ELECTROMAGNETIC TOPOLOGY:  
JUNCTION CHARACTERIZATION METHODS

by

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ABSTRACT

This paper deals with the electrical characterization of topological network junctions, using a formalism in accordance with Electromagnetic Topology. More precisely, it is shown how the characteristic impedance of the network in which the junction is located can be chosen to treat several physical cable connection and termination configurations.

It is possible to characterize, in the electromagnetic sense, any junction of order $N$ (with $N$ ports), by considering it as the connection of two junctions of lower order which it is easier to model analytically, or to measure its $Y$ or $S$ parameter.

A topological approach has been adopted to obtain the result, not only because the BLT equation is applied but also because the concept is used for breaking space down into topological volumes. Moreover, this method is well suited for numerical treatment.

The case of junctions called "forks", which represent the separation of a conductor into other conductors, is given as an example.

Keywords (NASA thesaurus): Electromagnetic Topology—BLT equation—Scattering coefficients—Electromagnetic radiation—Transmission lines—Electromagnetic wave transmission—Complex systems.

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(a) Electromagnetic Topology.

- Interaction diagram: graphic representations of the breakdown of a problem into several problems included in one another and limited to so-called topological volumes; their surface is called "topological surface".

- Interaction graph: graphic representation of the interactions between the various topological volumes. Each volume is symbolized by a so-called volume node. The channel of penetration of one volume into another is symbolized by a so-called surface node. The nodes are interconnected by edges expressing the electromagnetic interactions.

(b) Topological Network of Lines

A network consists of tubes interconnecting the junctions. The tubes are propagation lines. The junctions express the scattering of a signal. There are surface junctions associated with the surface nodes of an interaction graph and volume junctions associated with the volume nodes. The signal transmitted on the network is called wave. It can be electrical (case of physical lines) or electromagnetic (case of fictitious propagation channel of an interference).

- Order of a junction: number of ports of the junction (from the standpoint of an electric circuit, it is the number of conductors connected to the junction). As tubes can include several conductors, the number of tubes connected to a junction is necessarily less than or equal to the order of the junction.

(c) Matrix and Vector Notations

A matrix $A$ is denoted $(A)$ and a vector $V$ is denoted $(V)$. Several quantities of this type are defined herein:

- $(S)$ = matrix of the $S$ parameters of a junction.
- $(S_0)$ = matrix of the microwave $S$ parameters of a junction (generally referenced to a characteristic scalar impedance $Z_0 = 50 \Omega$).
- $(Y)$ = matrix of the $Y$ parameters of a junction.
- $(V)$ = column vector of the input voltages of the junction.
- $(I)$ = column vector of the input currents in the junction.
- $(Z_c)$ = characteristic impedance matrix.
- $(\Gamma)$ = propagation matrix.

(d) Supermatrix and Supervector Notations

A supermatrix is generally a large matrix in which the row and column indexes have been grouped into superindexes in order to break the matrix down into blocks. This allows a supermatrix to be considered as a matrix of matrices. Similarly, a supervector is considered a vector of vectors.

The notation of a supermatrix $A$ is $[A]$ (in brackets); similarly, the notation of a supervector $V$ is $[V]$.

We also use blocks of supermatrix $A$ denoted $[A]_i, j$ where $I$ and $J$ are superindexes in Roman numerals. Similarly, a block of supervector $V$ is denoted $[V]_i$.

When we wish to specify the size of a supermatrix or a matrix block, we write $[A]_{n \times m}$ where $n$ is the number of rows and $m$ is the number of columns. Thus, we define a zero block by $[0]_{n \times m}$ and a unit block by $[1]_{n \times m}$.

Similarly, the size of a vector block is defined by writing $[V]_n$ where $n$ is the number of components. A zero vector block is thus denoted $[0]_n$.

The main supermatrices and supervectors used in this paper are:

- $[Y]$: admittance supermatrix;
- $[S]$: supermatrix of topological $S$ parameters or scattering supermatrix;
- $[S_0]$: supermatrix of microwave $S$ parameters or microwave scattering supermatrix;
- $[W(z)]$: wave supervector propagating on the network;
- $[H^{o}]$: source supervector applied to the junctions.

I. - INTRODUCTION

AND STATEMENT OF THE PROBLEM

Electromagnetic topology is a method used to approach a complex electromagnetic coupling problem. One of its original features is to break space down into several volumes in which the problem is solved locally independently of the neighboring volumes [1, 2, 4]. The formalism best suited for describing the interactions between volumes and the path of interference through the volumes is inspired from that of multiconductor transmission line networks generalized to topology.

The signals propagating in these networks are waves whose mathematical expression contains electromagnetic information (fields $E$ and $H$) or electrical information (voltage $V$ and current $I$). A network consists of a set of tubes through which propagate the waves, interconnected by junctions insuring distribution of the waves [3].
On each tube can be defined a propagation matrix \( (\Gamma) \) connecting the waves to each end and to each junction, a scattering matrix \( (S) \) (called matrix of parameters \( S \)). Each of these matrices is used to fill the blocks of supermatrices \( [S] \) and \( [\Gamma] \) of the topological network \([4]\).

The BLT (Baum-Liu-Tesche) equation is used to determine the signals in any point of the network by grouping the propagation and distribution equations. Its expression generalized to waves is as follows \([4]\):

\[
\{ [1] - [S][\Gamma] \} \{W(0)\} = [S]\{W^{xS}\} \tag{1}
\]

where
- \([1]\) is the unit matrix;
- \([S]\) is the network distribution supermatrix;
- \([\Gamma]\) is the network propagation supermatrix;
- \([W(0)]\) is the supervector of the waves leaving the junctions;
- \([W^{xS}]\) is the supervector of the source waves applied to the junctions.

The concept of supermatrix means that all the calculations of equation (1) can be made by blocks.

In this paper, our analysis is limited to networks or portions of networks for which the voltage and current quantities can be defined unambiguously. We then propose to determine the topological \( S \) parameters of any junction with \( N \) ports.

It may not always be easy to measure or model the electric properties of such junctions. This is why, after describing a general method for determining the \( S \) parameters, we propose to approach this problem by considering the junction of order \( N \) as the union of two junctions of an order lower than \( N \) and whose \( S \) parameters are known (or easier to determine).

II. GENERAL METHOD FOR DETERMINING THE \( S \) PARAMETERS OF A JUNCTION

II.1. REVIEW OF THE SCATTERING EQUATION OF THE JUNCTION

A junction of order \( N \) is characterized by a scattering matrix relating the outgoing waves with the incoming waves. In the case of a multiconductor line, the following expressions \([2, 5]\) are generally associated with the incoming and outgoing waves:

\[
(V) + (Z_j)(I) \quad \text{for the incoming wave}
\]

\[
(V) - (Z_j)(I) \quad \text{for the outgoing wave}
\]

Where \((V)\) and \((I)\) are vectors with dimension \(N\) of the voltages and currents incoming in each port of the junction, \((Z_j)\) is a matrix with dimension \(N \times N\) called characteristic matrix of the junction, which characterizes propagation on the lines connected to each port.

The scattering equation is defined by the following relation \([2]\):

\[
(V) - (Z_j)(I) = (S)(V) + (Z_j)(I) \tag{2}
\]

\((S)\) is a matrix with dimension \(N \times N\) called junction scattering matrix.

II.2. IMPORTANCE OF \((Z_j)\) FOR THE DETERMINATION OF \((S)\)

Equation (2) shows that matrix \((S)\) is referenced to a characteristic impedance matrix \((Z_j)\) which expresses the propagation and electrical coupling properties of the lines connected to the \(N\) ports of the junction.

In network topology, the lines are grouped into one or more tubes, according as they are electrically coupled or not \([4]\).

We will use two simple examples to illustrate how the \( S \) parameters of a given junction can have a different expression depending on the topological network used.

Let us take the example of a junction \( J \) of order \( N \). Let us consider that this junction is the electrical termination of \( N \) lines which we will assume to be electrically coupled.

The topological representation of the junction in a network is that shown in Figure 1, where only a single tube is to be considered. Matrix \((Z_j)\) to be considered in (2) is then full so as to express the coupling of the \(N\) lines connected to the \(N\) ports of the junction. It has the form:

\[
(Z_j) = \begin{pmatrix}
Z_{11} & \cdots & Z_{1N} \\
\vdots & \ddots & \vdots \\
Z_{N1} & \cdots & Z_{NN}
\end{pmatrix}
\]

Let us now consider that two groups of coupled lines \((N1\) and \(N2\) lines respectively) are connected to \(N1\) and \(N2\) ports of this same junction \( J \). Assuming that these two groups do not interact, we can represent them topologically as two tubes, \( I \) and \( II \), connected to \( J \) (see Fig. 2).
Matrix \( Z_c \) that we must consider in (2) is structured as follows:

\[
(Z_c) = \begin{pmatrix} (Z_{c1}) & \underbrace{(0)_{N1 \times N2}} \\ \underbrace{(0)_{N2 \times N1}} & (Z_{cII}) \end{pmatrix}
\]

where \( (Z_{c1}) \) is a matrix block with dimension \( N1 \times N1 \) corresponding to the characteristic impedance matrix of tube I and \( (Z_{cII}) \) with dimension \( N2 \times N2 \) corresponds to tube II. The notation \( (0)_{N1 \times N2} \) and \( (0)_{N2 \times N1} \) expresses two null matrix blocks with dimension \( N1 \times N2 \) and \( N2 \times N1 \).

II.3. — ELECTRICAL CHARACTERIZATION OF A JUNCTION

As was seen, the expression of matrix \( (S) \) depended on a characteristic impedance matrix \( (Z_c) \). This representation is therefore not an intrinsic representation of the electrical properties of a junction. With this in mind, matrix \( (Y) \) of the admittance parameters appears better suited. It is defined by the following relation:

\[
(I) = (Y)(V)
\]

(3)

However, the \( Y \) parameters are not iterative, which means that if several junctions are cascaded, the output quantities of a junction become the input quantities of the next junction. For topological processing, a characterization closer to the topological \( S \) parameters may therefore be preferred.

Let us assume that each of the ports is referenced to a unique scalar characteristic impedance \( Z_{c0} \). Equation (2) becomes:

\[
(V) - Z_{c0}(I) = (S_0)[(V) + Z_{c0}(I)]
\]

(4)

It can be seen that if each of vectors \( (V) \) and \( (I) \) is divided by \( 2\sqrt{Z_{c0}} \), expression (4) corresponds exactly to the definition of the microwave \( S_0 \) parameters [5]. Combining this equation with (3) yields the expression for the \( Y \) parameters:

\[
(Y) = \frac{1}{Z_{c0}}[(1) - (S_0)]^{-1}[(1) - (S_0)]
\]

(5)

Equations (4) and (5) therefore show that a junction can be characterized independently of the network by conventional methods for determining the admittance \( Y \) parameters or the microwave \( S_0 \) parameters.

The evaluation of the characteristic impedance of the medium \( (Z_c) \) then allows the topological \( S \) parameters of the network to be found by the following equation:

\[
(S) = [(1) - (Z_c)(Y)][(1) + (Z_c)(Y)]^{-1}
\]

(6)

determined from equations (2) and (3).

III. — CHARACTERIZATION OF A JUNCTION OF ORDRE \( N \)

USING TWO JUNCTIONS OF ORDERS \( M1 \) AND \( M2 \)

III.1. — STATEMENT OF THE PROBLEM

It is not always easy to electrically characterize a junction by measurement or analytic modeling, especially when the number of ports is large. This is why we had the idea of breaking down a junction \( J \) of order \( N \) into an arrangement of two other junctions, \( J_a \) and \( J_b \), for which the expression of the \( S \) parameters is known and with orders \( M1 \) and \( M2 \) respectively.

The principle consists of sharing \( k \) ports between \( J_a \) and \( J_b \). It is demonstrated that \( k \) is equal to:

\[
k = \frac{M1 + M2 - N}{2}
\]

In order to express the \( S \) parameters of \( J \) as a function of those of \( J_a \) and \( J_b \), we adopted a topological approach consisting of comparing two networks containing \( J \) and \( J_a \), \( J_b \) respectively. To facilitate our reasoning, we assumed that all the \( S \) parameters used here were referenced to a single scalar characteristic impedance \( Z_{c0} \) (typically 50 Ohms).

Once junction \( J \) of order \( N \) is constructed, its \( S \) parameters can be referenced to any characteristic impedance matrix \( (Z_c) \) using equations (4), (5) and (6).

Figure 3 shows the two topological networks.

Network 1 includes junction \( J \) of order \( N \). Tubes \( T_1 \) connected to the junction are assumed to contain only one conductor. At the end of each of tubes \( T_1 \), we included a terminal junction \( J_p \).

Network 2 includes two junctions, \( J_a \) and \( J_b \), of orders \( M1 \) and \( M2 \). Tube \( T_0 \) interconnects \( J_a \) and \( J_b \). It contains the lines interconnecting the \( k \) ports of the two junctions. The dimension of the two wave vectors \( [W]^a \) and \( [W]^b \) propagating on \( T_0 \) is therefore equal to \( K = 2k \).
correspond to a terminal impedance equal to the reference characteristic impedance $Z_0$.

The identification of the networks also assumes that there is no propagation on tube $T_0$ (by definition, a junction must be small compared with the wavelength used).

III.2. SYNTHETIC REPRESENTATION OF THE PROBLEM

We can simplify the representation of networks 1 and 2 of Figure 3 by advancing purely topological considerations. Figures 4a and b shows networks 1 and 2 with space broken down into a volume $V_0$ separated from an external volume $V_{ext}$ by a surface $S_0$. Under these conditions, junctions $J_a$, $J_b$ and $J$ can be assimilated to surface junctions and junctions $J_i$ to volume junctions.

We are then able to determine the topological graph corresponding to the above topological diagram. The graph shows the interactions between volumes. Each volume is represented by a so-called “volume” node; each surface is represented by a so-called “surface” node. The interaction between volumes is expressed by arcs connecting the volume nodes to the surface nodes.

The topological graph corresponding to the volume breakdown defined above is illustrated in Figures 5a
and $b$. It shows a volume node $N_{II}$ corresponding to the source located in $V_{III}$ and two surface nodes $N_I^b$ and $N_I$. Nodes $N_{II}$ and $N_I^b$ in Figure 5a and nodes $N_{III}$ and $N_I$ in Figure 5b interact. Node $N_I^b$ interacts with itself (expressing the interaction between junctions $J_a$ and $J_b$).

We can then define a new representation of topological networks 1 and 2 similar to the topological graphs of Figures 5a and $b$, thereby allowing a better synthesis of the volume interactions and providing more condensed analytic formulas. A junction $J_{II}$ grouping all the terminal junctions $J_a$ is made to correspond to volume node $N_{II}$. Similarly, $J_{III}$ of Figure 6a corresponds to surface node $N_{III}$ expressing the grouping of junctions $J_a$ and $J_b$. Junction $J_{I}$ of Figure 6b associated with surface $N_I$ represents junction $J_{II}$.

The reconfiguration of the two topological networks defined in Figure 3 leads us to group all the vector quantities in supervectors. Thus, the group of source vectors ($W^{(s)}$), allows us to define a supervector denoted [$W_{III}$] located in junction $J_{II}$. Similarly, on each of the tubes, we define wave supervectors [$W_{III}$] and [$W_I$] for the incident and reflected waves respectively on junctions $J_{II}$ and $J_{I}$. Wave $W_{III}$ shown only in Figure 6a expresses the propagation in $V_{II}$.

two quantities can be grouped to obtain [$W(0)$], i.e.:

$$ [W(0)] = \begin{bmatrix} [W]_I \\ [W]_{II} \end{bmatrix} \quad (7) $$

Similarly, supervector [$W^{(S)}$] is expressed by:

$$ [W^{(S)}] = \begin{bmatrix} [0]_N \\ [W^{(S)}]_{III} \end{bmatrix} \quad (8) $$

It can therefore be seen that supervectors $[\Gamma]$ and $[S^r]$ can also be treated by blocks according to indexes $I$ and $II$.

Supermatrix $[\Gamma]$ is a unit matrix since the propagation effect on the network is ignored. We can therefore write:

$$ [\Gamma] = \begin{bmatrix} [\Gamma]_{II, I} & [0]_{N \times N} \\ [0]_{N \times N} & [\Gamma]_{II, II} \end{bmatrix} \quad (9) $$

where:

$$ [\Gamma]_{I, I} = [\Gamma]_{II, II} = [1]_{N \times N} \quad (10) $$

Supermatrix $[S]$ which we shall denote $[S^r]$ for case 6b is obtained by writing the characteristic wave-wave matrix ($W, W$) for the network [4] relative to indexes $I$ and $II$:

$$ (W, W) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (11) $$

This matrix has the advantage of indicating the location of the nonzero blocks of $[S]$; we can therefore write:

$$ [S] = \begin{bmatrix} [0]_{N \times N} & [S]_{II, II} \\ [S^r]_{III, I} & [0]_{N \times N} \end{bmatrix} \quad (12) $$

Considering the assumption of no reflection on the terminal junctions and since $[S^r]_{III, I}$ corresponds to the matrix of the $S$ parameters of junction $II$, we have:

$$ [S^r]_{III, I} = [0]_{N \times N} \quad (13) $$

III.3. — WRITING OF THE BLT EQUATION FOR THE NETWORK INCLUDING THE JUNCTION OF ORDER $N$ (Fig. 6b)

Let us go back to equation (1) and attempt to express supervectors [$W(0)$] and [$W^{(S)}$] with dimension $2N$ as well as the propagation supermatrix $[\Gamma]$ and a scattering supermatrix $[S]$ with dimension $2N \times 2N$.

Tube 1 is the support for two waves, $W_I$ and $W_{III}$ which can be represented by [$W_I$] and [$W_{III}$]. These
hence

$$[W]_{II} = [0]_N$$  \hspace{1cm} (14)$$

(see Appendix I).

The BLT equation for the network is then summarized by a single relation:

$$[W]_I = [S']_{II} [W^{esc}]_{II}$$  \hspace{1cm} (15)$$

III.4. - WRITING OF THE NETWORK INCLUDING TWO JUNCTIONS OF ORDERS M1 AND M2 LESS THAN N (Fig. 6a)

The network studied is that of Figure 6a. It includes three wave supervectors, $[W]_I$, $[W]_{II}$, $[W]_{III}$. Indexes $I$ and $II$ are associated with vectors of dimension $N$; index $III$ is associated with vectors of dimension $K = M1 + M2 - N$ (see Sec. III,1).

These indexes can be used to define all the quantities of relation (1).

We therefore have:

$$[W](0) = \begin{bmatrix} [W]_I \\ [W]_{II} \\ [W]_{III} \end{bmatrix}$$  \hspace{1cm} (16)$$

$$[W^{esc}] = \begin{bmatrix} [0]_N \\ [W^{esc}]_{II} \\ [0]_K \end{bmatrix}$$  \hspace{1cm} (17)$$

The absence of any propagation effect in the network means that $[\Gamma]$ has the following structure:

$$[\Gamma] = \begin{bmatrix} [\Gamma]_{II} & [0]_{N \times N} & [0]_{N \times K} \\ [0]_{N \times N} & [\Gamma]_{III} & [0]_{N \times K} \\ [0]_{K \times N} & [0]_{N \times K} & [\Gamma]_{III, III} \end{bmatrix}$$  \hspace{1cm} (18)$$

where

$$[\Gamma]_{II} = [\Gamma]_{II, II} = [1]_{N \times N}$$  \hspace{1cm} (18)$$

and

$$[\Gamma]_{III, III} = [1]_{K \times K}$$  \hspace{1cm} (19)$$

The scattering supervector is constructed from the characteristic $(W, W)$ matrix of the network, i.e.:

$$(W, W) = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$  \hspace{1cm} (20)$$

hence:

$$[S] = \begin{bmatrix} [0]_{N \times N} & [S]_{II, II} & [S]_{II, III} \\ [S]_{III, I} & [0]_{N \times N} & [0]_{N \times K} \\ [0]_{K \times N} & [S]_{III, II} & [S]_{III, III} \end{bmatrix}$$  \hspace{1cm} (21)$$

The assumption of the absence of reflection in the terminal junctions and the fact that $[S]_{II, I}$ is identified with the matrix of the $S$ parameters of $J_{II}$ is expressed by:

$$[S]_{II, I} = [0]_{N \times N}$$  \hspace{1cm} (22)$$

hence

$$[W]_{II} = [0]_N$$  \hspace{1cm} (23)$$

(see Appendix I).

The BLT equation of the network is then summarized by two relations:

$$[S]_{II, I}[W^{esc}]_{II} = [W]_I - [S]_{II, III}[V]_{III}$$  \hspace{1cm} (24)$$

$$[S]_{III, II}[W^{esc}]_{II} = ([1]_{K \times K} - [S]_{III, III})[W]_{II}$$  \hspace{1cm} (25)$$

The elimination of $[W]_{II}$ from (24) and (25) leads us to the relation:

$$[S]_{II, I} + [S]_{II, III} ([1]_{K \times K} - [S]_{III, III})^{-1} [S]_{III, II} [W^{esc}]_{II} = [W]_I$$  \hspace{1cm} (26)$$


Equations (15) and (26) both relate supervectors $[W^{esc}]_{II}$ and $[W]_I$.

They must be identical by construction of the two networks of Figures 6a and b.

We infer that:

$$[S']_{II, II} = [S]_{II, II} + [S]_{II, III} ([1]_{K \times K} - [S]_{III, III})^{-1} [S]_{III, II}$$  \hspace{1cm} (27)$$

where $[S']_{II, II}$ is the distribution matrix for the junction with $N$ ports and $[S]_{II, II}$, $[S]_{II, III}$, $[S]_{III, II}$ and $[S]_{III, III}$ are expressed as a function of the $S_0$ parameters of the two junctions with $M1$ and $M2$ ports (since, as was seen in Sec. II.2, we can reason considering that all the ports are decoupled; it is only a posteriori that supermatrix $[S']$ is referenced to the characteristic impedance supermatrix $[Z]$ of the real case to be treated).

Equation (27) has the same form independently of the number of ports of the system. It can therefore be used recurrently to treat the problem with $N$ ports from problems with three ports.

III.6. - APPLICATION: CHARACTERIZATION OF A JUNCTION OF ORDER 4 USING TWO JUNCTIONS OF ORDER 3

III.6.1. - Definition of the Application

The junction to be characterized is the so-called "four-port fork" function consisting of the separation
of a conductor into three other conductors.

The diagram of Figure 7 shows this fork with an electric cell $C_i$ on each conductor $l$. This expresses an imperfect electrical connection in the connection node. Figure 8 represents an elementary cell $C_i$ by two impedances, $Z'_i$ and $Z_i$ (used, for instance, to express a choke effect), separated by a parallel impedance $Z''_i$ (used for instance to express a capacitive coupling).

The branch points $P_1, P_2, P_3, P_4, P_5$ and $N_a, N_b$ are used to identify the location of the cells on the different conductors.

As an example, we chose a junction such that all its cells $C_i$ exhibited purely resistive impedances, an ideal case reproduced in the laboratory:

Table I gives the values of $Z_i, Z'_i$ and $Z''_i$:

<table>
<thead>
<tr>
<th>Cell</th>
<th>$Z_i$ in $\Omega$</th>
<th>$Z'_i$ in $\Omega$</th>
<th>$Z''_i$ in $\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>0</td>
<td>0</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$C_2$</td>
<td>16.6</td>
<td>16.6</td>
<td>118.44</td>
</tr>
<tr>
<td>$C_3$</td>
<td>33.17</td>
<td>33.17</td>
<td>20.57</td>
</tr>
<tr>
<td>$C_4$</td>
<td>41.05</td>
<td>41.05</td>
<td>10.17</td>
</tr>
<tr>
<td>$C_5$</td>
<td>16.6</td>
<td>16.6</td>
<td>68.19</td>
</tr>
</tbody>
</table>

The microwave $S_i$ parameters (with respect to a characteristic impedance of 50 $\Omega$) were measured by

network analyzer and calculated by a numerical code.

Identical results were found with both methods:

$$(S_i) = \begin{pmatrix}
-0.340 & 0.357 & 0.048 & 0.024 \\
0.357 & -0.039 & 0.026 & 0.013 \\
0.048 & 0.026 & -0.018 & 0.013 \\
0.024 & 0.013 & 0.013 & -0.002
\end{pmatrix}$$

We propose to find these values by breaking the four-port fork into two three-port forks and then applying equation (27).

III,5.2. Breakdown of the Four-Port Fork into Two Three-Port Forks

III,6.2.1. Principle of Characterization of a Three-Port Fork

The three-port fork consists of separating a conductor into two other conductors. As for the four-port fork, we symbolized in Figure 9a any electrical imperfections by cells $C_i$ of the same type as those of Figure 8. The global electrical diagram is shown in Figure 9b. This allows us to establish simple analytical formulas for the $Y$ parameters of the junction (see Appendix 2). The $Y$ parameters can then be transformed into $S$ parameters by equation (6).

It should be noted that these analytic expressions, relatively simple in the case of three-port junctions, become rapidly very complicated to determine when the number of ports is increased. This shows the

![Fig. 9. (a) Model of a three-port fork. (b) Global electrical diagram of a three-port fork.](image-url)
advantage of breaking down junctions with a large number of ports to lower order junctions.

III,6.2.2. Presentation of the First Three-Port Fork

The first three-port fork \( J_a \) that we chose for the breakdown has the properties given in Table II.

<table>
<thead>
<tr>
<th>Cell</th>
<th>( Z_1 ) in ( \Omega )</th>
<th>( Z'_1 ) in ( \Omega )</th>
<th>( Z''_1 ) in ( \Omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_1 )</td>
<td>0</td>
<td>0</td>
<td>( \infty )</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>0</td>
<td>0</td>
<td>( \infty )</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>16.6</td>
<td>16.6</td>
<td>118.44</td>
</tr>
</tbody>
</table>

The corresponding \( S_0 \) parameters are with respect to 50 \( \Omega \):

\[
(S_0) = \begin{pmatrix}
-0.297 & 0.703 & 0.380 \\
0.703 & -0.297 & 0.380 \\
0.380 & 0.380 & -0.027
\end{pmatrix}.
\]

They can be determined by direct measurement or by numerical or analytic computations (see Appendix 2).

III,6.2.3. Presentation of the Second Three-Port Fork

The second three-port fork \( J_b \) has the following characteristics (Table III).

<table>
<thead>
<tr>
<th>Cell</th>
<th>( Z_1 ) in ( \Omega )</th>
<th>( Z'_1 ) in ( \Omega )</th>
<th>( Z''_1 ) in ( \Omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_1 )</td>
<td>16.6</td>
<td>16.6</td>
<td>68.19</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>33.17</td>
<td>33.17</td>
<td>20.57</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>41.05</td>
<td>41.05</td>
<td>10.17</td>
</tr>
</tbody>
</table>

Parameters \( S_0 \) (with respect to 50 \( \Omega \)) are:

\[
(S_0) = \begin{pmatrix}
-0.085 & 0.067 & 0.033 \\
0.067 & -0.017 & 0.013 \\
0.033 & 0.013 & -0.001
\end{pmatrix}.
\]

III,6.2.4. Placing in Common of the Two Three-Port Forks

What remains to be done is to place in common junctions \( J_a \) and \( J_b \) to recover the scheme of Figure 7. For this purpose, we can locate the junctions in this figure according to the various ports \( P_i \). We summarized these properties in Table IV.

III,6.3. Topological Processing of the Grouping of the Two Junctions

The next step consists of filling the matrix blocks of equation (27).

For this purpose, we must identify the two networks described in Figures 10 a and b.

![Figure 10](image)

**Fig. 10.** (a) Network with two three-port junctions. (b) Network with one four-port junction.

The first network corresponds to the interconnection of \( J_a \) and \( J_b \) on ports 2 and 1 respectively and the second to \( J_a \) a junction with four ports.

In the two figures, ports 1, 2, 3 and 4 and waves \( W_1 \) to \( W_6 \) are common, which complies with the principles stated in Sec. III.1.

It can then be demonstrated that the different blocks of (27) are written (see Appendix 3)

\[
[S]_{I,I} = \begin{bmatrix}
-0.297 & 0.380 & 0 & 0 \\
0.380 & -0.027 & 0 & 0 \\
0 & 0 & -0.001 & 0.013 \\
0 & 0 & 0.013 & -0.017
\end{bmatrix}
\]

<table>
<thead>
<tr>
<th>Junction</th>
<th>Port 1 location</th>
<th>Port 2 location</th>
<th>Port 3 location</th>
<th>Separation: node</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_a )</td>
<td>( P_1 )</td>
<td>( P_3 )</td>
<td>( P_2 )</td>
<td>( N_a )</td>
</tr>
<tr>
<td>( J_b )</td>
<td>( P_2 )</td>
<td>( P_3 )</td>
<td>( P_4 )</td>
<td>( N_b )</td>
</tr>
</tbody>
</table>
[according to equation (41) of Appendix 3]

\[
[S]_{III,III} = \begin{bmatrix}
0 & 0.703 \\
0 & 0.380 \\
0.033 & 0 \\
0.067 & 0
\end{bmatrix}
\]

It is interesting to note that the BLT equation (topological description) proved the most efficient for conducting this work aimed at calculating an input term in topological formalism. Electromagnetic topology thus appears not only as a formalism but also as a methodology for approaching electromagnetic coupling problems in global systems such as those encountered in modern aircraft.

[according to equation (42) of Appendix 3]

\[
[S]_{III,II} = \begin{bmatrix}
0.703 & 0.380 & 0 & 0 \\
0 & 0 & 0.033 & 0.067
\end{bmatrix}
\]

[according to equation (43) of Appendix 3]

\[
[S]_{III,II} = \begin{bmatrix}
0 & -0.297 \\
-0.85 & 0
\end{bmatrix}
\]

[according to equation (44) of Appendix 3].

By equation (27) yields the matrix of \([S]_\ell\) parameters of the four-port junction given in Sec. III,6.1.

CONCLUSION

In aeronautics, electric circuits occur as main cable paths and branches connecting numerous equipment items. The cable branches result in impedance discontinuities on the conductors, affecting the signals.

Branches are conventionally represented as junctions with \(N\) ports whose scattering parameters are determined. They can be computed analytically when the number of ports remains small (less than 3) but are almost impossible to compute in the practical case encountered in aeronautics.

In this paper, we used electromagnetic topology to develop signal scattering in a junction with \(N\) ports by the \(S\) parameters. The formulation of a complex problem by the topological method [1, 2, 4] consists of breaking it down into elementary problems expressed in a suitable form (matrix of \(S\) parameters, propagation matrix, etc.).

It appeared that using the formalism of BLT equation led to a recurrent formulation of the scattering matrix for a junction with \(N\) ports from scattering matrices for a junction with a lower order.

The difficulty of the computation does not increase with the complexity of the junction to be described and the method proved to be well suited to numerical processing of complex problems.

APPENDIX 1

DEMONSTRATION OF THE ABSENCE OF REFLECTED WAVES ON TERMINATION JUNCTIONS \(J_{II}\) OF THE NETWORK IN FIGURES 6a and 6b

Using the notations of Sec. III,3, the BLT equation for the network of Figure 6b can be broken down into two matrix equations.

\[
[S]_{II, I} [W]_{II} = [W]_{II} - [S]_{II, I} [W]_{II}
\]

\[
[0]_{N} = -[S]_{II, I} [W]_{II} + [W]_{II}
\]

Similarly, using the notations of Sec. III,4, the BLT equation for the network of Figure 6a can be broken down into three matrix equations.

\[
[S]_{III, II} [W]_{III} = [W]_{II} - [S]_{III, II} [W]_{III}
\]

\[
[0]_{N} = -[S]_{III, II} [W]_{III} + [W]_{III}
\]

\[
[S]_{III, II} [W]_{III} = -[S]_{III, II} [W]_{III}
\]

\[
+(1)[N \times N - [S]_{III, II}] [W]_{III}
\]

For these two networks, the BLT equation therefore leads to two identical equations, (29) and (31). The absence of reflection in termination junctions \(J_{II}\) is expressed by:

\[
[S]_{III, I} = [0]_{N \times N}
\]

and

\[
[S]_{II, I} = [0]_{N \times N}
\]

Accordingly, from (29) and (31), we infer equations (14) and (23) of Secs. II,3 and III,4

\[
[W]_{II} = [S]_{II, I} [W]_{I} = [0]_{N}
\]

or

\[
[W]_{II} = [S]_{II, I} [W]_{I} = [0]_{N}
\]

Equations (15), (24) and (25) of Secs. III,3 and III,4 are then easily inferred.
APPENDIX 2

Y PARAMETERS OF A THREE-PORT FORK

Let us return to the electrical diagram of Figure 9 b. The matrix of Y parameters is defined by:

\[
\begin{pmatrix}
I_1 \\
I_2 \\
I_3
\end{pmatrix} =
\begin{pmatrix}
Y_{11} & Y_{12} & Y_{13} \\
Y_{21} & Y_{22} & Y_{23} \\
Y_{31} & Y_{32} & Y_{33}
\end{pmatrix}
\times
\begin{pmatrix}
V_1 \\
V_2 \\
V_3
\end{pmatrix}
\] (33)

where \(I_i, 1 \leq i \leq 3\), is the input current on port \(i\) and \(V_i\) is the input voltage on this port.

We then find:

\[
Y_{ii} = \frac{1 + Z_{Ti} / Z''_{i}}{R_i + Z_T (1 + Z_{i}/Z''_{i})}
\] (34)

and

\[
Y_{ji} = A \times B \times C
\] (35)

where

\[
A = \frac{-1}{1 + (Z_j + Z''_{j} / Z_{ji}) (Z_k + Z''_{k} / Z_{ki})}
\]

\[
B = \frac{1}{(1 + Z_{j}/Z''_{j}) (Z_{Ti} + Z_{i}/Z''_{i})}
\]

\[
C = 1 - \frac{Z_{ji}/Z''_{i}}{Z_{ii}''}
\]

with

\[1 \leq i \leq 3; \quad 1 \leq j \leq 3; \quad 1 \leq k \leq 3\]

and

\[i \neq j; \quad j \neq k; \quad i \neq k\]

for the extradiagonal terms.

The notations of (34) and (35) are as follows:

- whatever \(i \in [1,3]\)

\[
Z_{i}' / Z_i = \frac{Z_{i}'' Z_i}{Z_{i}'' Z_i}
\] (36)

\[
Z_{Ti} = Z_i + (Z_j + Z''_{j} / Z_{ji}) (Z_k + Z''_{k} / Z_{ki})
\] (37)

It should be noted that equations (34) and (35) are not valid for zero series impedances \(Z_i\) and \(Z_{i}''\).

These relations also allow us to determine the \(S\) parameters of the junction using equation (6).

APPENDIX 3

DETERMINATION OF THE DIFFERENT MATRIX BLOCKS OF EXPRESSION (27) IN THE CASE OF EXPRESSION OF THE \(S_0\) PARAMETERS OF A FOUR-PORT JUNCTION \(J\) AS A FUNCTION OF THOSE OF TWO THREE-PORT JUNCTIONS, \(J_a\) AND \(J_b\)

For this purpose, we identified the two networks of Figures 10 a and b called network 1 and network 2 respectively.

Let \((S_0)\) be the matrix of the \(S_0\) parameters of \(J_a\), \((S_0_b)\), the matrix of the \(S_0\) parameters of \(J_b\), \((S_0)\) the matrix of the \(S_0\) parameters of \(J\).

To place ourselves in a case similar to that discussed in Sec. III.6.3, we showed on each tube of the network of Figure 10 a the corresponding port of junctions \(J_a\) and \(J_b\) as was also the case for Figure 10 b.

1. We then write the wave-wave matrix \((W, W)_2\) for network 2:

\[
(W, W)_2 = \begin{bmatrix}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

The scattering supermatrix \([S']\) of network 2 therefore has the form:

\[
[S'] = \begin{bmatrix}
0_{4 \times 4} & [S']_{I, II} \\
[1_{4 \times 4}] & 0_{4 \times 4}
\end{bmatrix}
\] (38)

where superindexes \(I\) and \(II\) group the following wave indexes, \(N_i, i \in [1,8]\):

\[I = \{1, 2, 3, 4\}\]

\[II = \{5, 6, 7, 8\}\]

It can then be seen that block \([S']_{I, II}\) of equation (27) identifies with \((S_0)\), in the case of Figure 10 b and we have:

\[
[S']_{I, II} = \begin{bmatrix}
S_{011} & S_{012} & S_{013} & S_{014} \\
S_{021} & S_{022} & S_{023} & S_{024} \\
S_{031} & S_{032} & S_{033} & S_{034} \\
S_{041} & S_{042} & S_{043} & S_{044}
\end{bmatrix}
\] (39)
2. Now let us write the wave-wave matrix \((W. W)_1\)
of network 1:

\[
(W. W)_1 = \begin{bmatrix}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

The scattering supermatrix \([S]\) of network 1 therefore has the form:

\[
[S] = \begin{bmatrix}
[S]_{I,II} & [S]_{I,III} \\
[S]_{III,II} & [S]_{III,III}
\end{bmatrix}
\]

where superindexes \(I, II, III\) contain the following wave indexes \(W_i \in \{1, 10\}:

\[
I = \{1, 2, 3, 4\} \\
II = \{5, 6, 7, 8\} \\
III = \{9, 10\}.
\]

From Figure 10a it can be seen that the matrix blocks included in relation (27) are expressed as a function of the following \(S_{0a}\) and \(S_{0}\) parameters of \(J_a\) and \(J_s:\n
\[
[S]_{I,II} = \begin{bmatrix}
S_{0a11} & S_{0a12} & 0 & 0 \\
S_{0a12} & S_{0a22} & S_{0a23} & 0 \\
0 & 0 & S_{0b22} & S_{0b23} \\
0 & 0 & S_{0b32} & S_{0b33}
\end{bmatrix}
\]

\[
[S]_{I,III} = \begin{bmatrix}
0 & S_{0a2} \\
S_{0a2} & 0 \\
S_{0b2} & S_{0b3}
\end{bmatrix}
\]

\[
[S]_{III,II} = \begin{bmatrix}
S_{0a2} & S_{0a3} & 0 & 0 \\
S_{0b2} & S_{0b3} & 0 & 0 \\
0 & 0 & S_{0b32} & S_{0b33}
\end{bmatrix}
\]

\[
[S]_{III,III} = \begin{bmatrix}
0 & S_{0a3} \\
0 & 0 \\
S_{0b32} & S_{0b33}
\end{bmatrix}
\]

REFERENCES


