s))[L-z']\}(V_n^{(s)}(z', s))_+ dz' \quad (22)$$

If we attempt to identify equation (22) with the topological network propagation equation (4), taking (17) into account, it is shown that we have on each tube:

$$(\Gamma_n)=\{\exp-(\gamma_{cn,m}(s).L\} \quad (23)$$

propagation matrix, and

$$(U_n^{(s)}(s))_+ = \int_0^L \{\exp-(\gamma_{cn,m}(s))[L-z']\} \times (V_n^{(s)}(z', s))_+ dz' \quad (24)$$

source wave vector.

III.2.2.3. – Description of the Entire Network

Equations (17), (23) and (24) allow the quantities of the entire network to be grouped as forward wave supervectors denoted $[(V(z, s)]_+$, propagation supermatrix $[\Gamma]$ and scattering supermatrix $[S]$.

The scattering equation is then written:

$$[(V(0, s))]_+ = [S][(V(L, s))]_+ \quad (25)$$

and the propagation equation is written:

$$[(V(L, s))]_+ = [\Gamma][(V(0, s))]_+ + [(U^{(s)}(s))]_+ \quad (26)$$

Combining equations (25) and (26) then gives the BLT equation in its known form as established by **C. Baum, T. K. Liu, F. Tesche** [3]:

$$\{[1]-[S][\Gamma]\}[(V(0, s))]_+ = [S][(U^{(s)}(s))]_+ \quad (27)$$

III,3. – SOLUTION OF THE BLT EQUATION

III,3.1. – Block Matrix Computation: Advantage of the Characteristic Matrices

Let us go back to the BLT equation defined in (6) and set:

$$[I]=[1]-[S][\Gamma] \quad (28)$$

interaction supermatrix.

Solving the BLT equation then consists of isolating unknown $[W(0)]$ by inverting the interaction matrix. We can write:

$$[W(0)]=[I]^{-1}[S][W^{(s)}] \quad (29)$$

However, care must be taken when applying the conventional matrix inversion methods. In effect, supermatrices $[I]$ and $[S]$ must be considered as large, poorly conditioned matrices (they include many null values).

This is why it is preferable to solve (29) by working directly on the matrix blocks. Concretely, a method must be found to allow only the nonzero blocks of the supermatrices to be kept in the memory. For this purpose, we introduce the characteristic matrices of the network which allow the network geometry to be numerically described at the same time as indicating the location of the nonzero blocks of the supermatrices used in (6).

The computation must be conducted in several steps. The characteristic matrices are used to group the coefficients of the propagation and scattering matrices in blocks. Then, for each computation, the characteristic matrices are tested to check which blocks must be used, thereby avoiding computations on null matrix blocks.

The topological description of the problem expressed numerically thus allows the computations to be simplified and structured.

III,3.2. – *Definition of the Characteristic Matrices*

III.3.2.1. – Characteristic Matrices

Characteristic matrices are matrices of indicators whose values correspond to a code used to describe the topological network. Typically, the values of these coefficients are 0, +1, −1. These matrices therefore give the geometric relations existing between the three constituents of the network: junctions, waves, tubes.

The coding depends on the choice of each user and a small number of characteristic matrices can completely describe the network (although the computations may be facilitated by redundant indicators). We therefore chose the following characteristic matrices:

- the Junction-Junction matrix: $(J.J)$;
- the Wave-Wave matrix: $(W.W)$;
- the Junction-Wave matrix: $(J.W)$.

Finally, to completely describe an aggression, it is necessary to know the tubes on which electromagnetic couplings of external sources occur: we therefore introduce the characteristic Wave-Source vector $(W.S)$.

III,3.2.2. – The Junction-Junction Matrix $(J.J)$

This matrix is defined by the following code: given two different junctions J_v and $J_{v'}$

$$\begin{cases} (J.J)_{vv'}=1 \text{ if junctions } J_v \text{ and } J_{v'} \text{ are connected by a tube} \\ (J.J)_{vv'}=0 \quad \text{else.} \end{cases}$$

For instance, the $(J.J)$ matrix corresponding to the network described in Figure 9*b* is as follows:

$(J.J)$ = (junction number ≡ column index; junction number ≡ row index)

	6	4	5	7	8	9
6	0	1	0	0	0	0
4	1	0	1	0	0	0
5	0	1	0	1	0	0
7	0	0	1	0	1	1
8	0	0	0	1	0	0
9	0	0	0	1	0	0

It can be noted that this matrix is sufficient to fully describe the tree structure of the network.

III,3.2.3. – The Wave-Wave Matrix $(W.W)$

Given two waves, W_i and W_j, the matrix indicators are given by:

$(W.W)_{ij}=1$ if there is a junction J such that wave W_i leaves J and wave W_j arrives on J

$(W.W)_{ij}=0$ else.

In the example of Figure 9*b*, the Wave-Wave matrix is written:

$(W.W)$ = (wave number ≡ column index; wave number ≡ row index)

	1	2	3	4	5	6	7	8	9	10
1	0	1	0	0	0	0	0	0	0	0
2	1	0	0	1	0	0	0	0	0	0
3	1	0	0	1	0	0	0	0	0	0
4	0	0	1	0	0	1	0	0	0	0
5	0	0	1	0	0	1	0	0	0	0
6	0	0	0	0	1	0	0	1	0	1
7	0	0	0	0	1	0	0	1	0	1
8	0	0	0	0	0	0	1	0	0	0
9	0	0	0	0	1	0	0	1	0	1
10	0	0	0	0	0	0	0	0	1	0

This matrix is very important for the construction of the scattering supermatrix $[S]$. The nonzero coefficients are in effect significant of the location of the nonzero blocks in this supermatrix.

III,3.2.4. – The Junction-Wave Matrix $(J.W)$

This matrix is used to determine the junction to which each wave is related. We can adopt the following convention: given a wave W_i and a junction J_v

$$\begin{cases} (J.W)v_i=1 & \text{if } W_i \text{ leaves } J_v \\ (J.w)v_i=-1 & \text{if } W_i \text{ arrives on } J_v \\ (J.w)v_i=0 & \text{else} \end{cases}$$

Let us go back to the example of Figure 9*b*. The characteristic $(J.W)$ matrix is as follows:

$(J.W)$ = (wave number i ≡ column index; Junction number v ≡ row index.)

	1	2	3	4	5	6	7	8	9	10
6	1	−1	0	0	0	0	0	0	0	0
4	−1	1	1	−1	0	0	0	0	0	0
5	0	0	−1	1	1	−1	0	0	0	0
7	0	0	0	0	−1	1	1	−1	1	−1
8	0	0	0	0	0	0	−1	1	0	0
9	0	0	0	0	0	0	0	0	−1	1

III,3.2.5. – The Wave-Source Vector $(W.S)$

This characteristic matrix is used to determine the tubes on which there exists an electromagnetic coupling due to an external source. We use the following convention: consider a wave W_i:

$$\begin{cases} (W.S)_i=1 \text{ if there is a source coupled on the propagation tube of } W_i \\ (W.S)_i=0 \text{ else.} \end{cases}$$

This characteristic vector is used to determine the location of the blocks of nonzero vectors of supervector $[W^{(s)}]$.

In the example of Figure 9*b*, $(W.S)$ equals:

$(W.S)$ = (wave number ≡ row index)

1	1
2	1
2	1
4	1
5	1
6	1
7	1
8	1
9	1
10	1

IV. – CONCLUSION

Electromagnetic topology was developed in the USA by **C. E. Baum.** To our knowledge, no paper making a synopsis of the knowledge required for practical use of this method has yet been published. Herein, we detailed the steps in the topological processing of a problem: the topological breakdown and writing of the BLT equation.

The first operation, undoubtedly the most important and the most difficult, consists of breaking space down into regions to obtain a hierarchy of the channels of penetration of the energy into the structure.

Depending on the subject investigated (currents on shieldings or signals induced on cables), we show that there are several possibilities of breakdown, using a simplified example.

The second operation consists of establishing an equation which governs the scattering and propagation of the interference signals in any point of the system: this equation, established and applied by C. E. Baum in the framework of multiconductor lines, is presented herein in a form allowing all types of wave propagtion to be taken into account.

It appeared that electromagnetic topology, although using a very complicated mathematical formalism, represents a real simplification of the problem. In effect, it may appear unsuitable for processing a simple case, but in complex cases where conventional methods are inoperative, it continues to be usable with no greater difficulty. This good suitability to processing of complex systems is mainly related to two characteristics: the ease of computerization and the modularity.

The modularity is due to the capacity of the method to break a problem down into several elementary problems and manage their interactions. This breakdown requires creating a library of "elementary coupling modules" to identify and express the widest variety of electromagnetic problems. Certain of these modules are already formalized (diffusion through a wall, coupling of fields through openings, etc.). Unfortunately, the presentation of the results often lends itself poorly to their integration in the topological method and they therefore have to be translated. Other modules remain to be developed and will be the subject of future work.

V. – MATHEMATICAL APPENDICES

APPENDIX 1

Calculation pf the Principal Value of a Matrix

Let us consider a diagonalizable matrix (A); we shall call the principal value of (A) a matrix (B) such that $(B)^2=(A)$.

In effect, let us diagonalize (A); let $(A)_d$ be the diagonal matrix. There exists a unit matrix P such that:

$$(A)=(P)^{-1}(A_d)(P)$$

where

$$(A_d)_{ij}=\begin{cases}\lambda_i & \text{if } i=j\\ 0 & \text{else}\end{cases}$$

where $(P)^{-1}$ is the inverse matrix of (P).

Let us set

$$(B)=(P)^{-1}(A_d)^{1/2}(P)$$

where

$$(A_d)_{i,j}^{1/2}=\begin{cases}\lambda_i^{1/2} & \text{if } i=j\\ 0 & \text{else}\end{cases}$$

It is verified that $(B)^2=(A)$. In effect:

$$(B)^2=(P)^{-1}(A_d)^{1/2}(P).(P)^{-1}(A_d)^{1/2}(P)$$
$$=(P)^{-1}(A_d)(P)=(A)$$

APPENDIX 2

Exponential of a Matrix

Let us consider matrices (A), (A_d) and (P) of Appendix 1.

We have

$$(\exp(A))=\exp\{(P)^{-1}(A_d)(P)\}$$
$$=\left(\sum_{i=1}^{n}(P)^{-1}(A_d)^n(P)\right)$$
$$=(P)^{-1}(\exp(A_d))(P)$$

where

$$(\exp(A_d))_{ij}=\begin{cases}e^{\lambda i} & \text{if } i=j\\ 0 & \text{else.}\end{cases}$$

APPENDIX 3

Integral of a Matrix

We consider a matrix (A) denoted $(a_{ij}(x))$ depending on a variable (x).

The integral of matrix (A), denoted $\int(A)\,dx$, is the matrix B whose elements are $B_{ij}=\int a_{ij}(x)\,dx$.

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