S-parameter determination with a pair of current injection and measurement probes

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Abstract

This paper presents a technique to determine scattering parameters with a set of two current probes. One is used to inject the signal. The other is used to receive the signal. In electromagnetic topology, this technique is particularly useful when it is impossible to make in-situ measurements. Usually, the measurement of scattering parameters requires to disconnect the loads of the system under test to substitute reference loads. In our case, the method we propose only requires that the ports of the device under test present a wire extension.

Based on the definition of topological parameters, the technique and the associated numerical treatment are described assuming that the probes behave like ideal transformers. The injection and measurement probes can be characterized with scattering parameter measurements performed on a simple calibration device. In addition, the knowledge of the loads connected to the device under test is required, but the loads can have any value. They can even be a part of a complex network. Supposing that the probes can be modeled as voltage transformers. The theory deals with relating the scattering parameter measurement made at the level of the probes with those defined at the level of the device under test ports.

Validations of the theory applying perfect models of probes are presented. They are carried out on simple transmission lines. Good agreements are obtained between 1 MHz and 100 MHz, which, for example, gives the user enough information and accuracy to come back to the per-unit-length electrical parameters of the lines. An additional test, using a set of non clamping probes, is presented to demonstrate how higher frequency results can be obtained when improving the calibration and the performance of the probes.

Key words:
Scattering Parameters ; Current Injectors and Current Probes ; Electromagnetic Topology ; Cable Networks ; Transmission Lines ; Electromagnetic Compatibility ; Electromagnetic Coupling.
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1. Introduction

In Electromagnetic Topology (EMT) ([1], [2]), the coupling on a system may be described by the resolution of the BLT (from "Baum" "Liib" and "Tesche") equation on a network ([3], [4]). The network is made of tubes connected by junctions. The tubes provide the propagation of the signals onto the network whereas the junctions provide their scattering into the different connected tubes. This scattering is characterized by the so-called "scattering parameter" (S-parameter) matrices [5]. In the past ten years, this kind of modeling has proved its efficiency for predicting electromagnetic (EM) coupling on complex wiring systems ([6], [7]). The applications have always pointed out the convenience of being able to introduce measured data in the BLT equation. The measurements provide a way to replace difficult modeling of junctions. Also, they may provide useful data to calibrate the modeling. This is why it seemed particularly useful to be able to measure scattering parameters of a system, in situ, that is to say directly on its host structure.

A classical method to measure S-parameters deals with directly connecting the ports of the device under test (DUT) to the cables of a network analyzer. The method we present here deals with measuring the S-parameters with a pair of a current injector probe and a current measurement probe. The main drawback of the first mentioned method is that, on real structures, because the equipment requires specific connectors, it is often quite impossible to connect the cables of a network analyzer. For instance, this drawback is obvious when one wants to measure S-parameters on a wiring system. Moreover, the connection of a network analyzer imposes a scalar impedance (generally 50 Ω) at the input of the device under test [8]. On the contrary, with clamping current probes, appropriate measurement technique would not require to disconnect or modify the connections of the DUT.

In a first section, we demonstrate the current probe method. First, we recall the general definition of scattering parameters. Then, we present how to characterize the injection and measurement probes thanks to scattering parameters. The measurement procedure and the associated numerical treatment are then described in detail.

In a second section, we validate the formalism with the CRIPTE numerical code. Then we show applications on simple wiring examples: a one-wire transmission line and a two-wire transmission line.

Finally, in a last section, we analyze the frequency limitation of the technique using a set of high frequency nonclamping current probes.

This paper is an improvement of [9]. The improvements especially deal with some corrections applied in theoretical formulas, numerical validations achieved with the CRIPTE code and new measurements carried out to extend the frequency application domain of the method.

2. Demonstration of the procedure

2.1 Scattering parameter measurement

The measurement of scattering parameters with a network analyzer is performed with two measurement cables:

- a cable A for the injection of the signal (channel A),
- a cable B for the measurement of the signal (channel B).

The principle of the measurement of S-parameter with a network analyzer is summarized on figure 1 (measurement of $S_{ij}$, the reflection parameter, and $S_{ji}$, the transmission parameter, on two test-ports "j" and "i").
The injection is supposed to be on port "j" and the reception on port "i". \( V_j \) and \( V_i \), \( I_j \) and \( I_i \), represent respectively the voltages and the currents on those two ports.

![S-parameter measurement principle](image)

**Fig. 1 : S-parameter measurement principle**

Cable A and cable B of the network analyzer bring a \( Z_e \) input impedance at the level of port "j" and port "i". The measurement of the scattering parameter is said to be "referred" to \( Z_e \). One will pay attention to the fact that the other ports, whereas not involved in the measurement, have to be loaded by the same \( Z_e \) impedance. The injection with cable A is modeled applying a voltage generator \( E \), in series with the \( Z_e \) input impedance at the level of port "j". This generator is related to the voltage and current developed at this port by:

\[
V_j + Z_e \cdot I_j = E
\]

Hence, the classical definition of reflection and transmission S-parameters can also be expressed as a function of \( E \). Particularly, one will notice that the transmission parameter is equal to two times the transfer function between the output voltage and the input generator.

\[
S_{ij} = \frac{V_i - Z_e \cdot I_i}{V_j + Z_e \cdot I_j} = 1 - \frac{2 \cdot Z_e \cdot I_j}{E} \tag{2}
\]

\[
S_{iy} = \frac{V_i - Z_e \cdot I_i}{V_j + Z_e \cdot I_j} = \frac{2 \cdot V_i}{V_j + Z_e \cdot I_j} = -2 \cdot \frac{Z_e \cdot I_i}{E} = 2 \cdot \frac{V_i}{E} \tag{3}
\]

### 2.2 Characterization of the injection current probe

Generally, people talk of "current injectors" because those devices are designed to induce current on cables. This is true only when the cables under study are loaded at each end by a low value impedance. As a matter of facts, an injection probe rather behaves as a *voltage transformer*. The induction of the probe can be described by a voltage generator \( E^n \), applied on the excited wire.

This generator may be understood as the tangential component of the electric field created in the middle of the probe [10]. So this generator can be approximated as constant for all the wires inside the cable. For instance, this is the model applied to determine the response of a shielded cable, excited with a current injector, whatever the loads at each end of the shield are [6]. Of course, this approximation is valid if the cross section of the cable is small compared to the center space of the injector and if the *insertion impedance*, induced by the probe on the excited wire, can be neglected. In addition, the injection level must be small enough for the probe to operate linearly.

Figure 2 presents a simple procedure to characterize the injector. The injector is clamped on a *test wire*. The test wire can be chosen as a simple wire whose length is small enough to allow high frequency characterization. S-parameter measurements are carried out considering a three-port system when the port A of the network analyzer is connected to the injection probe. The three ports are:

- port 1 and port 2, at both ends of the test wire,
- port 3, on the injection probe.
As mentioned below, all the ports are loaded with the same $Z_0$ impedance, the reference impedance of the network analyzer. When cable A, or cable B, is connected to one of the ports of the DUT, this reference load is brought by the network analyzer itself. The remaining port, not connected to the network analyzer, must be connected to a real impedance equal to $Z_0$.

![Diagram of the current injector with S-parameters]

**Fig. 2: Characterization of the current injector with S-parameters**

On each of the three ports, "$E^p$, a voltage, $V_0$, and a current, $I_0$, are defined. Because the test wire has a small length, the propagation can be neglected. Except for their signs, the $V_1$ and $V_2$ voltages are equal. We can write:

$$E^m + 2V_1 = 0$$  (4)

The injection being made on port 3, we can define $\beta$ as the transfer function between the voltage induced by the injector on the test wire, $E^m$, and the voltage generator, $E^p$, applied on the injector by the network analyzer. Hence, according to (3) and (4), we have the following relation:

$$S_{13} = \frac{2V_1}{V_1 + Z_0 \cdot I_3} = \frac{2V_1}{E^p} = \frac{-E^m}{E^p} = -\beta$$  (5)

This characterization procedure can be completed using a voltage amplifier in order to increase the value of $\beta$. But in this case, the user has to account for the amplifier in the calibration, what requires some care not to damage the network analyzer.

Another way to obtain more signal is also to fix one of the ports on the wire at a short circuit value. Then, the relation between $S_{13}$ and $\beta$ only varies by a factor of 2.

### 2.3 Characterization of the measurement probe

The characterization of the measurement probe is similar to the characterization of the injection probe and relies on the same test-wire device used for the injection characterization. Now the injection (connection of cable A) is made on port 1 or port 2 of the test wire. The S-parameters are measured with the cable B connected to the port 3. On figure 3, we have taken the same labeling as the one presented in figure 2.
Fig. 3: Characterization of the current reception probe with S-parameters

As we did for \( \beta \), we define the \( \alpha \) factor as the transfer function between the current generated on port 1 of the test wire, \( I_1 \), and the current induced on port 3, \( I_3 \). This coefficient can also be expressed as a function of the \( S_{31} \) parameter:

\[
S_{31} = \frac{2 \cdot V_j}{E} = \frac{-2 \cdot Z_o \cdot I_1}{I_3} = \frac{I_2}{I_3} = \frac{I_p}{I_{wire}} = \frac{1}{\alpha}
\]  

(6)

The setup being fully reciprocal provided the same type of current probe is used for the injection and the measurement, we have \( S_{31} = S_{13} \) and \( \alpha \) and \( \beta \) are ideally related by:

\[
\alpha = \frac{1}{\beta}
\]  

(7)

Figure 4 represents \( \beta \) and \( \alpha^{-1} \) transfer functions determined on two similar clamping probes, used at the same time for injection and measurement. Their characteristics are flat in low frequency (up to 100 MHz), but present a slight difference pointing out the difficulty of being provided two similar probes.

Fig. 4: Example of \( \beta \) and \( \alpha^{-1} \) transfer functions determined on two identical current probes.
2.4 S-parameter measurement procedure using current probes

2.4.1 General ideas

We have previously seen how S-parameter measurements achieved by a network analyzer could be related to physical voltages and currents. The reference load of those measurements is said "scalar" because the loads applied on each port are the same and are equal to the internal load brought by the network analyzer. However, the same kind of definition can be derived if the reference load is given under the form of a matrix, as it is generally the case in EMT.

The purpose of the two following sections is to establish the relation between the S-parameters measured at the level of the current probes and the S-parameters defined at the level of the ports of the DUT. To make the demonstration progressive, first, we present the result when the impedance load of the DUT is the same on all the ports (scalar case). Then we derive the general definition when the impedance load of the DUT is a matrix associated to a network of different loads (matrix case).

2.4.2 Scalar case

Let us suppose that cable A of the network analyzer is connected to an injection probe (probe A) and that cable B is connected on a reception probe (probe B). Those two probes will be respectively clamped on two ports "j" and "i" of the DUT to make an Sij measurement. Moreover, in the scalar case, we will suppose that the DUT is loaded by the same scalar impedance Zc on each of its ports.

Figure 5 presents the experimental setup. One will notice that a wiring extension is required on the ports to make the measurement possible. In the general case, the non zero length of the port will be a limitation to extend the frequency range of the method. Nevertheless, with this property, its application on long wiring systems becomes natural.

![Diagram](image)

*Fig. 5: Principle of S-parameter measurement in the scalar case (Sij measurement)*

**Determination of Sij:**

In this case, the injection and reception probes are respectively located on port "j" and "i". We will call Zc, the reference measurement load of the network-analyzer. Generally, this impedance is equal to 50 Ω. We will call Sij, the transmission S-parameter between port A and port B of the network analyzer, expressed as a function of the following values:

- Vj and Vb, Ij and Ib, respectively the voltages and currents developed on those two ports and
- Ei, the voltage generator applied on port A.
From (1) and (3), we have:

\[ S \rho_{ij} = \frac{2 \cdot V_B}{V_A + Z_\rho \cdot I_A} = \frac{-2 \cdot Z_\rho \cdot I_B}{E_A} \]  

(8)

Moreover, let us call \( S_{ij}^{DUT} \) the S-parameter of the DUT, referenced to the scalar impedance \( Z_\epsilon \), and determined according to (3):

\[ S_{ij}^{DUT} = \frac{-2 \cdot Z_\epsilon \cdot I_j}{E_j} \]  

(9)

Meanwhile, from (6) and (7), it is possible to express \( I_B \) and \( E_A^\rho \) as a function of \( I_i \), the current induced on port "i", and \( E_j^{inj} \), the voltage generator induced on port "j". This is achieved introducing \( \alpha \) and \( \beta \) transfer functions. Using (9), we find:

\[ S \rho_{ij} = \frac{-2 \cdot Z_\rho \cdot I_j}{E_j^{inj}} \frac{\beta}{\alpha} = S_{ij}^{DUT} \cdot \frac{Z_\epsilon \cdot \beta}{Z_\rho \cdot \alpha} \]  

(10)

Hence, (10) allows to express the S-parameter at the level of the DUT as a function of the transmission parameter measured between the current probes.

\[ S_{ij}^{DUT} = S_{ij}^{meas} \cdot \frac{Z_\rho \cdot \alpha}{Z_\epsilon \cdot \beta} \]  

(11)

**Determination of \( S_{ii} \):**

In this case, the injection and measurement probes are applied on the same port "i". As previously, from (1) and (2), it is possible to derive \( S_{ii}^{meas} \), the transmission S-parameter measured between the two probes.

\[ S \rho_{ii} = \frac{2 \cdot Z_\rho \cdot I_B}{E_A^\rho} \]  

(12)

Then, (5) ad (6) are used to relate \( I_B \), the current measured with probe B, and \( E_A^\rho \), voltage generator applied with probe A, as a function of \( I_i \), the current induced on port "i" with \( E_i^{inj} \), the voltage generator induced on this same port "i".

\[ S \rho_{ii} = \frac{2 \cdot Z_\rho \cdot I_i}{E_i^{inj}} \frac{\beta}{\alpha} \]  

(13)

The scattering parameters at the level of the DUT can be derived from (2). Once more, we recall that those parameters are referenced to the scalar impedance \( Z_\epsilon \).

\[ S_{ii}^{DUT} = 1 - \frac{2 \cdot Z_\epsilon \cdot I_i}{E_i^{inj}} \]  

(14)

Hence, (13) and (14) allow to relate \( S \rho_{ii} \) and \( S_{ii}^{DUT} \). We find:

\[ S \rho_{ii} = \frac{\beta}{\alpha} \cdot \frac{Z_\rho}{Z_\epsilon} \cdot (S_{ii}^{DUT} - 1) \]  

or:

\[ \boxed{S \rho_{ii} = \frac{\beta}{\alpha} \cdot \frac{Z_\rho}{Z_\epsilon} \cdot (S_{ii}^{DUT} - 1)} \]  

or:

\[ \boxed{S \rho_{ii} = \frac{\beta}{\alpha} \cdot \frac{Z_\rho}{Z_\epsilon} \cdot (S_{ii}^{DUT} - 1)} \]  

or:

\[ \boxed{S \rho_{ii} = \frac{\beta}{\alpha} \cdot \frac{Z_\rho}{Z_\epsilon} \cdot (S_{ii}^{DUT} - 1)} \]
\[ S^{DUT}_n = I + \frac{\alpha}{\beta} \cdot \frac{Z_c}{Z_o} \cdot S^n_i \]  

(16)

2.4.3 Matrix case: direct procedure

Compared to the scalar case, the matrix case is more general in the sense that the reference loads are not necessary equal. More, the reference load system can be considered as a network of loads involving possible differential impedance values. In a topological way ([1], [2]), the problem is equivalent to connect the junction equivalent to the DUT to another junction whose impedance parameters (Z-parameters) are equal to a \([Z_c]\) matrix (figure 6). Consequently, if one knows the S-parameters of the network connected to the DUT (easily related to Z-parameters as shown in section 2.6), a procedure, similar to the one presented in the scalar case, can be generalized. Particularly, this network can be another cable network whose equivalent S-parameter have been determined by a compacting process [3].

![Diagram of S-parameter measurement in the matrix case](image)

Fig. 6: Principle of S-parameter measurement in the matrix case (\(S_0\) measurement)

Derivation of S-parameters at the level of the current probes:

When one injects on a port "j" with probe A, the S-parameter measurement may be represented in a matrix way:

\[
\begin{bmatrix}
0 \\
\vdots \\
- - \\
E_j^p \\
\vdots \\
0 \\
\end{bmatrix} \cdot \begin{bmatrix}
S^p \\
\vdots \\
- - \\
I_j^p \\
\vdots \\
0 \\
\end{bmatrix} = 2 \cdot Z_o \cdot \begin{bmatrix}
0 \\
\vdots \\
- - \\
I_j^p \\
\vdots \\
0 \\
\end{bmatrix}
\]

(17)

\(Z_o\) still represents the reference impedance of the network analyzer. \(I_j^p\) represents the current measured on probe B at the level of port "j". \(E_j^p\) represents the injected voltage with probe A, at the level of port "j".

Now, if we suppose that each port "j" of the DUT is excited one at a time with the current injector A, (17) can be generalized in the following matrix form:

\[
\begin{bmatrix}
E_j^p & 0 & 0 \\
\vdots & \vdots & \vdots \\
0 & E_j^p & 0 \\
\vdots & \vdots & \vdots \\
0 & 0 & E_n^p \\
\end{bmatrix} \cdot \begin{bmatrix}
S^p \\
\vdots \\
- - \\
I_j^p \\
\vdots \\
I_n^p \\
\end{bmatrix} = 2 \cdot Z_o \cdot \begin{bmatrix}
I_j^p \\
\vdots \\
- - \\
I_j^p \\
\vdots \\
I_n^p \\
\end{bmatrix}
\]

(18)

where \(I_j^p\) is the current measured on current probe B clamped on port "j", when the current injection of probe A is applied on port "j". (18) will be summarized by:
\[
[S^p] \cdot [E^p] = -2 \cdot Z_o \cdot [I^p]
\] (19)

**Derivation of S-parameters at the level of the DUT:**

At the level of the DUT, we have to consider an excitation provided by the voltage generator \( E_j^{m} \) induced by the current injector on port "j" and the set of the induced currents, \( I_j^{DUT} \) on all the ports "i". They are related by the matrix relation:

\[
[S^{DUT}] \cdot \begin{bmatrix}
0 \\
- \frac{Z_j}{E_j^{m}} \\
- \\
- \\
0
\end{bmatrix} = \begin{bmatrix}
0 \\
- \frac{Z_j}{E_j^{m}} \\
- \\
- \\
0
\end{bmatrix} \cdot \begin{bmatrix}
0 \\
- \frac{Z_j}{E_j^{m}} \\
- \\
- \\
0
\end{bmatrix} - 2 \cdot [Z_e] \cdot \begin{bmatrix}
0 \\
- \\
- \\
- \\
0
\end{bmatrix}
\] (20)

If now, all the ports are excited, one at a time, (20) can be generalized as:

\[
[S^{DUT}] \cdot \begin{bmatrix}
E_i^{DUT} & 0 & 0 \\
- \frac{Z_i}{E_i^{DUT}} & 0 & 0 \\
- & - \frac{Z_i}{E_i^{DUT}} & 0 \\
0 & 0 & E_n^{DUT}
\end{bmatrix} = \begin{bmatrix}
E_i^{DUT} & 0 & 0 \\
- \frac{Z_i}{E_i^{DUT}} & 0 & 0 \\
- & - \frac{Z_i}{E_i^{DUT}} & 0 \\
0 & 0 & E_n^{DUT}
\end{bmatrix} - 2 \cdot [Z_e] \cdot \begin{bmatrix}
I_i^{DUT} \\
I_i^{DUT} \\
I_i^{DUT} \\
I_n^{DUT}
\end{bmatrix}
\] (21)

We will summarize that matrix equation by:

\[
[S^{DUT}] \cdot [E^{m}] = [E^{m}] - 2 \cdot [Z_e] \cdot [I^{DUT}]
\] (22)

**Derivation of S-parameters at the level of the DUT as respect with S-parameters at the level of the probes:**

As in the scalar case, the relation between \([S^{DUT}]\) and \([S^p]\) is simply obtained writing (5) and (6) in a matrix form:

\[
\beta \cdot [E^p] = [E^{m}]
\] (23)

and

\[
\alpha \cdot [I^p] = [I^{DUT}]
\] (24)

and introducing those relations in (19), we obtain the matrix relation:

\[
[S^{DUT}] = [I_d] + \frac{Z_e}{Z_o} \cdot \frac{\alpha}{\beta} \cdot [S^p]
\] (25)

with \([I_d]\) being the identity matrix. One will notice that (25) is nothing but the generalization of (11) and (16), derived for the scalar case.
2.5 Modified procedure

The procedure previously described always requires to measure the transmission parameter between an injection probe and a measurement probe. The procedure can be simplified for the measurement of the reflection parameter $S_{ii}$ at the level of port "i". Indeed, on probe A, clamped to that port, the network analyzer is able to measure a $S^p_{ii}$ parameter. Now on, (19) takes the form:

$$\begin{bmatrix} S^p \end{bmatrix} \cdot \begin{bmatrix} E^p \end{bmatrix} = \begin{bmatrix} E^p \end{bmatrix} - 2 \cdot Z_0 \cdot \begin{bmatrix} I^p \end{bmatrix}$$  \hspace{1cm} (26)

Consequently, this "modified" procedure supposes that probe A is used both for the injection and the measurement. Meanwhile, (22) is not modified. Assuming that the pair of probes have the same $\alpha$ and $\beta$ transfer functions, the combination of (22), (23) and (24) with (26) leads to a modified expression of the relation between $[S^{DUT}]$ and $[S^p]$. We find:

$$[S^{DUT}] = \begin{bmatrix} I_d \end{bmatrix} + \frac{Z_0}{\beta^2} \cdot \begin{bmatrix} I^p \end{bmatrix}$$  \hspace{1cm} (27)

This procedure may be considered as more simple than the direct one because it requires the clamping of only one probe on each port. Especially, this procedure avoids the possible coupling of the injection and the measurement probe when they are inserted on the same port.

Nevertheless, the modified procedure relies on an important assumption that all the current probes have the same transfer functions, which is difficult to achieve for usual probes (see figure 4). The modified procedure has to be considered with care. Moreover, the measurement of the S-parameters made in the direct method always relies on a transmission parameter measurement, which does not necessarily require the use of a network analyzer. Considering those two reasons, the direct procedure has always been considered in all the validations presented in the following sections.

2.6 Relations between Z-impedance, Y-parameters and S-parameters

In (25) and (26), $[S^{DUT}]$ is a S-parameter matrix referenced to a characteristic impedance matrix, $[Z_c]$. In the theory of electromagnetic topology, such S-parameters are called "topological" scattering parameters ([11], [4]). They are the one used in the BLT equation and have the property to be equal to the actual reflection and transmission coefficients of the junction. Nevertheless, in order to use the same junctions in different networks, where they may be connected to tubes having different characteristic impedance matrices, other forms are generally used to characterize the junction. Here, we present 3 types of parameters allowing to get rid of the $[Z_c]$ reference matrix:

- the $[Z]$ matrix of the Z-parameters,
- the $[Y]$ matrix of the Y-parameters,
- the $[S_{30}]$ matrix of the S-parameters, referenced on a usual scalar impedance equal to 50 $\Omega$. In this case, the S-parameters are called "S$_{30}$ parameter". This matrix is nothing but a particular form of a topological matrix. The following relations are the one computed into the CRIPTE code [12] :

$$[Z] = \left(\begin{bmatrix} I_d \end{bmatrix} - [S^{DUT}] \right)^{-1} \cdot \left(\begin{bmatrix} I_d \end{bmatrix} + [S^{DUT}] \right) \cdot [Z_c]$$  \hspace{1cm} (26)

$$[Y] = [Z]^{-1} = [Z_c]^{-1} \cdot \left(\begin{bmatrix} I_d \end{bmatrix} - [S^{DUT}] \right) \cdot \left(\begin{bmatrix} I_d \end{bmatrix} + [S^{DUT}] \right)^{-1}$$  \hspace{1cm} (27)
\[
[S_{50}] = ([Z] - 50 \cdot [I_d]) \cdot ([Z] + 50 \cdot [I_d])^{-1}
\]  \hspace{1cm} (28)

3. Validation of the method

3.1 General objectives

The validation procedure we have used in all our examples is the following:

**Step 1**: we have determined the topological S-parameters on different wiring systems, using the current probe method.

**Step 2**: all those S-parameters have been transformed in \( S_{50} \) parameters, using (26) and (28).

**Step 3**: this way, the comparison with \( S_{50} \) parameters directly measured on the ports of the DUT has been made possible.

First, we present a validation of the theory carried out by simulating the measurement procedure with the CRIPTED code.

Then, we show results of the application of the experimental procedure on canonical wiring systems.

The two first systems considered have been two transmission lines (a one-wire and a two-wire transmission line). The current probe used were the ones whose \( \alpha \) and \( \beta \) transfer functions are the ones presented on figure 4. One will notice that the response of the probe is flat up to 100 MHz only. But, the main advantage of those probes is that they can be opened. Moreover, our objective was to verify that the obtained S-parameters were precise enough in low frequency. For instance, from those parameters, one should be able to determine the **per unit length electrical parameters of the lines**. Indeed, it is possible to derive those parameters based on S-parameter measurements at one end of a line, when the other end is alternatively loaded by a short-circuit and by an open circuit ([13], [14]).

The last test we have carried out intended to analyze the frequency limit of this method using a high frequency current probe. Of course, to guarantee a flat response in high frequency, it is not possible to open such a current probe, and the method we emphasized in this document looses a part of its convenience.

3.2 Validation of the theory

To validate the theory we have chosen to simulate the procedure using the CRIPTED code [12]. The interest of the numerical simulation relies on the fact that it allows one to simulate perfect injection and measurement current probes. The example we have considered is a single wire transmission line on an infinite metallic plane. The line can be loaded by two lumped elements \( Z_l \) and \( Z_i \) (figure 7).
The three load configurations having been tested as respect with the value of $Z_1$ and $Z_2$ are described in table 1.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$Z_1$</th>
<th>$Z_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50 Ω</td>
<td>50 Ω</td>
</tr>
<tr>
<td>2</td>
<td>16 Ω</td>
<td>10 Ω</td>
</tr>
<tr>
<td>3</td>
<td>16 Ω</td>
<td>120 Ω</td>
</tr>
</tbody>
</table>

Table 1: Load configurations tested on the one-wire transmission line

In the CRIPTE code, the simulation of the measurement at the level of the probe is automatically achieved by performing the compacting of a sub-network. This is why we have chosen to calculate all the S-parameters we needed at the level of the probes with only one compacting calculation. Because the model of probes does not modify significantly the impedance of the DUT, and in order to reduce the number of calculations, we have applied an injection probe, noted "I", and a measurement probe, noted "M", on both ports, 1 and 2. The matrix, obtained in only one compacting calculation, is 4x4 and contains the 2x2 matrix we need. Figure 8 gives an example of an $S_{11}$ measurement. The injection on port 1 and the measurement on port 2 are represented by the black probes.

The topological network used for the demonstration is described on figure 9. The scattering parameters of the transmission line have previously been calculated and stored in the matrix $S^{DUT}$. The junction representing the DUT is connected to the junctions representing the injection probe and the measurement probes. The two loads connected at both ends of the DUT are modeled with two junctions having the Z-parameter value of $Z_1$ and $Z_2$. All the tubes involved in this topological network are assumed to have a zero length because the ports of the DUT and the size of the probes are supposed infinitesimal.
The $S$-parameters of the ideal probes are stored in $S_0^a$ and $S_0$, respectively for the injection and the measurement. Port 3 is the injection or measurement port, whereas port 1 and port 2 are the ports on the internal wire. The current is supposed to flow from port 1 to port 2. This is important for the connection of the junctions to the network.

The determination of the parameters of the current probe relies on imposed values:

- $S_{11} = S_{12} = -\alpha$ or $\beta$ \hspace{1cm} (27)
- $S_{21} = S_{22} = \alpha$ or $\beta$ \hspace{1cm} (28)

The other values are obtained considering the following properties:

- the system is reciprocal, which implies symmetric matrices,
- the system is lossless, which implies that the matrices are unitary matrix (their transpose is equal to their conjugate) [8]. This property is summarized by (29) and (30).

$$\sum_{n=1}^{j} S_{jn} \cdot S_{jn}^* = 1 \hspace{1cm} (29)$$

$$\sum_{n=1}^{j} S_{in} \cdot S_{jn}^* = 0 \hspace{0.5cm} i \neq j \hspace{1cm} (30)$$

Hence the scattering matrices of the two probes can be determined as follows:

$$S_0^a = \begin{pmatrix}
\frac{1}{2} \left( l - \sqrt{1 - 2 \cdot \beta^2} \right) & \frac{1}{2} \left( l + \sqrt{1 - 2 \cdot \beta^2} \right) \\
\frac{1}{2} \left( l + \sqrt{1 - 2 \cdot \beta^2} \right) & \frac{1}{2} \left( l - \sqrt{1 - 2 \cdot \beta^2} \right)
\end{pmatrix}$$

$$S_0^a = \begin{pmatrix}
\frac{1}{2} \left( l - \sqrt{1 - 2 \cdot \alpha^2} \right) & \frac{1}{2} \left( l + \sqrt{1 - 2 \cdot \alpha^2} \right) \\
\frac{1}{2} \left( l + \sqrt{1 - 2 \cdot \alpha^2} \right) & \frac{1}{2} \left( l - \sqrt{1 - 2 \cdot \alpha^2} \right)
\end{pmatrix}$$

One will notice that the mismatching due to the insertion of the probe is not relevant if $\beta$ and $\alpha'$ have a low value. Indeed, in this case, the following parameters can be approximate by:

$$S_{11} \cong S_{22} \cong \frac{l}{2} \cdot \beta^2 \equiv_\beta \rightarrow 0 \hspace{1cm} (33)$$

$$S_{21} \cong S_{21} \equiv l - \frac{l}{2} \cdot \beta^2 \equiv_\beta \rightarrow l \hspace{1cm} (34)$$

$$S_{11} \equiv l - \beta^2 \equiv_\beta \rightarrow l \hspace{1cm} (35)$$

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Relation (33) and relation (34) mean that the transmission is perfect on the wire and there is no reflection. (35) means that the impedance seen from port 3 is close to an open circuit.

Figure 10 presents the results obtained in the three configurations in superimposition with the directly measured $S_{30}$ scattering parameters. The agreement is perfect. This result fully validates the formalism described in the previous sections and helps us to realize the difference existing between a perfect case and a realistic case.

![Graph showing S30 parameter obtained in all the configurations on the one-wire transmission line when simulating perfect current probes](image)

**Fig. 10** : $S_{30}$ parameter obtained in all the configurations on the one-wire transmission line when simulating perfect current probes

### 3.3 Application of the procedure on real wiring systems

#### 3.3.1 Application on a one-wire transmission line

The first example we have considered is similar to the one we have tested with the CRIPTE code, a single wire transmission line, involving at each end the same three load-configurations presented on table 1.

Figure 11 gives an example of an S-parameter measured directly at the level of the current probes. Such input measurements are treated to determine $[S^{MT}]$, at the level of the ports of the line. One will notice that the first resonance happens after 100 MHz. After this resonance, the value of the parameters begins to increase, due to the characteristics of the current probes that are not flat anymore in this frequency range (see figure 4).
Figure 12 presents $S_{11}$ and $S_{12}$ parameters obtained after treatment in configuration 1. We have superimposed the direct $S_{xy}$-parameter measurements coming from the network analyzer (directly connected on port 1 and port 2 of the line) and the $S_{xy}$-parameters coming from the current probe measurements (using injection probe A and reception probe B).

Figure 13 and 14 present the same kind of plots in configuration 2 and configuration 3 respectively. For the three configurations, one will notice the good agreement between 1 MHz and 100 MHz. This frequency range is exactly the one required for the usual determination of the per unit length electrical parameters of a transmission line. Under 1 MHz, the plots of $S_{11}$ obtained with the current probe method presents some noise because of the low value of the reflection coefficient. From 100 MHz, the assumption that the current injector behaves as an ideal voltage transformer is not valid anymore. Related to that, the size of the current probes cannot be neglected at those high frequencies.

Figure 12: $S_{xy}$-parameter obtained in configuration 1 (50 Ω and 50 Ω) on the one-wire transmission line

Figure 11: Example of $S$-parameters measured at the level of the current probes on the one-wire transmission line.
Fig. 13: $S_{11}$-parameter obtained in configuration 2 (16 $\Omega$ and 10 $\Omega$) on the one-wire transmission line

Fig. 14: $S_{11}$-parameter obtained in configuration 3 (16 $\Omega$ and 120 $\Omega$) on the one-wire transmission line

3.3.2 Application on a two-wire transmission line

The second example deals with a two-wire cable transmission line. As previously, different load-configurations have been considered applying 4 impedance values $Z_1$, $Z_2$, $Z_3$, $Z_4$ on port 1, 2, 3, and 4 respectively (figure 15).
Fig. 15: Application of the method on a two-wire transmission line ($S_{12}$-measurement example)

The current probe we have used is the same as in the one-wire transmission line case. The two load-configurations tested are reported in table 2.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$Z_1$</th>
<th>$Z_2$</th>
<th>$Z_3$</th>
<th>$Z_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50 Ω</td>
<td>50 Ω</td>
<td>50 Ω</td>
<td>50 Ω</td>
</tr>
<tr>
<td>2</td>
<td>50 Ω</td>
<td>120 Ω</td>
<td>10 Ω</td>
<td>16 Ω</td>
</tr>
</tbody>
</table>

Table 2: Load configurations tested on the two-wire transmission line

Figure 16 and figure 17 present the $S_{22}$-parameters obtained after the treatment of the topological $S$-parameters determined with the current probes. They are superimposed with the $S_{22}$-parameters directly measured with the network analyzer on the four ports of the DUT.

Compared to the results obtained for the one-wire transmission line, the agreement is not as good for the reflection coefficients whereas it remains fully comparable for the other coefficients (again between 1 MHz and 100 MHz). The discrepancy obtained on $S_{11}$ coefficients does not have a real explanation at the present moment. Nevertheless, an error in the calibration of the network analyzer might be the origin of that result. Nevertheless, pulling out a wire in a bundles creates a mismatching which may reverse the current as shown in [15]. Such a reversing sign of the current would affect significantly the value of $S_{11}$. 

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Fig. 16: $S_{11}$-parameter obtained in configuration 1 (50 Ω, 50 Ω, 50 Ω, and 50 Ω) on the two-wire transmission line.
3.3.3 Application of the method with high frequency performance current probes

In that section we wanted to emphasize the fact that the limitation of the technique is not in the theory but in the quality of the probes. For that purpose, we have used a set of probes, whose $\alpha$ and $\beta$ transfer functions present a flat response up to 4 GHz (figure 18). The high-quality measurements are made possible thanks to a specific calibration device which keeps a very precise 50 $\Omega$ built-in characteristic impedance. Of course, to reach such a performance in high frequency, the possibility to open such a probe is excluded. The two transfer functions were obtained applying the S-parameter measurements described in section 2.2 on the calibration test provided with the current probes.
Fig. 18: $\beta$ and $\alpha^{-1}$ transfer functions determined on two identical current probes.  
(Courtesy Voss Scientific)

The DUT considered in the test was made of a single wire laying on a metallic plane (figure 19). Load configuration with two 50 $\Omega$ at both ends of the cable have only been considered. It must be noticed that the geometry is far from a straight regular transmission line. This configuration reminds us that the technique can be applied to cases other than transmission lines. As in the previous transmission line cases, the $S_{21}$-parameters obtained after treatment of the current probe measurements have been compared to the ones coming from the direct measurements on the DUT ports.

Fig. 19: DUT considered for the application of high frequency performance current probes  
(Courtesy of "Voss Scientific").

Figure 20 and figure 21 present the results obtained for $S_{11}$ and $S_{12}$. Because the usual application of the probes is for low current measurements, their injection transfer function is not really effective. Therefore, an amplifier in the 100 MHz to 1 GHz frequency range has been required in the injection link. In addition, the network analyzer used for the measurements is a high frequency one (from 50 MHz to several tens of GHz). The combination of the frequency characteristics of the amplifier and the network analyzer explain the non regular plots observed in low frequency up to 400 MHz.
The numerical treatment is particular good for $S_{21}$ up to 1 GHz. Unfortunately, the results are not as satisfactory for $S_{11}$. Again, a possible coupling between the injection and measurement probe is suspected. The application of the indirect method should be tested in the future to confirm this hypothesis. However, both measurements offer promising results for the extension of the method in higher frequencies.

![Graphs showing $S_{21}$ determination with high frequency performance probes](image)

**Fig. 20: $S_{21}$ determined with the high frequency performance probes**

**Fig. 21: $S_{21}$ determined with the high frequency performance probes**

### 4. Possible evolution of the method

The $S$-parameter measurement technique and the numerical treatment we have presented in this document offer the capability to determine $S$-parameters on a system without connecting the reference load of the network analyzer on each port of the DUT.

The results obtained on simple cables make us think that the method is precise enough to come back to the per-unit-length values of the electrical parameters of more complex cables. Indeed, in [16], it has already been applied to determine the per-unit-length parameters of a shielded cable bundle inside the cockpit of the EMPTAC aircraft. The results are good for the determination of the inductance parameters because the other end of the cable was loaded with a short circuit, allowing a significant amplitude of the current on wires. The results are not as good in the case the conductance matrix determination because the other end of the cable had to be in an open circuit. Nevertheless, the results obtained have demonstrated their usefulness to calibrate the electrical parameter calculations performed with the numerical code LAPLACE [12].

It is also possible to think of applying the method to measure in-situ-$S$-parameter of sub-networks, as it can be performed numerically in the CRIPTE code. Such measured S-parameters could be introduced in complex topological networks, involving measurement and modeled data at the same time. Moreover, a similar technique could be developed to determine the Thevenin generator of a DUT. This kind of method is the one applied in [17], to come back to the voltage generators of a two-port system, using a LISN device.

Nevertheless, the validity of this technique is limited to the frequency range of the probes. The results we obtained using high frequency performance probes (non clamping ones) demonstrate that the limitation is not in the method but in the technology. In addition, such probes would be perfectly suited for the test-wiring method [9], [18]. Indeed, in this method, they are used again as a voltage transformer to solve an inverse problem providing the real sources distributed along the wiring.
5. References


