Interaction Notes

Note 595

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Aspects of Random-Lay Multiconductor Cable Propagation Which Are Not Statistical

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Abstract

Many practical multiconductor transmission lines can be considered as random-lay cables in which the wires assume various positions in the transmission-line cross section. This paper considers some of the properties of propagation on such structures. In particular some aspects of such propagation are deterministic in nature. Such conclusions are based on low- and high-frequency considerations, and symmetry. Furthermore, reciprocity and energy conservation place restrictions on the allowable statistical distributions.

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1. Introduction

The interaction of electromagnetic fields with complex electrical and electronic systems gives us a very difficult or even intractable problem to fully analyze. Part of the problem lies in the very description of the system itself which can have various uncertainties (e.g., wire location, joint corrosion, etc.). Two copies of the same system type (aircraft model, ship model, etc.) may have significant differences in their responses. How, then, should one approach the interaction analysis in such cases?

One approach to this problem is statistical in nature. One makes certain parameters of the problem (position coordinates, impedance values, etc.) random variables, and from this attempts to find distribution functions for the electromagnetic response parameters (voltages, currents, fields). This has been applied with some success to cavities, including mode-stirred chambers and related system response [8, 12].

There has been some attempt at a statistical description for the response of random nonuniform multiconduction transmission lines (random NMTLs, or RNMTLs) [1, 4]. This is the subject of the present paper. As we shall see, there are certain aspects of this problem which are deterministic rather than statistical.

Here we consider the physics of the problem to obtain constraints on RNMTL response. After summarizing the general product-integral formulae, various constraints are imposed. These include reciprocity and conservation of energy. There are also special cases for low- and high-frequency response. Then some implications of geometrical symmetry are explored.

2. Telegrapher Equation and Product Integrals

An N-conductor (plus reference) NMTL has the telegrapher equations [7, 10]

$$\frac{d}{dz}\left(\tilde{V}_n(z,s)\right) = -\left(\tilde{Z}'_{n,m}(z,s)\right) \bullet \left(\tilde{I}_n(z,s)\right) = \left(\tilde{V}_n^{(s)'}(z,s)\right)$$
$$\frac{d}{dz}\left(\tilde{I}_n(z,s)\right) = -\left(\tilde{Y}'_{n,m}(z,s)\right) \bullet \left(\tilde{V}_n(z,s)\right) + \left(\tilde{I}_n^{(s)'}(z,s)\right)$$

$\left(\tilde{V}_n(z,s)\right) \equiv$ voltage vector

$\left(\tilde{I}_n(z,s)\right) \equiv$ current vector (positive current convention in direction of increasing z)

$\left(\tilde{V}_n^{(s)'}(z,s)\right) \equiv$ longitudinal voltage source per unit length

$\left(\tilde{I}_n^{(s)'}(z,s)\right) \equiv$ transverse current source per unit length

$z(\text{real}) \equiv$ position coordinate along the line

$\sim \;\equiv$ two-sided Laplace transform over time, t

$s = \Omega + j\omega \equiv$ Laplace-transform variable or complex frequency (2.1)

$\left(\tilde{Z}'_{n,m}(z,s)\right) = \left(\tilde{Z}'_{n,m}(z,s)\right)^T \equiv$ per-unit-length (series) impedance matrix (N x N)

$\left(\tilde{Y}'_{n,m}(z,s)\right) = \left(\tilde{Y}'_{n,m}(z,s)\right)^T \equiv$ per-unit-length (parallel) admittance matrix (N x N)

where reciprocity has been used. In solving these equations we construct [3]

$$\left(\tilde{\gamma}_{n,m}(z,s)\right) = \left[\left(\tilde{Z}'_{n,m}(z,s)\right) \bullet \left(\tilde{Y}'_{n,m}(z,s)\right)\right]^{1/2} \text{ (positive real (p.r.) square root)}$$
$$\equiv \text{propagation matrix}$$

$$\begin{aligned}\left(\tilde{Z}_{c_{n,m}}(z,s)\right) &= \left(\tilde{Z}_{c_{n,m}}(z,s)\right)^T \equiv \left(\tilde{Y}_{c_{n,m}}(z,s)\right)^{-1} \\ &= \left(\tilde{\gamma}_{n,m}(z,s)\right) \bullet \left(\tilde{Y}'_{n,m}(z,s)\right)^{-1} \\ &= \left(\tilde{\gamma}_{n,m}(z,s)\right)^{-1} \bullet \left(\tilde{Z}'_{n,m}(z,s)\right) \\ &\equiv \text{characteristic impedance matrix}\end{aligned} \tag{2.2}$$

These equations are solved by construction of a first-order supervector/supermatrix differential equation which can take various forms. One such form [7] is

$$\frac{d}{dz}\begin{pmatrix} \left(\tilde{V}_n(z,s)\right) \\ \left(\tilde{Z}_{n,m}(s)\right) \cdot \left(\tilde{I}_n(z,s)\right) \end{pmatrix}$$

$$= -\begin{pmatrix} \left(0_{n,m}\right) & \left(\tilde{Z}'_{n,m}(s)\right) \cdot \left(\tilde{Y}_{n,m}(s)\right) \\ \left(\tilde{Z}_{n,m}(s)\right) \cdot \left(\tilde{Y}'_{n,m}(z,s)\right) & \left(0_{n,m}\right) \end{pmatrix} \odot \begin{pmatrix} \left(\tilde{V}_n(z,s)\right) \\ \left(\tilde{Z}_{n,m}(s)\right) \cdot \left(\tilde{I}_n(z,s)\right) \end{pmatrix}$$

$$+ \begin{pmatrix} \left(\tilde{V}_n^{(s)'}(z,s)\right) \\ \left(\tilde{Z}_{n,m}(s)\right) \cdot \left(\tilde{I}_n^{(s)'}(z,s)\right) \end{pmatrix}$$

$$\left(\tilde{Z}_{n,m}(s)\right) = \left(\tilde{Z}_{n,m}(s)\right)^T = \left(\tilde{Y}_{n,m}(s)\right)$$

≡ normalizing impedance matrix chosen at our convenience (p.r., but not a function of z) (2.3)

This is solved via a supermatrizant differential equation

$$\frac{d}{dz}\left(\left(\tilde{U}_{n,m}(z,z_0;s)\right)_{\upsilon,\upsilon'}\right) = \left(\left(\tilde{\Gamma}_{n,m}(z,s)\right)_{\upsilon,\upsilon'}\right) \odot \left(\left(\tilde{U}_{n,m}(z,z_0;s)\right)_{\upsilon,\upsilon'}\right)$$

$$\left(\left(\tilde{\Gamma}_{n,m}(z,s)\right)_{\upsilon,\upsilon'}\right) = -\begin{pmatrix} \left(0_{n,m}\right) & \left(\tilde{Z}'_{n,m}(z,s) \cdot \left(\tilde{Y}_{n,m}(s)\right)\right) \\ \left(\tilde{Z}_{n,m}(s) \cdot \left(\tilde{Y}'_{n,m}(s)\right)\right) & \left(0_{n,m}\right) \end{pmatrix} \qquad (2.4)$$

$$\left(\left(\tilde{U}_{n,m}(z_0,z_0;s)\right)_{\upsilon,\upsilon'}\right) = \left(\left(1_{n,m}\right)_{\upsilon,\upsilon}\right) = \begin{pmatrix} \left(1_{n,m}\right) & \left(0_{n,m}\right) \\ \left(0_{n,m}\right) & \left(1_{n,m}\right) \end{pmatrix} \text{(boundary condition)}$$

which is solved as the product integral

$$\left(\left(\tilde{U}_{n,m}(z,z_0;s)\right)_{\upsilon,\upsilon'}\right) = \prod_{z_0}^{z} e^{\left(\left(\tilde{\Gamma}_{n,m}(z',s)\right)_{\upsilon,\upsilon'}\right)dz'} \qquad (2.5)$$

In terms of this the voltage and current are solved as

$$\begin{pmatrix} \left(\tilde{V}_n(z,s)\right) \\ \left(\tilde{Z}_{n,m}(s)\right) \cdot \left(\tilde{I}_n(z,s)\right) \end{pmatrix} = \left(\left(\tilde{U}_{n,m}(z,z_0;s)\right)_{\upsilon,\upsilon'}\right) \odot \begin{pmatrix} \left(\tilde{V}_n(z_0,s)\right) \\ \left(\tilde{Z}_{n,m}(s)\right) \cdot \left(\tilde{I}_n(z_0,s)\right) \end{pmatrix}$$

$$+\int_{z_0}^{z} \left(\left(\tilde{U}_{n,m}(z,z';s)\right)_{\upsilon,\upsilon'}\right) \odot \begin{pmatrix} \left(\tilde{V}_n^{(s)'}(z,s)\right) \\ \left(\tilde{Z}_{n,m}(s)\right) \cdot \left(\tilde{I}_n^{(s)'}(z,s)\right) \end{pmatrix} dz' \qquad (2.6)$$

There are various other forms of supermatrizant differential equations depending on how one normalizes the voltage and current variables (e.g., [10]). As one can see, it is the properties of the supermatrizant that one needs to understand, including from a statistical point of view. For simplicity we neglect the distributed sources in the present discussion, instead concentrating on the propagation along the NMTL.

3. Implications of Reciprocity and Conservation of Energy

In formulating a statistical description of the supermatrizant there are fundamental properties which constrain the forms of the distribution functions. For example, the real power at any frequency (for $s = j\omega$) going to a load must not exceed that launched into the NMTL. Among other things, this implies that the distribution functions should be bounded (i.e., not extend to infinite voltages/currents). From reciprocity, we know that impedance and admittance matrices must be symmetric, even when described statistically.

From a deterministic point of view, such properties have been considered in some detail in [9] for the super-matrizant form in (2.4). In [9 (section 2)) we find various symmetries in the matrizant blocks $(\tilde{U}_{n,m}(z, z_0; s))_{\upsilon,\upsilon'}$ which should be preserved for $s = j\omega$ in a statistical description. The special case of lossless NMTLs [9 (Section 5)] leads to special constraints on the matrizant blocks based on Foster's theorem for reactance functions. This is extended [9 (Section 6)] to show that the matrizant blocks are even or odd functions of s (=$j\omega$ in this case). Finally, some bounds based on 2-norms (related to real power) are established in [9 (Section 7)].

While the details are not repeated here, these constraints should guide the construction of a statistical description of RNMTLs.

4. Low-Frequency Solution

At low-frequencies we have

$$\left(\tilde{\gamma}_{n,m}(z,s)\right) = \left(0_{n,m}\right) \text{ as } s \to 0 \tag{4.1}$$

Provided $(\tilde{Y}'_{n,m}(z,s))$ is capacitive at low frequencies (dielectric medium) and/or $(\tilde{Z}'_{n,m}(z,s))$ is inductive at low frequencies (perfectly conducting wires). In this typical case we have

$$\begin{aligned} \left(\left(\tilde{\Gamma}_{n,m}(z,s)\right)_{\upsilon,\upsilon'}\right) &= \left(\left(0_{n,m}\right)_{\upsilon,\upsilon'}\right) \\ \left(\left(\tilde{U}_{n,m}(z,z_0;s)\right)_{\upsilon,\upsilon'}\right) &= \left(\left(1_{n,m}\right)_{\upsilon,\upsilon'}\right) \text{ as } s \to 0 \end{aligned} \tag{4.2}$$

provided both conditions above are met.

Thus, at low frequencies, the matrizant is independent of the variation of the wire positions, radii, etc., which might be functions of z. As such the RNMTL is deterministic at low frequencies. Statistical distributions for the matrizant blocks should go to this result as $j\omega \to 0$.

One can understand this result from simple circuit considerations. Essentially it says that voltages and currents are the same at both ends of the transmission line, a simple continuity condition.

5. High-Frequency Solution

Turning to high frequencies we have the results of [10]. For this purpose we have renormalized the voltage and current vectors as

$$
\begin{aligned}
\left(\tilde{\mathrm{v}}_n(z,s)\right) &\equiv \left(\tilde{y}_{c_{n,m}}(z,s)\right) \cdot \left(\tilde{V}_n(z,s)\right) \\
\left(\tilde{i}_n(z,s)\right) &\equiv \left(\tilde{z}_{c_{n,m}}(z,s)\right) \cdot \left(\tilde{I}_n(z,s)\right) \\
\left(\tilde{z}_{c_{n,m}}(z,s)\right) &\equiv \left(\tilde{Z}_{c_{n,m}}(z,s)\right)^{1/2} \text{ (p.r. square root)} \\
\left(\tilde{y}_{c_{n,m}}(z,s)\right) &\equiv \left(\tilde{Y}_{c_{n,m}}(z,s)\right)^{1/2} \text{ (p.r. square root)} \\
&= \left(\tilde{z}_{c_{n,m}}(z,s)\right)^{-1}
\end{aligned}
\tag{5.1}
$$

These are then combined as

$$
\left(\tilde{\mathrm{v}}_{s_n}(z,s)\right)_{\substack{1\\2}} = \left(\tilde{\mathrm{v}}_{n}(z,s)\right) \pm \left(\tilde{i}_{n}(z,s)\right) \tag{5.2}
$$

giving waves propagating in the $\pm$ directions.

For a leading high-frequency term we have

$$
\begin{aligned}
\left(\tilde{g}_{n,m}(z,s)\right) &= -\left(\tilde{y}_{c_{n,m}}(z,s)\right) \cdot \left(\tilde{\gamma}_{n,m}(z,s)\right) = \left(\tilde{z}_{c_{n,m}}(z,s)\right) = \left(\tilde{g}_{n,m}(z,s)\right)^T \\
\left(\left(\tilde{g}_{n,m}(z,s)\right)_{\upsilon,\upsilon'}\right) &= \begin{pmatrix} \left(\tilde{g}_{n,m}(z,s)\right) & \left(0_{n,m}\right) \\ \left(0_{n,m}\right) & -\left(\tilde{g}_{n,m}(z,s)\right) \end{pmatrix} = \left(\left(\tilde{g}_{n,m}(z,s)\right)_{\upsilon,\upsilon'}\right)^T \\
\left(\left(\tilde{G}_{n,m}(z,z_0;s)\right)_{\upsilon,\upsilon'}\right) &= \prod_{z_0}^{z} e^{\left(\left(g_{n,m}(z',s)\right)_{\upsilon,\upsilon'}\right)dz'} \\
&= \left[\prod_{z_0}^{z} e^{\left(\tilde{g}_{n,m}(z',s)\right)dz'}\right] \oplus \left[\prod_{z_0}^{z} e^{-\left(\tilde{g}_{n,m}(z',s)\right)dz'}\right]
\end{aligned}
\tag{5.3}
$$

Launching a wave at z_0 gives a boundary condition

$$\begin{pmatrix} (\tilde{\text{v}}_n(z_0,s))_1 \\ (\tilde{\text{v}}_n(z_0,s))_2 \end{pmatrix} = \begin{pmatrix} (\tilde{\text{v}}_n(z_0,s)) \\ (0_n) \end{pmatrix} \tag{5.4}$$

This gives a general solution

$$\begin{gathered} \left(\tilde{V}_n(z,s)\right) = \left(\tilde{z}_{c_{n,m}}(z,s)\right) = \left(\tilde{G}_{n,m}(z,z_0;s)\right) \cdot \left(\tilde{y}_{c_{n,m}}(z_0,s)\right) \cdot \left(\tilde{V}_n(z_0,s)\right) \\ \text{as } s \to \infty \text{ in RHP} \\ \left(\tilde{G}_{n,m}(z,z_0;s)\right) = \prod_{z_0}^{z} e^{\left(\tilde{g}_{n,m}(z',s)\right)dz'} \end{gathered} \tag{5.5}$$

As the high-frequency leading term. For statistical analysis then one needs the statistical properties of $(\tilde{g}_{n,m}(z,s))$, and hence of $(\tilde{z}_{c_{n,m}}(z,s))$ and $(\tilde{\gamma}_{n,m}(z,s))$. Note that for the above result as the leading term we have assumed that the line parameters vary smoothly between z_0 and z. This avoids high-frequency reflections at step-like discontinuities.

There is a very special case of interest, i.e., the case of equal modal speeds. This corresponds to the case of perfectly conducting wires in a transversely uniform medium (z variation being allowed). In this case we have the result

$$\begin{aligned} \left(\tilde{V}_n^{(0)}(z,s)\right) &= \tilde{G}(z,z_0;s)\left(\tilde{z}_{c_{n,m}}(z,s)\right) \cdot \left(\tilde{y}_{c_{n,m}}(z_0,s)\right) \cdot \left(\tilde{V}_n(z_0,s)\right) \\ \left(\gamma_{n,m}(z,s)\right) &= \gamma(z,s)\ \left(1_{n,m}\right) \\ \tilde{G}(z,z_0;s) &= e^{-\int_{z_0}^{z} \tilde{\gamma}(z',s)dz'} \equiv \text{uniform delay term} \end{aligned} \tag{5.6}$$

The importance of this result is that it uses only parameters at z_0 and z; it is *independent of the parameters at intermediate positions*. So, no matter how the wire positions, etc., may vary from one end to the other in some random fashion, the result is independent of this and can be considered deterministic. We need know only the end conditions.

Of course, this result is only the leading term as $s \to \infty$. There are correction terms discussed in [10], these being not so simple. However, a statistical description of an RNMTL should be consistent with this as $s \to \infty$.

6. Case of N Conductors Plus Reference with C_N Symmetry

6.1. Basic considerations

Now let us consider some implications of geometrical symmetry in the transmission-line cross section. For this purpose let us consider a "baby problem" that will illustrate some symmetry implications. Specifically, let us consider an N-conductor MTL with C_N symmetry as in Fig. 6.1. Here we have cross section coordinates

$$x = \Psi \cos(\phi) \quad , \quad y = \Psi \sin(\phi) \tag{6.1}$$

By C_N symmetry we mean that rotation by

$$\phi_n = 2\pi \frac{n}{N} \quad , \quad n = 1, 2, \cdots, N \tag{6.2}$$

leaves the geometry unchanged. The N wires (radius r_0) are located at $(\Psi, \phi) = (b, \phi_n)$, and the reference conductor is a cylindrical shield on $\Psi = a$. The dielectric is assumed uniform for simplicity, but more elaborate variation is allowed for C_N symmetry.

As discussed in previous papers [5, 6], such a configuration can be readily modeled. We begin with the per-unit-length parameters.

$$\begin{aligned}
&\left(\tilde{Z}'_{n,m}(s)\right) = \left(L'_{n,m}\right) = s\mu\left(f_{g_{n,m}}\right) \\
&\left(\tilde{Y}'_{n,m}(s)\right) = c\left(C'_{n,m}\right) = s\varepsilon\left(f_{g_{n,m}}\right)^{-1} \\
&\left(\tilde{\gamma}_{n,m}(s)\right) = s\left[\mu\varepsilon\right]^{1/2}\left(1_{n,m}\right) \\
&\left(L'_{n,m}\right) \equiv \text{inductance-per-unit-length matrix} \\
&\left(C'_{n,m}\right) \equiv \text{capacitance-per-unit length matrix} \\
&\quad \mu \equiv \text{permeability} \\
&\quad \varepsilon \equiv \text{permittivity} \\
&\left(f_{g_{n,m}}\right) = \left(f_{g_{n,m}}\right)^T \equiv \text{geometric-impedance-factor matrix} \\
&\qquad \equiv \text{real, positive-definite matrix}
\end{aligned} \tag{6.3}$$

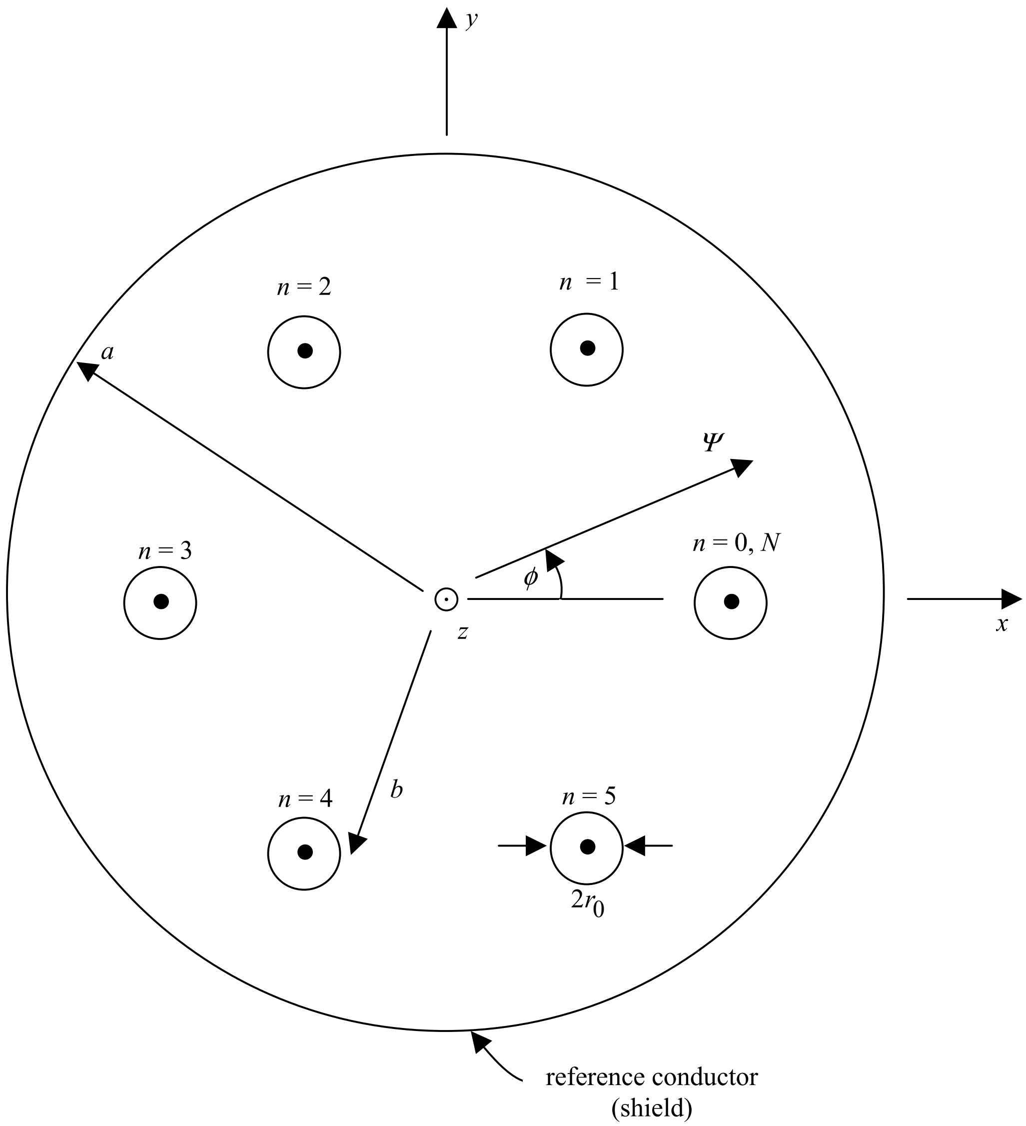


Figure 6.1. MTL With C_N Symmetry: Example for $N = 6$.

From this we also have

$$\left(Z_{c_{n,m}}\right) = \left(Y_{c_{n,m}}\right)^{-1} = \left[\frac{\mu}{\varepsilon}\right]^{1/2}\left(f_{g_{n,m}}\right) \equiv \text{characteristic impedance matrix} \tag{6.4}$$

The modes of propagation on such an MTL can be described in various ways, particularly for the case of equal modal speeds. The more general case of C_N symmetry (including reciprocity, but nonuniform dielectric allowing multiple modal speeds) is based on the circulant nature of the matrices [5, 11]. In this case we have

$$\begin{aligned} Z_{c_{n,m}} &= \text{function of } m\text{-}n + MN \\ M &= \text{any integer} \end{aligned} \tag{6.5}$$

Such matrices have eigenvectors

$$\left(x_n\right) = N^{-1/2}\left(e^{j2\pi\beta n/N}\right) \text{ for } \beta = 1,2,\cdots,N \tag{6.6}$$

with

$$\text{number of distinct eigenvalues } = \begin{cases} N/2 \text{ for } N \text{ even} \\ [N+1]/2 \text{ for } N \text{ odd} \end{cases} \tag{6.7}$$

These eigenvalues give the impedances for the associated modes. Instead of complex eigenvectors, the degeneracy associated with reciprocity makes the matrices bicirculant (circulant in columns as well as rows) and allows real eigenvectors (doubly degenerate) in the form [5]

$$\left(w_n^{(1)}\right)_\beta = \left[\frac{2}{N}\right]^{1/2}\begin{pmatrix} \cos\left(\frac{2\pi\beta}{N}\right) \\ \vdots \\ \cos\left(\frac{2\pi[N-1]\beta}{N}\right) \\ 1 \end{pmatrix}$$

$$\beta = \begin{cases} 1,2,\cdots,\frac{N}{2}-1 \text{ if } N = \text{even} \\ 1,2,\cdots,\frac{N-1}{2} \text{ if } N = \text{odd} \end{cases}$$

$$\left(w_n^{(2)}\right)_\beta = \left[\frac{2}{N}\right]^{1/2}\begin{pmatrix} \sin\left(\frac{2\pi\beta}{N}\right) \\ \vdots \\ \sin\left(\frac{2\pi[N-1]\beta}{N}\right) \\ 0 \end{pmatrix}$$

$$\left(w_n\right)_{N/2} = N^{-1/2}\begin{pmatrix}-1\\+2\\-1\\\vdots\\-1\\+1\end{pmatrix} \text{ for } N \text{ even}$$

$$\left(w_n\right)_0 = \left(w_n\right)_N = N^{-1/2}\begin{pmatrix}1\\1\\1\\\vdots\\1\end{pmatrix} \text{ (common mode)} \tag{6.8}$$

These modes are important for the subsequent discussion.

6.2. Permutation of Wire Positions

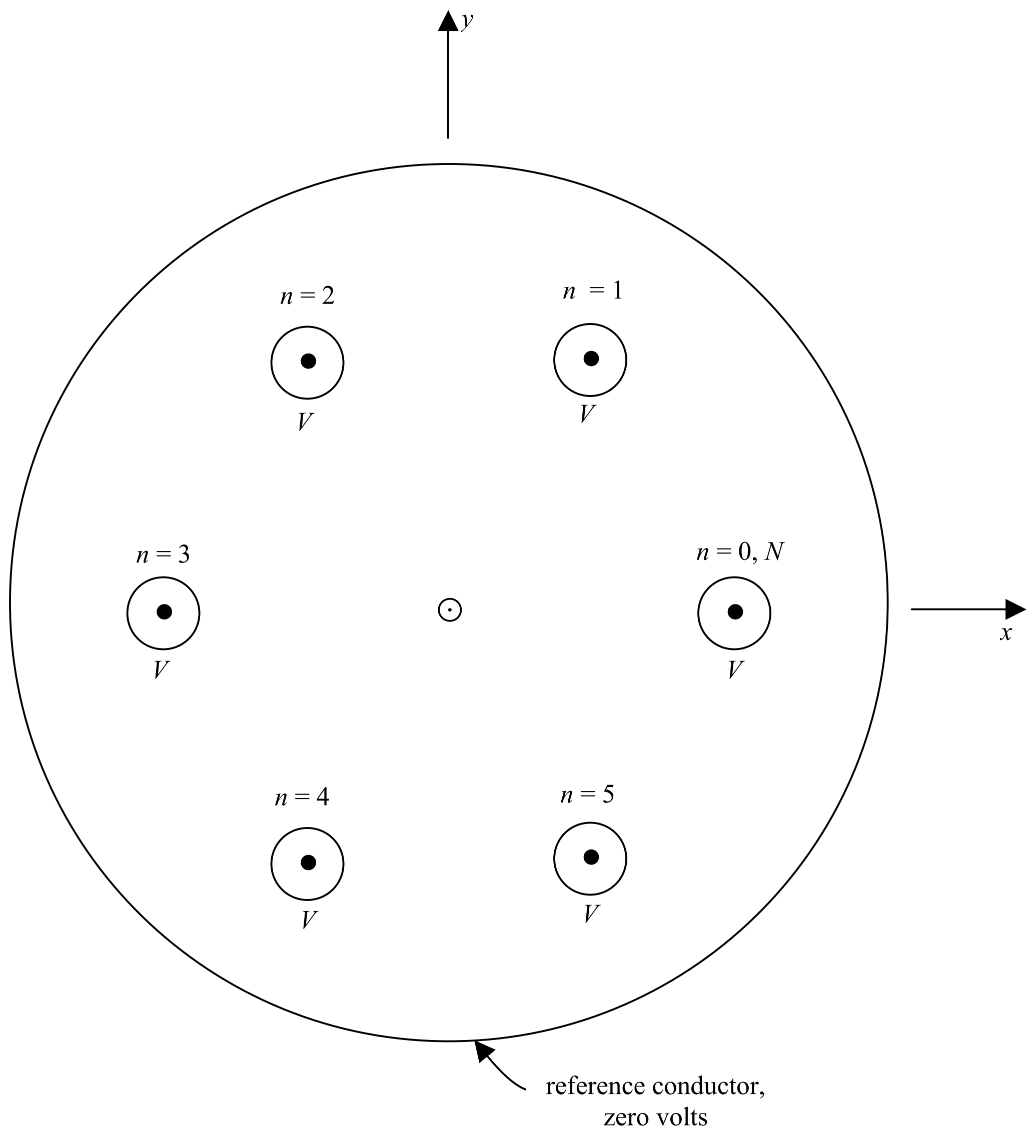
In order to randomize this particular type of MTL, let us permute the wire positions, leaving C_N symmetry both before and after the wire interchange. We neglect the details of the physical interchange, assuming that wavelengths are large compared to the distance over which the interchange takes place. This allows us to enforce voltage and current continuity on each wire through the position interchange.

This kind of permutation is discussed in [4], where permutation matrices and their properties are discussed. One can interchange two wires, followed by another interchange, etc. In the general case one can consider this as any reordering of the numbers 1 through *N*. Here we consider just a few of these to find the implications for the various eigenmodes (eigenvectors) incident on such wire interchanges.

6.3. Common Mode

Consider the common mode, illustrated in Fig. 6.2. All conductors have the same voltage *V*. They also have the same currents (say *I*) with positive convention in the +z direction. The modal impedance is simply *V/(NI).*

Now interchange two of the wires, say 1 and 2. After the interchange we still have *V* and *I* on each of the conductors with boundary conditions matched before and after the interchange. Therefore, the common mode propagates through such a wire interchange with no reflection (again, neglecting the details in the vicinity of the interchange). This applies to any permutation of the N wires.



All N conductors have the same voltage, V,
and equal associated currents.

Figure 6.2. Common Mode on MTL.

This leads to a fundamental conclusion:

> The common mode is unaffected by wire-position interchanges. As such randomization of the wire positions does not affect the common mode. This mode propagates in a deterministic fashion.

Let us note that the common mode plays an important role in MTL response [2].

6.4. Differential Modes

The case of differential modes is more complicated. Let us consider the first several values of N to illustrate this. The case of $N = 1$ has only a common mode, this being already considered.

6.4.1. $N = 2$

The case of $N = 2$ is illustrated in Fig. 6.3. An interchange of the wires is a way of thinking of a twisted shielded pair. There is only one differential mode and it is continuous through the interchange.

This mode is not affected by randomly interchanging the wires.

6.4.2. $N = 3$

The case of $N = 3$ is illustrated in Fig. 6.4. There are now two differential modes as a case of double degeneracy. As such any linear combination of the two modes is also an eigenmode. In Fig. 6.4A we take $\beta = 1$ in the cosine functions in (6.8) for illustration. Interchanging wires 0 and 1 merely rotates the mode imploying no discontinuity. In Fig. 6.4B we take the sine functions for $\beta = 1$ to describe the mode. In this case, interchanging wires 0 and 2 produces the same mode (reflected or rotated), implying no discontinuity. As the general case of the mode is merely a linear combination of the two, it propagates through the interchange position with no reflection.

This mode is then also not affected by randomly interchanging the wires.

6.4.3. $N = 4$

Going on the $N = 4$ we come to a case where randomizing the wire interchanges will give a statistical result, at least in part. To illustrate this Fig. 6.5A considers the $\beta = 1$ modes (the cosine mode, and its rotation, the sine mode). Interchanging wires 0 and 1 produces the configuration as indicated, which is not an eigenmode, but a mixture of modes for $\beta = 1$ and 2. This produces reflection (and changed transmission) at the interchange position. The same occurs on interchange of any two adjacent wires. However, interchanging opposite wire (0 and 2, or 1 and 3) produces no change. So, some wire changes produce reflections while others do not.

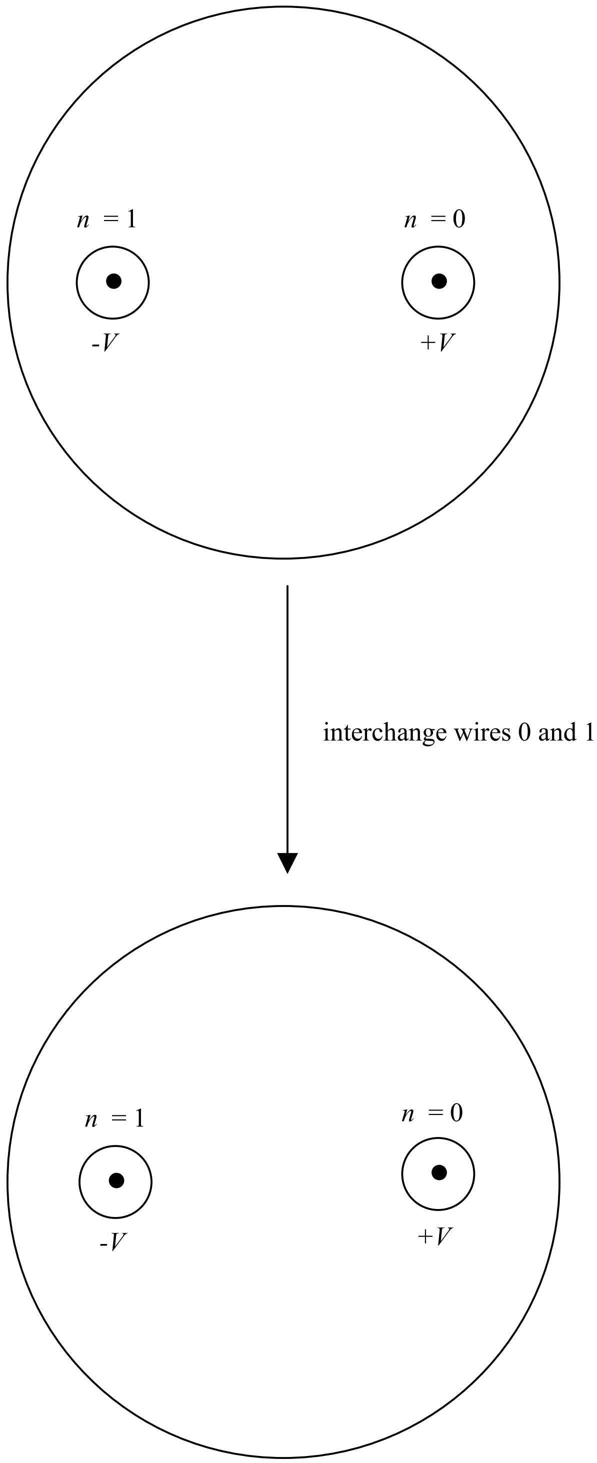
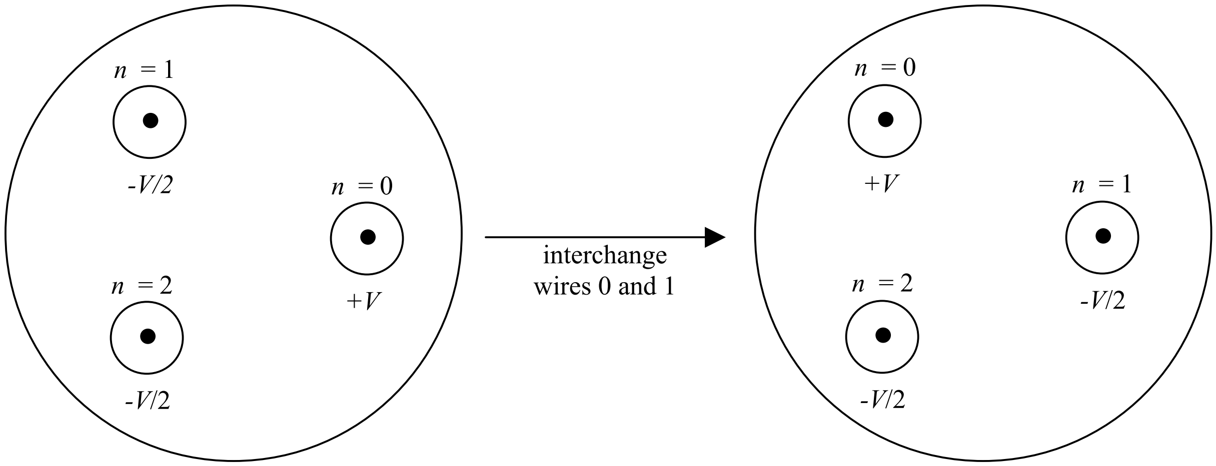
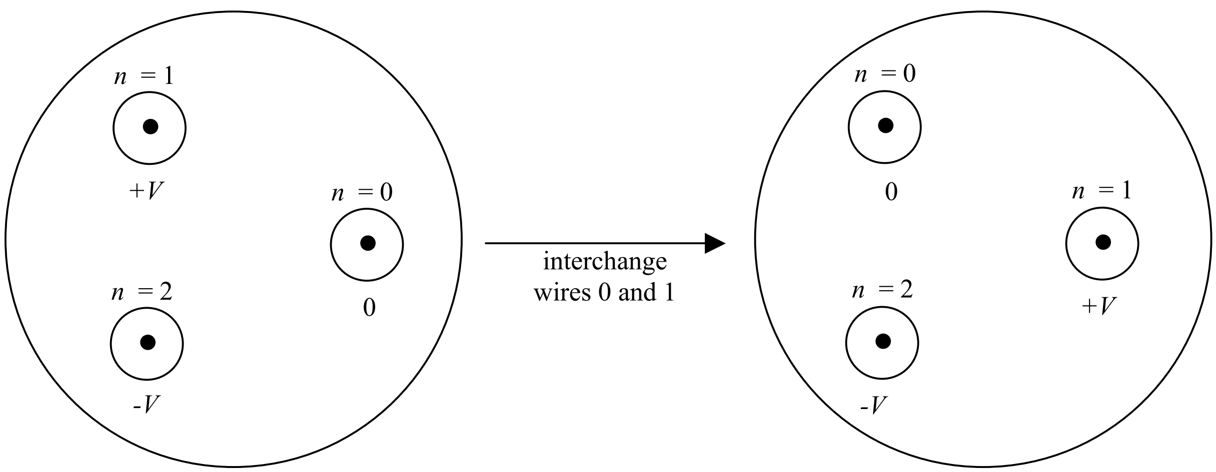


Figure 6.3. Case of $N = 2$

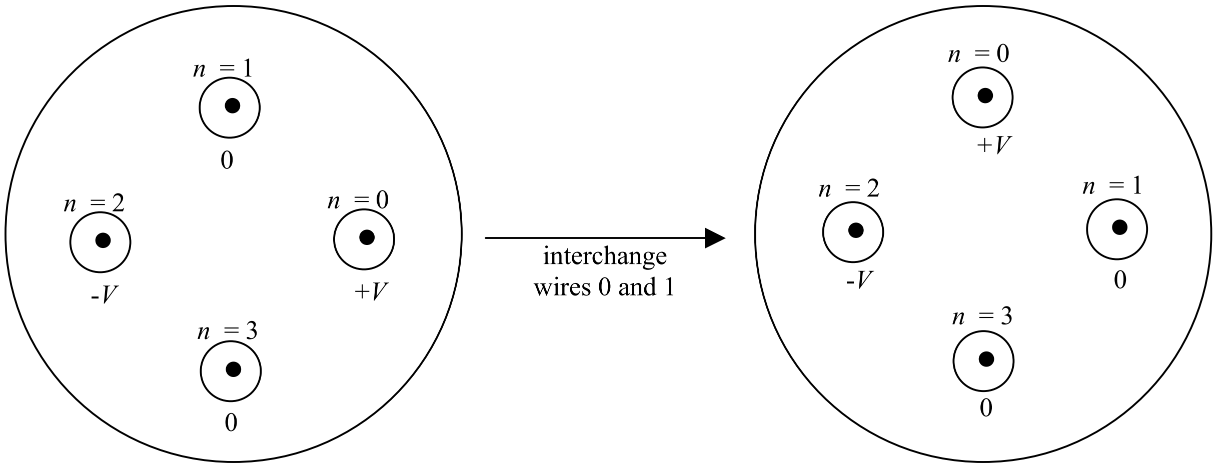


A. $\beta = 1$ cosine mode

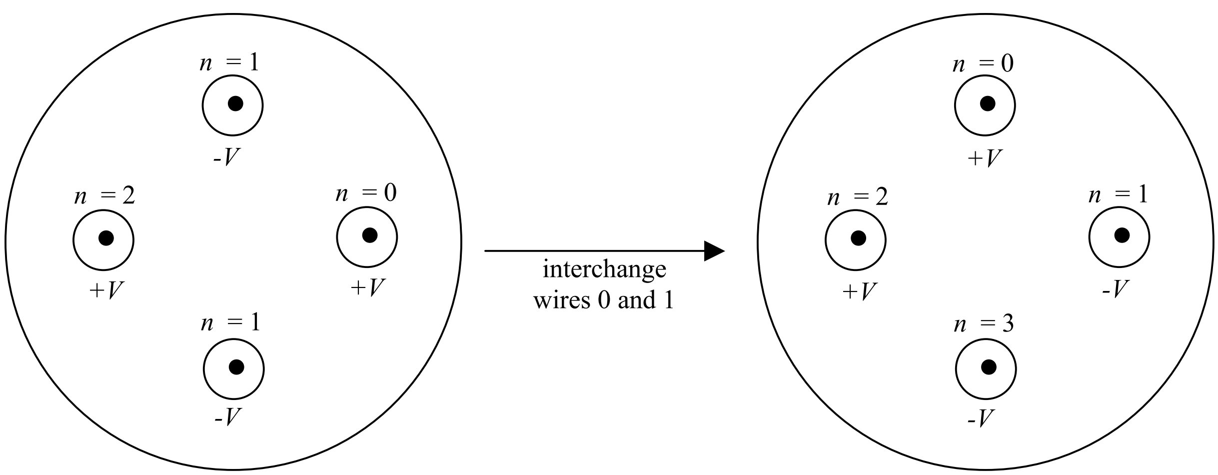


B. $\beta = 1$ sine mode

Figure 6.4. Case of $N = 3$



A. $\beta = 1$ cosine mode



B. $\beta = 2$ cosine mode

Figure 6.5. Case of $N = 4$

Similarly, in Fig. 6.5B we have the $\beta = 2$ mode. On interchanging wires 0 and 1 (or any pair of adjacent wires) a new configuration is produced which is a mixture of modes, again giving reflections. However, as before, if opposite wires are interchanged no reflection occurs.

Such behavior should be reflected in the statistical propagation on such a random MTL.

6.4.4. General Case

As we can now see propagation on a random C_N MTL (with constant a, b, r_0) is somewhat complicated, involving both deterministic and statistical properties. This is a simple case illustrating what symmetry can introduce into the NMTL propagation. There may be other special cases to consider before one can develop an adequate statistical description for an RNMTL.

7. Concluding Remarks

We can now see that the statistical description of propagation on an RNMTL has both deterministic and statistical aspects. At low-frequencies circuit considerations lead to a deterministic description. At high frequencies the special case of equal modal speeds leads to a result involving only the ends of the NMTL, regardless of the (smooth) variation of the parameters in between the ends. This leaves intermediate frequencies as a domain for statistical analysis.

Symmetry in the transmission-line cross section is another important consideration. As we have seen a special case of C_N symmetry has led to some significant results. The common mode is unaffected by wire interchange, including in a random manner. Some differential modes are similarly unaffected, while others are subject to statistical considerations. Perhaps such symmetry analysis can be extended to more general configurations.

Conservation theorems of electromagnetics, including reciprocity and energy are also important. They place constraints on allowable statistical distributions for RNMTLs. So there are many factors to be considered when formulating statistical properties of RNMTLs.

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