Interaction Notes

Note 489

May 1990

ELECTROMAGNETIC TOPOLOGY: JUNCTION CHARACTERIZATION METHODS

by

J.P. Parmantier(*), J.C. Alliot(**)
G. Labaune(***), P. Degauque(****)

ABSTRACT

This paper deals with the electrical characterization of topological network junctions, using a formalism in accordance with Electromagnetic Topology. More precisely, it is shown how the characteristic impedance of the network in which the junction is located can be chosen to treat several physical cable connection and termination configurations.

It is possible to characterize, in the electromagnetic sense, any junction of order N (with N ports), by considering it as the connection of two junctions of lower order which it is easier to model analytically, or to measure its Y or S parameter.

A topological approach has been adopted to obtain the result, not only because the BLT equation is applied but also because the concept is used for breaking space down into topological volumes. Moreover, this method is well suited for numerical treatment.

The case of junctions called "forks", which represent the separation of a conductor into other conductors, is given as an example.

Keywords (NASA thesaurus): Electromagnetic Topology — BLT equation — Scattering coefficients — Electromagnetic radiation — Transmission lines — Electromagnetic wave transmission — Complex systems.

(*) Dassault Aviation (now at ONERA)
(**) ONERA
(***) ONERA (now at Thomson CSF/RCM)
(****) Université de Lille-1

TERMINOLOGY INDEX NOTATIONS

(a) Electromagnetic Topology.

– *Interaction diagram:* graphic representations of the breakdown of a problem into several problems included in one another and limited to so-called topological volumes; their surface is called "topological surface".

– *Interaction graph:* graphic representation of the interactions between the various topological volumes. Each volume is symbolized by a so-called volume node. The channel of penetration of one volume into another is symbolized by a so-called surface node. The nodes are interconnected by edges expressing the electromagnetic interactions.

(b) Topological Network of Lines

A network consists of tubes interconnecting the junctions. The tubes are propagation lines. The junctions express the scattering of a signal. There are surface junctions associated with the surface nodes of an interaction graph and volume junctions associated with the volume nodes. The signal transmitted on the network is called wave. It can be electrical (case of physical lines) or electromagnetic (case of fictitious propagation channel of an interference).

– *Order of a junction:* number of ports of the junction (from the standpoint of an electric circuit, it is the number of conductors connected to the junction). As tubes can include several conductors, the number of tubes connected to a junction is necessarily less than or equal to the order of the junction).

(c) Matrix and Vector Notations

A matrix A is denoted (A) and a vector V is denoted (V). Several quantities of this type are defined herein:

(S) = matrix of the S parameters of a junction.

(S_0) = matrix of the microwave S parameters of a junction (generally referenced to a characteristic scalar impedance $Z_{c0} = 50\ \Omega$).

(Y) = matrix of the Y parameters of a junction.

(V) = column vector of the input voltages of the junction.

(I) = column vector of the input currents in the junction.

(Z_c) = characteristic impedance matrix.

(Γ) = propagation matrix.

(d) Supermatrix and Supervector Notations

A supermatrix is generally a large matrix in which the row and column indexes have been grouped into superindexes in order to break the matrix down into blocks. This allows a supermatrix to be considered as a matrix of matrices. Similarly, a supervector is considered a vector of vectors.

The notation of a supermatrix A is $[A]$ (in brackets); similarly, the notation of a supervector V is $[V]$.

We also use blocks of supermatrix A denoted $[A]_{I,J}$ where I and J are superindexes in Roman numerals. Similarly, a block of supervector V is denoted $[V]_I$.

When we wish to specify the size of a supermatrix or a matrix block, we write $[A]_{n \times m}$ where n is the number of rows and m is the number of columns. Thus, we define a zero block by $[0]_{n \times m}$ and a unit block by $[1]_{n \times m}$.

Similarly, the size of a vector block is defined by writing $[V]_n$ where n is the number of components. A zero vector block is thus denoted $[0]_n$.

The main supermatrices and supervectors used in this paper are:

$[Y]$: admittance supermatrix;

$[S]$: supermatrix of topological S parameters or scattering supermatrix;

$[S_0]$: supermatrix of microwave S parameters or microwave scattering supermatrix;

$[W(z)]$: wave supervector propagating on the network;

$[W^{(s)}]$: source supervector applied to the junctions.

I. – INTRODUCTION AND STATEMENT OF THE PROBLEM

Electromagnetic topology is a method used to approach a complex electromagnetic coupling problem. One of its original features is to break space down into several volumes in which the problem is solved locally independently of the neighboring volumes [1, 2, 4]. The formalism best suited for describing the interactions between volumes and the path of interference through the volumes is inspired from that of multiconductor transmission line networks generalized to topology.

The signals propagating in these networks are waves whose mathematical expression contains electromagnetic information (fields **E** and **H**) or electrical information (voltage V and current I). A network consists of a set of tubes through which propagate the waves, interconnected by junctions insuring distribution of the waves [3].

On each tube can be defined a propagation matrix (Γ) connecting the waves to each end and to each junction, a scattering matrix (S) (called matrix of parameters S). Each of these matrices is used to fill the blocks of supermatrices $[S]$ and $[\Gamma]$ of the topological network ([4]).

The BLT (Baum-Liu-Tesche) equation is used to determine the signals in any point of the network by grouping the propagation and distribution equations. Its expression generalized to waves is as follows [4]:

$$\{[1]-[S][\Gamma]\}[W(0)]=[S][W^{(S)}] \tag{1}$$

where

[1] is the unit matrix;

$[S]$ is the network distribution supermatrix;

$[\Gamma]$ is the network propagation supermatrix;

$[W(0)]$ is the supervector of the waves leaving the junctions;

$[W^{(S)}]$ is the supervector of the source waves applied to the junctions.

The concept of supermatrix means that all the calculations of equation (1) can be made by blocks.

In this paper, our analysis is limited to networks or portions of networks for which the voltage and current quantities can be defined unambiguously. We then propose to determine the topological S parameters of any junction with N ports.

It may not always be easy to measure or model the electric properties of such junctions. This is why, after describing a general method for determining the S parameters, we propose to approach this problem by considering the junction of order N as the union of two junctions of an order lower than N and whose S parameters are known (or easier to determine).

II. – GENERAL METHOD FOR DETERMINING THE S PARAMETERS OF A JUNCTION

II,1. – REVIEW OF THE SCATTERING EQUATION OF THE JUNCTION

A junction of order N is characterized by a scattering matrix relating the outgoing waves with the incoming waves. In the case of a multiconductor line, the following expressions [2, 5] are generally associated with the incoming and outgoing waves:

$(V)+(Z_c)(I)$ for the incoming wave

$(V)-(Z_c)(I)$ for the outgoing wave.

Where (V) and (I) are vectors with dimension N of the voltages and currents incoming in each port of the junction, (Z_c) is a matrix with dimension $N \times N$ called characteristic matrix of the junction, which characterizes propagation on the lines connected to each port.

The scattering equation is defined by the following relation [2]:

$$(V)-(Z_c)(I)=(S)((V)+(Z_c)(I)) \tag{2}$$

(S) is a matrix with dimension $N \times N$ called junction scattering matrix.

II,2. – IMPORTANCE OF (Z_c) FOR THE DETERMINATION OF (S)

Equation (2) shows that matrix (S) is referenced to a characteristic impedance matrix (Z_c) which expresses the propagation and electrical coupling properties of the lines connected to the N ports of the junction.

In network topology, the lines are grouped into one or more tubes, according as they are electrically coupled or not [4].

We will use two simple examples to illustrate how the S parameters of a given junction can have a different expression depending on the topological network used.

Let us take the example of a junction J of order N. Let us consider that this junction is the electrical termination of N lines which we will assume to be electrically coupled.

The topological representation of the junction in a network is that shown in Figure 1, where only a single

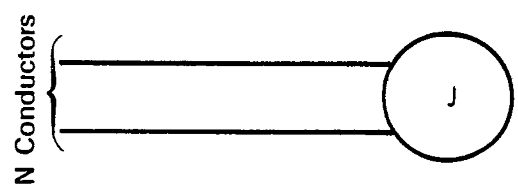

Fig. 1.

tube is to be considered. Matrix (Z_c) to be considered in (2) is then full so as to express the coupling of the N lines connected to the N ports of the junction. It has the form:

$$(Z_c)=\begin{pmatrix} Z_{c_{11}} & \dots & Z_{c_{1,N}} \\ \vdots & & \vdots \\ Z_{c_{N,1}} & \dots & Z_{c_{N,N}} \end{pmatrix}.$$

Let us now consider that two groups of coupled lines ($N1$ and $N2$ lines respectively) are connected to $N1$ and $N2$ ports of this same junction J. Assuming that these two groups do not interact, we can represent them topologically as two tubes, I and II, connected to J (*see* Fig. 2).

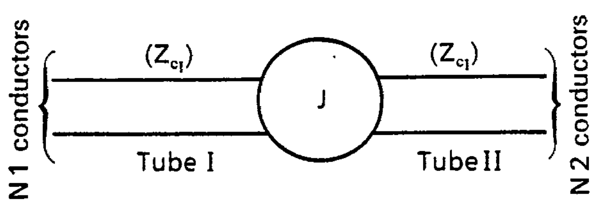

Fig. 2.

Matrix (Z_c) that we must consider in (2) is structured as follows:

$$(Z_c) = \begin{pmatrix} (Z_{cI}) & (0)_{N1\times N2} \\ (0)_{N2\times N1} & (Z_{cII}) \end{pmatrix}$$

where (Z_{cI}) is a matrix block with dimension $N1 \times N1$ corresponding to the characteristic impedance matrix of tube I and (Z_{cII}) with dimension $N2 \times N2$ corresponds to tube II. The notation $(0)_{N1\times N2}$ and $(0)_{N2\times N1}$ expresses two null matrix blocks with dimension $N1 \times N2$ and $N2 \times N1$.

II,3. – ELECTRICAL CHARACTERIZATION OF A JUNCTION

As was seen, the expression of matrix (S) depended on a characteristic impedance matrix (Z_c). This representation is therefore not an intrinsic representation of the electrical properties of a junction. With this in mind, matrix (Y) of the admittance parameters appears better suited. It is defined by the following relation:

$$(I) = (Y)(V) \qquad (3)$$

However, the Y parameters are not iterative, which means that if several junctions are cascaded, the output quantities of a junction become the input quantities of the next junction. For topological processing, a characterization closer to the topological S parameters may therefore be preferred.

Let us assume that each of the ports is referenced to a unique scalar characteristic impedance Z_{c0}. Equation (2) becomes:

$$(V) - Z_{c0}(I) = (S_0)[(V) + Z_{c0}(I)] \qquad (4)$$

It can be seen that if each of vectors (V) and (I) is divided by $2\sqrt{Z_{c0}}$, expression (4) corresponds exactly to the definition of the microwave S_0 parameters [5]. Combining this equation with (3) yields the expression for the Y parameters:

$$(Y) = \frac{1}{Z_{c0}}[(1) + (S_0)]^{-1}.[(1) - (S_0)] \qquad (5)$$

Equations (4) and (5) therefore show that a junction can be characterized independently of the network by conventional methods for determining the admittance Y parameters or the microwave S_0 parameters.

The evaluation of the characteristic impedance of the medium (Z_c) then allows the topological S parameters of the network to be found by the following equation:

$$(S) = [(1) - (Z_c)(Y)][(1) + (Z_c)(Y)]^{-1} \qquad (6)$$

determined from equations (2) and (3).

III. – CHARACTERIZATION OF A JUNCTION OF ORDRE N USING TWO JUNCTIONS OF ORDERS $M1$ AND $M2$

III,1. – STATEMENT OF THE PROBLEM

It is not always easy to electrically characterize a junction by measurement or analytic modeling, especially when the number of ports is large. This is why we had the idea of breaking down a junction J of order N into an arrangement of two other junctions, J_a and J_b, for which the expression of the S parameters is known and with orders $M1$ and $M2$ respectively.

The principle consists of sharing k ports between J_a and J_b. It is demonstrated that k is equal to:

$$k = \frac{M1 + M2 - N}{2}.$$

In order to express the S parameters of J as a function of those of J_a and J_b, we adopted a topological approach consisting of comparing two networks containing J and J_a, J_b respectively. To facilitate our reasoning, we assumed that all the S parameters used here were referenced to a single scalar characteristic impedance Z_{c0} (typically 50 Ω).

Once junction J of order N is constructed, its S parameters can be referenced to any characteristic impedance matrix (Z_c) using equations (4), (5) and (6).

Figure 3 shows the two topological networks.

Network 1 includes junction J of order N. Tubes T_i connected to the junction are assumed to contain only one conductor. At the end of each of tubes T_i, we included a terminal junction J_i.

Network 2 includes two junctions, J_a and J_b, of orders $M1$ and $M2$. Tube T_0 interconnects J_a and J_b. It contains the lines interconnecting the k ports of the two junctions. The dimension of the two wave vectors $[W]_a$ and $[W]_b$ propagating on T_0 is therefore equal to $K = 2k$.

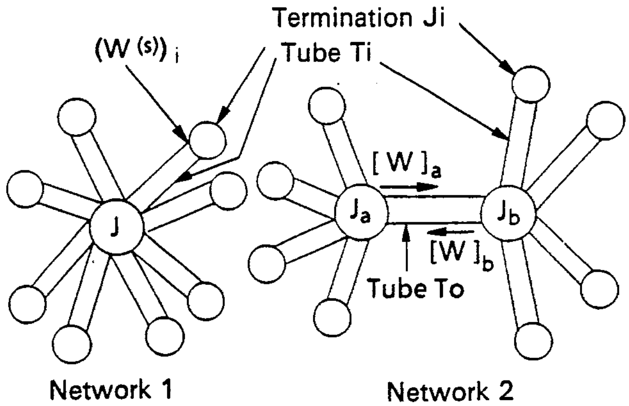

Fig. 3.

The other tubes T_i not interconnecting J_a and J_b are assumed identical to those of network 1 as is the case for junctions J_i.

It should be noted that grouping the lines in tube T_0 allows a better synthesis of the grouping of J_a and J_b, although all the lines of networks 1 and 2 are assumed decoupled.

To express the S parameters of J as a function of the S parameters of J_a and J_b, the BLT equations for the two networks are written assuming that the source wave generators $(W^{(s)})_i$, identical in both cases, are located in junctions J_i. We shall also assume that terminal junctions J_i insure optimum energy transfer of sources $(W^{(S)})_i$ on the network. They actually correspond to a terminal impedance equal to the reference characteristic impedance Z_{c0}.

The identification of the networks also assumes that there is no propagation on tube T_0 (by definition, a junction must be small compared with the wavelength used).

III,2. – SYNTHETIC REPRESENTATION OF THE PROBLEM

We can simplify the representation of networks 1 and 2 of Figure 3 by advancing purely topological considerations. Figures 4 *a* and *b* shows networks 1 and 2 with space broken down into a volume V_0 separated from an external volume V_{ext} by a surface S_0. Under these conditions, junctions J_a, J_b and J can be assimilated to surface junctions and junctions J_i to volume junctions.

We are then able to determine the topological graph corresponding to the above topological diagram. The graph shows the interactions between volumes. Each volume is represented by a so-called "volume" node; each surface is represented by a so-called "surface" node. The interaction between volumes is expressed by arcs connecting the volume nodes to the surface nodes.

The topological graph corresponding to the volume breakdown defined above is illustrated in Figures 5 *a*

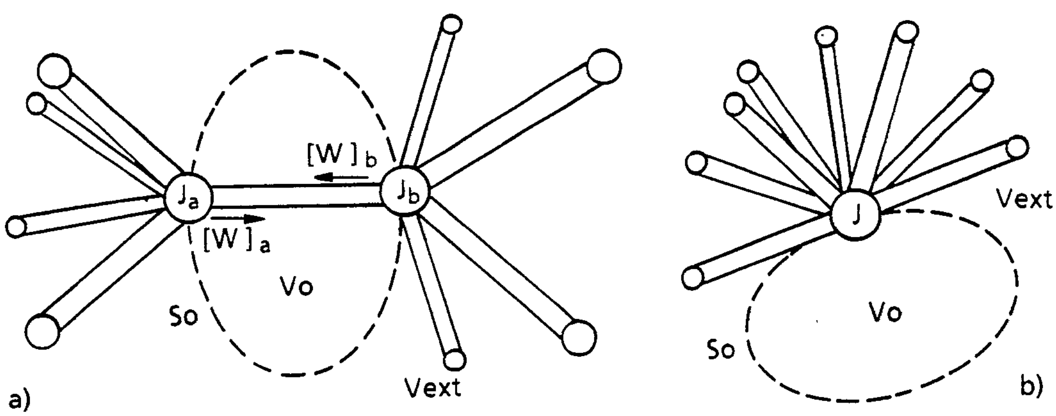

Fig. 4.

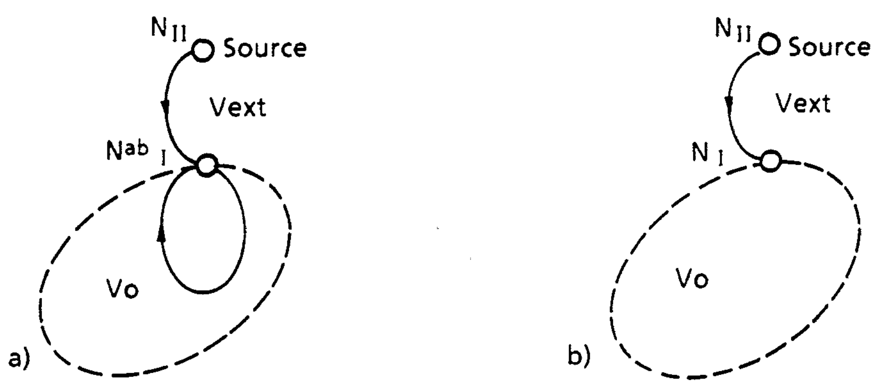

Fig. 5.

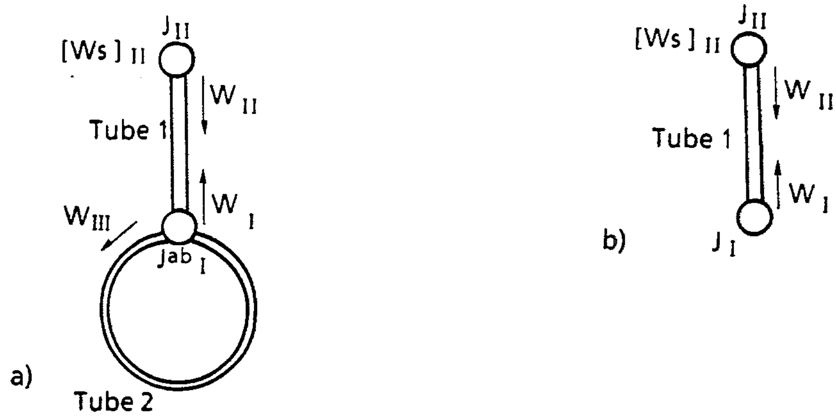

Fig. 6.

and b. It shows a volume node N_{II} corresponding to the source located in V_{ext} and two surface nodes N_I^{ab} and N_I. Nodes N_{II} and N_I^{ab} in Figure 5 a and nodes N_{II} and N_I in Figure 5 b interact. Node N_I^{ab} interacts with itself (expressing the interaction between junctions J_a and J_b).

We can then define a new representation of topological networks 1 and 2 similar to the topological graphs of Figures 5 a and b, thereby allowing a better synthesis of the volume interactions and providing more condensed analytic formulas. A junction J_{II} grouping all the terminal junctions J_i is made to correspond to volume node N_{II}. Similarly, J_I^{ab} of Figure 6 a corresponds to surface node N_I^{ab} expressing the grouping of junctions J_a and J_b. Junction J_I of Figure 6 b associated with surface N_I represents junction J.

The reconfiguration of the two topological networks defined in Figure 3 leads us to group all the vector quantities in supervectors. Thus, the group of source vectors $(W^{(s)})_i$ allows us to define a supervector denoted $[W_S]_{II}$ located in junction J_{II}. Similarly, on each of the tubes, we define wave supervectors $[W]_{II}$ and $[W]_I$ for the incident and reflected waves respectively on junctions J_I^{ab} and J_I. Wave W_{III} shown only in Figure 6 a expresses the propagation in V_0.

III,3. – WRITING OF THE BLT EQUATION FOR THE NETWORK INCLUDING THE JUNCTION OF ORDER N (Fig. 6 b)

Let us go back to equation (1) and attempt to express supervectors $[W(0)]$ and $[W^{(S)}]$ with dimension $2N$ as well as the propagation supermatrix $[\Gamma]$ and a scattering supermatrix $[S]$ with dimension $2N * 2N$.

Tube 1 is the support for two waves, W_I and W_{II}, which can be represented by $[W]_I$ and $[W]_{II}$. These two quantities can be grouped to obtain $[W(0)]$, *i.e.*:

$$[W(0)]=\begin{bmatrix} [W]_I \\ [W]_{II} \end{bmatrix} \tag{7}$$

Similarly, supervector $[W^{(S)}]$ is expressed by:

$$[W^{(S)}]=\begin{bmatrix} [0]_N \\ [W^{(S)}]_{II} \end{bmatrix} \tag{8}$$

It can therefore be seen that supermatrices $[\Gamma]$ and $[S']$ can also be treated by blocks according to indexes I and II.

Supermatrix $[\Gamma]$ is a unit matrix since the propagation effect on the network is ignored. We can therefore write:

$$[\Gamma]=\begin{bmatrix} [\Gamma]_{I,I} & [0]_{N\times N} \\ [0]_{N\times N} & [\Gamma]_{II,II} \end{bmatrix} \tag{9}$$

where:

$$[\Gamma]_{I,I}=[\Gamma]_{II,II}=[1]_{N\times N} \tag{10}$$

Supermatrix $[S]$ which we shall denote $[S']$ for case 6 b is obtained by writing the characteristic wave-wave matrix $(W.W)$ for the network [4] relative to indexes I and II:

$$(W.W)=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{11}$$

This matrix has the advantage of indicating the location of the nonzero blocks of $[S]$; we can therefore write:

$$[S]=\begin{bmatrix} [O]_{N\times N} & [S]_{I,II} \\ [S']_{II,I} & [0]_{N\times N} \end{bmatrix} \tag{12}$$

Considering the assumption of no reflection on the terminal junctions and since $[S']_{II,I}$ corresponds to the matrix of the S parameters of junction II, we have:

$$[S']_{II,I}=[0]_{N\times N} \tag{13}$$

hence

$$[W]_{II} = [0]_N \qquad (14)$$

(*see* Appendix I).

The BLT equation for the network is then summarized by a single relation:

$$[W]_I = [S']_{I,II}[W^{(S)}]_{II} \qquad (15)$$

III,4. – WRITING OF THE NETWORK INCLUDING TWO JUNCTIONS OF ORDERS $M1$ AND $M2$ LESS THAN N (Fig. 6*a*)

The network studied is that of Figure 6*a*. It includes three wave supervectors, $[W]_I$, $[W]_{II}$, $[W]_{III}$. Indexes I and II are associated with vectors of dimension N; index III is associated with vectors of dimension $K = M1 + M2 - N$ (*see* Sec. III,1).

These indexes can be used to define all the quantities of relation (1).

We therefore have:

$$[W](0)] = \begin{bmatrix} [W]_I \\ [W]_{II} \\ [W]_{III} \end{bmatrix} \qquad (16)$$

$$[W^{(S)}] = \begin{bmatrix} [0]_N \\ [W^{(S)}]_{II} \\ [0]_K \end{bmatrix}. \qquad (17)$$

The absence of any propagation effect in the network means that $[\Gamma]$ has the following structure:

$$[\Gamma] = \begin{bmatrix} [\Gamma]_{I,I} & [0]_{N\times N} & [0]_{N\times K} \\ [0]_{N\times N} & [\Gamma]_{II,II} & [0]_{N\times K} \\ [0]_{K\times N} & [0]_{K\times N} & [\Gamma]_{III,III} \end{bmatrix}$$

where

$$[\Gamma]_{I,I} = [\Gamma]_{II,II} = [1]_{N\times N} \qquad (18)$$

and

$$[\Gamma]_{III,III} = [1]_{K\times K} \qquad (19)$$

The scattering supermatrix is constructed from the characteristic $(W.W)$ matrix of the network, *i.e.*:

$$(W.W) = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \qquad (20)$$

hence:

$$[S] = \begin{bmatrix} [0]_{N\times N} & [S]_{I,II} & [S]_{I,II} \\ [S]_{II,I} & [0]_{N\times N} & [0]_{N\times K} \\ [0]_{K\times N} & [S]_{III,II} & [S]_{III,III} \end{bmatrix} \qquad (21)$$

The assumption of the absence of reflection in the terminal junctions and the fact that $[S]_{II,I}$ is identified with the matrix of the S parameters of J_{II} is expressed by:

$$[S]_{II,I} = [0]_{N\times N} \qquad (22)$$

hence

$$[W]_{II} = [0]_N \qquad (23)$$

(*see* Appendix 1).

The BLT equation of the network is then summarized by two relations:

$$[S]_{I,II}[W^{(S)}]_{II} = [W]_I - [S]_{I,III}[V_{III}] \qquad (24)$$

$$[S]_{III,II}[W^{(S)}]_{II} = ([1]_{K\times K} - [S]_{III,III})[W]_{III} \qquad (25)$$

The elimination of $[W]_{II}$ from (24) and (25) leads us to the relation:

$$\{[S]_{I,II} + [S]_{I,III}([1]_{K\times K} - [S]_{III,III})^{-1}[S]_{III,II}\}[W^{(S)}]_{II} = [W]_I \qquad (26)$$

III,5. – EXPRESSION OF THE S PARAMETERS OF THE JUNCTION OF ORDER N AS A FUNCTION OF THE S PARAMETERS OF THE TWO JUNCTIONS OF ORDERS $M1$ AND $M2$

Equations (15) and (26) both relate supervectors $[W^{(S)}]_{II}$ and $[W]_I$.

They must be identical by construction of the two networks of Figures 6*a* and *b*.

We infer that:

$$[S']_{I,II} = [S]_{I,II} + [S]_{I,III}([1]_{K\times K} - [S]_{III,III})^{-1}[S]_{III,II} \qquad (27)$$

where $[S']_{I,II}$ is the distribution matrix for the junction with N ports and $[S]_{I,II}$, $[S]_{I,III}$, $[S]_{III,II}$ and $[S]_{III,III}$ are expressed as a function of the S_0 parameters of the two junctions with $M1$ and $M2$ ports (since, as was seen in Sec. II.2, we can reason considering that all the ports are decoupled; it is only a posteriori that supermatrix $[S']$ is referenced to the characteristic impedance supermatrix $[Z_c]$ of the real case to be treated).

Equation (27) has the same form independently of the number of ports of the system. It can therefore be used recurrently to treat the problem with N ports from problems with three ports.

III,6. – APPLICATION: CHARACTERIZATION OF A JUNCTION OF ORDER 4 USING TWO JUNCTIONS OF ORDER 3

III,6.1. – *Definition of the Application*

The junction to be characterized is the so-called "four-port fork" function consisting of the separation

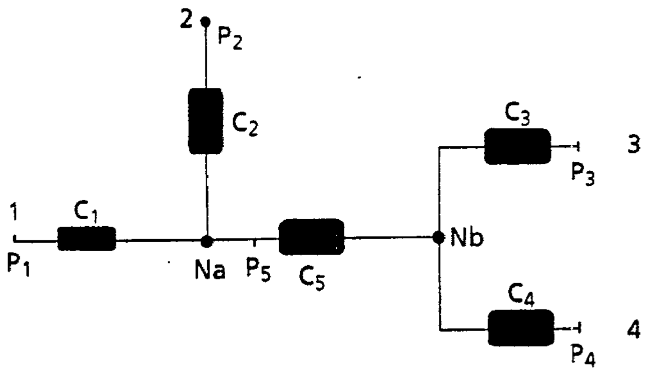

Fig. 7. – Model of a four-port fork.

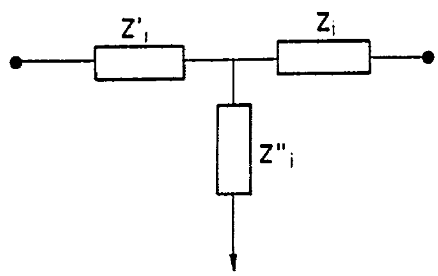

Fig. 8. – Model of an electric cell C_i.

of a conductor into three other conductors.

The diagram of Figure 7 shows this fork with an electric cell C_i on each conductor i. This expresses an imperfect electrical connection in the connection node. Figure 8 represents an elementary cell C_i by two impedances, Z'_i and Z_i (used, for instance, to express a choke effect), separated by a parallel impedance Z''_i (used for instance to express a capacitive coupling).

The branch points P_1, P_2, P_3, P_4, P_5 and N_a, N_b are used to identify the location of the cells on the different conductors.

As an example, we chose a junction such that all its cells C_i exhibited purely resistive impedances, an ideal case reproduced in the laboratory:

Table I gives the values of Z_i, Z'_i and Z''_i :

TABLE I

Cell	Z_i in Ω	Z'_i in Ω	Z''_i in Ω
C_1	0	0	∞
C_2	16.6	16.6	118.44
C_3	33.17	33.17	20.57
C_4	41.05	41.05	10.17
C_5	16.6	16.6	68.19

The microwave S_0 parameters (with respect to a characteristic impedance of 50 Ω) were measured by network analyzer and calculated by a numerical cade.

Identical results were found with both methods:

$$(S'_0) = \begin{pmatrix} -0.340 & 0.357 & 0.048 & 0.024 \\ 0.357 & -0.039 & 0.026 & 0.013 \\ 0.048 & 0.026 & -0.018 & 0.013 \\ 0.024 & 0.013 & 0.013 & -0.002 \end{pmatrix}$$

We propose to find these values by breaking the four-port fork into two three-port forks and then applying equation (27).

III,6.2. – *Breakdown of the Four-Port Fork into Two Three-Port Forks*

III,6.2.1. – Principle of Characterization of a Three-Port Fork

The three-port fork consists of separating a conductor into two other conductors. As for the four-port fork, we symbolized in Figure 9 *a* any electrical imperfections by cells C_i of the same type a those of Figure 8. The global electrical diagram is shown in Figure 9 *b*. This allows us to establish simple analytical formulas for the Y parameters of the junction (*see* Appendix 2). The Y parameters can then be transformed into S parameters by equation (6).

It should be noted that these analytic expressions, relatively simple in the case of three-port junctions, become rapidly very complicated to determine when the number of ports is increased. This shows the

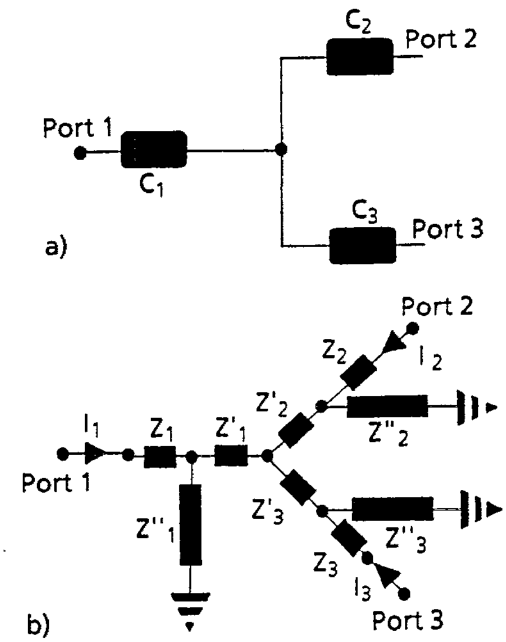

Fig. 9. – (*a*) Model of a three-port fork.
(*b*) Global electrical diagram of a three-port fork.

advantage of breaking down junctions with a large number of ports to lower order junctions.

III,6.2.2. – Presentation of the First Three-Port Fork

The first three-port fork J_a that we chose for the breakdown has the properties given in Table II.

TABLE II

Cell	Z_i in Ω	Z_i' in Ω	Z_i'' in Ω
C_1	0	0	∞
C_2	0	0	∞
C_3	16.6	16.6	118.44

The corresponding S_0 parameters are with respect to 50 Ω.

$$(S_{0_a}) = \begin{pmatrix} -0.297 & 0.703 & 0.380 \\ 0.703 & -0.297 & 0.380 \\ 0.380 & 0.380 & -0.027 \end{pmatrix}.$$

They can be determined by direct measurement or by numerical or analytic computations (*see* Appendix 2).

II,6.2.3. – Presentation of the Second Three-Port Fork

The second three-port fork J_b has the following characteristics (Table III).

TABLE III

Cell	Z_i in Ω	Z_i' in Ω	Z_i'' in Ω
C_1	16.6	16.6	68.19
C_2	33.17	33.17	20.57
C_3	41.05	41.05	10.17

Parameters S_0 (with respect to 50 Ω) are:

$$(S_{0_b}) = \begin{pmatrix} -0.085 & 0.067 & 0.033 \\ 0.067 & -0.017 & 0.013 \\ 0.033 & 0.013 & -0.001 \end{pmatrix}.$$

III,6.2.4. – Placing in Common of the Two Three-Port Forks

What remains to be done is to place in common junctions J_a and J_b to recover the scheme of Figure 7. For this purpose, we can locate the junctions in this figure according to the various ports P_i. We summarized these properties in Table IV.

III,6.3. – *Topological Processing of the Grouping of the Two Junctions*

The next step consists of filling the matrix blocks of equation (27).

For this purpose, we must identify the two networks described in Figures 10*a* and *b*.

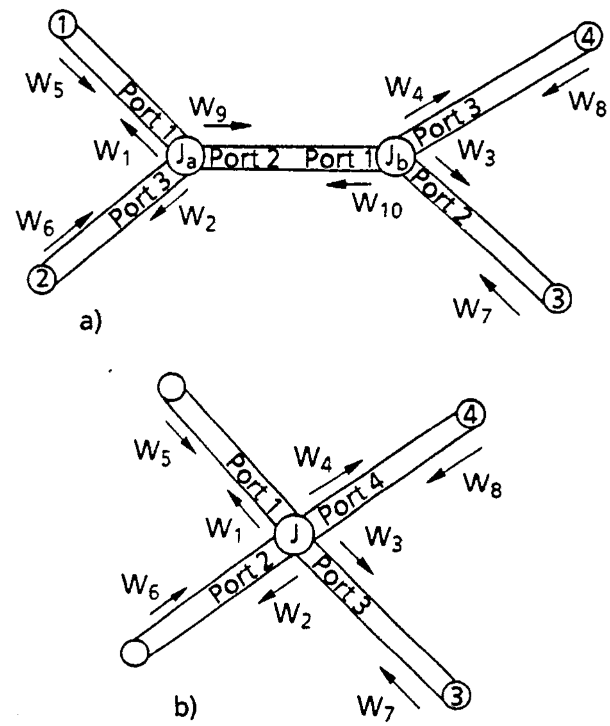

Fig. 10. – (*a*) Network with two three-port junctions.
(*b*) Network with one four-port junction.

The first network corresponds to the interconnection of J_a and J_b on ports 2 and 1 respectively and the second to J, a junction with four ports.

In the two figures, ports 1, 2, 3 and 4 and waves W_1 to W_8 are common, which complies with the principles stated in Sec. III.1.

It can then be demonstrated that the different blocks of (27) are written (*see* Appendix 3)

$$[S]_{I,II} = \begin{bmatrix} -0.297 & 0.380 & 0 & 0 \\ 0.380 & -0.027 & 0 & 0 \\ 0 & 0 & -0.001 & 0.013 \\ 0 & 0 & 0.013 & -0.017 \end{bmatrix}$$

TABLE IV

Junction	Port 1 location	Port 2 location	Port 3 location	Separation node
J_a	P_1	P_5	P_2	N_a
J_b	P_5	P_3	P_4	N_b

[according to equation (41) of Appendix 3]

$$[S]_{I,III}=\begin{bmatrix} 0 & 0.703 \\ 0 & 0.380 \\ 0.033 & 0 \\ 0.067 & 0 \end{bmatrix}$$

[according to equation (42) of Appendix 3]

$$[S]_{III,II}=\begin{bmatrix} 0.703 & 0.380 & 0 & 0 \\ 0 & 0 & 0.033 & 0.067 \end{bmatrix}$$

[according to equation (43) of Appendix 3]

$$[S]_{III,III}=\begin{bmatrix} 0 & -0.297 \\ -0.85 & 0 \end{bmatrix}$$

[according to equation (44) of Appendix 3].

It is easily verified that the determination of $[S']_{I,II}$ by equation (27) yields the matrix of $[S'_0]$ parameters of the four-port junction given in Sec. III,6.1.

CONCLUSION

In aeronautics, electric circuits occur as main cable paths and branches connecting numerous equipment items. The cable branches result in impedance discontinuities on the conductors, affecting the signals.

Branches are conventionally represented as junctions with N ports whose scattering parameters are determined. They can be computed analytically when the number of ports remains small (less than 3) but are almost impossible to compute in the practical case encountered in aeronautics.

In this paper, we used electromagnetic topology to develop signal scattering in a junction with N ports by the S parameters. The formulation of a complex problem by the topological method [1, 2, 4] consists of breaking it down into elementary problems expressed in a suitable form (matrix of S parameters, propagation matrix, etc.).

It appeared that using the formalism of BLT equation led to a recurrent formulation of the scattering matrix for a junction with N ports from scattering matrices for a junction with a lower order.

The difficulty of the computation does not increase with the complexity of the junction to be described and the method proved to be well suited to numerical processing of complex problems.

It is interesting to note that the BLT equation (topological description) proved the most efficient for conducting this work aimed at calculating an input term in topological formalism. Electromagnetic topology thus appears not only as a formalism but also as a methodology for approaching electromagnetic coupling problems in global systems such as those encountered in modern aircraft.

APPENDIX 1

DEMONSTRATION OF THE ABSENCE OF REFLECTED WAVES ON TERMINATION JUNCTIONS J_{II} OF THE NETWORK IN FIGURES 6 *a* and *b*

Using the notations of Sec. III,3, the BLT equation for the network of Figure 6 *b* can be broken down into two matrix equations.

$$[S']_{I,II}[W^{(S)}]_{II}=[W]_I-[S']_{I,II}[W]_{II} \qquad (28)$$

$$[0]_N=-[S']_{II,I}[W]_I+[W]_{II} \qquad (29)$$

Similarly, using the notations of Sec. III,4, the BLT equation for the network of Figure 6 *a* can be broken down into three matrix equations.

$$[S']_{I,II}[W^{(S)}]_{II}=[W]_I-[S]_{I,II}[W]_{II} -[S]_{I,III}[W]_{III} \qquad (30)$$

$$[0]_N=-[S]_{II,I}[W]_I+[W]_{II} \qquad (31)$$

$$[S]_{III,II}[W^{(S)}]_{II}=-[S]_{III,II}[W]_{II} +([1]_{K\times K}-[S]_{III,III})[W]_{III} \qquad (32)$$

For these two networks, the BLT equation therefore leads to two identical equations, (29) and (31). The absence of reflection in termination junctions J_{II} is expressed by:

$$[S']_{III,I}=[0]_{N\times N}$$

and

$$[S]_{II,I}=[0]_{N\times N}$$

Accordingly, from (29) and (31), we infer equations (14) and (23) of Secs. II,3 and III,4

$$[W]_{II}=[S']_{II,I}[W]_I=[0]_N \qquad (33)$$

or

$$[W]_{II}=[S]_{II,I}[W]_I=[0]_N.$$

Equations (15), (24) and (25) of Secs. III,3 and III,4 are then easily inferred.

APPENDIX 2

Y PARAMETERS OF A THREE-PORT FORK

Let us return to the electrical diagram of Figure 9 b. The matrix of Y parameters is defined by:

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{pmatrix} \times \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} \tag{33}$$

where I_i, $1 \leqq i \leqq 3$, is the input current on port i and V_i is the input voltage on this port.

We then find:

$$Y_{ii} = \frac{1 + Z_{T_i}/Z_i''}{R_i + Z_T\,(1 + Z_i/Z_i'')} \tag{34}$$

and

$$Y_{ji} = A \times B \times C \tag{35}$$

where

$$A = \frac{-1}{1 + (Z_j' + Z_j''//Z_j)/(Z_k' + Z_k''//Z_k)}$$

$$B = \frac{1}{(1 + Z_j/Z_j'')\,(Z_{T_i} + Z_i//Z_i'')}$$

$$C = 1 - \frac{Z_i//Z_i''}{Z_i''}$$

with

$$1 \leqq i \leqq 3; \qquad 1 \leqq j \leqq 3; \qquad 1 \leqq k \leqq 3$$

and

$$i \neq j; \qquad j \neq k; \qquad i \neq k$$

for the extradiagonal terms.

The notations of (34) and (35) are as follows:

– whatever $l \in [1,3]$

$$Z_l''//Z_l = \frac{Z_l''\,Z_l}{Z_l''\,Z_l} \tag{36}$$

$$Z_{T_i} = Z_i' + (Z_j' + Z_j''//Z_j)//(Z_k' + Z_k''//Z_k). \tag{37}$$

It should be noted that equations (34) and (35) are not valid for zero series impedances Z_i and Z_i'.

These relations also allow us to determine the S parameters of the junction using equation (6).

APPENDIX 3

DETERMINATION OF THE DIFFERENT MATRIX BLOCKS OF EXPRESSION (27) IN THE CASE OF EXPRESSION OF THE S_0 PARAMETERS OF A FOUR-PORT JUNCTION J AS A FUNCTION OF THOSE OF TWO THREE-PORT JUNCTIONS, J_a AND J_b

For this purpose, we identified the two networks of Figures 10 a and b called network 1 and network 2 respectively.

Let (S_{0_a}) be the matrix of the S_0 parameters of J_a, (S_{0_b}), the matrix of the S_0 parameters of J_b, (S_0') the matrix of the S_0 parameters of J.

To place ourselves in a case similar to that discussed in Sec. III,6.3, we showed on each tube of the network of Figure 10 a the corresponding port of junctions J_a and J_b as was also the case for Figure 10 b.

1. We then write the wave-wave matrix $(W.W)_2$ for network 2:

$$(W.W)_2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The scattering supermatrix $[S']$ of network 2 therefore has the form:

$$[S'] = \begin{bmatrix} [0]_{4\times4} & [S']_{I,\,II} \\ [1]_{4\times4} & [0]_{4\times4} \end{bmatrix} \tag{38}$$

where superindexes I and II group the following wave indexes, N_i, $i \in [1, 8]$:

$$I = \{1, 2, 3, 4\}$$
$$II = \{5, 6, 7, 8\}.$$

It can then be seen that block $[S']_{I,\,II}$ of equation (27) identifies with (S_0'), in the case of Figure 10 b and we have:

$$[S']_{I,\,II} = \begin{bmatrix} S_{0_{11}} & S_{0_{12}} & S_{0_{13}} & S_{0_{14}} \\ S_{0_{21}} & S_{0_{22}} & S_{0_{23}} & S_{0_{24}} \\ S_{0_{31}} & S_{0_{23}} & S_{0_{33}} & S_{0_{34}} \\ S_{0_{41}} & S_{0_{24}} & S_{0_{43}} & S_{0_{44}} \end{bmatrix} \tag{39}$$

2. Now let us write the wave-wave matrix $(W.W)_1$ of network 1:

$$
(W.W)_1 = \begin{array}{c c}
 & \begin{array}{cccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{array} &
\begin{bmatrix}
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0
\end{bmatrix}
\end{array}
$$

The scattering supermatrix $[S]$ of network 1 therefore has the form:

$$
[S] = \begin{bmatrix}
[0]_{4\times 4} & [S]_{I,II} & [S]_{I,III} \\
[S]_{II,I} & [0]_{4\times 4} & [0]_{4\times 2} \\
[0]_{2\times 4} & [S]_{III,II} & [S]_{III,III}
\end{bmatrix} \qquad (40)
$$

where superindexes I, II, III contain the following wave indexes W_i $i\in[1, 10]$:

$$I=\{1, 2, 3, 4\}$$
$$II=\{5, 6, 7, 8\}$$
$$III=\{9, 10\}.$$

From Figure $10a$ it can be seen that the matrix blocks included in relation (27) are expressed as a function of the following S_{0_a} and S_{0_b} parameters of J_a and J_b:

$$
[S]_{I,II} = \begin{bmatrix}
S_{0_{a11}} & S_{0_{a13}} & 0 & 0 \\
S_{0_{a31}} & S_{0_{a33}} & 0 & 0 \\
0 & 0 & S_{0_{b22}} & S_{0_{b23}} \\
0 & 0 & S_{0_{b32}} & S_{0_{b33}}
\end{bmatrix} \qquad (41)
$$

$$
[S]_{I,III} = \begin{bmatrix}
0 & S_{0_{a12}} \\
0 & S_{0_{a32}} \\
S_{0_{b21}} & 0 \\
S_{0_{b31}} & 0
\end{bmatrix} \qquad (42)
$$

$$
[S]_{III,II} = \begin{bmatrix}
S_{0_{a21}} & S_{0_{a23}} & 0 & 0 \\
0 & 0 & S_{0_{b12}} & S_{0_{b13}}
\end{bmatrix} \qquad (43)
$$

$$
[S]_{III,III} = \begin{bmatrix}
0 & S_{0_{a22}} \\
S_{0_{b11}} & 0
\end{bmatrix} \qquad (44)
$$

REFERENCES

[1] BAUM C.E. - *Sublayers Sets and Relative Shielding Order in Electromagnetic Topology.* Electromagnetics. Vol.2, (1982), pp 335-354.

[2] BAUM C.E., LIU T.K., and TESCHE F.M. - *On the Analysis of General Multiconductor Transmission Line Networks.* Interaction Notes 350, November 1978.

[3] YANG F.C. and BAUM C.E. - *Electromagnetic Topology Measurements and Norms of Scattering parameters of Subshields.* Electromagnetics. Vol .6, (1986), pp 47-59.

[4] PARMANTIER J.P., ALLIOT J.C., LABAUNE G., and DEGAUQUE P. - *Electromagnetic Coupling on Complex Systems :Topological Approach.* Interaction Notes 488, May 1990.

[5] RAMO S., WHINNERY J.R. and VAN DUZER T. - *Fields and Waves in Communication Electronics.* John Wiley & Sons.