Lightning Phenomenology Notes

Note 2

22 December 1981

Properties of Lightning-Leader Pulses

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Abstract

Recent measurements have shown that lightning leader pulses have very fast rise times. Characteristic times for the rise of as small as a little less than 30 ns have been observed. Considering the nature of these leader pulses, an approximate model is obtained. This model is based on an equivalent transmission line as sometimes used in antenna theory. A corona which increases the capacitance per unit length is assumed to form around the central arc channel. Letting the corona radius be governed by a simple breakdown model, a nonlinear wave equation is obtained for the arc. Solving this equation one obtains velocities of the order of 10^8 m/s and currents of the order of 15 kA; this agrees well with the data.

II. Equivalent Corona Radius

The concept of an equivalent corona radius Ψ_c to approximately describe the transmission of waves along wires was proposed by this author in order to model the experiments conducted at Kaman Sciences by Book et al. [4-7]. As indicated in fig. 2.1 there is a set of radii or equivalent radii

$$\begin{aligned}
\Psi_0 &\equiv \text{wire radius (or later, arc radius)} \\
\Psi_c &\equiv \text{equivalent corona radius} \\
\Psi_\infty &\equiv \text{equivalent "outer" reference conductor radius}
\end{aligned} \tag{2.1}$$

In this approximation one envisions that there is some reasonably well-defined (or at least an equivalent) corona region described by $\Psi_0 < \Psi < \Psi_c$ where Ψ is the cylindrical radius in a cylindrical (Ψ,ϕ,z) coordinate system with the z axis centered in the wire (or lightning arc). This corona region has some conductivity σ which may be a function of Ψ and t (time), and permittivity ε which may be taken as ε_0, the permittivity of free space.

In a transmission-line model one needs appropriate per-unit-length parameters. For present purposes let us approximate the longitudinal impedance per unit as an inductance per unit length

$$\begin{aligned}
L' &= \mu_0 f_L \\
f_L &\equiv \frac{1}{2\pi} \ln\left(\frac{\Psi_\infty}{\Psi_0}\right) \\
\mu_0 &\equiv \text{permeability of free space}
\end{aligned} \tag{2.2}$$

This corresponds to assuming that the longitudinal current is all on the wire (or later, arc).

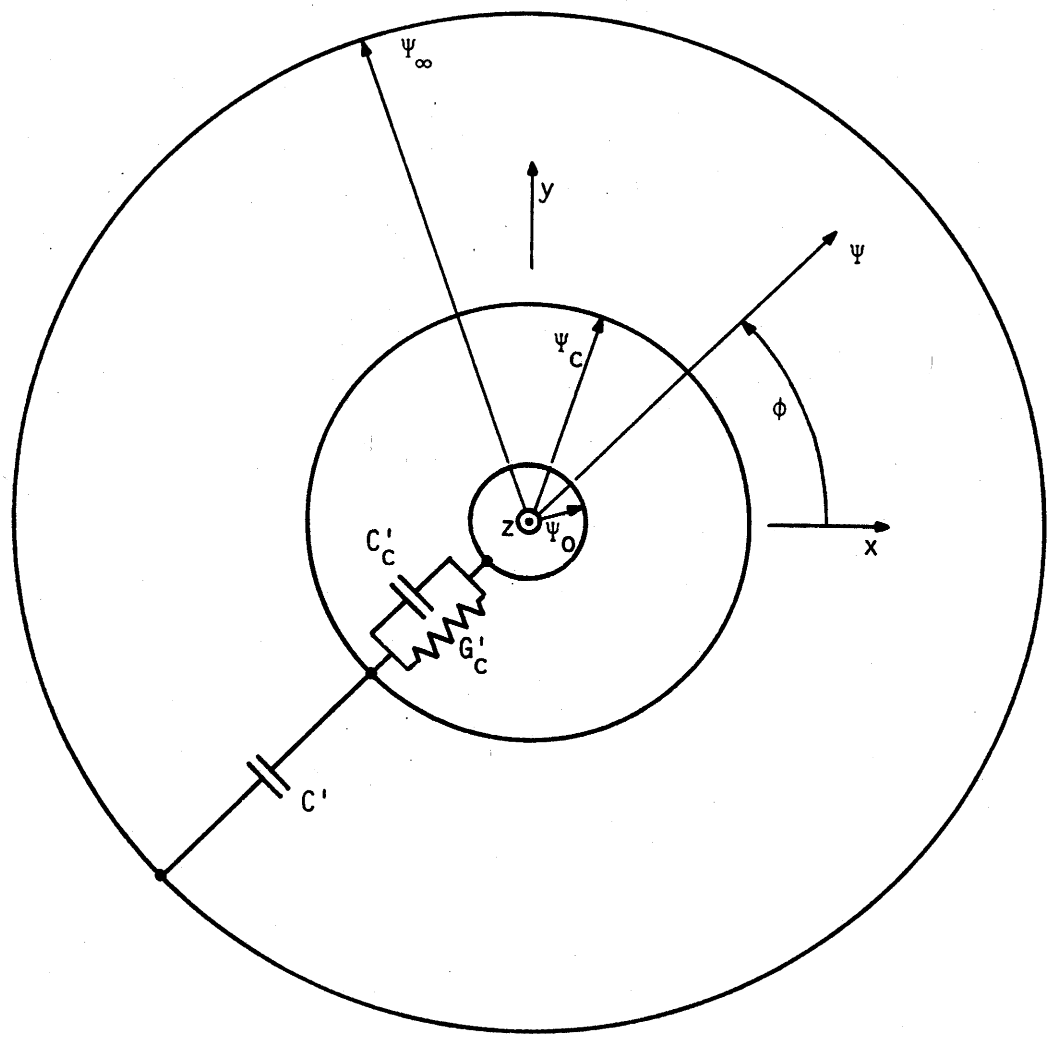


Fig. 2.1 Equivalent-corona-radius model

The transverse admittance per unit length is more complicated. Given an assumption of a rotationally symmetric corona (and strictly a coaxial geometry), then as in fig. 2.1 one can divide the transverse admittance per unit length according to the radial dependence of σ. For simplicity we assume that we have a definite corona boundary at $\Psi = \Psi_c$ and that for the region exterior to the corona we have a capacitance per unit length

$$C' = \frac{\varepsilon_0}{f_C}$$

$$f_C \equiv \frac{1}{2\pi} \ln\left(\frac{\Psi_\infty}{\Psi_c}\right) \tag{2.3}$$

$$\varepsilon_0 \equiv \text{permittivity of free space}$$

The geometrical factors for inductance and capacitance per unit length are related by

$$f_C = f_L - \Delta$$

$$\Delta = \frac{1}{2\pi} \ln\left(\frac{\Psi_c}{\Psi_0}\right) \tag{2.4}$$

so that as corona develops Δ grows from zero and gives a measure of the change in C'.

If we approximate the corona conductivity, σ, as uniform for $\Psi_0 < \Psi < \Psi_c$ then we have a parallel corona capacitance per unit length

$$C'_c = \frac{2\pi\varepsilon_0}{\ln\left(\frac{\Psi_c}{\Psi_0}\right)} = \frac{\varepsilon_0}{\Delta} \tag{2.5}$$

and conductance per unit length

$$G'_c = \frac{2\pi\sigma}{\ln\left(\frac{\Psi_c}{\Psi_0}\right)} = \frac{\sigma}{\Delta} \tag{2.6}$$

where the capacitance per unit length in the absence of corona is

$$C'_o = \left[C'^{-1} + C_c'^{+1}\right]^{-1} = \frac{\varepsilon_o}{f_L} = \frac{2\pi\varepsilon_o}{\ell n\left(\frac{\Psi_\infty}{\Psi_o}\right)} \tag{2.7}$$

For present purposes the corona conductance is assumed sufficiently large that G'_c can be considered to have zero voltage drop (compared to that across C'), and hence C'_c can also be neglected. Of course, σ is not assumed so large that $\Psi_c - \Psi_o$ is comparable to or greater than a skin depth since that would require changing the impedance per unit length away from that of a simple constant L'. In this approximation the significant current, I, is on the wire or arc of radius Ψ_o, while the significant charge per unit length, Q', is on the outer corona "boundary" of radius Ψ_c.

An approximate value for the corona radius can be obtained from

$$E_b = \frac{|Q'|}{2\pi\varepsilon_o\Psi_c} \equiv \text{effective breakdown electric field}$$

$$\Psi_c = \frac{|Q'|}{2\pi\varepsilon_o E_b} \tag{2.8}$$

$$\frac{d\Psi_c}{d|Q'|} = \frac{1}{2\pi\varepsilon_o E_b} = \frac{\Psi_c}{|Q'|}$$

This is an important approximation in that the corona radius is only a function of a single temporal/spatial variable, Q'.

Other kinds of equivalent corona radii are possible involving some average over the charge distribution. However, for present simplicity the above form is adopted. Perhaps future developments of this model will adopt other forms.

III. Transmission-Line Equations

For a simple transmission line with a single voltage/current pair of interest one has a set of coupled first order partial differential equations in space and time. In the classical linear, time-invariant, bilateral (reciprocal) case these take the form

$$\frac{\partial \tilde{V}}{\partial z} = -\tilde{Z}'\tilde{I} + \tilde{V}_s'$$

$$\frac{\partial \tilde{I}}{\partial z} = -\tilde{Y}'\tilde{I} + \tilde{I}_s'$$

$$\tilde{Z}' \equiv \text{longitudinal impedance per unit length} \qquad (3.1)$$

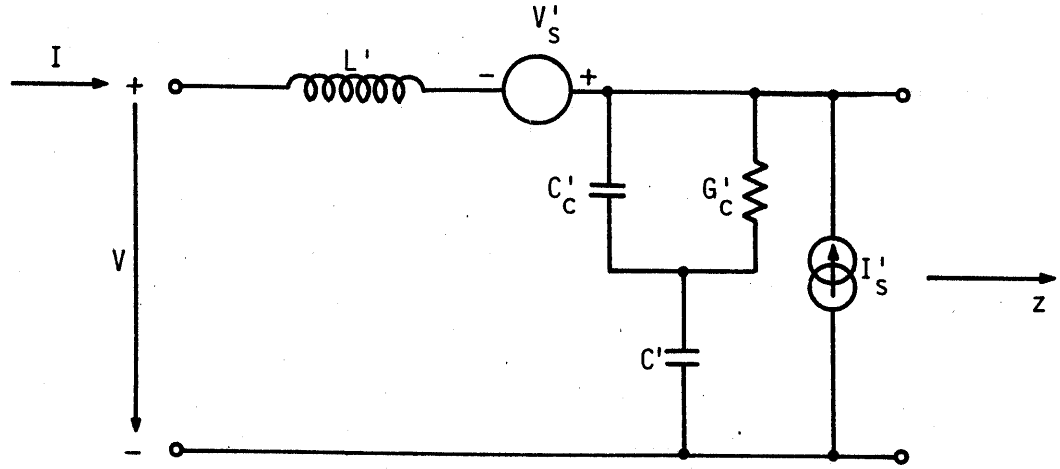
$$\tilde{Y}' \equiv \text{transverse admittance per unit length}$$

$$\tilde{V}_s' \equiv \text{longitudinal voltage source per unit length}$$

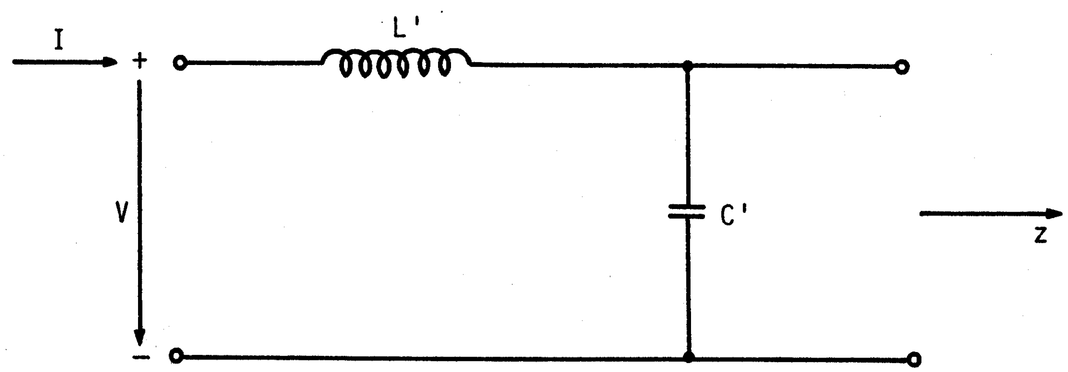
$$\tilde{I}_s' \equiv \text{transverse current source per unit length}$$

where a tilde ~ above a quantity indicates a function of the Laplace-transform variable or complex frequency, s.

In time domain (3.1) takes on a more complex form depending on the forms of $\tilde{Z}'$ and $\tilde{Y}'$ which become not impedance and admittance functions (per unit length) but more general operators. However, one can also include non-linear effects in time domain. Consulting fig. 3.1A note that one can construct a per-unit-length equivalent circuit using the equivalent circuit elements per unit length which have been discussed. Note the inclusion of the time-domain per-unit-length sources for generality; in the present problem these are set to zero. Using the approximation of sufficiently large σ replaces G_c' and C_c' by a short circuit. This leaves the simplified equivalent circuit per unit length in fig. 3.1B. Note that while L' is taken as invariant with respect to both time, t, and space, z, C' is a function of both of these parameters thereby making the equivalent transmission line nonuniform.



A. General case with uniform corona



B. Case without sources and with sufficiently large corona conductivity

Fig. 3.1 Per-unit-length transmission-line equivalent-circuit representation

With these approximations our time-domain transmission-line equations take the form

$$\frac{\partial V}{\partial z} = -L' \frac{\partial I}{\partial t}$$
$$\frac{\partial I}{\partial z} = -\frac{\partial}{\partial t}[C'V] \tag{3.2}$$

Using the definition of capacitance per unit length

$$C' = \frac{Q'}{V} \tag{3.3}$$

The second of (3.2) is the equation of continuity. Replacing V gives for (3.2)

$$\frac{\partial}{\partial z}\left(\frac{Q'}{C'}\right) = -L' \frac{\partial I}{\partial t}$$
$$\frac{\partial I}{\partial z} = -\frac{\partial Q'}{\partial t} \tag{3.4}$$

Using (2.3) and (2.8) we have our approximate form for C' as

$$C' = \frac{\varepsilon_0}{f_C}$$
$$f_C = \frac{1}{2\pi} \ln\left(\frac{Q'_\infty}{|Q'|}\right) \tag{3.5}$$
$$Q'_\infty \equiv 2\pi\varepsilon_0 E_b \Psi_\infty$$

so that C' is only a function of Q'. Then (3.4) has independent variables z and t, and dependent variables I and Q'.

A convenient form for (3.4) is found by eliminating I by operating on the two equations by partial derivatives with respect to z and t respectively giving

$$\frac{1}{L'} \frac{\partial^2}{\partial z^2}\left(\frac{Q'}{C'}\right) - \frac{\partial^2 Q'}{\partial t^2} = 0 \tag{3.6}$$

which is a nonlinear wave equation for Q' since C' is a function of Q' only (by hypothesis).

IV. Nonlinear Waves from Homogeneous Equations

The coupled set of first-order equations (3.4) and the corresponding wave equation (3.6) are homogeneous (sourceless) and admit solutions as nonlinear waves as pointed out by Chen [8,9] in the form

$$Q' \equiv Q'(\tau)$$

$$\tau = t \pm \frac{z}{v(Q')} \qquad (4.1)$$

$$v(Q') = \left\{ \frac{1}{L'} \frac{d}{dQ'} \left[\frac{Q'}{C'(Q')} \right] \right\}^{\frac{1}{2}}$$

These are waves propagating in the $\mp$ z direction depending on the sign chosen (and noting the symmetry). Note that the actual functional form of the dependence of C' on Q' does not need to be specified for this result to hold. Using the data of Book et al. [4,5] Chen observed that the observed pulse stretching (or increasing delay) at large amplitudes was consistent with an amplitude dependent velocity as in (4.1), and that the experimental velocities decreased in the experiment from 3×10^8 m/s to between 1×10^8 m/s and 2×10^8 m/s at the higher amplitudes. In addition it was noted that not all the details of the waveforms were consistent with this kind of a model, but the major portion of the waveform fit such a model rather well.

Equations (3.4) can be transformed into the nonlinear retarded time τ by noting

$$\tau = t \pm \frac{z}{v(Q'(\tau))}$$

$$\left.\frac{\partial}{\partial t}\right|_{\substack{z \\ \text{constant}}} = \left.\frac{\partial \tau}{\partial t}\right|_{\substack{z \\ \text{constant}}} \frac{d}{d\tau}$$

$$
\left.\frac{\partial}{\partial z}\right|_{\substack{t\\ \text{constant}}} = \left.\frac{\partial \tau}{\partial z}\right|_{\substack{t\\ \text{constant}}} \frac{d}{d\tau}
$$

$$
\left.\frac{\partial \tau}{\partial t}\right|_{\substack{z\\ \text{constant}}} = 1 \pm z \frac{d}{d\tau}\left[\frac{1}{v}\right] \left.\frac{\partial \tau}{\partial t}\right|_{\substack{z\\ \text{constant}}} \tag{4.2}
$$

$$
\left.\frac{\partial \tau}{\partial z}\right|_{\substack{t\\ \text{constant}}} = \pm\left\{\frac{1}{v} + z \frac{d}{d\tau}\left[\frac{1}{v}\right] \left.\frac{\partial \tau}{\partial z}\right|_{\substack{t\\ \text{constant}}}\right\}
$$

$$
\left.\frac{\partial \tau}{\partial t}\right|_{\substack{z\\ \text{constant}}} = \left\{1 \mp z \frac{d}{d\tau}\left[\frac{1}{v}\right]\right\}^{-1}
$$

$$
\left.\frac{\partial \tau}{\partial z}\right|_{\substack{t\\ \text{constant}}} = \pm \frac{1}{v}\left\{1 \mp \frac{d}{d\tau}\left[\frac{1}{v}\right]\right\}^{-1}
$$

These partial-derivative formulas can be substituted into (3.4); the second is satisfied and the first give the third equation of (4.1). The second of (3.4) becomes

$$
\frac{dI(\tau)}{d\tau} = \mp v(Q'(\tau)) \frac{dQ'(\tau)}{d\tau} \tag{4.3}
$$

noting now that I is a function of τ alone since Q' and v are functions of τ alone. Integrating (4.3) gives

$$
I(\tau) - I_o = \mp \int_{Q'_o}^{Q'(\tau)} v(q')\, dq'
$$

$$
\left.\begin{aligned} I_o &= I(\tau_o) \\ Q_o &= Q(\tau_o) \end{aligned}\right\} \text{ initial conditions at } \tau = \tau_o \tag{4.4}
$$

With given initial conditions then (4.1) and (4.4) give the complete solution to (3.4) or equivalently (3.6).

V. Application to Lightning Leader

Applying the foregoing to the arc in the atmosphere which begins the lightning event requires some careful consideration. Approximations are a key consideration. Figure 5.1 shows a view of the tip of the lightning leader. Several physical aspects can be considered.

The concept of a transmission-line model itself leads to problems. There is no physical return or reference conductor. However, it has been established that in the case of sufficiently thin conductors a transmission-line model is still approximately valid [10]. For this purpose thin means a small radius compared to other characteristic distances such as conductor length(s) and times of concern multiplied by the speed of light. Even if the arc channel has a small radius (say of the order of 1 mm) there is still the larger corona radius of concern.

Another set of problems concerns the leader tip. The pulse of concern is not propagating on a preexisting conducting wire. The charge near the arc tip must exceed some value to cause electrical breakdown of the air and thereby a propagating leader tip. This may give the key to our model. If the leader tip propagates faster than the wave on the equivalent transmission line (as in (4.1)) then the tip is in effect stretched, thereby reducing the local charge per unit length and the associated electric field, and hence stopping the breakdown or reducing the tip velocity. Conversely, if the tip is propagating slower than the wave behind it, charge will pile up increasing the breakdown rate and hence the tip velocity. Thus it would appear that the velocity behind the tip and the velocity of the tip are linked. Furthermore, since Q' should be above some value to maintain the breakdown velocity we can neglect small values of Q' in searching for appropriate solutions.

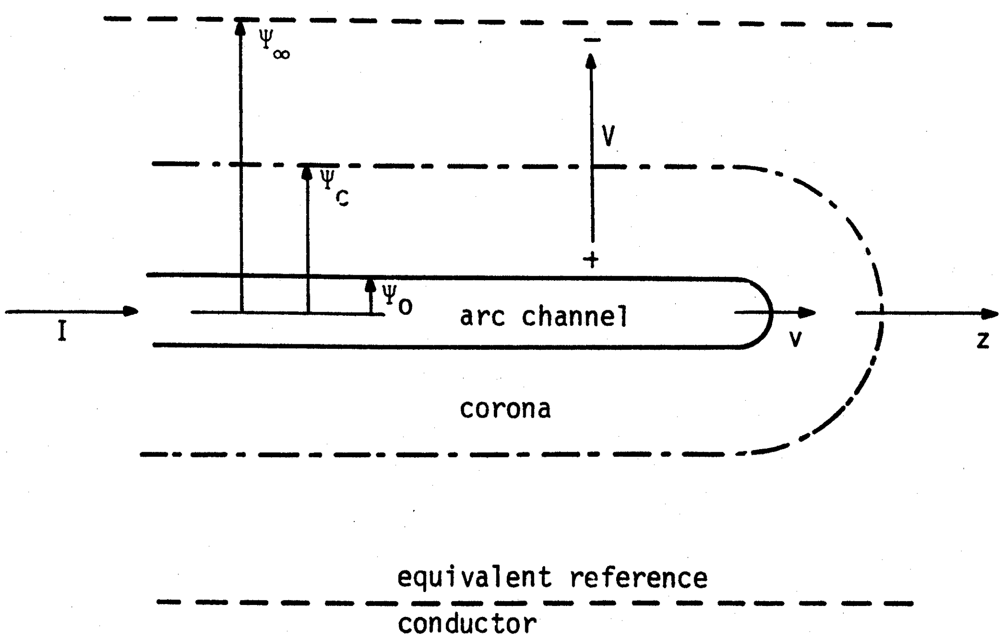


Fig. 5.1 Lightning-leader model

With the foregoing in mind let us look for what might be thought of as a shockwave with no charge or current in front of it and a uniform Q' and I behind it, i.e.,

$$\begin{aligned}
I(\tau) &= \mp\, v(Q'(\tau))Q'(\tau) \\
Q'(\tau) &= Q'u(\tau) \\
I(\tau) &= Iu(\tau) \\
u(\tau) &= \begin{cases} 1 \text{ for } \tau > 0 \\ 0 \text{ for } \tau < 0 \end{cases}
\end{aligned} \tag{5.1}$$

Note that this solution satisfies (4.4) as a special simple case.

Further aiding our physical arguments, there is some data that one can use [1]. Recent measurements of the electromagnetic fields from lightning show some very fast leader-like signals with characteristic times in the rise of the order of 30 ns or somewhat longer. Via the free-space dyadic Green's function, a far-field approximation, and reflection at the ground plane one can relate the fields back to the currents. For the distance back to the source acoustic arrays and videotape pictures are used. This results in a characteristic source parameter

$$\begin{aligned}
\overleftrightarrow{1}_t \cdot T(t) &\simeq \overleftrightarrow{1}_t \cdot \frac{\partial}{\partial t} \int_{V''} \vec{J}(\vec{r}\,'',t)\; dV'' \\
\overleftrightarrow{1}_t &\equiv \overleftrightarrow{1} - \vec{1}_r\vec{1}_r \equiv \text{transverse dyad} \\
\overleftrightarrow{1} &\equiv \text{identity} \\
\vec{1}_r &\equiv \text{unit radius vector from observer to source}
\end{aligned} \tag{5.2}$$

for which only the transverse components are observed. Note that for greater accuracy the current can be written as a function of retarded time.

Summarizing the data in [1], the magnitude of the transverse components of individual pulses of $\vec{T}$ is somewhat variable with the stronger ones in the 10^{11} Am/s to 10^{12} Am/s range. One example reached about 1.5 TAm/s, but appeared to be more like a return stroke.

For comparison to the data note that our simple form of leader in (5.1) gives a magnitude

$$|\vec{T}| \simeq |I| v(Q') \tag{5.3}$$

neglecting retarded time effects. In natural lightning one does not have a semiinfinite channel of current. The pulse event includes not only the leader tip but a propagation of the disturbance in the opposite direction as well, communicating the streamer motion back along the arc. This may increase the result in (5.3) by a factor of 2 or so.

Other physical parameters need to be estimated. Let us take

$$\begin{aligned} E_b &\simeq 2 \text{ MV/m} \\ \Psi_0 &\simeq 1 \text{ mm} \\ \Psi_\infty &\simeq 10 \text{ m} \end{aligned} \tag{5.4}$$

The breakdown field is only approximate and the reader may wish to try other numbers; it is altitude dependent, decreasing at higher altitudes. The arc radius Ψ_0 is very crude, but fortunately it only enters logarithmically. The equivalent outer reference conductor radius is also very crude, but again it only enters logarithmically. The present estimate is based on about a 30 ns observed characteristic time for the pulse rise giving a transit time out to this radius; one can also guess some characteristic length of arc significant in an individual pulse as being of this general order as well. Perhaps in the future these numbers can be refined somewhat.

VI. Velocity Equation

Take the corona radius model in section 2 and solve for the velocity in (4.1). Starting from the velocity formula

$$\begin{aligned}
\left[\frac{v(Q')}{c}\right]^2 &= \frac{1}{c^2}\,\frac{1}{L'}\,\frac{d}{dQ'}\left[\frac{Q'}{C(Q')}\right] \\
&= \frac{1}{f_L}\,\frac{d}{d|Q'|}\left[|Q'|\,f_c(Q')\right] \\
&= \frac{1}{f_L}\left\{f_C(Q') + |Q'|\,\frac{df_C(Q')}{d|Q'|}\right\} \\
&= \frac{1}{f_L}\left\{\frac{1}{2\pi}\,\ln\left(\frac{Q'_\infty}{|Q'|}\right) - \frac{1}{2\pi}\right\} \\
&= \frac{1}{f_L}\left\{f_L - \Delta - \frac{1}{2\pi}\right\} \\
&= 1 - \frac{\Delta + \frac{1}{2\pi}}{f_L}
\end{aligned} \tag{6.1}$$

$$c \equiv \frac{1}{\sqrt{\mu_o \varepsilon_o}}$$

we can see the decrease of the velocity from that of light. In terms of charge-per-unit-length parameters we have

$$\begin{aligned}
\left[\frac{v(Q')}{c}\right]^2 &= \frac{\ln\left(\frac{Q'_\infty}{|Q'|}\right) - 1}{\ln\left(\frac{Q'_\infty}{Q'_o}\right)} \\
&= \frac{\ln\left(\frac{Q'_\infty}{e|Q'|}\right)}{\ln\left(\frac{Q'_\infty}{Q'_o}\right)} \\
&= 1 - \frac{\ln\left(\frac{e|Q'|}{Q'_o}\right)}{\ln\left(\frac{Q'_\infty}{Q'_o}\right)}
\end{aligned}$$

$$Q'_o \equiv 2\pi\varepsilon_o E_b \Psi_o \tag{6.2}$$

$$Q'_\infty \equiv 2\pi\varepsilon_o E_b \Psi_\infty$$

and in terms of radius parameters we have

$$\left[\frac{v(Q')}{c}\right]^2 = \frac{\ln\left(\frac{\Psi_\infty}{\Psi_c}\right) - 1}{\ln\left(\frac{\Psi_\infty}{\Psi_o}\right)}$$

$$= \frac{\ln\left(\frac{\Psi_\infty}{e\Psi_c}\right)}{\ln\left(\frac{\Psi_\infty}{\Psi_o}\right)}$$

$$= 1 - \frac{\ln\left(\frac{e\Psi_c}{\Psi_o}\right)}{\ln\left(\frac{\Psi_\infty}{\Psi_o}\right)} \tag{6.3}$$

There are some special cases of interest if we choose Ψ_c as special values. For

$$\Psi_c = \Psi_o - \qquad \left[\frac{v}{c}\right]^2 = 1 \tag{6.4}$$

since there is no corona and (6.1) then do not apply. For

$$\Psi_c = \Psi_o +$$

$$\left[\frac{v}{c}\right]^2 = 1 - \frac{1}{2\pi f_L} = 1 - \frac{1}{\ln\left(\frac{\Psi_\infty}{\Psi_o}\right)} \tag{6.5}$$

showing some velocity decrease. Note that the velocity becomes imaginary if Ψ_c becomes too large. The critical value is given by

$$\left[\frac{v}{c}\right]^2 = 0$$
$$\frac{\Psi_c}{\Psi_\infty} = \frac{1}{e} \tag{6.6}$$

so our values of Ψ_c of interest are in the range

$$\Psi_o < \Psi_c < \frac{\Psi_\infty}{e} \tag{6.7}$$

VII. Solution of Equations

Summarizing our equations we have

$$|I| = v|Q'|$$

$$\left[\frac{v}{c}\right]^2 = 1 - \frac{\ell n\left(\frac{e|Q'|}{Q'_o}\right)}{\ell n\left(\frac{Q'_\infty}{Q'_o}\right)} \tag{7.1}$$

$$\Psi_c = \frac{|Q'|}{2\pi\varepsilon_o E_b}$$

with variables: $|I|$, $|Q'|$, v, and Ψ_c. This is an underdetermined system so far. There is also the data concerning $\vec{T}$ or $|I|v$.

To get a clue as to what is happening consider the following table in which Ψ_c is varied as a parameter:

Ψ_c	$\frac{v}{c}$	$\|Q'\|$	$\|I\|$	$\|I\|v$
0.1 m	0.62	11 μC/m	2.1 kA	0.33 TAm/s
0.2 m	0.56	22 μC/m	3.1 kA	0.52 TAm/s
0.5 m	0.47	55 μC/m	7.7 kA	1.1 TAm/s
1 m	0.38	110 μC/m	12 kA	1.4 TAm/s
2 m	0.26	220 μC/m	17 kA	1.2 TAm/s

From this we can see that as Ψ_c increases, v decreases, $|Q'|$ increases, $|I|$ increases, but $|I|v$ increases then decreases. Since $|I|v$ is related to the electromagnetic fields observed at a distance the maximum value is an interesting parameter.

The maximum $|I|v$ is found by differentiation with respect to an appropriate parameter. Let us use $|Q'|$ giving

$$|I|v = v^2|Q'|$$

$$= c^2 Q' \left\{ 1 - \frac{\ln\left(\frac{e|Q'|}{Q'_o}\right)}{\ln\left(\frac{Q'_\infty}{Q'_o}\right)} \right\}$$

$$\frac{d}{d|Q'|}\left[|I|v\right] = c^2 \left\{ 1 - \frac{\ln\left(\frac{e|Q'|}{Q'_o}\right)}{\ln\left(\frac{Q'_\infty}{Q'_o}\right)} + |Q'|\left[- \frac{1}{|Q'|} \frac{1}{\ln\left(\frac{Q'_\infty}{Q'_o}\right)} \right] \right\} \tag{7.2}$$

$$= c^2 \left\{ 1 - \frac{\ln\left(\frac{e^2|Q'|}{Q'_o}\right)}{\ln\left(\frac{Q'_\infty}{Q'_o}\right)} \right\}$$

Equating the derivative to zero gives a set of parameters associated with the maximum $|I|v$ as

$$|Q'_{assoc.}| = \frac{Q'_\infty}{e^2}$$

$$= \frac{2\pi\varepsilon_o \Psi_\infty E_b}{e^2}$$

$$\Psi_{c_{assoc.}} = \frac{\Psi_\infty}{e^2}$$

$$v_{assoc.} = c \left\{ \ln\left(\frac{Q'_\infty}{Q'_o}\right)\right\}^{\frac{1}{2}} \tag{7.3}$$

$$= c \left\{ \ln\left(\frac{\Psi_\infty}{\Psi_o}\right)\right\}^{\frac{1}{2}}$$

$$|I_{assoc.}| = v_{assoc.}|Q'_{assoc.}|$$

$$\left[|I|v\right]\Big|_{max.} = v^2_{assoc.}|Q'_{assoc.}|$$

Using our previously selected parameters (as in (5.4)) gives specific numbers as

$$
\begin{aligned}
|Q'_{assoc.}| &\simeq 150\ \mu C/m \\
\Psi_{c_{assoc.}} &\simeq 1.1\ m \\
v_{assoc.} &\simeq 1.0 \times 10^{8}\ m/s \\
|I_{assoc.}| &\simeq 15\ kA \\
\left[|I|v\right]\Big|_{max.} &\simeq 1.5\ TAm/s
\end{aligned}
\tag{7.4}
$$

which gives rather reasonable numbers for comparison to various experimental numbers including those in [1].

VIII. Summary

This equivalent nonlinear transmission-line model of the lightning leader agrees with some aspects of the data, giving approximate values of current, charge-per-unit length, and velocity. However, there are clear limitations to this model, including the lack of information concerning characteristic times of the rise and pulse width. Perhaps some improvements can be made to include non-zero resistance per unit length of the arc; this may be associated with the finite width of the pulses by preventing the current from maintaining its full value after some time giving a depletion wave to quench the pulse. There may be some similarity to a nonlinear relaxation oscillator.

One can think of possible improvements to the model by removing the transmission-line approximation. In antenna theory one often treats thin wires via an integral equation known as the Pocklington equation in which there are integrals over current and charge using a thin-wire approximation. Including the foregoing corona-radius model with current and charge-per-unit length on different radii, Ψ_0 and Ψ_c respectively, is a potential approach.

Another approach to removing the transmission-line approximation involves a Lorentz transformation to a coordinate frame moving in the z direction with velocity, v. In this frame the electromagnetic problem is quasistatic if all parameters of the original problem propagate with constant velocity, v, in the z direction and with constant amplitude.

The present development has evolved with the leader tip in mind. However, it may also apply to other aspects of a lightning event, e.g., a return stroke, with appropriate changes in the physical parameters in the model. Evidently considerable research is needed.

References

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