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A Time-Domain View of Choice of Transient Excitation Waveforms for Enhanced Response of Electronic Systems

Carl E. Baum
Air Force Research Laboratory
Directed Energy Directorate

Abstract

This paper revisits the choice of excitation waveforms for maximizing the response of electronic systems. Previous results have been based on frequency-domain considerations. Here these results are found for resonant responses directly in time domain by simple application of concepts of causality, linearity, and time-translation invariance of systems, combined with norms of the response waveforms. This gives a simple visualization of resonant buildup.

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1. Introduction

The selection of temporal waveforms of incident fields for maximizing the response of complex electronic systems (say, in terms of voltage at some pin leading into some box) has been treated in previous papers [1, 4, 6]. The purpose of the present paper is to revisit this problem and present the basic result from a different point of view, one which may be more appealing to some readers. The considerations here are directly in time domain.

From simple theoretical considerations and empirical observations [1] one can arrive at frequencies in the rough vicinity of 1 GHz as optimal for generic back-door interaction. This result depends on the characteristics of sources (narrowband) and antennas for radiating high-power microwave pulses, combined with the characteristic dimensions for resonances in the illuminated systems of interest. Subsequent papers [4, 6] go into more details of the interaction theory, showing the benefits of tuning to a system resonance and establishing the pulse width (number of cycles) to ring up the pin voltage to near maximum (based on the width (or Q) of the resonance).

In recent years, experimental results have been published (in the public domain) which are in agreement with the foregoing observations. In Germany, measurements were made on a "smart ammunition system" (missile) showing important resonant responses in the range of 200 MHz to 2 GHz with important system effects at field levels of about 30 V/m at around 700 MHz [7]. In the Netherlands, measurements were made on a personal computer showing important system effects as low as 30 V/m incident with frequencies from 1 GHz to 2.9 GHz [9].

Having discussed this matter at numerous scientific conferences, this general observation has become known as Baum's law:

> Electronic systems respond well to electromagnetic environments with frequencies around 1 GHz because the size of the human hand is resonant at about this frequency, and human beings build and operate things with their hands, thereby building things with similar characteristic dimensions.

Of course, this can also be stated (and has been stated) in various equivalent and/or abbreviated forms.

In the spirit of Prof. C. H. Papas, it is instructive to consider a "baby problem", which while simple, reveals the essential physics of the more general problem. This then gives, what to some people, is a simple way of viewing the problem. Essentially what we will do is compare the temporal response of a resonant system to multiple pulses at appropriate temporal spacing to the response to a single pulse.

2. Causal, Linear, Time-Invariant Systems

Consider some electromagnetic system which we can model by a transfer function $\tilde{T}(s)$ or convolution operator $T(t)\circ$, relating a response (say a voltage) $V_r(t)$ to an excitation or source $V_s(t)$ as

$$
\begin{aligned}
V_r(t) &= T(t)\circ V_s(t) \\
\tilde{V}_r(s) &= \tilde{T}(s)\,\tilde{V}_s(t) \\
&\quad \sim \;\equiv\; \text{two-sided Laplace transform over time t} \\
&\quad s \;=\; \Omega + j\omega \;=\; \text{Laplace-transform variable or complex frequency} \\
&\quad \circ \;=\; \text{convolution with respect to time}
\end{aligned}
\tag{2.1}
$$

Immediately we notice that implicit in this formulation we have assumed a linear system defined by

$$
\begin{aligned}
&a_1 V_s^{(1)}(t) + a_2 V_s^{(2)}(t) \;\text{ produces }\; a_1 V_r^{(1)}(t) + a_2 V_r^{(2)}(t) \\
&\quad a_n \equiv \text{real constants}
\end{aligned}
\tag{2.2}
$$

and a time-invariant system defined by

$$
\begin{aligned}
&V_s(t-\tau) \;\text{ produces }\; V_r(t-\tau) \\
&\quad \tau \equiv \text{any real time}
\end{aligned}
\tag{2.3}
$$

Otherwise the transfer function $\tilde{T}(s)$ is not defined via the Laplace/Fourier transform. Of course some systems change configuration at various times by discrete changes such as switches (whether electronic or mechanical), so the above applies throughout any time window of constant system configuration.

Let the system be causal, which requires that there be no response before the forcing function exists. In terms of the time-domain function $T(t)$ characterizing the transfer convolution operator this implies

$$
T(t) = 0 \;\text{ for }\; t < 0 \tag{2.4}
$$

Passive systems have this property, as do stable active systems (e.g., amplifiers).

An extended definition of causality comes from relativity theory, which states that no disturbance can propagate faster than c, the speed of light. By this is meant that a source which is zero (exactly) for all time before some time t_0 can produce *no* response at some distance r away before a time $t_0 + r/c$. Physical systems have nonzero physical dimensions, so (2.4) applies for t less than some time (perhaps small) greater than zero. As a practical matter one can sometimes replace c by a lesser speed, v, corresponding to propagation through a medium (e.g., dielectric) other than free space. Strictly speaking one must let frequency tend asymptotically to infinity when estimating v. However, due to limitations on the upper frequencies realistically contained in the pulses one may use an appropriate high-frequency value of v.

3. Norms

Norms are mathematical operators that convert vectors, matrices, functions, and operators into real, nonnegative scalars. In a sense they measure how large these mathematical entities are. For present purposes, let us consider norms of time domain waveforms. A more complete discussion is contained in [8].

For time-domain waveforms, say in terms of voltage, we have the norm properties

$$
\begin{aligned}
\|V(t)\| &= \begin{cases} 0 \ \ \textit{iff} \ \ V(t) \equiv 0 \ \text{ or has zero "measure" per the perticular norm} \\ > 0 \ \ \text{otherwise} \end{cases} \\
\|\alpha V(t)\| &= |\alpha| \ \|V(t)\| \quad , \quad \alpha \equiv \text{scalar} \\
\|V_1(t) + V_2(t)\| &\le \|V_1(t)\| + \|V_2(t)\|
\end{aligned}
\tag{3.1}
$$

This is totally general, leaving many possibilities for specific norms. Let us restrict the norms of interest to "natural" norms [5], based on the physical properties of systems of interest. One of these is time-translation invariance, i.e., a *time-invariant norm* as [3]

$$
\|V(t-\tau)\| = \|V(t)\| \quad , \quad \tau \ \text{real} \tag{3.2}
$$

so that the norm is the same, no matter at what time the waveform begins. From this we have the inequality

$$
\left\| \sum_{n=1}^{N} V(t-t_n) \right\| \le \sum_{n=1}^{N} \|V_1(t-t_n)\| = N \|V(t)\| \tag{3.3}
$$

If we consider only a portion of the waveform, say over a time window $t_1 \le t \le t_2$, we can have a *window* norm

$$
\|V(t)\|^{(t_1,t_2)} \equiv \|V(t)[u(t-t_1) - u(t-t_2)]\| \tag{3.4}
$$

Note that this is zero if $V(t)$ is zero throughout the window. Note that

$$
\begin{aligned}
\|V(t)\| &\equiv \|V(t)[1-u(t-t_1)] + V(t)[u(t-t_2) - u(t-t_2)] + V(t)u(t-t_2)\| \\
&\le \|V(t)\|^{(-\infty,\, t_1)} + \|V(t)\|^{(t_1,\, t_2)} + \|V(t)\|^{(t_2,\, \infty)}
\end{aligned}
\tag{3.5}
$$

implying

$$\|V(t)\|^{(t_1, t_2)} \leq \|V(t)\| \tag{3.6}$$

For window norms one needs to be careful to restrict the norm type to one that does not include a derivative (with respect to time) norm [2] since the unit step function has an unbounded derivative. One could also define a window norm with smooth edges if desired.

A commonly used norm, to which the window concept can be applied, is the p-norm defined by

$$\|V(t)\|_p \equiv \left[\int_{-\infty}^{\infty} |f(t)|^p \, dt \right]^{\frac{1}{p}} \quad , \quad 1 \leq p < \infty \tag{3.7}$$

with the special case of $p = \infty$ given by

$$\|V(t)\|_\infty = \sup_t |V(t)| \tag{3.8}$$

which is also called the peak norm. Commonly used values of p are 1, 2, and ∞. Note that the p-norm is time invariant. In this paper our emphasis is on the ∞-norm for peak voltages.

4. Repetition of Excitation

Returning to the excitation and response discussed in Section 2, consider a canonical excitation pulse as in Fig. 4.1. For simplicity let $V_s(t)$ be a unipolar pulse of full width T_s. Bipolar pulses can also be constructed from this by time shift T_s or larger with sign reversal.

Let this $V_s(t)$ excite a first-order pole pair as in Fig. 4.2 given by

$$\begin{aligned} V_r(t) &= V_0 \operatorname{Re}\left[e^{s_0 t + j\phi_0}\right] u(t) \\ &= V_0 e^{\Omega_0 t} \cos(\omega_0 t + \phi_0)\, u(t) \\ \Omega_0 &< 0 \\ s_0 &= \Omega_0 + j\omega_0 \end{aligned} \tag{4.1}$$

For simplicity we neglect other poles which may be present in the response and consider the case of a dominant pole pair which is often the case in transient responses. Important parameters of this response are the period

$$T = \frac{1}{f_0} = \frac{2\pi}{\omega_0} \tag{4.2}$$

and the successive positive and negative peaks V_n with

$$|V_n| \le V_0 \quad , \quad |V_{n+1}| < |V_n| \;\text{ for } n = 1,\, 2,\, 3, \cdots \tag{4.3}$$

where ϕ_0 is restricted to $-\pi/2 \le \phi_0 \le 0$ for simplicity in the example (not an essential restriction for the general result). Note that one can also have some other contributions to $V_r(t)$ at early time, such as by convolution of the $V_s(t)$ waveform (normalized to unit time integral or area) with the damped sinusoid unless $T_s \ll T/2$. (This is the class -2 form of the coupling coefficient as discussed in the singularity expansion method (SEM) [11].) This could affect V_1 (reducing it), but this is not essential to the discussion.

For our causal, linear, time-invariant system consider applying the excitation twice giving

$$V_s(t) + V_s(t-\tau) \text{ produces } V_r(t) + V_r(t-\tau) \tag{4.4}$$

Choosing τ as T the period we have

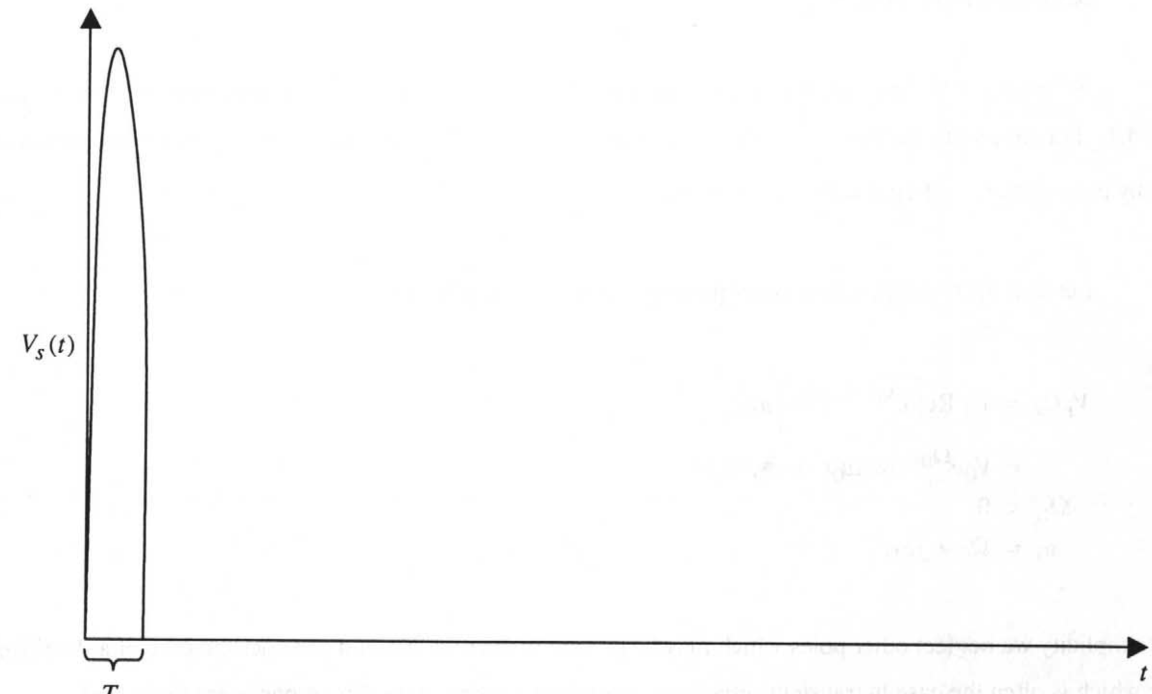

Fig. 4.1 Narrow, Unipolar Excitation Pulse

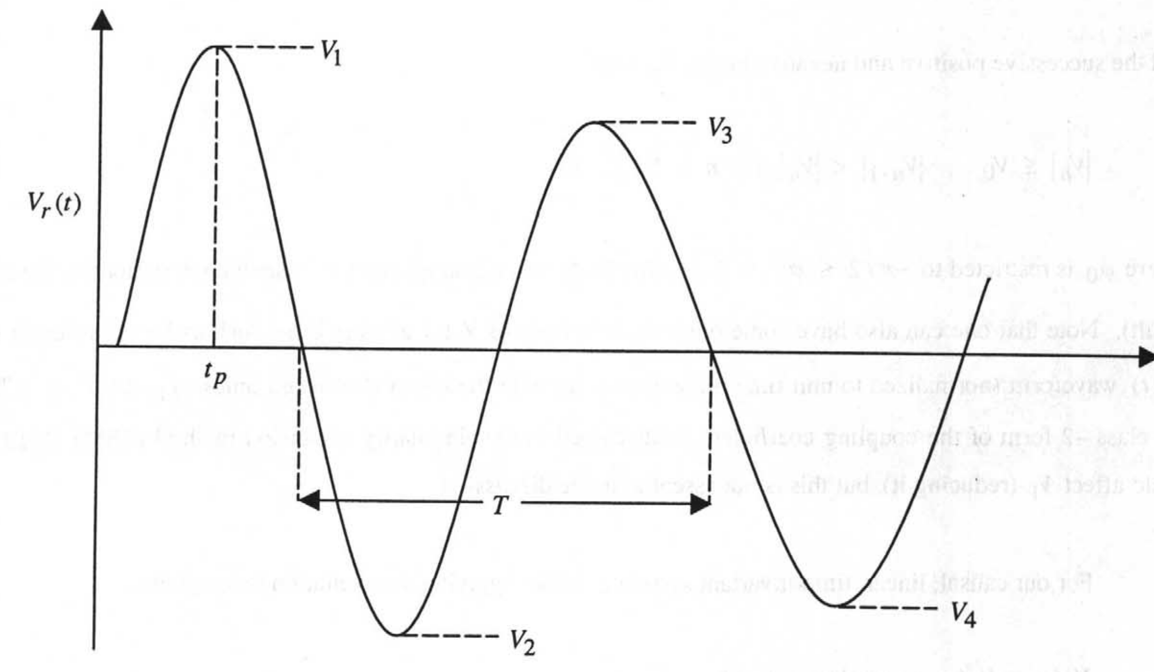

Fig. 4.2 First-Order Pole Pair (Resonance) Response

$$\|V_s(t) + V_s(t-T)\|_\infty = V_1 \tag{4.5}$$

since the two pulses (by choice) do not overlap. However, the response has

$$\|V_r(t) + V_r(t-T)\|_\infty = V_1 + V_3 \tag{4.6}$$

For small damping we have

$$V_1 \simeq V_3 \simeq V_0 \tag{4.7}$$

So the response peak voltage has doubled (implying four tiems the peak power) for no increase in the excitation peak.

From the norm discussion in Section 3 we have

$$\begin{aligned} \|V_r(t) + V_r(t+T)\|_\infty &\leq 2\|V_r(t)\| = 2V_1 \\ \|V_r(t) + V_r(t+T)\|_\infty &\geq \|V_r(t) + V_r(t+T)\|_\infty^{(T,\infty)} = V_1 + V_3 \end{aligned} \tag{4.8}$$

giving both upper and lower bounds.

Applying the excitation N times at successive intervals T gives (from (3.3))

$$\left\|\sum_{n=1}^{N} V_r\left(t-[n-1]T\right)\right\|_\infty \leq N\|V_r(t)\| = N\,V_1 \tag{4.9}$$

Here note that as N becomes large the norm does not grow without bound as $N \to \infty$ due to the damping of $V_r(t)$. Rather (4.6) generalizes to [10]

$$\begin{aligned} \left\|\sum_{n=1}^{N} V_r\left(t-[n-1]T\right)\right\|_\infty &= \sum_{n=1}^{N} V_{2n-1} = V_1 \sum_{n=1}^{N} e^{\Omega_o[n-1]T} \\ &= V_1 \sum_{n=0}^{N} e^{n\Omega_o T} = V_1 \frac{1-e^{N\Omega_o T}}{1-e^{\Omega_o T}} \end{aligned} \tag{4.10}$$

For large N we have

$$\left\| \sum_{n=1}^{N} V_r\left(t-[n-1]T\right) \right\|_\infty \rightarrow \frac{V_1}{1-e^{\Omega_o T}} \simeq \frac{V_1}{[-\Omega_o T]} \text{ for small } |\Omega_0 T| \tag{4.11}$$

remembering that $\Omega_o < 0$.

Here we have repeated $V_r(t)$ in a positive sense, but this is not essential. One can alternate signs with a spacing of $T/2$ for a similar resonance build up.

5. Concluding Remarks

So now we can see that by repetitive application of an excitation waveform one can increase the response provided one does this judiciously. Once the first response peak has been reached the introduction of a second excitation cannot reduce the peak voltage (∞-norm), but may increase this by as much as a factor of 2 for a resonant response. This generalizes to successive repetitions of the excitation with the response peak growing initially proportional to N (number of excitations), but saturating depending on the damping of the resonance.

The present result is a reinterpretation of previous results based on Laplace transforms (complex frequency domain). Here simple concepts of superposition (linearity) combined with time-translation and causality provide the result, giving a visualization of the process. While the argument is cast in terms of an excitation voltage $V_s(t)$, this is simply related to an incident electric field via a dot product with the usual effective height vector as used in antenna theory.

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