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Parsimony in Signature-Based Target Identification

Carl E. Baum
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Abstract

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1. Introduction

In signature-based target identification [20] the scattering dyadic of each type of target is characterized by a set of functions based on a particular scattering model (e.g., complex exponentials for the late-time response, or delta, step, ramp, etc., for the early-time response). Each of the functions is characterized by a small number of parameters (e.g., complex natural frequencies s_α) including a scaling coefficient (scalar, dyadic) to adjust the amplitude (perhaps including vector orientation). Each of the targets is represented in a target library by an appropriate set of such functions with parameter values particular to the individual target types (e.g., a particular type of aircraft such as a 707). The approach is to associate these parameter values with the characteristics of the electromagnetic waves scattered (usually backscattered, but not necessarily so) from the target by some appropriate radar.

The problem then is to distinguish one target from another by the differences in the parameters in the scattering model(s) inferred from the measured scattered fields. Given the presence of noise in any measurement there is some ambiguity in the declaration of a particular target type because of errors in the parameter estimation. Particularly as the parameter values for one target type approach those of another type the discrimination becomes increasingly difficult.

Let us distinguish between two types of parameters. One type (fixed parameters) assumes particular values (scalar, vector, dyadic) for each target type. A second type, which we might call variable parameters is adjusted as part of the process of fitting the scattering model to the data. Such variable parameters are typically the coefficients of the fitting functions, which adjust the amplitudes (not necessarily scalars) of the fitting functions to best fit the data. Since we wish to discriminate between various targets we would like it to be difficult to fit the wrong target parameters to the data. So we would like to reduce the number of variable parameters (and the range of their variation) as much as we can. This is aided by constraining these scaling coefficients (e.g., pole residues) to values appropriate to the target type to the degree practical.

This leads to a *principle of parsimony* as [25]: *"Use as few feature variables as possible to provide consistent classification."* In the present context feature variables are interpreted as variable parameters, or parameter values that aapply to multiple target types (at least approximately).

2. Fitting with General Function Sets

To illustrate the problem of fitting data with too many parameters, consider some target response $f^{(n)}(t)$ (before or after decomvolution for impulse response) along with some noise $v^{(n)}(t)$. We have some parametric scattering model for which we have a set of functions $\{g_\ell^{(m)}(t)\}$ for $\ell = 1, 2, \cdots$ (perhaps infinitely many) appropriate to the mth target type. Then we try to approximate $f^{(n)}(t)$ by these functions as

$$f^{(n)}(t) + v^{(n)}(t) \approx \sum_{\ell=1}^{L} a_\ell^{(n,m)} g_\ell^{(m)}(t) \tag{2.1}$$

where L is available for us to choose. The $a_\ell^{(n,m)}$ are chosen to minimize an appropriate norm of the difference of the two sides in (2.1).

Then define

$$N_{n,m}^{(L)} \equiv \min_{a_\ell^{(n,m)}} \left\| f^{(n)}(t) + v^{(n)}(t) - \sum_{\ell=1}^{L} a_\ell^{(n,m)} g_\ell^{(m)}(t) \right\| \tag{2.2}$$

where the norm $\| \ \|$ can be defined in various ways [3, 17], including the use of various weighting functions [18] if desired. While the above functions are written as functions of time t, they can be transformed to complex frequency ($s = \Omega + j\omega$) domain or to a wavelet/window-Fourier-transform domain [9, 31] and the norm can be defined in such terms. Having chosen our norm we define the set of $a_\ell^{(n,m)}$ that minimizes the difference as in (2.2) as the set $\left\{ a_{\ell,L}^{(n,m)} \middle| \ell = 1, 2, \cdots, L \right\}$. There may be more than one such set in which case we choose one of these sets at our convenience. Then we have

$$N_{n,m}^{(L)} \equiv \left\| f^{(n)}(t) + v^{(n)}(t) - \sum_{\ell=1}^{L} a_{\ell,L}^{(n,m)} g_\ell^{(m)}(t) \right\| \tag{2.3}$$

What now if we try again with $L + 1$ functions? We obtain some set $\left\{ a_{\ell,L+1}^{(n,m)} \middle| \ell = 1, 2, \cdots, L, L+1 \right\}$. This gives

$$N_{n,m}^{(L+1)} \equiv \min_{a_\ell^{(n,m)}} \left\| f^{(n)}(t) + v^{(n)}(t) - \sum_{\ell=1}^{L+1} a_\ell^{(n,m)} g_\ell^{(m)}(t) \right\|$$
$$= \left\| f^{(n)}(t) + v^{(n)}(t) - \sum_{\ell=1}^{L+1} a_{\ell,L+1}^{(n,m)} \; g_\ell^{(m)}(t) \right\| \tag{2.4}$$

Now one choice of the $a_\ell^{(n,m)}$ (before minimization) is just the $a_{\ell,L}^{(n,m)}$ with $a_{L+1}^{(n,m)} = 0$. This gives $N_{n,m}^{(L)}$ as in (2.3). this choice might give the minimum in (2.4), but generally gives something larger than the minimum. Hence we can conclude

$$N_{n,m}^{(L+1)} \leq N_{n,m}^{(L)} \tag{2.5}$$

This does not necessarily imply that $N_{n,m}^{(L)} \to 0$ as $L \to \infty$, but it does show that adding more functios in our target-signature set with adjustable weights makes the mth target-type signature more closely match the data from the nth target type (with or without noise).

Suppose however, that the $g_\ell^{(m)}(t)$ for $\ell = 1, 2, \cdots \vec{1}, \infty$ form a complete set on the support of interest (time interval, frequency ω interval, or even some ω, t phase space). Then any reasonably well behaved function can be approximated by this set and

$$N_{n,m}^{(L)} \to 0 \text{ as } L \to \infty \tag{2.6}$$

However, the $g_\ell^{(m)}(t)$ are for the mth target type and we are approximating the waveform for the nth target (plus noise). In this case minimization of the norm for large L cannot distinguish between the nth and mth targets. So it is important that the $g_\ell^{(m)}(t)$ *not be complete* for the domain of time, freq., etc., for successful target discrimination. We want the $g_\ell^{(m)}(t)$ to apply to *only* the mth target (for all m in our library). Note that a complete set of functions need not be orthogonal (zero inner products on the support). Of course a non-orthogonal set can be converted to an orthogonal one (Gram-Schmidt orthogonalization [26]).

Alternately, if the function set $g_\ell^{(m)}(t)$ is complete, it is important that L be limited (parsimony) so that only terms that are dominant for target m are included. Similarly the set $g_\ell^{(n)}(t)$ needs to include only terms that are dominant for target n. This is also a question of how to best order the $g_\ell^{(m)}(t)$, i.e., which is labelled by $\ell = 1$, etc. Presumably they should be placed in the order of decreasing dominance.

In (2.2) and (2.3) the norms do not take account of varying signal strength as the same target is measured at various distances from the radar. Using the far-field approximation (incident and scattered fields varying as 1/r with the same waveforms) one can normalize these expressions as the dimensionless expressions

$$W_{n,m}^{(L)} = \frac{N_{n,m}^{(L)}}{\left\| f^{(n)}(t) + v^{(n)}(t) \right\|} \tag{2.7}$$

for which we also have

$$W_{n,m}^{(L+1)} \leq W_{n,m}^{(L)} \tag{2.8}$$

However, this does not account for variation in the strengths of the signals between different targets (large vs. small scatterers). So one may wish to instead normalize as (one factor of r for the incident wave, one for the scattered)

$$U_{n,m}^{(L)} = r^2 \, N_{n,m}^{(L)} \tag{2.9}$$

assuming one has a measure of the range r to the target. In this form the combinations $r^2 \, a_\ell^{(n,m)}$ also give information concerning the strength of the various scattering modes and can perhaps be constrained (parsimony) to give better target discrimination. Combining these ideas we can define

$$M_{n,m}^{(L)} \equiv \min_{b_\ell^{(n,m)}} \frac{\left\| r^2 \left[f^{(n)}(t) + v^{(n)}(t) \right] - \sum_{\ell=1}^{L} b_\ell^{(n,m)} g_\ell^{(m)}(t) \right\|}{\left\| r^2 \left[f^{(n)}(t) + v^{(n)}(t) \right] \right\|} \tag{2.10}$$

so that the $b_\ell^{(n,m)}$ are now range invariant. The $b_\ell^{(n,m)}$ are, however, in general still aspect (polarization, angle of incidence) dependent.

While the discussion in this section has been in terms of scalar functions with scalar coefficients, vector and dyadic forms are readily considered in the same expressions, with appropriate attention to the norm used. Such forms are appropriately used with multiple radar measurements to give the scattering dyadic of the target which is then fit by the above procedure.

3. Exponential Functions

A common set of functions used for representing a time-domain signal $f(t)$ is exponential functions (in general complex). This is readily seen through the two-sided Laplace (or Fourier) transform as

$$\begin{aligned}
\tilde{f}(s) &= \int_{-\infty}^{\infty} f(t) e^{-st} dt \\
\sim &\equiv \text{two-sided Laplace transform} \\
s &= \Omega + j\omega = \text{Laplace-transform variable or complex frequency} \\
f(t) &= \frac{1}{2\pi j} \int_{Br} \tilde{f}(s) e^{st} ds \\
Br &\equiv \text{Bromwich contour parallel to } j\omega \text{ axis}
\end{aligned} \tag{3.1}$$

Here the Bromwich contour is taken to the right of any singularities of $\tilde{f}(s)$ in the s plane. So already we have a restriction that $f(t)$ must be passive, i.e. e^{st} with $\text{Re}[s] > 0$ is not allowed in representing $f(t)$. However, as discussed in the previous section, our signal from the nth target includes noise $v^{(n)}(t)$ which does not necessarily include this constraint. This implies some constraint (parsimony) on the allowable functions to represent a target and discriminate against noise. Note that the Bromwich contour extends over $-\infty < \omega < \infty$ implying an infinite set of functions to represent any target in the library (not parsimonious).

The integral over the Bromwich contour can be represented by a sum as

$$\begin{aligned}
f(t) &= \frac{1}{2\pi j} \int_{Br} \tilde{f}(s) e^{st} ds \approx \frac{1}{2\pi j} \sum_{q} \tilde{f}(s_q) e^{s_q t} u(t) \, \Delta s \\
\Delta s &= s_{q+1} - s_q
\end{aligned} \tag{3.2}$$

where now the s_q all lie on or to the left of the $j\omega$ axis. In this form we can see the effect of a finite sum of exponentials. As the number of such functions $\rightarrow \infty$ any well-behaved passive $f(t)$ can be represented. For a limited number of s_q some function $f^{(n)}(t)$ can be better approximated than $f^{(m)}(t)$.

Instead of an infinite interval in time $(t \rightarrow \infty)$ where the signal is eventually lost in the noise, one might consider a finite interval of time $t_1 \leq t \leq t_2$. The transform $\tilde{f}(s)$ of $f(t)$ can then be replaced by a Fourier series with $s = j\omega$. The ω_q are now discrete, but generally infinite in number. Furthermore, as is well known any reasonably behaved $f(t)$ (not necessarily passive, and including noise) can be accurately represented by such a Fourier series. So this is also not a good choice.

Following [27] we can deform the Bromwich contour into the left half plane. In Fig. 3.1A we see the singularities of a passive system lying in the left half plane (LHP). Since our $f(t)$ is real valued the singularities not on the negative Ω axis must occur in complex conjugate pairs, both for their locations and amplitudes (i.e., pole residues). As we deform our contour to the left these singularities are isolated to give separate functions which for poles (shown as first order, but not necessarily so) gives a representation as

$$\begin{aligned} \tilde{f}(s) &= \sum_{\alpha} R_{\alpha}[s-s_{\alpha}]^{-1} + \text{ entire function} \\ f(t) &= \sum_{\alpha} R_{\alpha}\ e^{s_{\alpha}t}u(t) + \text{ entire function in temporal form} \end{aligned} \tag{3.3}$$

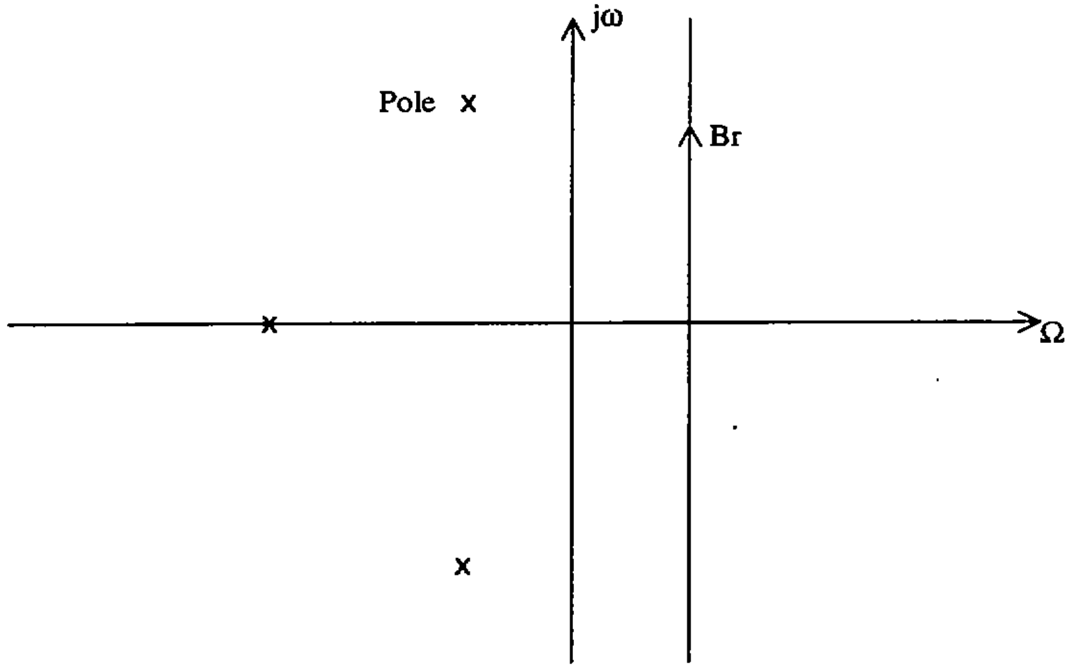
where the entire functions corresponds to the singularity as $s \to \infty$. In time domain it is an early-time contribution to the response [4]. Hence the pole series is used to represent the response for late times after the incident and scattered waves have had time to transit over the target. Already we see some parsimony at work. The poles corresponding to the mth target do not well approximate those belonging to the nth target for $m \neq n$. Furthermore, restricting the functions to discrete poles in the LHP makes it more difficult to approximate the noise $v^{(n)}(t)$.

So discrete pole locations in the LHP are more parsimoneous than a continuous distribution (or large number) of such locations on the $j\omega$ axis. Even more parsimoneous would be some restrictions on the residues R_{α}.

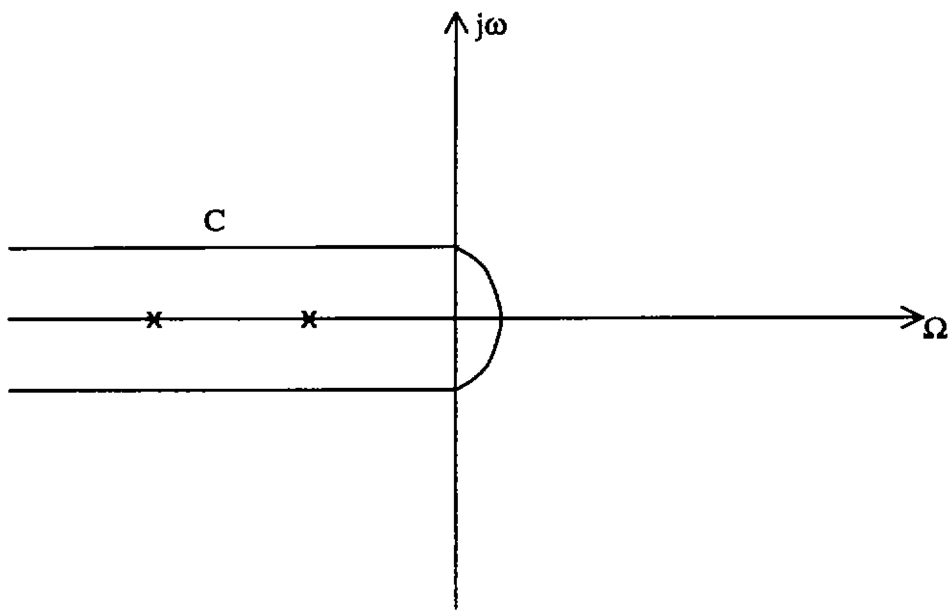
Carrying the contour deformation yet further, suppose that our targets of interest have no singularities away from the negative Ω axis. Then the contour collapses as in Fig. 3.1B to include only singularities there. While this can include in principle a branch cut there [4], there is an important class of targets which can be well-approximated by first order poles there. These are the diffusion poles in highly (but not perfectly) conducting metal targets of finite linear dimensions [6]. In this case the response takes the form

$$\begin{aligned} \tilde{f}(s) &= \sum_{\alpha} R_{\alpha}\,[s-\Omega_{\alpha}]^{-1} + R^{(\infty)} \\ f(t) &= \sum_{\alpha} R_{\alpha}\, e^{\Omega_{\alpha}t}\ u(t) + R^{(\infty)}\delta(t) \\ & R_{\alpha} \text{ real} \end{aligned} \tag{3.4}$$

where the entire function is now a constant in frequency domain or a delta function in time domain.



A. Bromwich contour: singularities in the left half plane



B. Deformation of contour: singularities on the negative Ω axis

Fig. 3.1. Inversion of Two-Sided Laplace Transform

Comparing (3.4) to (3.3) we can see that (3.4) is more parsimonious in the sense that it cannot represent poles in the third and fourth quadrants of the s plane far from the negative Ω axis. This is seen from analytic continuation of $\tilde{f}(s)$ from the $j\omega$ axis to the left. If $\tilde{f}(s)$ contains a more general complex pole as in (3.3) analytic continuation will lead to it. Functions as in (3.4) correspond to analytic continuation from the $j\omega$ axis of functions which contain none of the more general poles in (3.3).

For this special class of targets appropriate to magnetic singularity identification (MSI) the form in (3.4) is more general corresponding to a decomposition of the magnetic-polarizability dyadic $\overleftrightarrow{M}(s)$ [6]. The dyadic residues rotate together with the target allowing constraints on them [13, 15, 16], thereby giving more aspect independent parameters (besides the Ω_α) to aid in the target discrimination. This is then even more parsimonious.

4. Relation to Prony-Like Fitting of Data

Much work has been done extracting damped sinusoids out of data. See [19] for a review. Since then some improvement has been made with what is called matrix pencil [21]. As discussed in previous sections, this is like fitting the data with a large set of exponentials which is always possible with enough of them. Even noise is readily fit this way. Restricting the number of terms and only considering those with large coefficients (residues) helps.

A more robust procedure may consist of fitting the data with only the specific exponential sets corresponding to targets in the library to see which fits best. Of course, if the real target is not in the library, consideration of how good is the fit (size of residuals) may be needed to establish this fact. Filters such as the E/K pulse [9, 19] are one way of doing this. One has $E^{(m)}(t)$ for the mth target type which annihilates the late-time response (except for noise) when it is convolved with the response of the nth target only when $n = m$. This is achieved by setting

$$\tilde{E}^{(m)}(s_{\alpha}^{(m)}) = 0 \tag{4.1}$$

for all significant natural frequencies $s_{\alpha}^{(m)}$ of the mth target type.

Prony-like fitting may still be needed to generate the natural-frequency sets for the target library from experimental data. Of course, this can be achieved from many careful measurements under more ideal conditions. Such measurements may be eventually supported by accurate numerical computations of natural frequencies from integral and/or differential scattering equations. Preliminary estimates of the $s_{\alpha}^{(m)}$ can perhaps be refined by adjusting them to optimally annihilate the late time response for many target aspects with one $E^{(m)}(t)$.

5. Use of Polarization with Target Symmetry

When an electromagnetic plane wave impinges on a target from some particular angle of incidence, one often has a choice of polarization of the electric field. With two choices one can reconstruct the backscattering dyadic. If one has the target impulse response dyadic (2 x 2) for this particular angle of incidence, one can rotate the target about this incidence direction (or equivalently rotate the radar) to match the stored data and thereby also orient the target by an angle (real, 0 to 2π) about the incidence direction. Of course the rotation can be accomplished by a rotation of the scattering dyadic in a computer. In the process one can identify this rotation angle(say ψ) modulo π since the scattering dyadic is invariant to a rotation by π (sign reversal of incident and scattered fields). From a parsimony point of view this is a single real parameter varying over a restricted interval. The constraint of a known angle of incidence has greatly reduced the number of fitting parameters. The amplitude is also assumed to be constrained by independent knowledge of the distance to the target.

5.1 Target symmetry plane passing through observer: R_ψ symmetry

A special case of interest is that of a target with a symmetry plane (such as a typical fixed-wing aircraft) [1, 2, 29]. Referring to Fig. 5.1, let this symmetry plane lie along the direction of incidence at the target. With the usual h, v radar coordinates we have the directions (unit vectors)

$$\vec{1}_i = -\vec{1}_h \times \vec{1}_v \equiv \text{direction of incidence} \tag{5.1}$$

with $(\vec{1}_i, \vec{1}_v, -\vec{1}_i)$ forming a right handed system. Note that

$$\vec{1}_0 = -\vec{1}_i \equiv \text{direction of scattering (to observer)} \tag{5.2}$$

The target symmetry plane S_ψ is rotated from the vertical by an angle ψ giving R_ψ symmetry characterized by a reflection dyadic

$$\begin{aligned}
\overleftrightarrow{R}_y &= \overleftrightarrow{1} - 2\vec{1}_\psi \vec{1}_\psi \quad , \quad \vec{1}_\psi \perp S_\psi \\
\overleftrightarrow{1} &= \vec{1}_x \vec{1}_x + \vec{1}_y \vec{1}_y + \vec{1}_z \vec{1}_z \\
&= \vec{1}_h \vec{1}_h + \vec{1}_v \vec{1}_v + \vec{1}_i \vec{1}_i \\
&= \vec{1}_h \vec{1}_h + \vec{1}_v \vec{1}_v + \vec{1}_o \vec{1}_o \\
&\equiv \text{identity (three dimensional)}
\end{aligned} \tag{5.3}$$

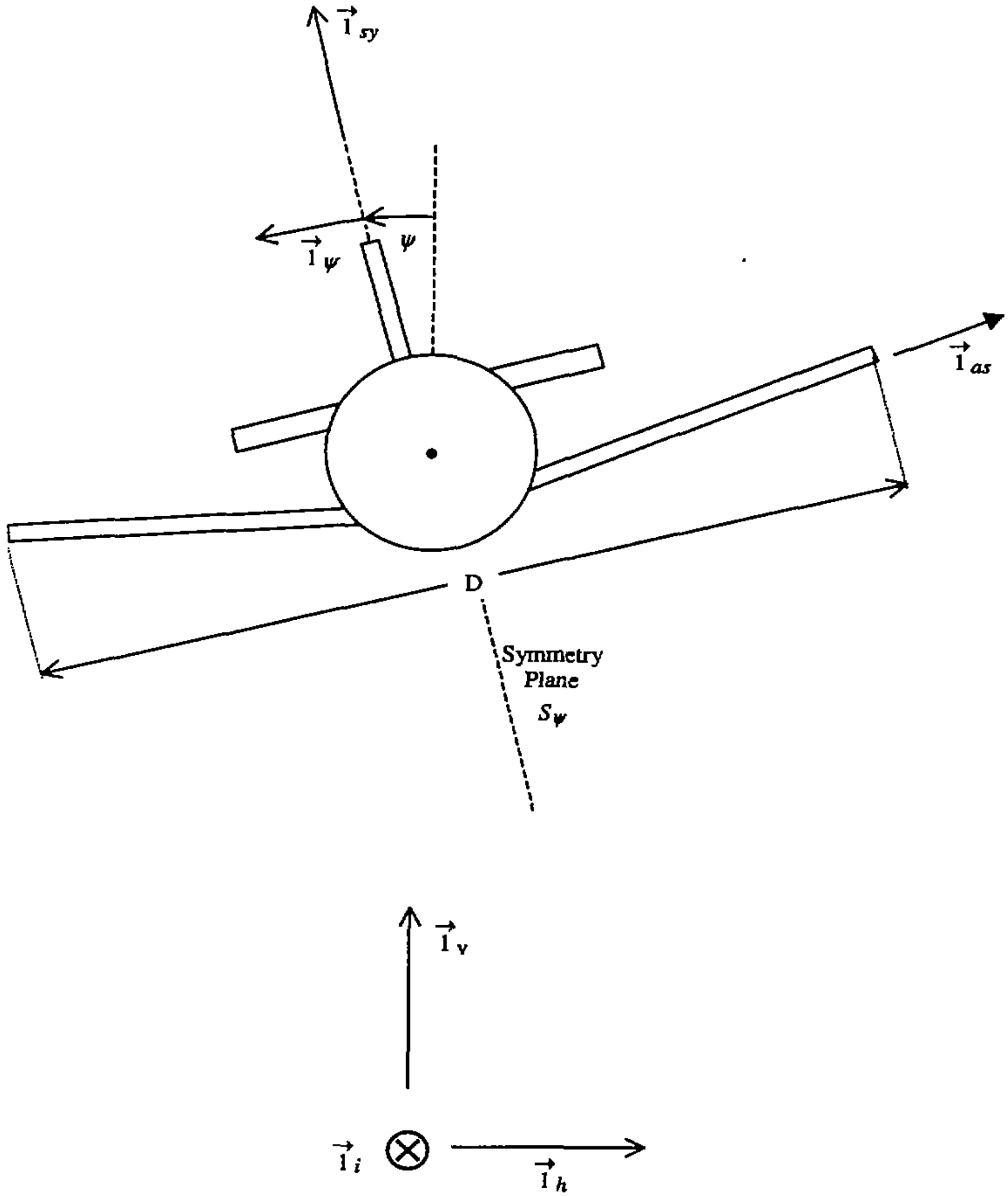


Fig. 5.1. Symmetry Plane Through Scatterer and Observer: R_ψ Symmetry

If one measures the scattering dyadic $\overleftrightarrow{\Lambda}(\overrightarrow{1}_i,s)$ or $\overleftrightarrow{\Lambda}(\overrightarrow{1}_i,t)$ (delta-function response) by removing the antenna characteristics and range dependence, this will have three separate terms in the 2 x 2 scattering dyadic ($\overrightarrow{1}_i$ coordinate removed) since the fields of concern only have h and v components and we have

$$\begin{aligned}
\overleftrightarrow{\Lambda}(\overrightarrow{1}_i,t) &= \begin{pmatrix} \Lambda_{h,h}(\overrightarrow{1}_i,t) & \Lambda_{h,v}(\overrightarrow{1}_i,t) \\ \Lambda_{v,h}(\overrightarrow{1}_i,t) & \Lambda_{v,v}(\overrightarrow{1}_i,t) \end{pmatrix} \\
&= \overleftrightarrow{\Lambda}^T(\overrightarrow{1}_i,t) \quad \text{(reciprocity)} \\
\Lambda_{v,h}(\overrightarrow{1}_i,t) &= \Lambda_{h,v}(\overrightarrow{1}_i,t)
\end{aligned} \tag{5.4}$$

Defining a rotation matrix [30] as

$$\begin{aligned}
\left(C_{n,m}(\psi)\right) &= \begin{pmatrix} \cos(\psi) & -\sin(\psi) \\ \sin(\psi) & \cos(\psi) \end{pmatrix} \\
\left(C_{n,m}(\psi)\right)^T &= \left(C_{n,m}(\psi)\right)^{-1} = \left(C_{n,m}(-\psi)\right) \quad \text{(orthogonal)}
\end{aligned} \tag{5.5}$$

gives a positive (counterclockwise) rotation of the coordinates as in Fig. 5.1. Suppose we have measured $\overleftrightarrow{\Lambda}$ in the h, v coordinate system. Then we can compute

$$\begin{aligned}
\overleftrightarrow{\Lambda}'(\overrightarrow{1}_i,t) &= \left(C_{n,m}(-\psi)\right) \cdot \overleftrightarrow{\Lambda}(\overrightarrow{1}_i,t) \cdot \left(C_{n,m}(\psi)\right) \\
&= \begin{pmatrix} \Lambda_{as,as}(\overrightarrow{1}_i,t) & 0 \\ 0 & \Lambda_{sy,sy}(\overrightarrow{1}_i,t) \end{pmatrix}
\end{aligned} \tag{5.6}$$

where we have rotated the scattering into the as, sy system corresponding to the antisymmetric part ($\perp R_\psi$) and symmetric part ($// R_\psi$). These two parts only (parsimony) are needed to characterize this scattering, being reduced to vectors parallel to $\overrightarrow{1}_{as}$ and $\overrightarrow{1}_{sy}$ on the symmetry plane. Note that this decomposition is frequency/time independent. Natural frequencies decompose into two sets $s_\alpha^{(as)}$ and $s_\alpha^{(sy)}$, etc. If ψ is not known a priori from some other measurement, that rotating the data by varying ψ discovers ψ to make (5.6) hold, thereby learning the roll angle. Note, however, that adding integer multiples of $\pi/2$ to ψ also produces this diagonalization. $\overleftrightarrow{\Lambda}$ generally has more symmetry than the target, in this case two symmetry planes giving the symmetry group

$$C_{2a} = R_{\psi} \otimes R_{\psi+\frac{\pi}{2}} \tag{5.7}$$

There is generally some error in target alignment such that the symmetry plane does not pass exactly through the observer (radar). This means that $\overleftrightarrow{\Lambda}$ will generally not be perfectly diagonal but the off-diagonal components will have some minimum (norm sense over time/frequency) for particular ψ. The smallness of this minimum may be a measure of target alignment along the direction of incidence.

Noting this alignment error one may wish to restrict the range of frequencies. If the maximum transverse dimension target is d one may wish to restrict radian wavelength λ such that

$$\lambda > D\tan(\chi) \simeq D\chi \tag{5.8}$$

where χ is the angular error. If say D is 10 m and χ is 10^{-2} radians then $\lambda > 10$ cm, restricting frequencies to less than about 400 MHz, with even lower uppermost frequencies as χ is increased. This is related to glint or angle noise in traditional radar systems [28].

5.2 Body-of-revolution target, including the nearby media: O_2 symmetry

A yet higher degree of symmetry is that of a body of revolution with axial symmetry planes giving $O_2 = C_{\infty a}$ symmetry [11]. Here the axis of revolution $\vec{1}_a$ is taken as perpendicular to the earth surface S_e which we take as the $z = 0$ plane, as in Fig. 5.2. The earth constitutive parameters are allowed to vary with z (layering) provided the O_2 symmetry is preserved, including the earth.

In this case the h, v coordinates are established with the traditional convention that $\vec{1}_h$ is parallel to S_e and $\vec{1}_v$ lies in the plane of incidence containing $\vec{1}_i$ and the rotation axis. In this case we have [11, 24]

$$\Lambda_{h,v}(\vec{1}_i,t) = 0 \tag{5.9}$$

for all $\vec{1}_i$, making it appropriate for synthetic aperture radar (SAR) as the radar is moved with respect to the target. This is called the vampire signature due to the lack of reflection in the h, v "mirror".

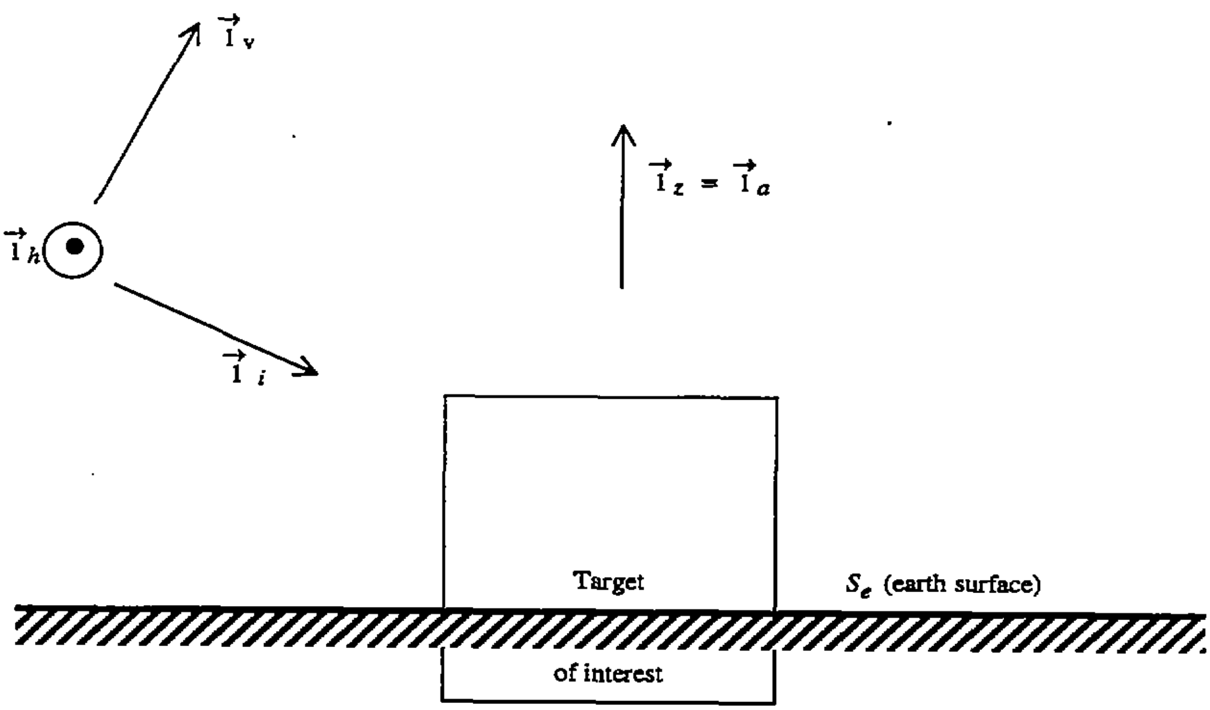


Fig. 5.2. Body of Revolution Including the Nearby Media: O_2 Symmetry

Here we use the nulling of a parameter (the cross polarization) as an identifier of a class of targets. However, it does not discriminate among the various target types with this symmetry. One can go on to consider other details of the $\Lambda_{h,h}$ and $\Lambda_{v,v}$ (only two remaining now) to achieve yet further discrimination [11].

6. Parsimony in Magnetic-Singularity Identification

For magnetic singularity identification (MSI) we have the general form of the magnetic polarizability dyadic as [6, 16]

$$
\begin{aligned}
\overleftrightarrow{M}(s) &= \overleftrightarrow{M}(\infty) + \sum_{\alpha} M_{\alpha} \overrightarrow{M}_{\alpha} \overrightarrow{M}_{\alpha} \left[s - s_{\alpha}\right]^{-1} \\
&= \overleftrightarrow{M}(0) + \sum_{\alpha} M_{\alpha} \overrightarrow{M}_{\alpha} \overrightarrow{M}_{\alpha} \frac{s}{s_{\alpha}} \left[s - s_{\alpha}\right]^{-1} \\
\overrightarrow{M}_{\alpha} \cdot \overrightarrow{M}_{\alpha} &= 1 \, , \; \overrightarrow{M}_{\alpha} \; = \text{ real unit vector for } \alpha\text{th mode} \\
M_{\alpha} &= \text{ real scalar} \\
s_{\alpha} &< 0 \text{ (all negative real natural frequencies)} \\
\overleftrightarrow{M}^{\binom{\infty}{0}} &= \sum_{\upsilon=1}^{3} M_{\upsilon}^{\binom{\infty}{0}} \overrightarrow{M}_{\upsilon}^{\binom{\infty}{0}} \overrightarrow{M}_{\upsilon}^{\binom{\infty}{0}} \\
&\equiv \begin{cases} \text{entire function (constant dyadic for } s = \infty) \\ \text{DC response (for } s = 0) \end{cases} \\
\overrightarrow{M}_{\upsilon_1}^{\binom{\infty}{0}} \overrightarrow{M}_{\upsilon_2}^{\binom{\infty}{0}} &= \overrightarrow{1}_{\upsilon_1,\upsilon_2} \text{ (orthonormal)} \\
M_{\upsilon}^{\binom{\infty}{0}} &\equiv \text{ real eigenvalues (not necessarily distinct)} \begin{cases} \text{nonpositive for } s = \infty \\ \text{nonnegative for } s = 0 \end{cases}
\end{aligned}
\qquad (6.1)
$$

This applies to highly, but not perfectly, conducting targets. The s_{α} correspond to exponential decays in time domain. The frequencies of interest are rather low, corresponding to diffusion into the metal targets. The incident fields are now not in the form of a plane wave, but the near fields of loops. Similarly the scattered near fields are sensed by loops. As such three-dimensional information concerning the target is available.

The s_{α}, M_{α}, and $M_{\upsilon}^{\binom{\infty}{0}}$ are all aspect independent while the $\overrightarrow{M}_{\alpha}$ and $\overrightarrow{M}_{\upsilon}^{\binom{\infty}{0}}$ rotate with the target, i.e. are fixed in a target-based coordinate system. As discussed in [16] by rotating the MSI signature in the target library and moving the target location (say under the earth surface) with respect to the observer location one can attempt to match the library entries to the data. In so doing there are six real parameters to adjust: three Euler angles for target orientation, two angles for direction to target, and one for distance to the target. Thereby the aspect-dependent parameters are constrained by their mutual orientation relationships.

The situation is further simplified in the case of target symmetry [7, 13, 15]. For a target with C_N symmetry with $N \geq 3$ (N-fold rotation axis with no assumption of symmetry planes) the magnetic polarizability dyadic becomes

$$\begin{aligned}
\overleftrightarrow{\tilde{M}}(s) &= \tilde{M}_a(s) \overrightarrow{1}_a \overrightarrow{1}_a + \tilde{M}_t(s) \overleftrightarrow{1}_t \\
\overrightarrow{1}_t &= \overrightarrow{1} - \overrightarrow{1}_a \overrightarrow{1}_a \equiv \text{transverse dyadic} \\
\tilde{M}_q(s) &= \tilde{M}_q(\infty) + \sum_{\alpha} M_{q_\alpha}\left[s - s_{q_\alpha}\right]^{-1} \\
&= \tilde{M}_q(0) + \sum_{\alpha} M_{q_\alpha} \frac{s}{s_{q_\alpha}}\left[s - s_{q_\alpha}\right]^{-1} \\
q &= a, t
\end{aligned} \tag{6.2}$$

where the rotation axis is taken as $\overrightarrow{1}_a$. Here the unit vectors have all conveniently lined up to give two distinct sets, each with common orientations (common aspect dependence) of the unit vectors characterizing the pole residues. In this case there are only needed two real scaling parameters (more parsimonious) to multiply the axial $\tilde{M}_a$ and transverse $\tilde{M}_t$ functions characterized by constrained poles and residues.

7. Effects of Variation in Embedding Media

For targets in a uniform well-characterized medium one can use the a priori knowledge of this medium to constrain the target signatures. In particular, aspect independent parameters can be considered constants, i.e., not variables depending on the medium parameters. Such is the case for targets effectively in free space, e.g., flying aircraft and missiles.

For targets in a variable medium such as earth the situation is more complicated since the signature in the scattering (e.g., natural frequencies) can be significantly affected by the constitutive parameters of the nearby earth [14]. In this case we are concerned primarily with frequencies such that wavelengths in the external medium are of the order of the target dimensions (used in electromagnetic singularity identification (EMSI)). If one has independent knowledge of the earth parameters (particularly the permittivity ε), say by a nearby measurement, then one can attempt to compute the effect of such parameters on the library signatures (e.g., natural frequencies) before fitting these to the radar data. Alternatively, one can use one or more of these parameters as variable fitting parameters with the library signatures to best fit the data (less parsimonious).

For targets that can be approximated as perfectly conducting in a uniform isotropic earth, there are exact scaling relationships for natural frequencies and associated modes and residues [5], making the shifting of library parameters fairly simple. For dielectric targets in such a medium the situation is more complicated. However, if the target permittivity is less than that of the surrounding earth there are applicable perturbation formulas simplifying the situation somewhat [8, 10].

Realistically, earth is not uniform. In particular the earth surface can be near the target of interest, significantly changing its signature [22, 23]. If the target is not too close to the interface (earth surface, either above or below) perturbation formulas can also be used [12]. In this case, distance from the interface is the parameter to be adjusted.

So introduction of variable parameters associated with the external medium makes the identification of the target less parsimonious, and hence more difficult. Other information which can restrict these parameters (say from other measurements) can help the situation.

8. Concluding Remarks

Parsimony in target identification then seeks to constrain the representations of the target scattering (the target signatures) so as to make it difficult for the target signature of the mth target type represent that of the nth for $m \neq n$. This implies that there be as few variable fitting parameters as possible.

It is generally helpful to have aspect-independent parameters to the extent feasible because these can be constrained as a priori constants (e.g., natural frequencies), not having to assume a large number of different values for the various possible (a priori unknown) directions of incidence and polarizations. To the extent that direction of incidence and polarization with respect to the target orientation are known from other measurements, one can constrain the various scaling constants for the target-signature functions, making it harder to fit the data with the wrong target type.

Target symmetry also plays a useful role in parsimony. Symmetry planes allow the 2 x 2 backscattering dyadic to be diagonalized, reducing the number of elements to be considered from three to two and giving orientation information in the process. For low-frequency MSI characterized by the magnetic-polarizability dyadic we have found that the number of scaling constants can be reduced from six in the general nonsymmetrical case to two for targets characterized by C_N symmetry for $N \geq 3$.

Part of the problem in target identification is the corruption of the scattering data by noise. In fitting the data with target signatures one is also fitting the noise with such signature functions. Of course we would like the fit to the noise to be poor. Furthermore, if we could distinguish the functional form of the noise (random?) and model it or remove it we might reduce this signature-fitting problem.

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