Interaction Notes

Note 545

27 July 1998

Splitting of Degenerate Natural Modes for Buried targets with Almost – O_2 Symmetry

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Abstract

For buried targets and associated lossy dielectric half space with O_2 symmetry (vertical rotation axis and axial symmetry planes, infinite in number) there is a two-fold degeneracy of the natural modes (for $m \geq 1$ in the $\cos(m\phi)$, $\sin(m\phi)$ decomposition), both with the same natural frequency. If the symmetry is not perfect, there is a splitting of these natural frequencies. Such asymmetry can come from many sources including target tilting and ground inhomogeneities.

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For buried targets and associated lossy dielectric half space with O_2 symmetry (vertical rotation axis and axial symmetry planes, infinite in number) there is a two-fold degeneracy of the natural modes (for $m \geq 1$ in the $\cos(m\phi)$, $\sin(m\phi)$ decomposition), both with the same natural frequency. If the symmetry is not perfect, there is a splitting of these natural frequencies. Such asymmetry can come from many sources including target tilting and ground inhomogeneities.

1. Introduction

A recent paper [4] has introduced a target signature based on continuous two-dimensional rotational symmetry with axial symmetry planes ($C_{\infty a} = O_2$ symmetry). For wavelengths on the order of the target dimensions, this is sufficient to give zero backscattering cross polarization in the usual radar h, v coordinates where the axis of rotation is perpendicular to the $\vec{1}_h$ direction. Furthermore this property remains as the radar is moved around the target. For on or under the ground surface, this also allows for a vertically stratified earth which maintains the above symmetry. Note that it is the lack of something (the h, v backscattering) which is the signature. As such this can be referred to as a vampire signature, a vampire not seeing its reflection in a mirror (in this case the h, v mirror), at least according to legend. This signature has now been experimentally observed [8]. Note that for SAR (synthetic aperture radar) measurements the O_2 symmetry is important since measurements are made from many azimuthal angles around the target.

As this signature is exploited, one should consider various theoretical and experimental aspects. No measurement is perfect; various sources of noise (errors) are present. One needs to quantify the amount of cross polarization present in the data, and compare it to the h, h and v, v components. This can be approached via the norms discussed in [6]. Furthermore, the target may not possess the O_2 symmetry in perfect form, particularly when one considers the target's relation to the nearby soil which may not be uniform (e.g., rocks) and may not have the ground surface perfectly perpendicular to the target's rotation axis. This last problem is considered in this paper.

2. Ideal Case of Perfect O_2 Symmetry

Summarizing from [4] we have a target such as illustrated in fig. 2.1. In general, the backscattering dyadic takes the form (2 × 2 considering only transverse coordinates)

$$\overleftrightarrow{\tilde{\Lambda}}_b(\overrightarrow{1}_i, s) = \begin{pmatrix} \tilde{\Lambda}_{b_{h,h}}(\overrightarrow{1}_i, s) & \tilde{\Lambda}_{b_{h,v}}(\overrightarrow{1}_i, s) \\ \tilde{\Lambda}_{b_{v,h}}(\overrightarrow{1}_i, s) & \tilde{\Lambda}_{b_{v,v}}(\overrightarrow{1}_i, s) \end{pmatrix}$$

$$\overleftrightarrow{\tilde{\Lambda}}_b(\overrightarrow{1}_i, s) = \overleftrightarrow{\tilde{\Lambda}}_b^T(\overrightarrow{1}_i, s) \quad \text{(reciprocity)}$$

$$\overrightarrow{1}_i \equiv \text{direction of incidence (radar to target)}$$

$$\overrightarrow{1}_0 \equiv \text{direction of scattering (target to radar)} \qquad (2.1)$$

$$= -\overrightarrow{1}_i$$

Here we have

$$\overrightarrow{1}_z \equiv \overrightarrow{1}_{sa} \equiv \text{rotation axis of target and ground} \qquad (2.2)$$

For the h, v coordinates aligned as in fig. 2.1., with $\overrightarrow{1}_v$ and $\overrightarrow{1}_i$ parallel to a symmetry plane of the target (and the radar) the backscattering dyadic reduces to

$$\overleftrightarrow{\tilde{\Lambda}}_b(\overrightarrow{1}_i, s) = \begin{pmatrix} \tilde{\Lambda}_{b_{h,h}}(\overrightarrow{1}_i, s) & 0 \\ 0 & \tilde{\Lambda}_{b_{h,v}}(\overrightarrow{1}_i, s) \end{pmatrix} \qquad (2.3)$$

$$\tilde{\Lambda}_{b_{v,h}}(\overrightarrow{1}_i, s) = 0 = \tilde{\Lambda}_{b_{h,v}}(\overrightarrow{1}_i, s)$$

This is the vampire signature. As one rotates the radar around the O_2 symmetric target (keeping $\overrightarrow{1}_h \perp \overrightarrow{1}_z$) this signature is retained.

As discussed in [4] the eigenmodes and natural modes are doubly degenerate for $m \geq 1$ in the cos(mϕ), sin(mϕ) of all these modes due to the O_2 symmetry. As illustrated in fig. 2.2, these two modes can be taken as symmetric and antisymmetric [12] with respect to some plane (the y, z plane in this case). One mode can be taken

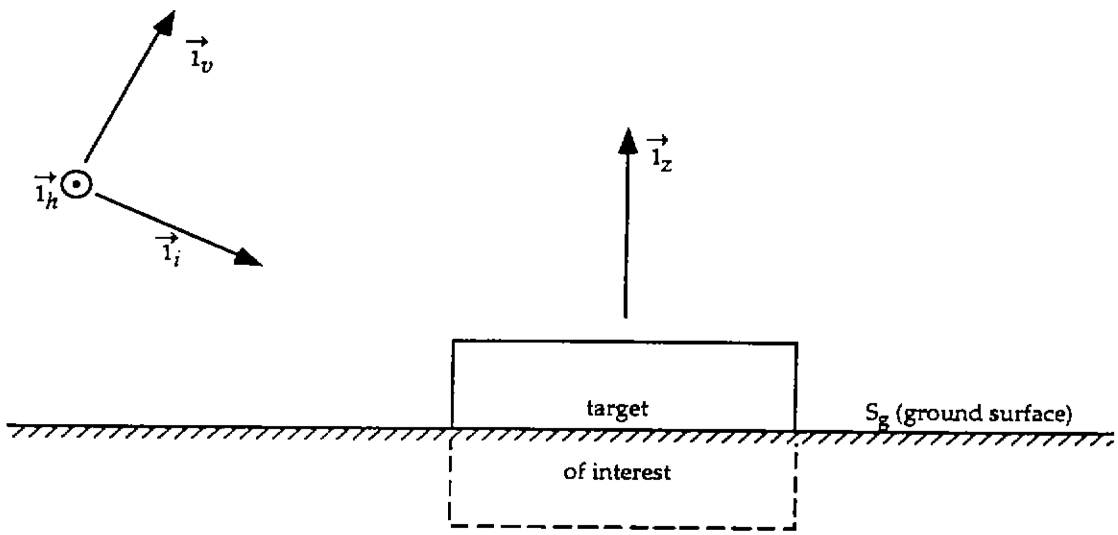
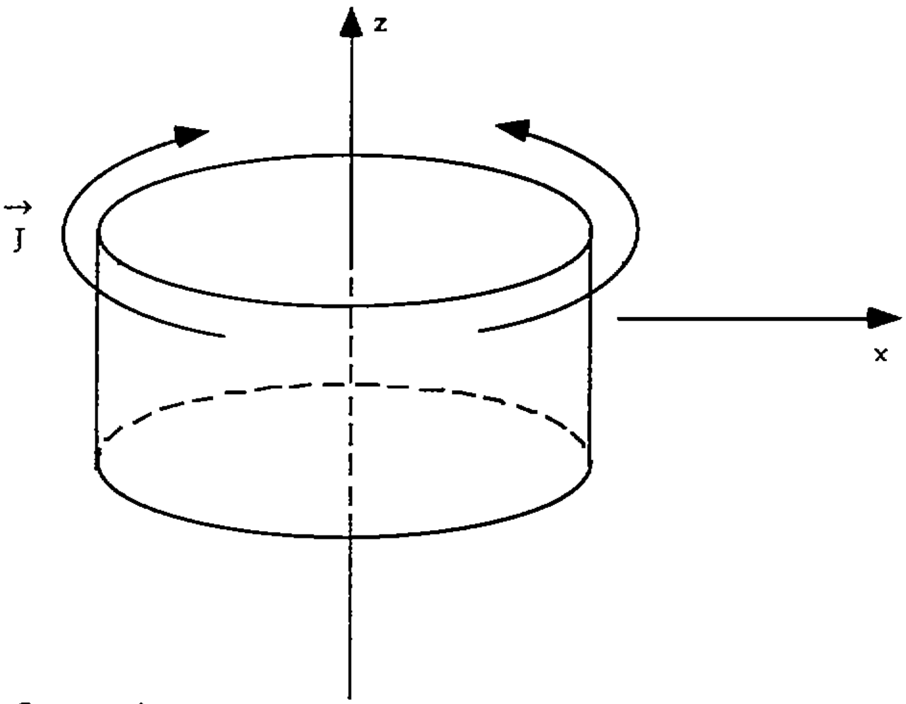
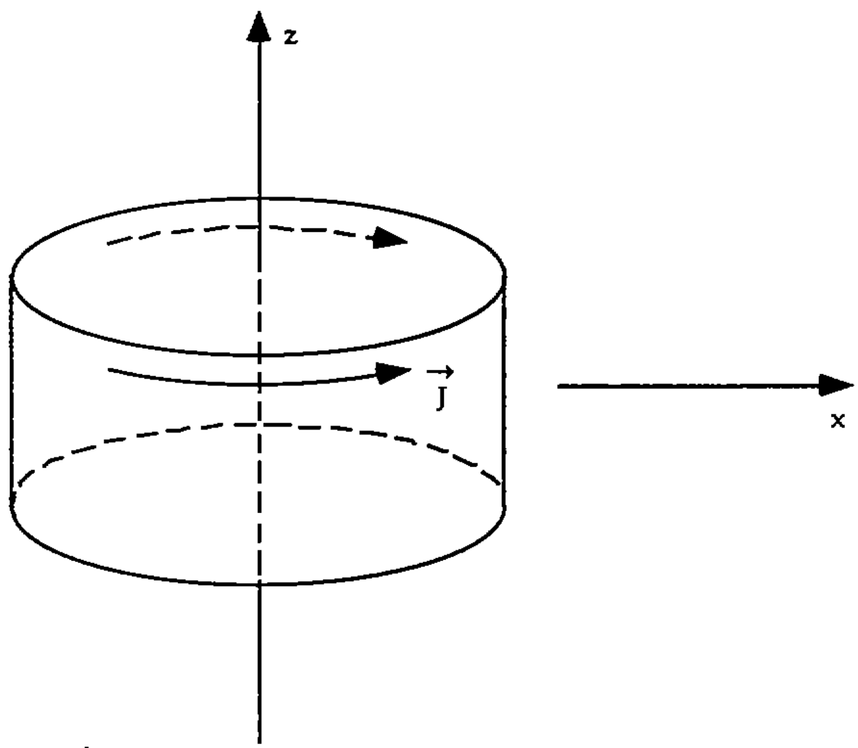


Fig. 2.1. Target and Radar Coordinates.



A. Symmetric



B. Antisymmetric

Fig. 2.2. Modal Degeneracy for $C_{\infty v}$ targets for $m = 1$.

as a 90° rotation of the other mode to form a convenient basis. These two modes share common natural frequencies (associated with a doubly degenerate eigenvalue of the scattering integral equation [3]). For $m \geq 1$, the natural frequencies, being doubly degenerate, appear in both h, h and v, v polarizations, albeit with generally different residues.

For the case of $m = 0$, there is only a single natural mode to consider for each natural frequency (except for accidental degeneracies not associated with the symmetry [4]). Each such natural frequency appears in exactly one polarization (h, h or v, v, but not both).

So, beyond the zero crosspol one can analyze the h,h and v,v scattering to find sets of natural frequencies with properties as just described. Thereby one can proceed further in the identification of the target.

3. Target Axis of Revolution Tilted with Respect to Ground Surface

Suppose now that our target is now tilted somewhat with respect to the ground surface as indicated in fig. 3.1. While the target itself still has O_2 symmetry, the combined target with environment does not. However, in tilting the target we are still left with a symmetry plane S which contains the target axis of revolution (direction $\pm \vec{1}_a$) and is perpendicular to the ground surface ($z = 0$ plane). For convenience we can take the coordinate origin ($\vec{r} = \vec{0}$) on the target axis of revolution, and S as the y, z plane ($x = 0$).

Define the usual cylindrical (Ψ, θ, ϕ) and spherical (r, θ, ϕ) coordinate systems via

$$
\begin{aligned}
x &= \Psi\cos(\phi) \quad , \quad y = \Psi\sin(\phi) \\
z &= r\cos(\theta) \quad , \quad \Psi = r\sin(\theta)
\end{aligned}
\tag{3.1}
$$

Our radar is located at some azimuth ϕ_r. The radar is assumed to have a vertical symmetry plane aligned with a plane of constant $\phi = \phi_r$. As one rotates the radar around the target (varying ϕ_r) one will find two values of ϕ_r, say ϕ_1 and ϕ_2, for which ϕ_r corresponds to the symmetry plane S of the target. In these two cases the backscatter cross polarization will be zero. Only a common symmetry plane is required for this property to hold [2, 4, 11].

With this symmetry alignment the v, v part is symmetric and the h, h part antisymmetric. Whereas in [4] these two parts have modes (eigenmodes and natural modes) that are pure 90° rotations (due to the O_2 symmetry), now such modes are in general slightly different (for small tilt angle ψ). Associated with this mode splitting a single natural frequency s_α is split into two closely spaced natural frequencies which we can designate $s_\alpha^{(sy)}$ and $s_\alpha^{(as)}$.

Now move our radar such that $\phi_r \neq \phi_1, \phi_2$. In this case the crosspol is in general nonzero. However, the natural frequencies, being solutions of the scattering equations with *no incident field*, remain the same and appear in general in all in-line and cross polarizations. Of course, for small ψ we may expect small crosspol but this may require detailed calculations and/or measurements. This kind of perturbation of natural frequencies has an analog in quantum mechanics where energy levels are perturbed by breaking symmetry as in the Stark and Zeeman effects [9, 10].

For $m = 0$, as discussed in [4], there is no double modal degeneracy. For O_2 symmetry (no tilting) such modes are ϕ independent with components expressed in cylindrical or spherical coordinates. Each mode is either symmetric (no ϕ component of electric current density) or antisymmetric (only ϕ component of electric current density), but not both. So certain natural frequencies $s_\alpha^{(sy)}$ appear in only v, v polarization and others $s_\alpha^{(as)}$ in only

h, h polarization. In this case, tilting the target a small angle ψ changes these modes a small amount and shifts the natural frequencies a small amount, but does not split the s_α into two where there was one.

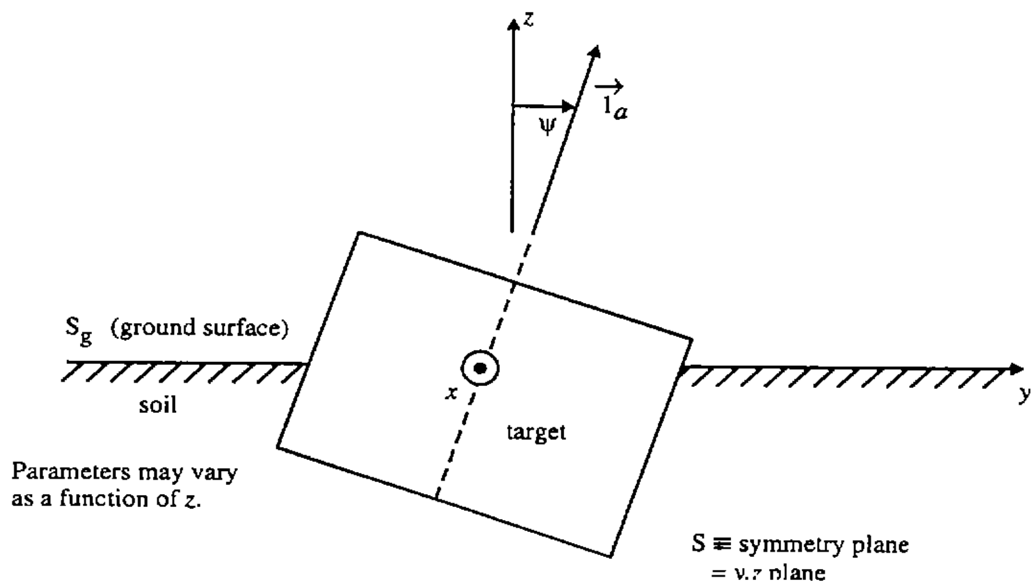


Fig. 3.1. Tilted Body of Revolution on/in Layered Earth.

4. More general Symmetry Breaking

The previous section considered a special kind of symmetry breaking, namely tilting, which left reflection symmetry R_x instead of O_2 symmetry. In this case the mode splitting for $m \geq 1$ can be described by symmetric and antisymmetric modes with respect to the symmetry plane S. However, there are more general types of symmetry breaking that one encounters. These also split the doubly degenerate modes and natural frequencies.

The deviations from ideal O_2 symmetry may include:

a. non-flat ground surface (including rough surface)
b. ground inhomogeneities other than vertical layering (e.g., rocks)
c. imperfections in the target (intentional or accidental)
d. non-ideal target placement (e.g., tilting as in Section 3)

A realistic situation may include all of the above.

A general approach to such problems is perturbation theory in which the deviation from O_2 symmetry is small in an appropriate sense. In this procedure, one first solves for the case with O_2 symmetry, finding natural frequencies and modes (say using a superscript 0). Then one writes the natural frequencies and modes as

$$\begin{aligned} s_\alpha &= s_\alpha^{(0)} + \Delta s_\alpha \\ \vec{j}_\alpha(\vec{r}) &= \vec{j}_\alpha^{(0)}(\vec{r}) + \Delta \vec{j}_\alpha(\vec{r}) \end{aligned} \tag{4.1}$$

the idea being to solve for the Δs_α for small excursions from O_2 symmetry. One appraoch to this consists in setting up an integral equation with incident field set to zero (natural frequencies being values of s which admit solutions (natural modes) $\vec{j}_\alpha(\vec{r})$ with no forcing function). This homogeneous integral equation is then cast in two forms: perturbed and unperturbed. The perturbed problem can involve a perturbed Green's function (e.g., accounting for medium inhomogeneities) and/or a perturbed domain of integration (presence or lack of soil at earth surface or change in medium parameters). Expanding the Green's function n a Taylor series about $s_\alpha^{(0)}$ the equations can be manipulated to obtain an equation for small Δs_α. At this point, one can note that there can be more than one Δs_α corresponding to the degree of degeneracy of the $\vec{j}_\alpha^{(0)}(\vec{r})$.

With the general problem being numerically complex, some simpler cases have been solved which can shed some light on the present problem. In [1] the perturbation of the thin-wire poles by a mirror thin-wire is considered. This shows the case of splitting double degeneracy for mirror objects in free space. One of the solutions (antisymmetric) corresponds to a thin wire in the presence of a perfectly conducting plane. In [7] the general perturbation formulas for a set of bodies are presented. This approach can be applied, for example, to the problem of a buried target in the presence of a rock (not too close) using the Green's function for auniform or layered-soil half space. In [5] the effect of the soil-half-space surface on the natural frequencies of a buried target is considered. Numerical results are presented for a thin wire showing how the perturbation of a natural frequency depends on the tilting of the wire with respect to the ground surface. These canonical results can be used to get some physical understanding of the perturbation process, and lead to additional canonical calculations.

5. Special Case of O_3 Symmetry

One can also have a target with O_3 symmetry (orthogonal symmetry in three dimensions). Such a target can be a perfectly conducting sphere, a uniform dielectric sphere, or a more general form as discussed in [3]. In this case any axis can be taken as the rotation axis $\vec{1}_a$. For such a target buried in the ground there is no sense in trying to tilt it. Just align $\vec{1}_a$ perpendicular to S_g and we evidently have a case of O_2 symmetry, the O_3 symmetry being broken by the ground surface. This remains O_2 as long as the ground retains O_2 symmetry (e.g., layered).

This type of target buried in an appropriate halfspace then retains the vampire signature, no matter what it's orientation or location (e.g., protruding through the ground surface).

6. Concluding Remarks

Symmetry analysis has shown how the two-fold modal degeneracy (for $m > 1$) for a buried target and environment with O_2 symmetry is split into two distinct modes and natural frequencies by breaking the O_2 symmetry. This is a qualitative result, not giving quantitative values for the natural-frequency shifts. However, it is a starting point for such a quantiative analysis. In particular the O_2 symmetry results can be used as a first term in a perturbation analysis which can be considerably simpler and give more physical insight than a purely numerical solution.

While some perturbation analyses exist for thin wires in the presence of lossy dielectric half spaces, such analyses need to be extended to O_2 targets such as conducting and dielectric circular disks and truncated circular cylinders, as well as more realistic target-representative shapes.

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