Interaction Note

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10 August 1998

Target-Vicinity Scattering Parameters

Carl E. Baum
Air Force Research Laboratory
Directed Energy Directorate

Abstract

In identifying a target based on information contained in the scattered field one would like to avoid the complications introduced by the medium around the target to the extent feasible. By concentrating on the properties of the Green functions with both source and observer on the target (i.e., the scattering integral-equation operator) we are able to define target-vicinity scattering parameters which are less sensitive to the clutter away from the target (by a few target dimensions). These are independent of the incident field and the distant measured scattered field. We are also able to solve a simpler scattering problem (the canonical problem) for a first approximation to these problems. The change or difference from this simpler problem can be addressed via perturbation theory.

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In identifying a target based on information contained in the scattered field one would like to avoid the complications introduced by the medium around the target to the extent feasible. By concentrating on the properties of the Green functions with both source and observer on the target (i.e., the scattering integral-equation operator) we are able to define target-vicinity scattering parameters which are less sensitive to the clutter away from the target (by a few target dimensions). These are independent of the incident field and the distant measured scattered field. We are also able to solve a simpler scattering problem (the canonical problem) for a first approximation to these problems. The change or difference from this simpler problem can be addressed via perturbation theory.

1. Introduction

The scattering of an electromagnetic wave from a target in a complex nonuniform medium, and using the scattered fields to identify the target is quite a challenging problem. Even with a finite library of target types (e.g., mine or unexploded ordnance (UXO) the various types of clutter (e.g., rocks rough ground surface, etc.) can greatly distort the signal one is trying to receive from the target. One would like to separate the clutter signals from the target signals to the extent possible.

One approach to this problem (as in fig. 1.1) is to separate out the scattering phenomena at the target (volume V_s) and in its immediate vicinity V_v, from the scattering of both incident and target-scattered fields at more distant positions (clutter, ground surface, etc.). So we would like to concentrate on parameters which are associated with the target itself, and not the propagation to and from the target through a scattering medium. We can call such parameters *target-vicinity parameters*. Of course, such parameters still have to be measured by the scattered field as received by our antennas.

In the analysis let us assume that the target vicinity V_v as in fig. 1.1 has a simple or "uncluttered" characteristic. It might be uniform and isotropic, or at least of a simpler character than that of the medium farther away. In our approach to this problem we will define a canonical or background problem and compare this to the more realistic problem with the clutter. By such we may be able to see what remains the same, or nearly so. This can even lead to a perturbation analysis to approximately quantify the changes.

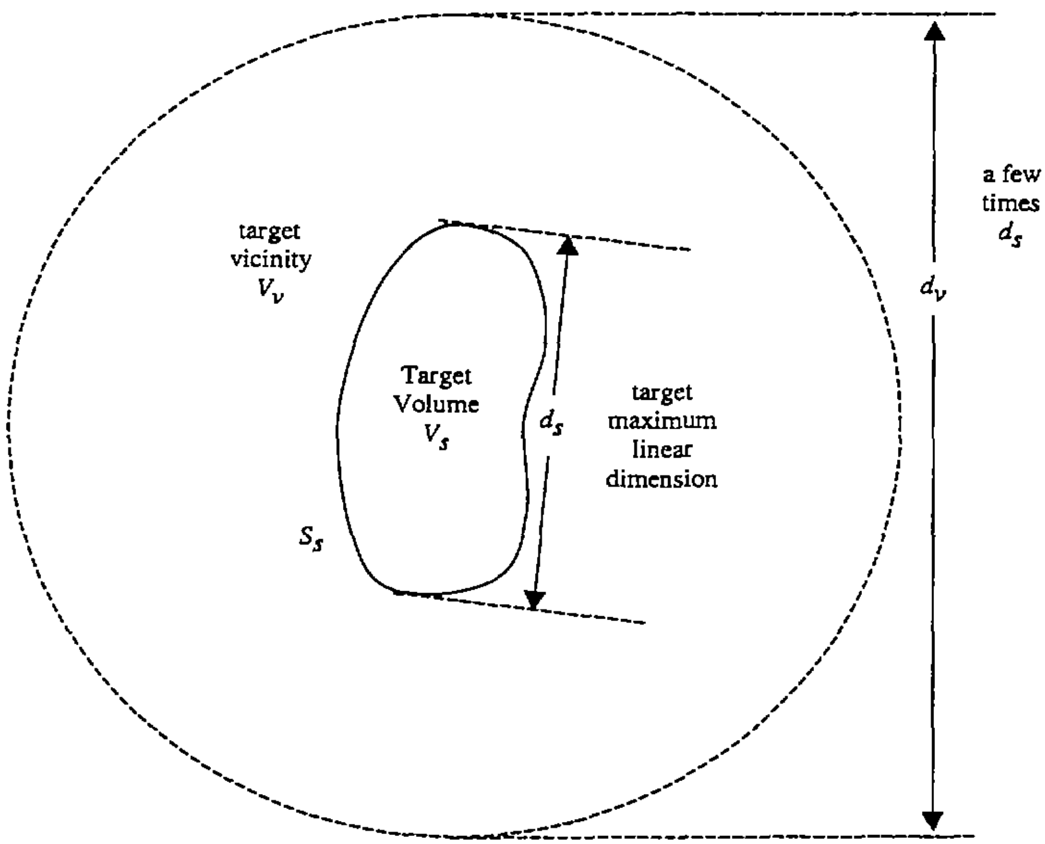


Fig. 1.1. Target and Vicinity.

2. Green Functions and Integral Equations

Here we are concerned with some of the general properties of Green functions and integral equations for scattering [9], not the details required for a numerical computation. For a set of electric- and magnetic-current-density sources we can write the resulting electric and magnetic fields as integrals (symmetric products) of the form (in complex-frequency domain)

$$\begin{aligned}
\tilde{\vec{E}}(\vec{r},s) &= \left\langle \tilde{\overleftrightarrow{D}}_{1,1}(\vec{r},\vec{r}\,';s); \tilde{\vec{J}}_e(\vec{r}\,',s)\right\rangle + \left\langle \tilde{\overleftrightarrow{D}}_{1,2}(\vec{r},\vec{r}\,';s); \tilde{\vec{J}}_h(\vec{r}\,',s)\right\rangle \\
\tilde{\vec{H}}(\vec{r},s) &= \left\langle \tilde{\overleftrightarrow{D}}_{2,1}(\vec{r},\vec{r}\,';s); \tilde{\vec{J}}_e(\vec{r}\,',s)\right\rangle + \left\langle \tilde{\overleftrightarrow{D}}_{2,2}(\vec{r},\vec{r}\,';s); \tilde{\vec{J}}_h(\vec{r}\,',s)\right\rangle \\
&\sim \;\equiv\; \text{Laplace transform (2-sided) over time } t \\
&s \equiv \Omega + j\omega = \text{Laplace-transform variable or complex frequency}
\end{aligned} \tag{2.1}$$

At this point (2.1) is merely a statement of linearity. The $\tilde{\overleftrightarrow{D}}_{p,q}$ represent the fields from discrete sources (spatial delta functions) and can be regarded as Green functions. Their detailed mathematical form depends on the properties (inhomogeneity, anisotropy, etc.) of the medium of concern, as well as radiation conditions (causality in time domain) where appropriate.

The Maxwell equations with constitutive relations give

$$\begin{aligned}
\nabla \times \tilde{\vec{E}}(\vec{r},s) &= -s\,\tilde{\overleftrightarrow{\mu}}(\vec{r},s)\cdot \tilde{\vec{H}}(\vec{r},s) \\
\nabla \times \tilde{\vec{H}}(\vec{r},s) &= [\tilde{\overleftrightarrow{\sigma}}(\vec{r},s) + s\,\tilde{\overleftrightarrow{\varepsilon}}(\vec{r},s)]\cdot \tilde{\vec{E}}(\vec{r},s)
\end{aligned} \tag{2.2}$$

where we have limited ourselves to the simple form of inhomogeneity and anisotropy above (avoiding, for example, chirality). Applying this to (2.1) gives (away from sources, $\vec{r} \neq \vec{r}\,'$)

$$\begin{aligned}
\tilde{\overleftrightarrow{D}}_{1,1}(\vec{r},s) &= [\tilde{\overleftrightarrow{\sigma}}(\vec{r},s) + s\,\tilde{\overleftrightarrow{\varepsilon}}(\vec{r},s)]^{-1} \cdot [\nabla \times \tilde{\overleftrightarrow{D}}_{2,1}(\vec{r},s)] \\
\tilde{\overleftrightarrow{D}}_{1,2}(\vec{r},s) &= [\tilde{\overleftrightarrow{\sigma}}(\vec{r},s) + s\,\tilde{\overleftrightarrow{\varepsilon}}(\vec{r},s)]^{-1} \cdot [\nabla \times \tilde{\overleftrightarrow{D}}_{2,2}(\vec{r},s)] \\
\tilde{\overleftrightarrow{D}}_{2,1}(\vec{r},s) &= -\frac{1}{s}\tilde{\overleftrightarrow{\mu}}^{-1}(\vec{r},s) \cdot [\nabla \times \tilde{\overleftrightarrow{D}}_{1,1}(\vec{r},s)] \\
\tilde{\overleftrightarrow{D}}_{2,2}(\vec{r},s) &= -\frac{1}{s}\tilde{\overleftrightarrow{\mu}}^{-1}(\vec{r},s) \cdot [\nabla \times \tilde{\overleftrightarrow{D}}_{1,2}(\vec{r},s)]
\end{aligned} \tag{2.3}$$

from which wave-like equations for the $\overleftrightarrow{D}_{p,q}$ can be derived. If one assumes reciprocal media (symmetric constitutive-parameter dyadics) then there is also symmetry on interchange of $\vec{r}$ and $\vec{r}\,'$.

For the simple case of uniform isotropic media (2.3) reduces via

$$\tilde{\gamma}^2(s) = s\tilde{\mu}(s)[\tilde{\sigma}(s) + \tilde{\varepsilon}(s)] \tag{2.4}$$

to have all four Green functions satisfy the usual wave equation with $\tilde{\gamma}$ as the propagation constant. In this case the dyadic Green functions can be written down in closed form [2, 4, 5]. This shows a singularity as $\vec{r} \rightarrow \vec{r}\,'$. This is an important phenomenon, which we use later when considering approximations. In the more general case there is still such a singularity but the complete representation of the Green functions is more complicated. In some physical situations with lots of clutter (rocks, etc.) one may regard these functions as numerically calculable or even measurable by appropriate experiments.

Applying (2.1) to scattering from a target we separate the fields into incident and scattered components, noting that the fields in (2.1) are now the scattered fields in

$$\begin{aligned} \tilde{\vec{E}}(\vec{r},s) &= \tilde{\vec{E}}^{(inc)}(\vec{r},s) + \tilde{\vec{E}}^{(sc)}(\vec{r},s) \\ \tilde{\vec{H}}(\vec{r},s) &= \tilde{\vec{H}}^{(inc)}(\vec{r},s) + \tilde{\vec{H}}^{(sc)}(\vec{r},s) \end{aligned} \tag{2.5}$$

The target contrast (relative to the background medium) is given by

$$\begin{aligned} \tilde{\overleftrightarrow{\varepsilon}}_c(\vec{r},s) &\equiv \frac{1}{s}\left[\left[\tilde{\overleftrightarrow{\sigma}}_s(\vec{r},s) + s\,\tilde{\overleftrightarrow{\varepsilon}}_s(\vec{r},s)\right] - \left[\tilde{\overleftrightarrow{\sigma}}(\vec{r},s) + s\,\tilde{\overleftrightarrow{\varepsilon}}(\vec{r},s)\right]\right] \\ \tilde{\overleftrightarrow{\mu}}_c(\vec{r},s) &\equiv \tilde{\overleftrightarrow{\mu}}_s(\vec{r},s) - \tilde{\overleftrightarrow{\mu}}(\vec{r},s) \end{aligned} \tag{2.6}$$

where a subscript, refers to the target (scatterer). The constitutive parameters of the medium within the target volume are those that would have been there were the target not present. They are used in defining the Green functions and we have some latitude in choosing them for our convenience. Note that the contrast permittivity has subsumed the conductivity within it for convenience. The electric- and magnetic-curent densities are related to the target contrast as

$$\tilde{\overleftrightarrow{J}}_e(\vec{r},s) = s\,\tilde{\overleftrightarrow{\varepsilon}}_c(\vec{r},s)\cdot\tilde{\vec{E}}(\vec{r},s)$$

$$\overleftrightarrow{J}_h(\vec{r},s) = s\,\overleftrightarrow{\mu}_c(\vec{r},s)\cdot\vec{H}(\vec{r},s) \tag{2.7}$$

Since the target volume V_s is restricted, only over this volume is the contrast nonzero. The symmetric products in (2.1) involve integrals only over V_s.

Include a normalizing impedance Z (ohms, perhaps frequency dependent) such as a wave impedance together with the magnetic field for convenience. Then the foregoing equations give the scattering equation

$$\begin{aligned}
&\left\langle (\overleftrightarrow{d}_{p,q}(\vec{r},\vec{r}\,';s)) \odot_{,} \begin{pmatrix} \vec{E}^{(inc)}(\vec{r}\,',s) \\ Z\vec{H}^{(inc)}(\vec{r}\,',s) \end{pmatrix} \right\rangle \\
&= \begin{pmatrix} \vec{E}^{(sc)}(\vec{r},s) \\ Z\vec{H}^{(inc)}(\vec{r},s) \end{pmatrix} - \left\langle (\overleftrightarrow{d}_{p,q}(\vec{r},\vec{r}\,';s)) \odot_{,} \begin{pmatrix} \vec{E}^{(sc)}(\vec{r}\,',s) \\ Z\vec{H}^{(sc)}(\vec{r}\,',s) \end{pmatrix} \right\rangle \\
&= \left\langle (\overleftrightarrow{\delta}_{p,q}(\vec{r},\vec{r}\,')) - (\overleftrightarrow{d}_{p,q}(\vec{r},\vec{r}\,';s)) \odot_{,} \begin{pmatrix} \vec{E}^{(sc)}(\vec{r}\,',s) \\ Z\vec{H}^{(sc)}(\vec{r}\,',s) \end{pmatrix} \right\rangle \\
&(\overleftrightarrow{d}_{p,q}(\vec{r},\vec{r}\,';s)) = s \begin{pmatrix} \overleftrightarrow{D}_{1,1}(\vec{r},\vec{r}\,';s)\cdot\overleftrightarrow{\varepsilon}_c(\vec{r}\,',s) & Z^{-1}\overleftrightarrow{D}_{1,2}(\vec{r},\vec{r}\,';s)\cdot\overleftrightarrow{\mu}_c(\vec{r}\,',s) \\ Z\overleftrightarrow{D}_{2,1}(\vec{r},\vec{r}\,';s)\cdot\overleftrightarrow{\varepsilon}_c(\vec{r}\,',s) & \overleftrightarrow{D}_{1,1}(\vec{r},\vec{r}\,';s)\cdot\overleftrightarrow{\mu}_c(\vec{r}\,',s) \end{pmatrix} \\
&(\overleftrightarrow{\delta}_{p,q}(\vec{r},\vec{r}\,')) = \begin{pmatrix} \overleftrightarrow{1}\,\delta(\vec{r}-\vec{r}\,') & \overleftrightarrow{0} \\ \overleftrightarrow{0} & \overleftrightarrow{1}\,\delta(\vec{r}-\vec{r}\,') \end{pmatrix} \\
&\overleftrightarrow{1} \equiv \vec{1}_x\vec{1}_x + \vec{1}_y\vec{1}_y + \vec{1}_z\vec{1}_z \equiv \text{3-dimensional identity}
\end{aligned} \tag{2.8}$$

Noting now that integration includes $\vec{r} = \vec{r}\,'$, one needs to be careful concerning the singularity there [2, 8]. Formally this integral equation is solved by inverting the integral operator involving the superdyadic

$$(\overleftrightarrow{g}_{p,q}(\vec{r},\vec{r}\,';s)) \equiv (\overleftrightarrow{\delta}_{p,q}(\vec{r},\vec{r}\,')) - (\overleftrightarrow{d}_{p,q}(\vec{r},\vec{r}\,';s)) \tag{2.9}$$

and, in turn, operating on the term involving the supervector (6-component) incident field.

Now (2.8) is quite general and special cases simplify it somewhat. If the permeability contrast $\tilde{\overleftrightarrow{\mu}}_c$ is zero, then (2.8) is reduced to a single 3×3 integral equation involving only $\tilde{\overleftrightarrow{D}}_{1,1}$ from (2.1) together with the permittivity contrast. If the target is perfectly conducting then we have on the target surface S_s

$$\overleftrightarrow{1}_s(\vec{r}) \cdot \left[\tilde{\vec{E}}^{(inc)}(\vec{r},s) + \tilde{\vec{E}}^{(sc)}(\vec{r},s) \right] = \vec{0}$$

$$\overleftrightarrow{1}_s(\vec{r}) = \overleftrightarrow{1} - \vec{1}_s(\vec{r})\, \vec{1}_s(\vec{r}) \tag{3.10}$$

$$\vec{1}_s(\vec{r}) \equiv \text{unit outward pointing normal to } S_s \text{ at } \vec{r} \text{ (on } S_s)$$

and (2.8) is again reduced to a single integral equation involving only $\tilde{\overleftrightarrow{D}}_{1,1}$ from (2.1), and the integration is now only over tangential components (2 components) on S_s instead of over the volume V_s.

At this point we can note that (2.8), as stated, includes the incident field which is not a property of the target. The target information is totally contained in the integral operator over V_s (or S_s) involving $(\tilde{\overleftrightarrow{g}}_{p,q})$. So while we need an incident field to illuminate the target and produce a scattered field containing information about the target, it is the information in the integral operator that we seek.

There are various ways to view this operator [2]. The natural frequencies and modes used in the singularity expansion method (SEM) are solutions of

$$\left\langle (\tilde{\overleftrightarrow{g}}_{p,q}(\vec{r},\vec{r}\,';s_\alpha)) \odot (\vec{e}_p(\vec{r}\,'))_{\alpha} \right\rangle = \vec{0} \tag{2.11}$$

This involves integration over the body, but the $\tilde{\overleftrightarrow{d}}_{p,q}$ contain information about the external medium. In any event the natural frequencies and modes are independent of the incident field. One can also consider the eigenvalues and eigenmodes of the operator used in the eigenmode expansion method (EEM) as

$$
\left\langle (\tilde{\overleftrightarrow{g}}_{p,q}(\vec{r},\vec{r}\,';s_\alpha)) \overset{\odot}{;} (\tilde{\vec{e}}_p(\vec{r}\,',s))_\beta \right\rangle = \tilde{\lambda}_\beta(s)\ (\tilde{\vec{e}}_p(\vec{r},s))_\beta
$$

$$
\left\langle (\tilde{\vec{\ell}}_p(\vec{r},s))_\beta \overset{\odot}{;} (\tilde{\overleftrightarrow{g}}_{p,q}(\vec{r},\vec{r}\,';s_\alpha)) \right\rangle = \tilde{\lambda}_\beta(s)\ (\tilde{\vec{\ell}}_p(\vec{r}\,',s))_\beta \tag{2.12}
$$

$$
\left\langle (\tilde{\vec{\ell}}_p(\vec{r}\,',s))_{\beta_1} \overset{\odot}{;} (\tilde{\vec{e}}_p(\vec{r}\,',s))_{\beta_2} \right\rangle = 1_{\beta_1,\beta_2} = \begin{cases} 1 \beta_1 = \beta_2 \\ 0 \ \text{otherwise} \end{cases}
$$

(biorthonormal)

These are also independent of the incident field and can be used to order the natural frequencies and modes. One can also look at the low-frequency properties leading to polarizability dyadics [4]. For high frequencies one could look at the asymptotic form of the dyadic operator as $s \to \infty$ in appropriate regions of the complex frequency plane.

3. Decomposition of Green Functions

Now write the Green functions in the form

$$
\begin{aligned}
\tilde{\overleftrightarrow{D}}_{p,q}(\vec{r},\vec{r}\,';s) &= \tilde{\overleftrightarrow{D}}^{(c)}_{p,q}(\vec{r},\vec{r}\,';s) + \tilde{\overleftrightarrow{D}}^{(\Delta)}_{p,q}(\vec{r},\vec{r}\,';s) \\
(\tilde{\overleftrightarrow{d}}_{p,q}(\vec{r},\vec{r}\,';s)) &= (\tilde{\overleftrightarrow{d}}^{(c)}_{p,q}(\vec{r},\vec{r}\,';s)) + (\tilde{\overleftrightarrow{d}}^{(\Delta)}_{p,q}(\vec{r},\vec{r}\,';s)) \\
(\tilde{\overleftrightarrow{g}}_{p,q}(\vec{r},\vec{r}\,';s)) &= (\tilde{\overleftrightarrow{g}}^{(c)}_{p,q}(\vec{r},\vec{r}\,';s)) + (\tilde{\overleftrightarrow{g}}^{(\Delta)}_{p,q}(\vec{r},\vec{r}\,';s)) \\
& c \Rightarrow \text{canonical or reference problem} \\
& \Delta \Rightarrow \text{difference or change} \\
(\tilde{\overleftrightarrow{g}}^{(\Delta)}_{p,q}(\vec{r},\vec{r}\,';s)) &= -\,(\tilde{\overleftrightarrow{d}}^{(\Delta)}_{p,q}(\vec{r},\vec{r}\,';s))
\end{aligned}
\tag{3.1}
$$

Here we have in mind some canonical or reference problem, which is simpler than the actual problem in important ways. In particular, we would like the integral operator (over V_s) to be approximately the same for the two problems so as to give, for example, nearly the same natural frequencies, natural modes, eigenvalues, and eigenmodes as discussed in Section 2.

Referring to fig. 1.1, let us consider what appropriate canonical problems might be. As a simple example, let us assume that the external medium (e.g., soil) is uniform and isotropic in the target vicinity with scalar constitutive parameters $\tilde{\sigma}(s)$, $\tilde{\varepsilon}(s)$ and $\tilde{\mu}(s)$. Extend these same constitutive parameters into V_s for defining the contrast as in (2.6). Furthermore, extend these same spatially independent constitutive parameters to all space, thereby defining the canonical problem as the target in a uniform isotropic medium for which the Green functions are well known. In the real problem there may be inhomogeneities (e.g., rocks) outside the target vicinity. Thinking of these are secondary scatterers then the difference Green functions are small (in V_s) if the scattering back at the target is small.

The canonical problem being defined in terms of the medium in the target vicinity, then the singularity at $\vec{r} = \vec{r}\,'$ (in V_s) is contained in the canonical Green functions and is absent from the difference Green functions. This is advantageous since we would like the canonical Green functions to dominate the target response. Viewing the difference Green functions as related to a multiple scattering problem, we can see the importance of such scatterers being sufficiently far away. If the target maximum linear dimension is d_s, then the linear dimension of the target vicinity d_v should be somewhat larger, how much larger to be determined. In fig. 1.1, the target vicinity (on which the canonical problem is based) is shown as bounded by a sphere, but other shapes may also be useful.

Note that there are cases in which this decomposition in (3.1) is not appropriate because the scattering back to the target is large. Such is the case in which the external medium is lossless and enclosed in a perfectly conducting cavity. Another example has a reflector in the external medium focussing the target scattering back onto the target. Recognizing that there are exceptions, let us continue.

In a more general sense the definition of the canonical problem can be based on symmetry [12]. Take the symmetry inherent in the media in the target vicinity and extend it through the target and on to infinity. The previous example was a simple case of a canonical uniform isotropic medium. In such a case the target can be rotated and translated in this medium without changing the canonical natural frequencies, natural modes, eigenvalues and eigenmodes. We can list some possibilities for the canonical medium in Table 3.1.

Table 3.1. Canonical Media and Associated Symmetry.

Canonical Medium		Symmetry
Uniform isotropic		All translations and rotations
Uniform anisotropic		All translations
Layered medium, i.e., variation in only one Cartesian coordinate z		
	isotropic -also allows anisotropy with axis parallel to z	Rotations parallel to and translations perpendicular to z axis
	anisotropic (general)	Translations perpendicular to z axis

Commenting on some of these cases the uniform isotropic medium can in principle be chiral, but this should not be significant for targets in the presence of soil. The layered medium includes as a special case the uniform isotropic half space such as might model soil in the presence of air. Concerning the symmetry an important consideration concerns its relation to the target symmetry. If the target symmetry group is a subgroup of the medium symmetry group then the medium symmetry does not destroy the target symmetry as far as its electromagnetic response. An example is a target with a two-dimensional rotation axis (discrete or continuous) parallel to the z axis defined by an isotropic soil half space in the presence of air.

4. Perturbation of Canonical Solution

If the difference term in (3.1) is sufficiently small we may consider a perturbation solution. Following the procedure used in quantum mechanics [1, 10, 11] we introduce a perturbation parameter h and write

$$
\begin{aligned}
(\tilde{\overleftrightarrow{g}}_{p,q}(\vec{r},\vec{r}\,';s)) &= (\tilde{\overleftrightarrow{g}}^{(c)}_{p,q}(\vec{r},\vec{r}\,';s)) + h(\tilde{\overleftrightarrow{g}}^{(\Delta)}_{p,q}(\vec{r},\vec{r}\,';s)) \\
h &= \begin{cases} 0 \Rightarrow & \text{unperturbed or canonical problem} \\ 1 \Rightarrow & \text{perturbed or actual problem} \end{cases}
\end{aligned}
\tag{4.1}
$$

Then the eigenvalues and eigenmodes are expanded in a series as

$$
\begin{aligned}
\tilde{\lambda}_\beta(s) &= \sum_{n=0}^{\infty} h^n\, \tilde{\lambda}^{(n)}_\beta(s) \\
(\tilde{\vec{e}}_p(\vec{r},s))_\beta &= \sum_{n=0}^{\infty} h^n (\tilde{\vec{e}}^{\,(n)}_p(\vec{r},s))_\beta \\
(\tilde{\vec{\ell}}_p(\vec{r},s))_\beta &= \sum_{n=0}^{\infty} h^n (\tilde{\vec{\ell}}^{\,(n)}_p(\vec{r},s))_\beta
\end{aligned}
\tag{4.2}
$$

The $n = 0$ terms correspond to the canonical problem, the $n = 1$ terms give the first order correction for the perturbed problem, and the $n \geq 2$ terms give successive higher order corrections.

Substituting these series in (2.12), collect terms according to the powers of h. For $n = 0$ we have

$$
\begin{aligned}
&\left\langle (\tilde{\overleftrightarrow{g}}^{(c)}_{p,q}(\vec{r},\vec{r}\,';s)) \;\odot\; (\tilde{\vec{e}}^{\,(0)}_{p,q}(\vec{r}\,',s))_\beta \right\rangle = \tilde{\lambda}^{(0)}_\beta(s)\,(\tilde{\vec{e}}^{\,(0)}_p(\vec{r},s))_\beta \\
&\left\langle (\tilde{\vec{\ell}}^{\,(0)}_{p,q}(\vec{r},s))_\beta \;\odot\; (\tilde{\overleftrightarrow{g}}_{p,q}(\vec{r},\vec{r}\,',s))_\beta \right\rangle = \tilde{\lambda}^{(0)}_\beta(s)\,(\tilde{\vec{\ell}}^{\,(0)}_p(\vec{r}\,',s))_\beta \\
&\left\langle (\tilde{\vec{\ell}}^{\,(c)}_p(\vec{r}\,',s))_{\beta_1} \;\odot\; (\tilde{\vec{e}}^{\,(0)}_p(\vec{r}\,',s))_{\beta_2} \right\rangle = 1_{\beta_1,\beta_2} \\
&\left\langle (\tilde{\vec{\ell}}^{\,(0)}_p(\vec{r},s))_\beta \;\odot\; (\tilde{\overleftrightarrow{g}}^{(c)}_{p,q}(\vec{r},\vec{r}\,';s)) \;\odot\; (\tilde{\vec{e}}^{\,(0)}_p(\vec{r}\,',s))_\beta \right\rangle = \tilde{\lambda}^{(0)}_\beta(s)
\end{aligned}
\tag{4.3}
$$

Note that these parameters are all associated with the canonical (unperturbed) problem only. The "difference" in the Green functions has not yet appeared. For $n = 1$ we have

$$
\begin{aligned}
&\left\langle (\overleftrightarrow{g}^{(c)}_{p,q}(\vec{r},\vec{r}\,';s)) \odot (\vec{e}^{(1)}_{p}(\vec{r}\,',s))_{\beta_2} \right\rangle + \left\langle (\overleftrightarrow{g}^{(\Delta)}_{p,q}(\vec{r},\vec{r}\,';s)) \odot (\vec{e}^{(0)}_{p}(\vec{r}\,',s))_{\beta} \right\rangle \\
&\quad = \tilde{\lambda}^{(0)}_{\beta}(s)\,(\vec{e}^{(1)}_{p}(\vec{r},s))_{\beta} + \tilde{\lambda}^{(1)}_{\beta}(s)\,(\vec{e}^{(0)}_{p}(\vec{r},s))_{\beta} \\
&\left\langle (\vec{\ell}^{(1)}_{p}(\vec{r},s))_{\beta} \odot (\overleftrightarrow{g}^{(c)}_{p,q}(\vec{r},\vec{r}\,';s)) \right\rangle + \left\langle (\vec{\ell}^{(0)}_{p}(\vec{r},s)) \odot (\overleftrightarrow{g}^{(\Delta)}_{p,g}(\vec{r},\vec{r}\,';s))_{\beta} \right\rangle \\
&\quad = \tilde{\lambda}^{(0)}_{\beta}(s)\,(\vec{\ell}^{(1)}_{p}(\vec{r}\,',s))_{\beta} + \tilde{\lambda}^{(1)}_{\beta}(s)\,(\vec{\ell}^{(0)}_{p}(\vec{r}\,',s))_{\beta} \\
&\left\langle (\vec{\ell}^{(0)}_{p}(\vec{r}\,',s))_{\beta_1} \odot (\vec{e}^{(1)}_{p,q}(\vec{r}\,',s))_{\beta_2} \right\rangle + \left\langle (\vec{\ell}^{(1)}_{p}(\vec{r}\,',s))_{\beta_1} \odot (\vec{e}^{(0)}_{p}(\vec{r}\,',s))_{\beta_2} \right\rangle = 0
\end{aligned}
\tag{4.4}
$$

Operating on the left of the first of these equations by $(\vec{\ell}^{(0)}_{\beta;p}(\vec{r},s)) \odot$ or on the right of the second of these by $\odot\, (\vec{r}^{(0)}_{\beta;p}(\vec{r}\,',s))$ gives

$$
\tilde{\lambda}^{(1)}_{\beta}(s) = \left\langle (\vec{\ell}^{(0)}_{p}(\vec{r},s))_{\beta} \odot (\overleftrightarrow{g}^{(\Delta)}_{p,q}(\vec{r},\vec{r}\,';s)) \odot (\vec{e}^{(0)}_{p}(\vec{r}\,',s))_{\beta} \right\rangle \tag{4.5}
$$

as our basic perturbation equation. Setting $h = 1$ gives our perturbation solution for the eigenvalues as

$$
\begin{aligned}
\tilde{\lambda}_{\beta}(s) &= \tilde{\lambda}^{(0)}_{\beta}(s) + \tilde{\lambda}^{(1)}_{\beta}(s) + \cdots \\
&\simeq \tilde{\lambda}^{(0)}_{\beta}(s) + \tilde{\lambda}^{(1)}_{\beta}(s)
\end{aligned}
\tag{4.6}
$$

For the special case that the Green functions are symmetric (e.g., the impedance (E field) integral equation [3]) the foregoing simplify somewhat as

$$
\begin{aligned}
&(\overleftrightarrow{g}^{(c)}_{p,q}(\vec{r},\vec{r}\,';s)) = (\overleftrightarrow{g}^{(c)}_{p,q}(\vec{r},\vec{r}\,';s))^{T} \\
&(\overleftrightarrow{g}^{(\Delta)}_{p,q}(\vec{r},\vec{r}\,';s)) = (\overleftrightarrow{g}^{(\Delta)}_{p,q}(\vec{r},\vec{r}\,';s))^{T} \\
&(\vec{\ell}_{p}(\vec{r},s))_{\beta} = (\vec{e}_{p}(\vec{r},s))_{\beta} \quad , \quad \left\langle (\vec{e}_{p}(\vec{r}\,',s))_{\beta_1} \odot (\vec{e}_{p}(\vec{r}\,',s))_{\beta_2} \right\rangle = 1_{\beta_1,\beta_2} \quad \text{(orthonormal)} \\
&(\vec{\ell}^{(n)}_{p}(\vec{r},s))_{\beta} = (\vec{e}^{(n)}_{p}(\vec{r},s))_{\beta}
\end{aligned}
$$

$$\tilde{\lambda}_{\beta}^{(0)}(s) = \left\langle (\vec{e}_p^{\,(0)}(\vec{r},s))_\beta \odot (\overleftrightarrow{g}_{p,q}^{(0)}(\vec{r},\vec{r}\,';s)) \odot (\vec{e}_p^{\,(0)}(\vec{r}\,',s))_\beta \right\rangle$$

$$\tilde{\lambda}_{\beta}^{(1)}(s) = \left\langle (\vec{e}_p^{\,(0)}(\vec{r},s))_\beta \odot (\overleftrightarrow{g}_{p,q}^{(\Delta)}(\vec{r},\vec{r}\,';s)) \odot (\vec{e}_p^{\,(0)}(\vec{r}\,',s))_\beta \right\rangle$$

$$\left\langle (\vec{e}_p^{\,(1)}(\vec{r}\,',s))_{\beta_1} \odot (\vec{e}_p^{\,(0)}(\vec{r}\,',s))_{\beta_2} \right\rangle + \left\langle (\vec{e}_p^{\,(0)}(\vec{r}\,',s))_{\beta_1} \odot (\vec{e}_p^{\,(1)}(\vec{r}\,',s))_{\beta_2} \right\rangle = 0 \tag{4.7}$$

$$\left\langle (\vec{e}_p^{\,(1)}(\vec{r}\,',s))_{\beta} \odot (\vec{e}_p^{\,(0)}(\vec{r}\,',s))_{\beta} \right\rangle = 0 \quad \text{(orthogonal)}$$

The perturbation analysis can be extended to natural frequencies as in [1]. Here we can define the unperturbed natural frequencies and modes by

$$\tilde{\lambda}_{\beta}^{(0)}(s_{\beta,\beta'}^{(0)}) = 0 \quad , \quad s_{\beta,\beta'}^{(0)} \equiv \text{natural frequencies}$$

$$\left.\begin{array}{l} (\vec{e}_p^{\,(0)}(\vec{r},s_{\beta,\beta'}^{(0)}))_\beta \equiv (\vec{e}_p^{\,(0)}(\vec{r}))_{\beta,\beta'} \\ (\vec{\ell}_p^{\,(0)}(\vec{r},s_{\beta,\beta'}^{(0)}))_\beta \equiv (\vec{\ell}_p^{\,(0)}(\vec{r}))_{\beta,\beta'} \end{array}\right\} \equiv \text{natural modes}$$

$$\left\langle (\overleftrightarrow{g}_{p,g}^{(c)}(\vec{r},\vec{r}\,';s_{\beta,\beta'}^{(0)})) \odot (\vec{e}_p^{\,(0)}(\vec{r}\,'))_{\beta,\beta'} \right\rangle = (\vec{0}_p) \tag{4.8}$$

$$\left\langle (\vec{\ell}_p^{\,(0)}(\vec{r}))_{\beta,\beta'} \odot (\overleftrightarrow{g}_{p,g}^{(c)}(\vec{r},\vec{r}\,';s_{\beta,\beta'}^{(0)})) \right\rangle = (\vec{0}_p)$$

The perturbed natural frequencies are written as

$$s_{\beta,\beta'} = s_{\beta,\beta'}^{(0)} + \Delta s_{\beta,\beta'} \tag{4.9}$$

The unperturbed eigenvalues are expanded in a Taylor series about $s_{\beta,\beta'}^{(0)}$ as

$$\tilde{\lambda}_{\beta}^{(0)}(s_{\beta,\beta'}) = \tilde{\lambda}_{\beta}^{(0)}(s_{\beta,\beta'}^{(0)}) + \left[\frac{d}{ds}\tilde{\lambda}_{\beta}^{(0)}(s)\Big|_{s=s_{\beta,\beta'}^{(0)}} \right] \Delta s_{\beta,\beta'} + \cdots$$

$$= \left[\frac{d}{ds} \tilde{\lambda}_{\beta}^{(0)}(s) \Big|_{s = s_{\beta,\beta'}^{(0)}} \right] \Delta s_{\beta,\beta'} + \cdots \tag{4.10}$$

Noting that

$$\tilde{\lambda}_{\beta}(s_{\beta,\beta'}) = 0 \tag{4.11}$$

from (4.6) we have

$$\begin{aligned} 0 &\simeq \tilde{\lambda}_{\beta}^{(0)}(s_{\beta,\beta'}) + \tilde{\lambda}_{\beta}^{(1)}(s_{\beta,\beta'}) \\ &\simeq \left[\frac{d}{ds} \tilde{\lambda}_{\beta}^{(0)}(s) \Big|_{s=s_{\beta,\beta'}^{(0)}} \right] \Delta s_{\beta,\beta'} + \tilde{\lambda}_{\beta}^{(1)}(s_{\beta,\beta'}) \end{aligned} \tag{4.12}$$

Approximating $\tilde{\lambda}_{\beta}^{(1)}$ by its leading term we have

$$\Delta s_{\beta,\beta'} = - \frac{\tilde{\lambda}_{\beta}^{(1)}(s_{\beta,\beta'}^{(0)})}{\frac{d}{ds} \tilde{\lambda}_{\beta}^{(0)}(s) \Big|_{s= s_{\beta,\beta'}^{(0)}}} \tag{4.13}$$

Note here that the first derivative of the unperturbed eigenvalue is assumed nonzero. If this is not the case one can go to the second derivative etc. in (4.10).

This result can be related back to the integral equation. Differentiating the first equation in (4.3) with respect to s and operating on the left with $(\vec{\ell}_{\beta,p}^{(0)}(\vec{r},s)) \odot$ gives

$$\begin{aligned} &\frac{d}{ds} \tilde{\lambda}_{\beta}^{(0)}(s) \Big|_{s= s_{\beta,\beta'}^{(0)}} \\ &= \left\langle (\vec{\ell}_p^{(0)}(\vec{r}, s_{\beta,\beta'}^{(0)}))_{\beta} \odot \frac{d}{ds} (\overleftrightarrow{g}_{p,g}^{(c)}(\vec{r}, \vec{r}\,'; s)) \Big|_{s= s_{\beta,\beta'}^{(0)}} \odot (\vec{e}_p^{(0)}(\vec{r}\,', s_{\beta,\beta'}^{(0)}))_{\beta} \right\rangle \\ &= \left\langle (\vec{\ell}_p^{(0)}(\vec{r}))_{\beta,\beta'} \odot \frac{d}{ds} (\overleftrightarrow{g}_{p,g}^{(c)}(\vec{r}, \vec{r}\,'; s)) \Big|_{s= s_{\beta,\beta'}^{(0)}} \odot (\vec{e}_p^{(0)}(\vec{r}\,'))_{\beta,\beta'} \right\rangle \end{aligned} \tag{4.14}$$

The eigenvalue perturbation is

$$\begin{aligned}\tilde{\lambda}^{(1)}_{\beta}(s^{(0)}_{\beta,\beta'}) &= \left\langle (\vec{\tilde{\ell}}^{(0)}_{p}(\vec{r}, s^{(0)}_{\beta,\beta'}))_{\beta} \odot ; (\overleftrightarrow{\tilde{g}}^{(\Delta)}_{p,g}(\vec{r}, \vec{r}\,'; s^{(0)}_{\beta,\beta'})) \odot ; (\vec{\tilde{e}}^{(0)}_{p}(\vec{r}\,', s^{(0)}_{\beta,\beta'}))_{\beta} \right\rangle \\ &= \left\langle (\vec{\ell}^{(0)}_{p}(\vec{r}))_{\beta,\beta'} \odot ; (\overleftrightarrow{\tilde{g}}^{(\Delta)}_{p,g}(\vec{r}, \vec{r}\,'; s^{(0)}_{\beta,\beta'})) \odot ; (\vec{e}^{(0)}_{p}(\vec{r}\,'))_{\beta,\beta'} \right\rangle\end{aligned} \tag{4.15}$$

Note that the index set (β,β') (β'th natural frequency of the βth eigenvalue) can be replaced by a single index α if desired. Without reference to the eigenvalues, but only natural frequencies and modes we have

$$\begin{aligned}\left\langle (\overleftrightarrow{\tilde{g}}^{(c)}_{p,q}(\vec{r}, \vec{r}\,'; s_{\alpha})) \odot ; (\vec{e}^{(0)}_{p}(\vec{r}\,'))_{\alpha} \right\rangle &= (\vec{0}_{p}) \\ \left\langle (\vec{\ell}^{(c)}_{p}(\vec{r}))_{\alpha} \odot ; (\overleftrightarrow{\tilde{g}}^{(c)}_{p,q}(\vec{r}, \vec{r}\,'; s_{\alpha})) \right\rangle &= (\vec{0}_{p}) \\ \left\langle (\vec{\ell}^{(0)}_{p}(\vec{r}))_{\alpha} \odot ; (\vec{e}^{(0)}_{p}(\vec{r}\,'))_{\alpha} \right\rangle &= (\vec{0}_{p})\end{aligned} \tag{4.16}$$

$$\Delta s_{\alpha} \simeq - \frac{\left\langle (\vec{\ell}^{(0)}_{p}(\vec{r}))_{\alpha} \odot ; (\overleftrightarrow{\tilde{g}}^{(\Delta)}_{p,q}(\vec{r}, \vec{r}\,'; s_{\alpha})) \odot ; (\vec{e}^{(0)}_{p}(\vec{r}\,'))_{\alpha} \right\rangle}{\left\langle (\vec{\ell}^{(0)}_{p}(\vec{r}))_{\alpha} \odot ; \frac{d}{ds}(\overleftrightarrow{\tilde{g}}^{(c)}_{p,q}(\vec{r}, \vec{r}\,'; s_{\alpha}))\Big|_{s=s_{\alpha}} \odot ; (\vec{e}^{(0)}_{p}(\vec{r}\,'))_{\alpha} \right\rangle}$$

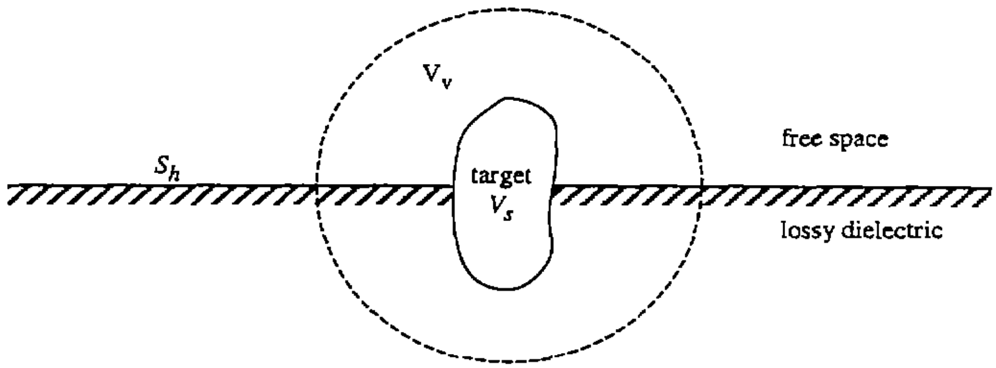
5. Target in Presence of Uniform Isotropic Half Space

To illustrate some of the concepts consider a target in the presence of a uniform, isotropic half space such as might model soil, i.e., a lossy dielectric (with magnetic properties if desired). As in fig. 5.1A, let the target stradle the planar boundary S_h between this medium and free space. In this case the target vicinity V_s contains both media. As discussed in Section 3 the canonical problem is formed by extending the media through the target making the half-space and its Green functions the canonical problem. In this form one can investigate various characteristics of the target in (or near) S_h, including tilting of a body-of-revolution target with respect to the surface normal [7].

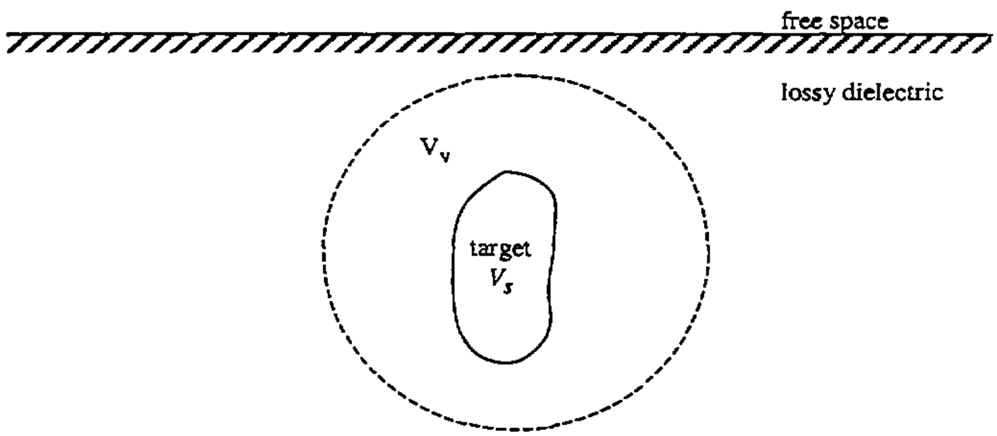
Now suppose, as in fig. 5.1B, that the target is in the lossy dielectric half space, and not too close to S_h. Then with the target vicinity V_v not intersecting with S_h, one can take the lossy dielectric medium to define a canonical problem by extending the medium throughout all space to give well-known relatively simple Green functions. Then the free half space (above S_h) can be defined as the difference or change to the problem. This can then be treated as a perturbation problem as in Section 4, which has been done in [6] for the case of a thin-wire target.

Conversely, as in fig. 5.1C, the target can be in the free half space. Then V_s is free space which can be extended through the target and to infinity to define the canonical problem. The lossy dielectric half space can now be taken as the difference problem. As a perturbation problem this has also been treated in [6] for the case of a thin-wire target.

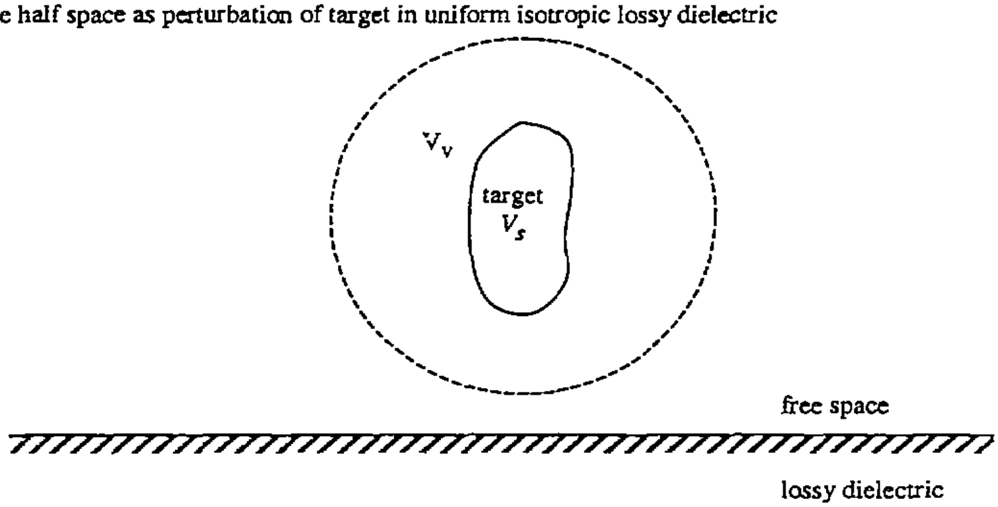
As a limiting case the lossy dielectric can be taken as a perfect conductor, in which case S_h is an image plane. This has been treated as a perturbation problem for a thin-wire target in [1].



A. Half space as canonical problem



B. Free half space as perturbation of target in uniform isotropic lossy dielectric



C. Lossy dielectric half space as perturbation of target in free space

Fig. 5.1. Target and Uniform Isotropic Half Space.

6. Concluding Remarks

So, by looking at the medium in V_v (the target vicinity), one can extend this medium through the target and on to infinity to define a canonical scattering problem. Then one can often treat the diference from this canonical problem as a difference problem containing the clutter. Such distant contributions can be considered as a small change to the Green functions *at the target*, where both source $\vec{r}\,'$ and observation $\vec{r}$ coordinats are in the target volume V_s. This is reflected in small changes to things like natural frequencies, natural modes, eigenvalues, eigenmodes, etc. that are properties of the integral-equation operator in the target domain V_s. Such are the target-vicinity scattering parameters. A useful way to evaluate the desired insensitivity of the target-scattering parameters to distant clutter is perturbation theory.

Of course, one must measure the scattered fields away from the target at some distance. So the target-vicinity parameters need to be observable in the distant scattered fields. For example, natural frequencies are generally observable in the far fields, independent of the incident field conditions and observer location. However, the strengths of these resonances (the residues) are functions of both of these. Thus there are questions concerning the incident fields and distant scattering location (waveforms, polarization, etc.) for optimizing the measurement of target-vicinity parameters.

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