Interaction Notes

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Quadrupole Terms in Magnetic Singularity Identification

Carl E. Baum
Air Force Research Laboratory
Directed Energy Directorate

Abstract

In magnetic singularity identification (MSI) of conducting and permeable scatterers one considers the low-frequency poles with real natural modes and frequencies to represent the magnetic-polarizability dyadic. This is an approximation neglecting the higher-order multipoles. This paper considers the magnetic quadrupole terms as a correction to the dipole-only representation. This leads to the concept of the effective center of a natural mode to minimize the quadrupole contribution. In the case of scatterers with certain symmetries there can also exist natural modes with no magnetic-dipole contribution, but with a quadrupole contribution. One potential use of the quadrupole information is in removing orientation ambiguities of the visually obscured scatterer. By judicious choice of the locations of the observer and the source of the incident magnetic field one can minimize or maximize the presence of the quadrupole terms in the scattering data.

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1. Introduction

In magnetic singularity identification (MSI) one considers the natural frequencies and modes associated with the diffusion of magnetic fields into highly conducting targets [2]. This is used in turn to express the magnetic polarizability dyadic to give a magnetic-dipole approximation to the scattered magnetic fields which are taken to be quasi static outside the target and fall off as r^{-3}. Higher order multipoles fall off even faster, but may still be significant close to the target.

A related question concerns the center of the target. Where does one best choose $\vec{r} = \vec{0}$? If this is not appropriately centered on an elementary magnetic dipole, then higher order multipoles enter the representation. An extended magnetic dipole (natural mode) in general also has higher-order multipoles. This paper explores the properties of such magnetic quadrupoles with a view to "minimizing" them where appropriate, and using this information to aid in target identification.

As usual we have the conventions

$$\begin{aligned}
&\sim \;\equiv\; \text{two-sided Laplace transform over time } t \\
&s = \Omega + j\omega = \text{Laplace-transform variable or complex frequency} \\
&\overleftrightarrow{1} = \vec{1}_x \vec{1}_x + \vec{1}_y \vec{1}_y + \vec{1}_z \vec{1}_z \equiv \text{three-dimensional identity} \qquad (1.1) \\
&\overleftrightarrow{1}_n = \overleftrightarrow{1} - \vec{1}_n \vec{1}_n \equiv \text{identity transverse to direction } \vec{1}_n \text{ where } n \text{ can take on various labels}
\end{aligned}$$

Concerning the external medium we have for an assumed uniform isotropic medium [2, 5]

$$\tilde{\gamma}_1(s) = \left[s\mu_0\left[\sigma_1 + s\varepsilon_1\right]\right]^{\frac{1}{2}} = \text{propagation constant}$$

$$\tilde{Z}_1(s) = \left[\frac{s\mu_0}{\sigma_1 + s\varepsilon_1}\right]^{\frac{1}{2}} = \text{wave impedance}$$

$a \equiv$ characteristic dimension (size) of target

$r >> a$ (r = distance to observer)

$|\tilde{\gamma}_1(s)|\, r << 1$ (allowing external scattered fields to be dominated by quasi-static terms)

$\mu_1 = \mu_0 \equiv$ permeability of free space (allowing one to ignore the earth/air interface when considering the magnetic field (quasi-static))

$$\left|\vec{H}^{(inc)}(\vec{0},s)\right| >> \left|\tilde{Z}_1(s)\right| \left|\vec{E}^{(inc)}(\vec{0},s)\right|$$ (dominance of incident magnetic field at target, due to near field of loop source(s)) (1.2)

$\vec{H}^{(sc)}(\vec{r},s) \equiv$ measured scattered magnetic field (neglecting scattered electric field)

We observe that σ_1 and ε_1 have negligible effects on the quasi-static magnetic field (external to the target).

Concerning the target we observe

$\overleftrightarrow{\sigma} = \overleftrightarrow{\sigma}^T \;, \; \overleftrightarrow{\mu} = \overleftrightarrow{\mu}^T$ assumed in dependent of s

$$\left\|\overleftrightarrow{\sigma}\right\| >> |s| \left\|\overleftrightarrow{\varepsilon}\right\| \Rightarrow$$ diffusion dominance in target (low frequencies of interest) (1.3)

which leads to first order poles with real negative s_α and real natural modes [2].

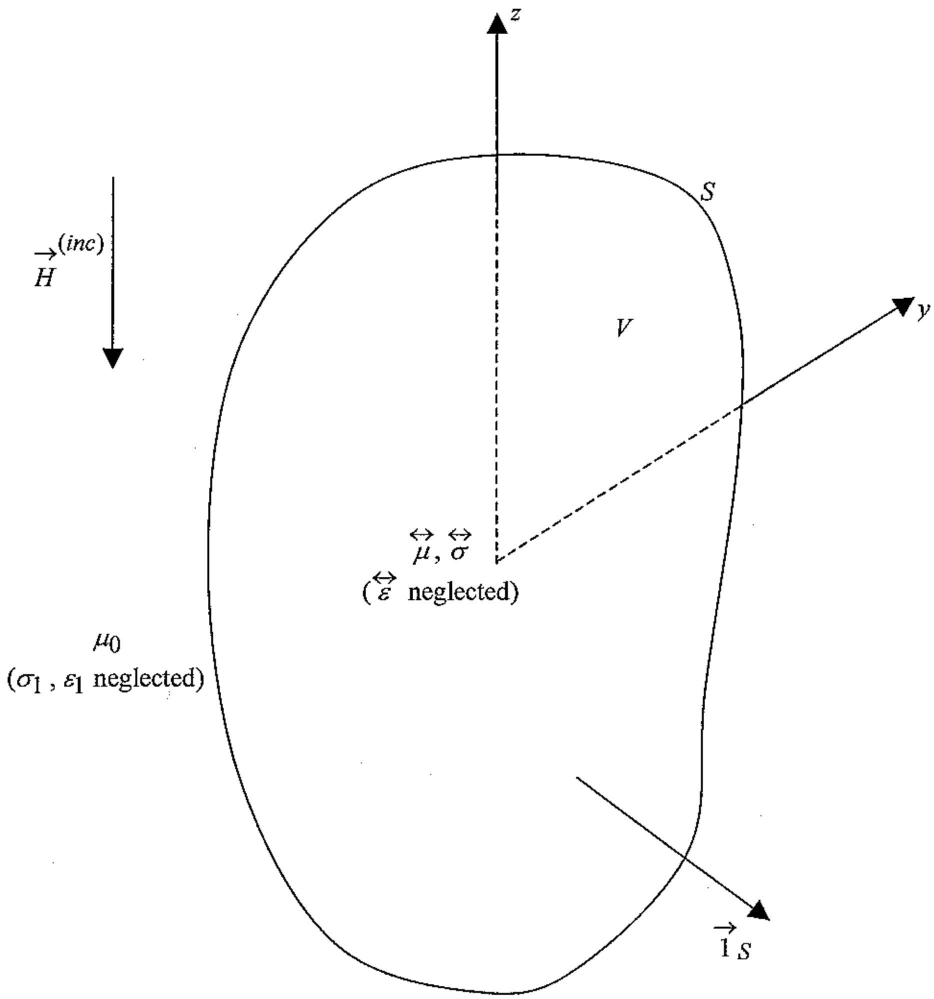


Fig. 1.1 Scatterer in Uniform, Isotropic Medium

2. Natural Modes in MSI

As discussed in [5, 9] one can formulate appropriate integral equations for the electric and magnetic currents in highly conducting targets. Summarizing we have

$$\tilde{\vec{E}}^{(sc)}(\vec{r},s) = -s\mu_0 \left\langle G_0(\vec{r},\vec{r}\,'), \overleftrightarrow{\sigma}(\vec{r}\,') \cdot \tilde{\vec{E}}^{(sc)}(\vec{r},s) \right\rangle$$

$$- s\left\langle \nabla G_0(\vec{r},\vec{r}\,') \underset{,}{\times} \left[\overleftrightarrow{\mu}(\vec{r}\,') - \mu_0 \overleftrightarrow{1} \right] \cdot \tilde{\vec{H}}^{(sc)}(\vec{r}\,',s) \right\rangle$$

$$\tilde{\vec{H}}^{(sc)}(\vec{r},s) = -\left\langle \nabla G_0(\vec{r},\vec{r}\,'), \nabla' \cdot \tilde{\vec{H}}(\vec{r}\,',s) \right\rangle$$

$$+ \left\langle \nabla G_0(\vec{r},\vec{r}\,') \underset{,}{\times} \overleftrightarrow{\sigma}(\vec{r}\,') \cdot \tilde{\vec{E}}(\vec{r}\,',s) \right\rangle$$

$$\tilde{\vec{E}}(\vec{r},s) = \tilde{\vec{E}}^{(inc)}(\vec{r},s) + \tilde{\vec{E}}^{(sc)}(\vec{r},s)$$

$$\tilde{\vec{H}}(\vec{r},s) = \tilde{\vec{H}}^{(inc)}(\vec{r},s) + \tilde{\vec{H}}^{(sc)}(\vec{r},s)$$

$$G_0(\vec{r},\vec{r}\,') = \frac{1}{4\pi \mid \vec{r} - \vec{r}\,' \mid} \quad \text{(transit times in external medium over target negligible)} \tag{2.1}$$

In terms of a supervector field (six components) (applying to total, incident, and scattered) we have

$$\left(\tilde{\vec{F}}_\ell(\vec{r},s) \right) = \left(\tilde{\vec{H}}(\vec{r},s) \;,\; N_e \tilde{\vec{E}}(\vec{r},s) \right)$$

$$\ell = \begin{cases} 1 \Rightarrow \text{ magnetic field} \\ 2 \Rightarrow \text{ normalized electric field} \end{cases} \tag{2.2}$$

$$N_e \equiv \text{normalizing scalar (e.g., } [s\mu a]^{-1})$$

The volume integral equations then take the form

$$\left(\tilde{\vec{F}}_\ell^{(sc)}(\vec{r},s) \right) = \left\langle \overleftrightarrow{\tilde{X}}_{\ell,\ell'}(\bullet) \underset{,}{\odot} \left(\tilde{\vec{F}}_\ell(\vec{r}\,',s) \right) \right\rangle \tag{2.3}$$

by appropriate manipulation of the terms in (2.1).

Setting the incident field to zero we have

$$\left\langle \tilde{\overleftrightarrow{X}}_{\ell,\ell'}(\cdot) - \overleftrightarrow{1}\,\delta(\overrightarrow{r}-\overrightarrow{r}')\,\underset{,}{\odot} \left(\overrightarrow{F}_{\ell}^{(sc)}(\overrightarrow{r}') \right)_{\alpha} \right\rangle = (\overrightarrow{0}_{\ell}) \tag{2.4}$$

as an equation for natural modes and natural frequencies s_α, by an appropriate numerical matricization of the continuous functions and operators. As shown in [2] we have

$$\begin{aligned} & s_\alpha < 0 \quad \text{(real and negative)} \\ & \left(\overrightarrow{F}_{\ell}^{(sc)}(\overrightarrow{r}) \right)_{\alpha} = \text{real vector (natural modes)} \end{aligned} \tag{2.5}$$

Furthermore, the coupling coefficients are also real and the poles are all first order. The coupling coefficients can be calculated from the formulas in [5] to complete the poles in the singularity-expansion-method (SEM) poles.

From the above natural-mode supervector we have

$$\begin{aligned} \overrightarrow{j}_{\alpha}(\overrightarrow{r}) &= \overleftrightarrow{\sigma}(\overrightarrow{r}) \cdot \overrightarrow{E}_{\alpha}^{(sc)}(\overrightarrow{r}) = N_{e_\alpha}^{-1}\,\overleftrightarrow{\sigma}(\overrightarrow{r}) \cdot \overrightarrow{F}_{2,\alpha}^{(sc)}(\overrightarrow{r}) \\ &= s_\alpha \left[\overleftrightarrow{\mu}(\overrightarrow{r}) - \mu_0 \overleftrightarrow{1} \right] \cdot \overrightarrow{F}_{1,\alpha}^{(sc)}(\overrightarrow{r}) \\ &\equiv \text{(electric-) current-density natural modes} \\ \overrightarrow{j}_{h_\alpha}(\overrightarrow{r}) &= s_\alpha \left[\overleftrightarrow{\mu}(\overrightarrow{r}) - \mu_0 \overleftrightarrow{1} \right] \cdot \overrightarrow{H}_{\alpha}^{(sc)}(\overrightarrow{r}) \\ &= s_\alpha \left[\overleftrightarrow{\mu}(\overrightarrow{r}) - \mu_0 \overleftrightarrow{1} \right] \cdot \overrightarrow{F}_{1,\alpha}^{(sc)}(\overrightarrow{r}) \\ &\equiv \text{magnetic-current-density natural modes} \end{aligned} \tag{2.6}$$

Note that through the normalization N_{e_α} (N_e evaluated at $s = s_\alpha$) the electric and magnetic modes are linked together, so that there is only one arbitrary coefficient determined in solving for $\left(\overrightarrow{F}_{\ell}^{(sc)}(\overrightarrow{r}) \right)_{\alpha}$.

The magnetic-dipole moments take the form

$$\vec{\tilde{m}}(s) = \vec{\tilde{m}}_e(s) + \vec{\tilde{m}}_h(s)$$

$$\vec{\tilde{m}}_e(s) = \frac{1}{2}\int_V \vec{r}\,' \times \vec{\tilde{J}}_e(\vec{r}\,',s)dV' = \frac{1}{2}\int_V \vec{r}\,' \times \left[\overleftrightarrow{\sigma}(\vec{r}\,') \cdot \vec{\tilde{E}}(\vec{r}\,',s) \right] dV' \tag{2.7}$$

$$\vec{\tilde{m}}_h(s) = \frac{1}{s\mu_0}\int_V \vec{\tilde{J}}_h(\vec{r}\,',s)\,dV' = \int_V \left[\frac{\overleftrightarrow{\mu}(\vec{r}\,')}{\mu_0} - \overleftrightarrow{1} \right] \cdot \vec{\tilde{H}}(\vec{r}\,',s)dV'$$

In terms of natural modes these become

$$\vec{m}_\alpha = \vec{m}_{e_\alpha} + \vec{m}_{h_\alpha}$$

$$\vec{m}_{e_\alpha} = \frac{1}{2}\int_V \vec{r}\,' \times \vec{j}_\alpha(\vec{r}\,')dV' \,, \quad \vec{m}_{h_\alpha} = \frac{1}{s_\alpha \mu_0}\int_V \vec{j}_{h_\alpha}(\vec{r}\,')dV' \tag{2.8}$$

From these the poles in the SEM form of the magnetic-polarizability dyadic can be calculated as in [5].

The quasi-static scattered magnetic field has the magnetic-dipole part given by

$$\vec{\tilde{H}}^{(d)}(\vec{r},s) = \frac{1}{4\pi r^3}\left[3\,\vec{1}_r\,\vec{1}_r - \overleftrightarrow{1} \right] \cdot \overleftrightarrow{\tilde{M}}(s) \cdot \vec{\tilde{H}}^{(inc)}(\vec{0},s)$$

$$\overleftrightarrow{\tilde{M}}(s) \equiv \text{magnetic-polarizability dyadic} \tag{2.9}$$

Where the coordinate origin $(\vec{r} = \vec{0})$ is centered in the target. (Later we consider optimal choices for this.) This dyadic takes the SEM form [2]

$$\overleftrightarrow{\tilde{M}}(s) = \overleftrightarrow{\tilde{M}}(\infty) + \sum_\alpha M_\alpha \vec{M}_\alpha \vec{M}_\alpha \left[s - s_\alpha \right]^{-1}$$

$$\frac{1}{s}\overleftrightarrow{\tilde{M}}(s) = \frac{1}{s}\overleftrightarrow{\tilde{M}}(0) + \sum_\alpha \frac{M_\alpha}{s_\alpha} \vec{M}_\alpha \vec{M}_\alpha \left[s - s_\alpha \right]^{-1}$$

$$\overleftrightarrow{M}(t) = \tilde{\overleftrightarrow{M}}(\infty)\,\delta(t) + \sum_{\alpha} M_\alpha \vec{M}_a \vec{M}_\alpha e^{s_\alpha t} u(t)$$

$$\int_{-\infty}^{t} \overleftrightarrow{M}(t')dt' = \left[\tilde{\overleftrightarrow{M}}(0) + \sum_{\alpha} \frac{M_\alpha}{s_\alpha} \vec{M}_a \vec{M}_\alpha e^{s_\alpha t} \right] u(t) \tag{2.10}$$

where all the constant scalars, vectors, and dyadics (symmetric) are real.

3. Magnetic Multipoles

Begin with our formula for the scattered magnetic field

$$\begin{aligned}\vec{\tilde{H}}^{(sc)}(\vec{r},s) &= \frac{1}{s\mu_0}\left\langle \nabla G_0(\vec{r},\vec{r}\,') \ , \ \nabla' \cdot \vec{\tilde{J}}_h(\vec{r}\,',s)\right\rangle \\ &\quad + \left\langle \nabla G_0(\vec{r},\vec{r}\,') \underset{;}{\times} \vec{\tilde{J}}_e(\vec{r}\,',s)\right\rangle \\ \vec{\tilde{J}}_e(\vec{r},s) &= \overleftrightarrow{\sigma}(\vec{r}) \cdot \vec{\tilde{E}}(\vec{r},s) \\ \vec{\tilde{J}}_h(\vec{r},s) &= s\left[\overleftrightarrow{\mu}(\vec{r}) - \mu_0 \overleftrightarrow{1}\right] \cdot \vec{\tilde{H}}(\vec{r},s)\end{aligned} \tag{3.1}$$

We need to expand this in negative powers of r.

For the gradient of the Green function we have, following [1]

$$\begin{aligned}\vec{g} &= \nabla\left[\frac{1}{4\pi|\vec{r}-\vec{r}\,'|}\right] = -\frac{\vec{r}-\vec{r}\,'}{4\pi|\vec{r}-\vec{r}\,'|^3} \\ &= \frac{1}{4\pi}\left[-\frac{\vec{1}_r}{r^2} + \frac{\vec{r}\,'}{r^3}\right]\left[1 - \frac{2}{r}\vec{1}_r \cdot \vec{r}\,' + \frac{r'^2}{r^2}\right]^{-\frac{3}{2}} \\ &= \frac{1}{4\pi}\left[-\frac{\vec{1}_r}{r^2} + \frac{\vec{r}\,'}{r^3}\right]\sum_{\ell=0}^{\infty}\binom{-\frac{3}{2}}{\ell}\left[-\frac{2}{r}\vec{1}_r \cdot \vec{r}\,' + \frac{r'}{r}\right]^{\ell}\end{aligned} \tag{3.2}$$

$$r = |\vec{r}\,| \ , \quad r' = |\vec{r}\,'|$$

$$\binom{p}{q} = \frac{\Gamma(p+1)}{\Gamma(q+1)\Gamma(p-q+1)} = \frac{p!}{q![p-q]!}$$

Collecting terms we can write

$$\vec{g} = \sum_{n=2}^{\infty}\vec{g}_n r^{-n}$$

$$\vec{g}_2 = -\frac{1}{4\pi}\vec{1}_r \quad \text{(monopole term, gives zero)}$$

$$\vec{g}_3 = \frac{1}{4\pi}\left[-3\,\vec{1}_r\,\vec{1}_r + \overleftrightarrow{1}\right]\cdot\vec{r}' \quad \text{(dipole term)}$$

$$\vec{g}_4 = \frac{1}{4\pi}\left[\frac{3}{2}r'^2 - \frac{15}{2}\left[\vec{1}_r\cdot\vec{r}'\right]^2\right]\vec{1}_r + 3\left[\vec{1}_r\cdot\vec{r}'\right]\vec{r}' \quad \text{(quadrupole term)} \tag{3.3}$$

Substituting these terms back in (3.1) gives us formulas for both dipole and quadrupole terms, the monopole term giving zero for the integrals.

4. Extended Target as an Ensemble of Elementary Magnetic Dipoles

An alternate formulation regards the electric and magnetic current densities in terms of a distribution of elementary magnetic dipoles and integrates over this with the dipole formula to obtain the scattered fields. For this purpose we have the magnetization (or magnetic-moment density) (see, e.g., [11])

$$\begin{aligned}\tilde{\vec{\chi}}(\vec{r}\,',s) &= \tilde{\vec{\chi}}_e(\vec{r}\,',s) + \tilde{\vec{\chi}}_h(\vec{r}\,',s) \\ \tilde{\vec{\chi}}_e(\vec{r}\,',s) &= \frac{1}{2}\vec{r}\,'\times\tilde{\vec{J}}_e(\vec{r}\,',s) = \frac{1}{2}\vec{r}\,'\times\left[\overset{\leftrightarrow}{\sigma}(\vec{r}\,')\cdot\tilde{\vec{E}}(\vec{r}\,',s)\right] \\ \tilde{\vec{\chi}}_h(\vec{r}\,',s) &= \frac{1}{2}\tilde{\vec{J}}_h(\vec{r}\,',s) = \left[\overset{\leftrightarrow}{\mu}-\mu_0\overset{\leftrightarrow}{1}\right]\cdot\tilde{\vec{H}}(\vec{r}\,',s)\end{aligned} \tag{4.1}$$

Here we list the spatial location as $\vec{r}\,'$, the coordinate of an elementary magnetic dipole. A magnetic dipole $\tilde{\vec{m}}(s)$ at this position has a magnetic field at $\vec{r}$ given by [2]

$$\tilde{\vec{H}}(\vec{r},s) = \frac{1}{4\pi}\left[3\frac{[\vec{r}-\vec{r}\,'][\vec{r}-\vec{r}\,']}{|\vec{r}-\vec{r}\,'|^5} - \frac{\overset{\leftrightarrow}{1}}{|\vec{r}-\vec{r}\,'|^3}\right]\cdot\tilde{\vec{m}}(s) \tag{4.2}$$

Then we have a form for the scattered magnetic field

$$\tilde{\vec{H}}^{(sc)}(\vec{r},s) = \left\langle\frac{1}{4\pi}\left[3\frac{[\vec{r}-\vec{r}\,'][\vec{r}-\vec{r}\,']}{|\vec{r}-\vec{r}\,'|^5} - \frac{\overset{\leftrightarrow}{1}}{|\vec{r}-\vec{r}\,'|^3}\right]\,;\,\tilde{\vec{\chi}}(\vec{r}\,s)\right\rangle \tag{4.3}$$

as an alternate formulation as compared to (3.1).

As in the previous section, let us expand the present form of Green function in inverse powers of r, with r^{-3} as the leading term. This will give the quadrupole term as the next term in the expansion. We have

$$|\vec{r}-\vec{r}'|^{3} = r^{-3}\left|\vec{1}_r - \frac{\vec{r}'}{r}\right|^{-3}$$

$$\left|\vec{1}_r - \frac{\vec{r}'}{r}\right|^{-3} = \left[1 - \frac{2\,\vec{1}_r \bullet \vec{r}'}{r} + \frac{r'^2}{r^2}\right]^{-\frac{3}{2}}$$

$$= 1 + 3\frac{\vec{1}_r \bullet \vec{r}'}{r} + O(r^{-2})$$

$$|\vec{r}-\vec{r}'|^{-5} = r^{-5}\left|\vec{1}_r - \frac{\vec{r}'}{r}\right|^{-5}$$

$$\left|\vec{1}_r - \frac{\vec{r}'}{r}\right|^{-5} = \left[1 - 2\frac{\vec{1}_r \bullet \vec{r}'}{r} + \frac{r'^2}{r^2}\right]^{-\frac{5}{2}}$$

$$= 1 + 5\frac{\vec{1}_r \bullet \vec{r}'}{r} + O(r^{-2}) \tag{4.4}$$

$$[\vec{r}-\vec{r}'][\vec{r}-\vec{r}'] = r^2\left[\vec{1}_r - \frac{\vec{r}'}{r}\right]\left[\vec{1}_r - \frac{\vec{r}'}{r}\right]$$

$$= r^2\left[\vec{1}_r\,\vec{1}_r - r^{-1}\left[\vec{1}_r\,\vec{r}' + \vec{r}'\,\vec{1}_r\right] + O(r^{-2})\right]$$

Substituting these in the basic expression we can write

$$\tilde{\vec{H}}^{(sc)}(\vec{r}, s) = \tilde{\vec{H}}^{(sc,d)}(\vec{r}, s) + \tilde{\vec{H}}^{(sc,q)}(\vec{r}, s) + O(r^{-5})$$

$$\tilde{\vec{H}}^{(sc,d)}(\vec{r}, s) = \vec{h}^{(d)}(\vec{r}) \bullet \int_V \tilde{\vec{\chi}}(\vec{r}, s)\, dV' \quad \text{(dipole term)}$$

$$\tilde{\vec{H}}^{(sc,q)}(\vec{r}, s) = \left\langle \overleftrightarrow{h}^{(q)}(\vec{r}, \vec{r}');\; \tilde{\vec{\chi}}(\vec{r}, s)\right\rangle \quad \text{(quadrupole term)}$$

$$\overleftrightarrow{h}^{(d)}(\vec{r}) = \frac{1}{4\pi r^3}\left[3\,\vec{1}_r\,\vec{1}_r - \overleftrightarrow{1}\right]$$

$$\overleftrightarrow{h}^{(q)}(\overrightarrow{r},\overrightarrow{r}') = \frac{3}{4\pi r^4}\left[\left[\overrightarrow{1}_r \cdot \overrightarrow{r}'\right]\left[5\,\overrightarrow{1}_r\,\overrightarrow{1}_r - \overleftrightarrow{1}\right] - \overrightarrow{1}_r\,\overrightarrow{r}' - \overrightarrow{r}'\,\overrightarrow{1}_r\right] \tag{4.5}$$

In this form we have the quadrupole term as a single integral. The magnetization can be viewed as a continuous distribution of magnetic dipoles, and can also be applied to sets of discrete magnetic dipoles. Our choice of optimum coordinate center $\overrightarrow{r} = \overrightarrow{0}$ is that which in some sense minimizes the quadrupole term. For the natural modes the effective center may possibly vary from mode to mode.

5. Two Displaced Magnetic Dipoles

Consider now a simple example of an extended source or scatter by taking two equal (parallel) dipoles as

$$\begin{aligned} \vec{m}_1(s) &= \frac{1}{2}\vec{m}_0(s) \quad \text{at} \quad \vec{r}\,'_0 \\ \vec{m}_2(s) &= \frac{1}{2}\vec{m}_0(s) \quad \text{at} \quad -\vec{r}\,'_0 \end{aligned} \tag{5.1}$$

The magnetic dipole is just

$$\vec{m}_0(s) = \vec{m}_1(s) + \vec{m}_2(s) \tag{5.2}$$

The quadrupole term is

$$\begin{aligned} \vec{H}^{(sc,q)}(\vec{r},s) &= \frac{3}{8\pi r^4}\left[\left[\vec{1}_r \cdot \vec{r}\,'_0\right]\left[5\,\vec{1}_r\vec{1}_r - \overleftrightarrow{1}\right] - \vec{1}_r\vec{r}\,'_0 - \vec{r}\,'_0\vec{1}_r\right]\cdot\vec{m}_0(s) \\ &\quad + \left[\left[\vec{1}_r \cdot [-\vec{r}\,'_0]\right]\left[5\,\vec{1}_r\vec{1}_r - \overleftrightarrow{1} - \vec{1}_r[-\vec{r}\,'_0] - [-\vec{r}\,'_0]\vec{1}_r\right]\cdot\vec{m}_0(s)\right] \\ &= \vec{0} \end{aligned} \tag{5.3}$$

The quadrupole term being zero, we have found that the optimal choice of coordinate origin between two identical dipoles is half way in between. Looking at the above result we can see that for parallel dipoles of unequal strength, but related by a real, positive constant we can still make the quadrupole term zero by an appropriately located effective center at a weighted position on the straight line between the two.

A related example is for antiparallel dipoles as

$$\begin{aligned} \vec{m}_1(s) &= \frac{1}{2}\vec{m}_0(s) \quad \text{at} \quad \vec{r}\,'_0 \\ \vec{m}_2(s) &= -\frac{1}{2}\vec{m}_0(s) \quad \text{at} \quad -\vec{r}\,'_0 \end{aligned} \tag{5.4}$$

In this case the net dipole moment is zero giving

$$\overset{\rightarrow}{\tilde{H}}^{(sc,d)}(\vec{r},s) = \vec{0} \tag{5.5}$$

We can have natural modes with this symmetry for appropriately symmetrical targets, which then have no dipole terms appearing in the magnetic polarizability dyadic (2.10). There is now a quadrupole term.

$$\overset{\rightarrow}{\tilde{H}}^{(sc,q)}(\vec{r},s) = \frac{3}{4\pi r^4}\left[\left[\vec{1}_r \cdot \vec{r}\,'_0\right]\left[5\,\vec{1}_r\,\vec{1}_r - \overset{\leftrightarrow}{1}\right] - \vec{1}_r\,\vec{r}\,'_0 - \vec{r}\,'_0\,\vec{1}_r\right] \cdot \overset{\rightarrow}{\tilde{m}}_0(s) \tag{5.6}$$

This is now the leading term. Falling off more rapidly with r than a dipole term it is seen closer to the target. This is fundamentally related to our original MSI assumption $r >> a$ in (1.2). Now we are looking closer to the target.

6. Effect of Symmetry on Magnetic-Dipole Moments of Natural Modes

Many properties of an electromagnetic scatterer are associated with geometric symmetries of the object [3, 4, 7, 8, 12, 13]. In particular the natural modes have the target symmetries incorporated in them. See the references for a detailed discussion of symmetries in the targets (including $\overleftrightarrow{\sigma}$ and $\overleftrightarrow{\mu}$ in the present case) and the associated groups with invariances on transformation by the group elements.

This phenomenon can be illustrated for the case of one or more symmetry planes as in [3]. To illustrate this consider the case that the scatterer has a symmetry plane which we take as the $z = 0$ plane. Then we have the reflection group for which we have the dyadic representation

$$\begin{gathered} R_z = \left\{ \overleftrightarrow{1}, \overleftrightarrow{R}_z \right\} , \quad \overleftrightarrow{R}_z^2 = \overleftrightarrow{1} \\ \overleftrightarrow{R}_z = \overleftrightarrow{1} - 2 \vec{1}_z \vec{1}_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = \overleftrightarrow{1}_z - \vec{1}_z \vec{1}_z \end{gathered} \tag{6.1}$$

All electromagnetic parameters can be composed in the form

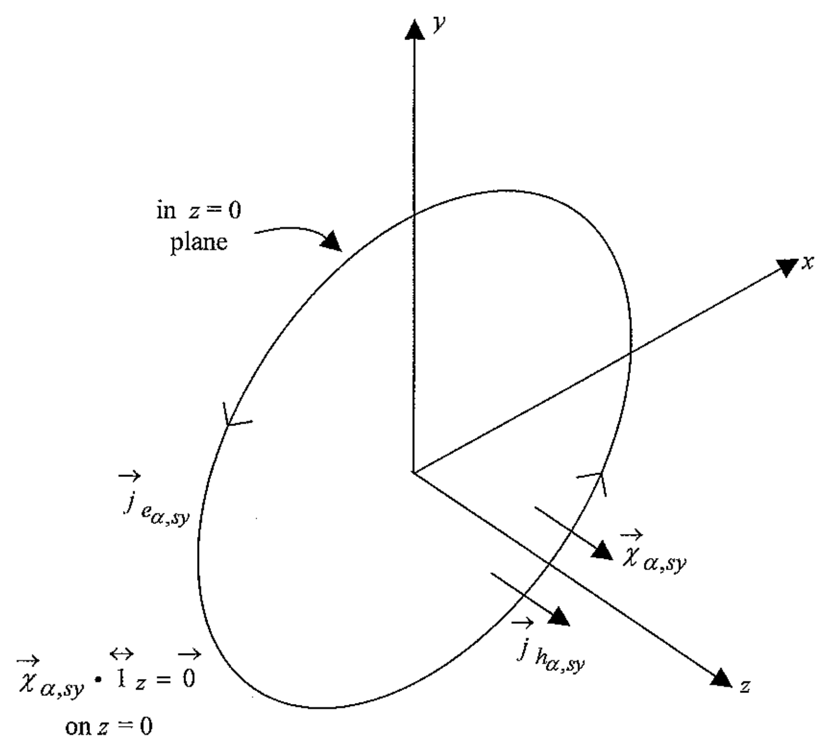
$$\begin{gathered} \vec{E}_{\substack{sy \\ as}}(\vec{r}^{(2)},t) = \pm \overleftrightarrow{R}_z \cdot \vec{E}_{\substack{sy \\ as}}(\vec{r}^{(1)},t) , \quad \vec{r}^{(2)} = \overleftrightarrow{R}_z \cdot \vec{r}^{(1)} \\ \vec{H}_{\substack{sy \\ as}}(\vec{r}^{(2)},t) = \mp \overleftrightarrow{R}_z \cdot \vec{H}_{\substack{sy \\ as}}(\vec{r}^{(1)},t) \end{gathered} \tag{6.2}$$

Remember that $\overleftrightarrow{\sigma}(\vec{r})$ and $\overleftrightarrow{\mu}(\vec{r})$ must be invariant to this transformation for this symmetry to apply. In this case the natural modes all separate according to the symmetric/antisymmetric decomposition.

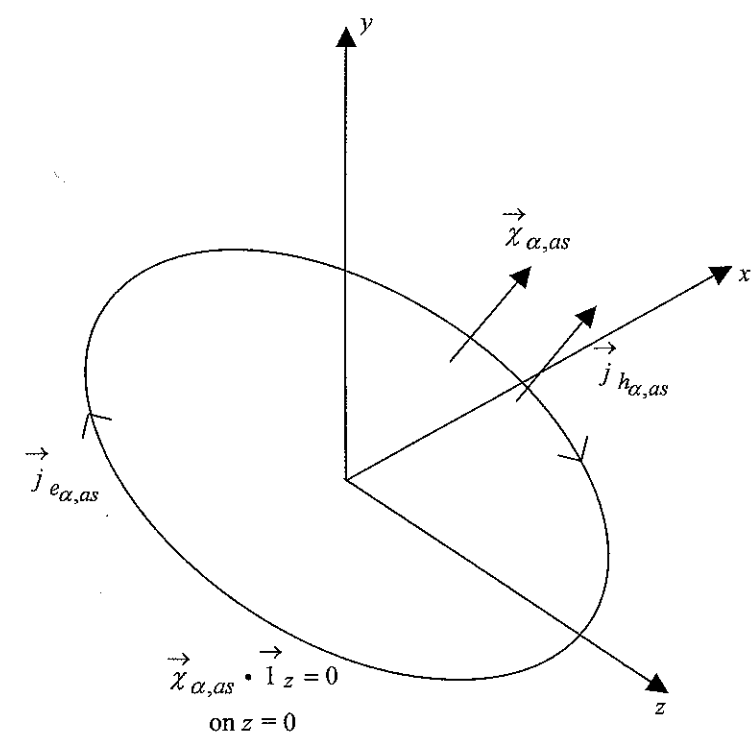
Referring to Fig. 6.1, let us consider the magnetization natural modes which we label $\vec{\chi}_{\alpha,sy}(\vec{r}\,')$ and $\vec{\chi}_{\alpha,as}(\vec{r}\,')$. These are magnetic parameters so that these have same symmetry properties as the magnetic field in (5.2), so that

$$\vec{\chi}_{\alpha,\substack{sy \\ as}}(\vec{r}^{(2)'}) = \mp \overleftrightarrow{R}_z \cdot \vec{\chi}_{\alpha,\substack{sy \\ as}}(\vec{r}^{(1)'}) , \quad \vec{r}^{(2)'} = \overleftrightarrow{R}_z \cdot \vec{r}^{(1)'} \tag{6.3}$$

These natural modes can be substituted in (4.5) to determine their dipole and quadrupole properties.



A. Symmetric magnetization



B. Antisymmetric magnetization

Fig. 6.1 Magnetization Natural Modes for Scatterer with a Symmetry Plane ($z = 0$)

Consider first the magnetic moment. For the symmetric mode this is

$$
\begin{aligned}
\vec{m}_{\alpha,sy} &= \int_{V_+} \vec{\chi}_{\alpha,sy}(\vec{r}\,')dV' + \int_{V_-} \vec{\chi}_{\alpha,sy}(\vec{r}\,')dV' \\
&= \int_{V_+} \left[\overset{\leftrightarrow}{1} - \overset{\leftrightarrow}{R}_z \right] \cdot \vec{\chi}_{\alpha,sy}(\vec{r}\,')dV' \\
&= 2\,\vec{1}_z \int_{V_+} \vec{1}_z \cdot \vec{\chi}_{\alpha,sy}(\vec{r}\,')dV' \\
&= m_{\alpha,sy}\,\vec{1}_z \quad \text{(a longitudinal term)} \qquad (6.4) \\
V &= V_+ \cup V_- \\
V_+ &\equiv \text{portion of } V \text{ for } z > 0 \\
V_- &\equiv \text{portion of } V \text{ for } z < 0
\end{aligned}
$$

which gives only a z component. For the antisymmetric mode this is

$$
\begin{aligned}
\vec{m}_{\alpha,as} &= \int_{V_+} \vec{\chi}_{\alpha,as}(\vec{r}\,')dV' + \int_{V_-} \vec{\chi}_{\alpha,as}(\vec{r}\,')dV' \\
&= \int_{V_+} \left[\overset{\leftrightarrow}{1} + \overset{\leftrightarrow}{R}_z \right] \cdot \vec{\chi}_{\alpha,as}(\vec{r}\,')dV' \qquad (6.5) \\
&= 2\,\overset{\leftrightarrow}{1}_z \cdot \int_{V_+} \vec{\chi}_{\alpha,as}(\vec{r}\,')dV' \quad \text{(a transverse term)}
\end{aligned}
$$

So the antisymmetric modes have no z component for the magnetic dipole moment. The above results from just one symmetry plane. Note also that $\vec{m}_{\alpha,sy}$ will be excited by only the symmetric part of the incident magnetic field, and $\vec{m}_{\alpha,as}$ only by the antisymmetric part.

This development can be extended to multiple symmetry planes. Consider, for example two symmetry planes: $x = 0$ and $y = 0$. Applying the above results consider the case that a mode is symmetric with respect to both R_x and R_y. By (6.4) then $\vec{m}_\alpha$ must be parallel to both $\vec{1}_x$ and $\vec{1}_y$, a contradiction. So $\vec{m}_\alpha$ cannot be symmetric to two perpendicular symmetry plane (and, by extension, three mutually perpendicular symmetry planes). However, we can have $\vec{m}_\alpha$ antisymmetric with respect to both R_x and R_y symmetries, noting that $\overset{\leftrightarrow}{1}_x$ and $\overset{\leftrightarrow}{1}_y$

(as in (6.5)) have the common element $\vec{1}_z \vec{1}_z$. In this case then $\vec{m}_\alpha$ (if nonzero) is symmetric with respect to R_z reflection, having a z component (as in (6.4)).

In the above there is the assumption that the various moment components are nonzero. If we allow components of $\vec{\chi}_\alpha$ to be nonzero while integrals over the appropriate components are zero, then such modes can exist with corresponding quadrupole (but not dipole) components. For example, consider that $\vec{\chi}_\alpha$ is antisymmetric with respect to all three symmetry planes. Then from (6.5) the dipole moment is zero. However, such as symmetry is also a special case of inversion symmetry

$$I = \left\{ \overleftrightarrow{1} , -\overleftrightarrow{1} \right\} \tag{6.6}$$

for which

$$\vec{\chi}_\alpha(-\vec{r}) = -\vec{\chi}_\alpha(\vec{r}) \ , \ \vec{m}_\alpha = \vec{0} \tag{6.7}$$

which is readily seen to integrate to zero for the dipole moment. Such a mode may generally have a quadrupole moment. Note that such a quadrupole mode is not excited by (not coupled to) a uniform incident magnetic field. This is seen by forming $\vec{\chi}_\alpha(\vec{r}) \cdot \vec{H}^{(inc)}(\vec{r},s)$ and integrating over V.

More general point symmetries are possible as discussed in [3, 8, 12]. For the magnetic-polarizability dyadic, and hence for the $\vec{\chi}_\alpha$ with nonzero magnetic moments, this is summarized in Table 6.1. The SEM form of $\overleftrightarrow{M}$ is given in (2.10).

A commonly encountered shape is a body of revolution (BOR) with axial symmetry planes giving $O_2 = C_{\infty a}$ symmetry. This type of scatterer is a special case of the above case of two symmetry planes. However, the added symmetry is convenient for understanding some of the properties of the natural modes and their dipole and quadrupole moments. In cylindrical (Ψ, ϕ, z) coordinate we let the z axis be the common axis for the infinite number of symmetry planes.

In cylindrical coordinates we then require

Table 6.1 Decomposition of Magnetic Polarizability Dyadic According to Target Point Symmetries.

Form of $\overset{\leftrightarrow}{\tilde{M}}(s)$	Symmety Types (Groups)	Symmetry Category
$\tilde{M}_z(s)\,\vec{1}_z\,\vec{1}_z + \overset{\leftrightarrow}{\tilde{M}}_t(s)$ $(\overset{\leftrightarrow}{\tilde{M}}_t(s)\cdot\vec{1}_z = \vec{0})$	R_z (single symmetry plane) C_2 (2-fold rotation axis)	1
$\tilde{M}_z(s)\,\vec{1}_z\,\vec{1}_z + \tilde{M}_x(s)\,\vec{1}_x\,\vec{1}_x + \tilde{M}_y(s)\,\vec{1}_y\,\vec{1}_y$	$C_{2a} = R_x \otimes R_y$ (two axial symmetry planes) D_2 (three 2-fold rotation axes)	2
$\tilde{M}_z(s)\,\vec{1}_z\,\vec{1}_z + \tilde{M}_t(s)\,\vec{1}_z$ $(\overset{\leftrightarrow}{1}_z = \overset{\leftrightarrow}{1} - \vec{1}_z\,\vec{1}_z \Rightarrow$ double degeneracy)	C_N for N ≥ 3 (N-fold rotation axis) S_N for N even and N ≥ 4 (N-fold rotation-reflection axis) D_{2d} (three 2-fold rotation axes plus diagonal symmetry planes)	3
$\tilde{M}(s)\,\overset{\leftrightarrow}{1}$ $(\overset{\leftrightarrow}{1} \Rightarrow$ triple degeneracy)	O_3 (generalized sphere) T, O, Y (regular polyhedra)	4

$$\begin{aligned}
\overset{\leftrightarrow}{\sigma}(\vec{r}) &= \overset{\leftrightarrow}{\sigma}^{(t)}(\Psi,z) + \sigma_{\phi,\phi}(\Psi,z)\,\vec{1}_\phi\,\vec{1}_\phi \\
\overset{\leftrightarrow}{\sigma}^{(t)}(\Psi,z)\cdot\vec{1}_\phi &= \vec{0} \\
\overset{\leftrightarrow}{\sigma}^{(t)}(\Psi,z) &= \overset{\leftrightarrow}{\sigma}^{(t)T}(\Psi,z) \\
&= \begin{pmatrix} \sigma_{\Psi,\Psi}(\Psi,z) & 0 & \sigma_{\Psi,z}(\Psi,z) \\ 0 & 0 & 0 \\ \sigma_{z,\Psi}(\Psi,z) & 0 & \sigma_{z,z}(\Psi,z) \end{pmatrix}
\end{aligned} \tag{6.8}$$

and similarly for $\overset{\leftrightarrow}{\mu}$. Here the form is clearly invariant to rotation about the z axis (varying ϕ). Full O_2 symmetry requires reflection planes containing the z axis which is given by the lack of Ψ,ϕ and ϕ,z components in the above. This can be compared to the form for O_3 symmetry in [3].

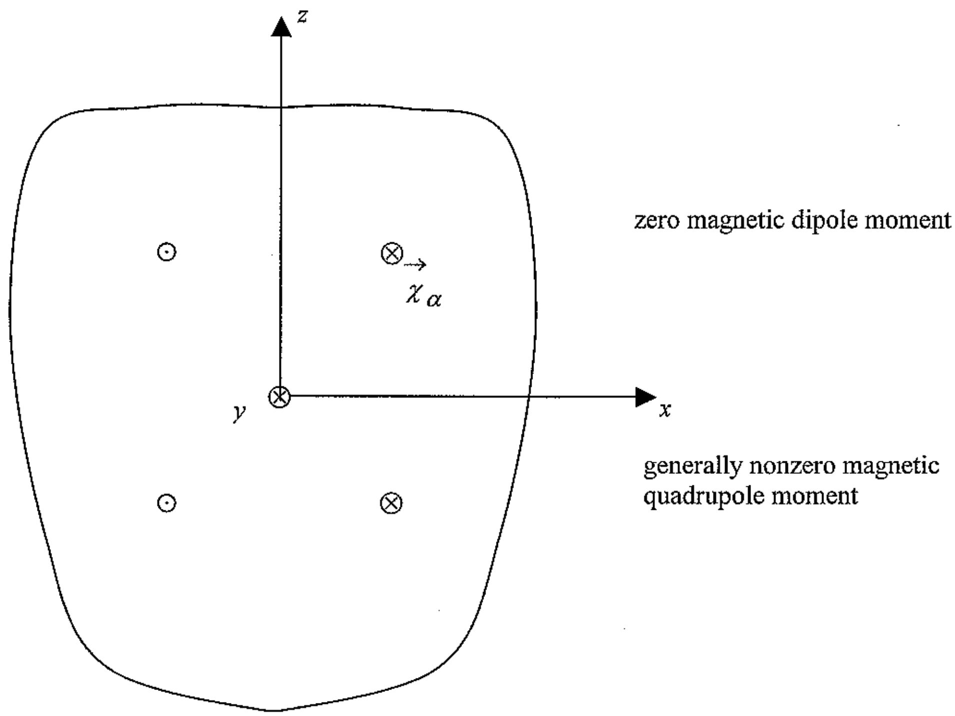
A BOR has the convenient property that the various eigenmodes and natural modes can be calculated by first expanding the scatterer response by separating out the ϕ dependence in the form of $\cos(m\phi)$ and $\sin(m\phi)$ where m = 0, 1, 2, ... (See, e.g., the discussion in [4].)

Consider the case of the $m = 0$ modes. These come in two kinds: E modes and H modes as indicated in Fig. 6.2. The E modes correspond to electric current parallel to the z axis, and magnetic current circulating around the z axis. These are associated with the suppressed electric-dipole moment. By pairing $\vec{\chi}_\alpha$ for ϕ and $\phi + \pi$, and noting the direction reversal, then the results of Section 5 give a net zero magnetic-dipole moment. However, there can be a nonzero magnetic-quadrupole moment. The H modes are more important and correspond to electric current circulating around the z axis with magnetic current parallel to this axis. There is a nonzero magnetic-dipole moment in this case. These latter magnetic-dipole moments are parallel to the z axis (longitudinal modes).

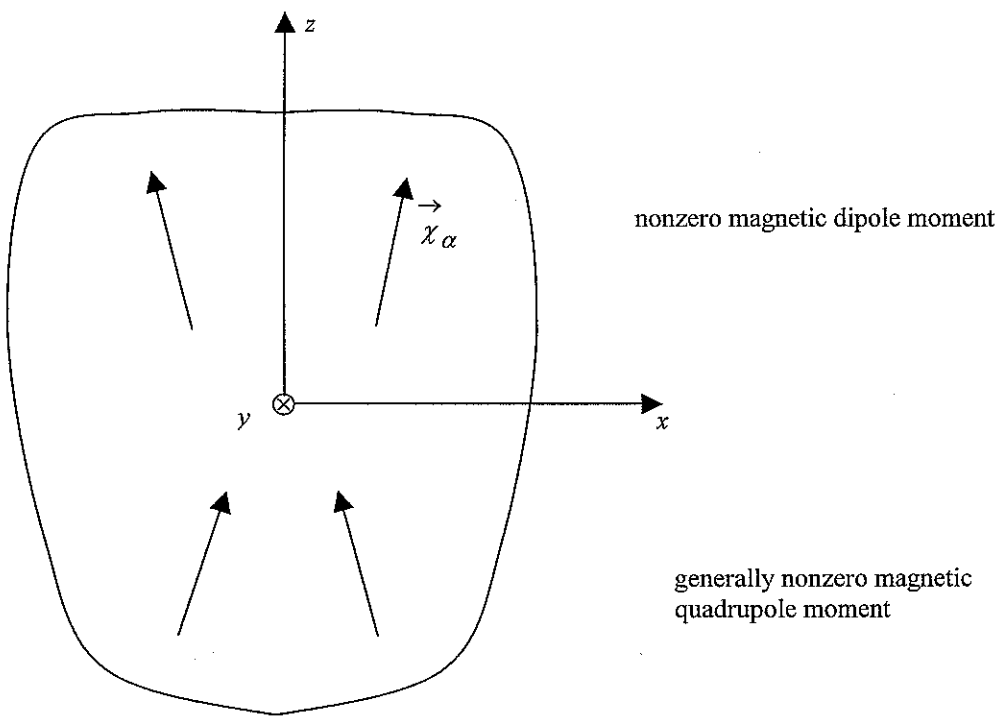
Increasing the symmetry, now add a transverse symmetry plane ($z = 0$), giving $O_{2t} = C_{\infty at}$ symmetry. The natural modes can now be divided into symmetric and antisymmetric modes with respect to this plane. Fig. 6.3 illustrates this for H modes with $m = 0$. As before the symmetric mode has a dipole moment. By pairing $\vec{\chi}_\alpha$ at $\vec{r}$ with that at $-\vec{r}$ we find from the discussion in Section 5 that the quadrupole term is zero. The antisymmetric mode has zero dipole moment, but a generally nonzero quadrupole term.

For a BOR the symmetry gives us an optimal coordinate origin on the z axis. However, there is still the question of where on the z axis. The examples in Fig. 6.2 illustrate this problem. The case of a dominant magnetic-dipole moment in Fig. 6.2B allows us to adjust the coordinate origin along the rotation axis to minimize the quadrupole term in some sense. However, this is not a simple scalar; the dyadic properties may not allow one to set this term to zero for all angles to a distant observer. Noting that there are in general many such m = 0 natural modes, the optimum choice of coordinate center may vary from mode to mode. This may be beneficial in orienting buried targets since magnetic dipoles with an assumed common "center" give an ambiguity ($\pm$) concerning the orientation of the rotation axis (i.e., which end is "up" [7]. For the case of a transverse symmetry plane the coordinate center is placed on the symmetry plane as well as the rotation axis, uniquely specifying it.)

Calculations and measurements of BORs for these low-frequency diffusion natural modes are found in [6, 9, 10]. In addition some of these are for m = 1 modes which give magnetic-dipole moments perpendicular to the z axis (transverse modes).

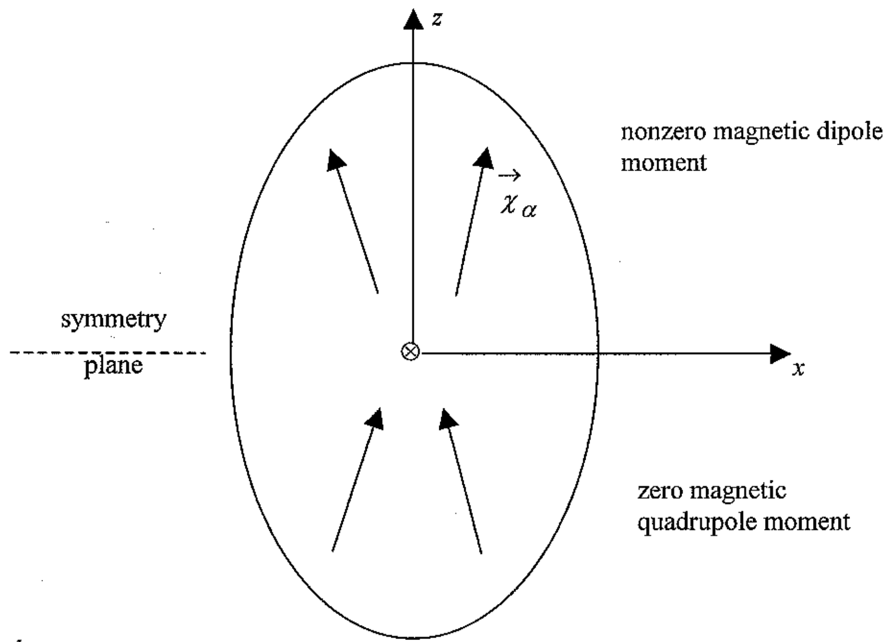


A. E modes: $\vec{J} \, // \, \vec{1}_z$ on z axis

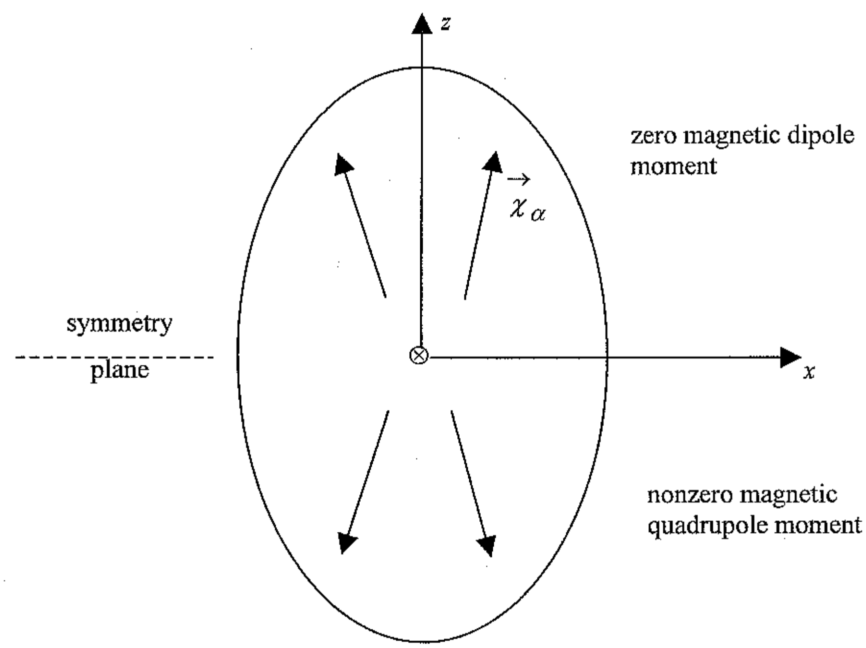


B. H modes: $\vec{J}$ circulating around z axis

Fig. 6.2 Natural Modes Exhibited on xz Plane for $O_2 = C_{\infty a}$ Scatterer: $m = 0$



A. Symmetric mode



B. Antisymmetric mode

Fig. 6.3 Natural Modes (H Modes) Exhibited on *xz* Plane for $O_{2t} = C_{\infty at}$ Scatterer: $m = 0$

7. Quadrupole Considerations

If one wishes to evaluate the quadrupole term, the kernal $\overset{\leftrightarrow}{h}^{(q)}$ in (4.5) can be used. However, this gives a more complicated angular variation and more complicated integral than the much simpler dipole term. In special cases of inversion symmetry where antiparallel $\vec{\chi}_{\alpha}$ pair up as in (5.6) the domain of integration is reduced. The O_{2t} symmetry for $m = 0$ is such a case as in (6.7). The rotation and reflection symmetries also give these symmetries to the associated scattered fields.

More generally, from (4.5) we have

$$\tilde{\vec{H}}_{\alpha}^{(sc,q)}(\vec{r}) = \left\langle \overset{\leftrightarrow}{h}^{(q)}(\vec{r},\vec{r}\,'); \vec{\chi}_{\alpha}(\vec{r}\,') \right\rangle$$
$$\overset{\leftrightarrow}{h}^{(q)}(\vec{r},\vec{r}\,') = \frac{3}{4\pi r^4}\left[\left[\vec{1}_r \cdot \vec{r}\,'\right]\left[5\,\vec{1}_r\,\vec{1}_r - \overset{\leftrightarrow}{1}\right] - \vec{1}_r\,\vec{r}\,' - \vec{r}\,'\,\vec{1}_r\right] \tag{7.1}$$

Now we would like to have some optimal choice of $\vec{r} = \vec{0}$ to minimize this term in some sense. Equivalently we can move the scatterer by some distance (vector) $\vec{r}_0$ giving

$$\tilde{\vec{H}}_{\alpha}^{(sc,q)}(\vec{r}) = \left\langle \overset{\leftrightarrow}{h}^{(q)}(\vec{r},\vec{r}\,'); \vec{\chi}_{\alpha}(\vec{r}\,' + \vec{r}_0) \right\rangle \tag{7.2}$$

Thus we need to evaluate

$$\left[5\,\vec{1}_r\,\vec{1}_r - \overset{\leftrightarrow}{1}\right] \cdot \int_V \left[\vec{1}_r \cdot \vec{r}\,'\right]\vec{\chi}_{\alpha}(\vec{r}\,' + \vec{r}_0)dV'$$
$$= \left[5\,\vec{1}_r\,\vec{1}_r - \overset{\leftrightarrow}{1}\right] \cdot \int_V \vec{1}_r \cdot \left[\vec{r}\,' - \vec{r}_0\right]\vec{\chi}_{\alpha}(\vec{r}\,')dV'$$
$$\vec{1}_r \int_V \vec{r}\,' \cdot \vec{\chi}_{\alpha}(\vec{r}\,' + \vec{r}_0)dV' = \vec{1}_r \int_V \left[\vec{r}\,' - \vec{r}_0\right] \cdot \vec{\chi}_{\alpha}(\vec{r}\,')dV' \tag{7.3}$$
$$\int_V \vec{r}\,'\,\vec{1}_r \cdot \vec{\chi}_{\alpha}(\vec{r}\,' + \vec{r}_0)dV' = \vec{1}_r \cdot \int_V \vec{\chi}_{\alpha}(\vec{r}\,')\left[\vec{r}\,' - \vec{r}_0\right]dV'$$

Neglecting coefficients (not functions of $\vec{r}\,'$, $\vec{r}_0$) we need to consider

$$\overleftrightarrow{Q}_1 = \int_V \left[\vec{r}\,' - \vec{r}_0\right] \vec{\chi}_\alpha(\vec{r}\,')dV' = \left[\int_V \vec{\chi}_\alpha(\vec{r}\,')\left[\vec{r}\,' - \vec{r}_0\right]dV'\right]^T$$

$$Q_2 = \int_V \left[\vec{r}\,' - \vec{r}_0\right] \cdot \vec{\chi}_\alpha(\vec{r}\,')dV' \tag{7.4}$$

Consider first the scalar term. Setting this to zero we have

$$Q_2 = 0$$

$$\vec{r}_0 \cdot \int_V \vec{\chi}_\alpha(\vec{r}\,')dV' = \vec{r}_0 \cdot \vec{m}_\alpha = \int_V \vec{r}\,' \cdot \vec{\chi}_\alpha(\vec{r}\,')dV' \tag{7.5}$$

Provided that the dipole term is nonzero, this is an equation for $\vec{r}_0$ which can generally be satisfied. If, however, $\vec{m}_\alpha = \vec{0}$, then this requires the quadrupole term (integral of $\vec{r}\,' \cdot \vec{\chi}_\alpha$) to be zero, not generally possible. So this gives one possible choice of $\vec{r}_0$ for a *displaced* magnetic dipole. Note that $\vec{r}_0$ having three components this gives a solution for $\vec{r}_0$ anywhere on a plane perpendicular to $\vec{\chi}_\alpha$ (conveniently real valued). For a BOR the center needs to be on the z axis giving a unique solution for

$$\vec{r}_0 = \left(x_0, y_0, z_0\right) = \left(0, 0, z_0\right) \tag{7.6}$$

provided $\vec{m}_\alpha$ is parallel to $\vec{1}_z$ ($m = 0$, H mode).

Setting the dyadic term to zero gives

$$\overleftrightarrow{Q}_1 = \overleftrightarrow{0}$$

$$\vec{r}_0 \int_V \vec{\chi}_\alpha(\vec{r}\,')dV' = \vec{r}_0\, \vec{m}_\alpha = \int_V \vec{r}\,'\, \vec{\chi}_\alpha(\vec{r}\,')dV' \tag{7.7}$$

Having in general nine components this condition is much harder to meet. With $\vec{m}_\alpha$ given (real valued) we have three coordinates of $\vec{r}_0$ to vary to match nine conditions. This happens in only special cases, not general $\vec{\chi}_\alpha$.

For a BOR with axial symmetry planes ($O_2 = C_{\infty a}$ symmetry) the natural mode takes the form for m = 0

$$\begin{aligned}
\vec{\chi}_\alpha(\vec{r}\,') &= \chi_{\Psi'_\alpha}(\Psi', z')\,\vec{1}_{\Psi'} + \chi_{z'_\alpha}(\Psi', z')\,\vec{1}_z \\
\vec{r}\,' &= \Psi'\,\vec{1}_{\Psi'} + z'\,\vec{1}_{z'} \\
\vec{1}_{\Psi'} &= \cos(\phi')\,\vec{1}_{x'} + \sin(\phi')\,\vec{1}_{y'}
\end{aligned} \tag{7.8}$$

The dyadic term then becomes

$$\begin{aligned}
\int_V \vec{r}\,'\vec{\chi}_\alpha(\vec{r}\,')dV' = \int_V \Big[& \Psi'\chi_{\Psi'}\,\vec{1}_{\Psi'}\,\vec{1}_{\Psi'} + z'\chi_{\Psi_\alpha}\,\vec{1}_{z'}\,\vec{1}_{\Psi'} \\
& + \Psi'\chi_{z'_\alpha}\,\vec{1}_{\Psi'}\,\vec{1}_{z'} + z'\chi_{z'_\alpha}\,\vec{1}_{z'}\,\vec{1}_{z'} \Big] dV' \, .
\end{aligned} \tag{7.9}$$

Noting that on integrating over ϕ' we have

$$\begin{aligned}
&\int_0^{2\pi} \vec{1}_{\Psi'} d\phi' = \vec{0} \;, \quad \int_0^{2\pi} d\phi' = 2\pi \\
&\int_0^{2\pi} \vec{1}_{\Psi'}\,\vec{1}_{\Psi'} d\phi' = \pi\left[\vec{1}_{x'}\,\vec{1}_{x'} + \vec{1}_{y'}\,\vec{1}_{y'}\right] = \pi\,\overleftrightarrow{1}_{z'} \\
&\hphantom{\int_0^{2\pi} \vec{1}_{\Psi'}\,\vec{1}_{\Psi'} d\phi' = \pi\left[\vec{1}_{x'}\,\vec{1}_{x'} + \vec{1}_{y'}\,\vec{1}_{y'}\right]} = \pi\,\overleftrightarrow{1}_z \\
&\vec{1}_{z'}\,\vec{1}_{z'} = \vec{1}_z\,\vec{1}_z \\
&dV' = \Psi'\,d\Psi'\,dz'\,d\phi
\end{aligned} \tag{7.10}$$

The dyadic term then becomes

$$\begin{aligned}
\int_V \vec{r}\,'\vec{\chi}_\alpha(\vec{r}\,')dV' &= \int_V \left[\Psi'\chi_{\Psi'_\alpha}\,\vec{1}_{\Psi'}\,\vec{1}_{\Psi'} + z'\chi_{z'_\alpha}\,\vec{1}_{z'}\,\vec{1}_{z'}\right]dV' \\
&= \pi\,\overleftrightarrow{1}_{z'} \int_{\Psi',z'} \Psi'^2\chi_{\Psi'_\alpha}(\Psi', z')d\Psi' dz' \\
&\quad + 2\pi\,\vec{1}_z\,\vec{1}_z \int_{\Psi',z'} \Psi' z'\chi_{z'_\alpha}(\Psi', z')d\Psi' dz'
\end{aligned} \tag{7.11}$$

with remaining integration over the intersection of a plane of constant ϕ' ($\Psi' \geq 0$) with V. For choice of optimum coordinate origin on the z axis we can null the second integral as

$$0 = \int\limits_{\Psi',z'} \Psi'[z' - z_0]\chi_{z'_\alpha}(\Psi', z')d\Psi' dz' \tag{7.12}$$

giving

$$z_0 \int\limits_{\Psi',z'} \Psi' \chi_{z'_\alpha}(\Psi', z')d\Psi' dz' = \int\limits_{\Psi',z'} \Psi' z' \chi_{z'_\alpha}(\Psi', z')d\Psi' dz' \tag{7.13}$$

which has a real-valued solution for z_0 provided the left integral is nonzero. However, the choice of z_0 has no effect on the integral proportional to $\overleftrightarrow{1}_z$ in (7.11).

Adding $z = 0$ as a symmetry plane, then the symmetric mode as in Fig. 6.3 (an H mode) has $\chi_{z'_\alpha}$ even in z' and $\chi_{\Psi'_\alpha}$ odd in z' making both integrals in (7.11) go to zero. This is consistent with the results in Section 6.

8. Concluding Remarks

In this paper the MSI theory is generalized to include magnetic quadrupole terms. As the incident-field source and/or the observer closely approach the scatterer (to dimensions related to the scatterer linear dimensions) such quadrupole terms can become significant.

Some quadrupole terms (especially for symmetrical scatterers) are not associated with magnetic dipoles. These can be minimized by making the incident magnetic field very uniform, thereby giving a zero residue to the s-plane pole. Alternately one can move the observer far enough away from the scatterer that the r^{-4} quadrupole terms are small compared to the r^{-3} dipole terms.

For quadrupole terms associated with dipole terms (same natural mode), these can be strongly excited, even by a uniform incident magnetic field. By appropriate choice of the coordinate origin, or equivalently choosing the effective center for the mode, one can minimize the quadrupole part, and in some cases (especially involving symmetry) make it go to zero. Note that each $\vec{m}_\alpha$ may have in general a different effective center. This may aid in orienting a target since the magnetic-polarizability dyadic (pure dipole with common effective center) is invariant to scatterer inversion and other transformations depending on the target symmetry.

Instead of minimizing the quadrupole contributions, one may wish in some cases to maximize them. This gives some additional information for target recognition, location, and orientation.

References

1. C. E. Baum, "Some Characteristics of Electric and Magnetic Dipole Antennas for Radiating Transient Pulses", Sensor and Simulation Note 125, January 1971.

2. C. E. Baum, "Low-Frequency Near-Field Magnetic Scattering from Highly, But Not Perfectly, Conducting Bodies", Interaction Note 499, November 1993; ch. 6, pp. 163-218, in C. E. Baum (ed.), *Detection and Identification of Visually Obscured Targets*, Taylor & Francis, 1998.

3. C. E. Baum, "The Magnetic Polarizability Dyadic and Point Symmetry", Interaction Note 502, May 1994; ch. 7, pp. 219-242, in C. E. Baum (ed.), *Detection and Identification of Visually Obscured Targets*, Taylor & Francis, 1998.

4. C. E. Baum, "Symmetry in Electromagnetic Scattering as a Target Discriminant", Interaction Ote 523, October 1996; pp. 295-307, in H. Mott and W. M. Boerner (eds.), Wideband Interferometric Sensing and Imaging Polarimetry, Proc. SPIE, Vol. 3120, 1997.

5. C. E. Baum, N. Geng, and L. Carin, "Integral Equations and Polarizability for Magnetic Singularity Identification", Interaction Note 524, March 1997.

6. L. S. Riggs, J. E. Mooney, D. E. Lawrence, J. T. Broach, and A. H. Trang, "On Identifying Conducting Objects using Low Frequency Magnetic Fields: Theory and Measurements", Interaction Note 542, July 1998.

7. C. E. Baum, "Application of Symmetry to Magnetic-Singularity Identification of Buried Targets", Interaction Note 543, June 1998.

8. C. E. Baum, "Use of Residue and Constant-Dyadic Information in Magnetic Singularity identification', Interaction Note 547, August 1998.

9. N. Geng, C. E. Baum, and L. Carin, "On the Low-Frequency Natural Response of Conducting and Permeable Targets", IEEE Trans. Geoscience and Remote Sensing, 1999, pp. 347-359.

10. L. Carin, H. Yu, Y Dalichaouch, and C. E. Baum, "On the Wideband EMI Response of a Rotationally Symmetric Permeable and Conducting Target", IEEE Trans. Geoscience and Remote Sensing, 2001 (in publication).

11. J. D. Jackson, *Classical Electrodynamics*, 2nd Ed., Wiley, 1975.

12. C. E. Baum and H. N. Kritikos, "Symmetry in Electromagnetics", ch. 1, pp. 1-90, in C. E. Baum and H. N. Kritikos (eds.), *Electromagnetic Symmetry*, Taylor & Francis, 1995.

13. C. E. Baum, "Target Symmetry and the Scattering Dyadic", ch. 4, pp. 204-236, in D. H. Werner and R. Mittra (eds.), *Frontiers in Electromagnetics*, IEEE Press, 2000.