Lightning Phenomenology Notes

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REVIEW OF PROPAGATION EFFECTS FOR ELECTROMAGNETIC PULSE TRANSMISSION

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Abstract

It is the purpose of this report to review the effects of propatation for a transient electromagnetic signal that thus enamated from a localized pulse source. The best example of the latter is a time-varying current in a lightning channel. The literature on the subject of transient electromagnetic fields is vast. But here we will focus on the effects caused by the presence of the earth's surface.

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I. INTRODUCTION

It is the purpose of this report to review the effects of propagation for a transient electromagnetic signal that has emanated from a localized pulse source. The best example of the latter is a time-varying current in a lightning channel.

The literature on the subject of transient electromagnetic fields is vast. But here we will focus on the effects caused by the presence of the earth's surface. Less attention will be paid to the influence of the atmosphere. For the most part the source of the fields will be regarded as a vertical electric dipole or current element endowed with a specified time-dependent moment. In most cases the problem boils down to performing an inverse Fourier or Laplace transformation of the time harmonic form. Here we will not elaborate on the analytical details nor will we carry out new numerical evaluations of the inverse transforms. Instead the significance, scope, and limitations of the existing results in the literature will be reviewed. This task requires that the known steady state or time harmonic form be clearly understood before the veracity of the transient counterpart be established.

II. FLAT EARTH MODEL

We consider first a homogeneous half-space model of the earth with a frequency independent conductivity σ and permittivity ε. The (magnetic) permeability μ_0 is also a constant and taken to be the same as its free-space value (i.e., $\mu_0 = 4\pi \times 10^{-7}$). The source is taken to be a vertical electrical dipole of current moment $M(i\omega)$ for a harmonic time factor $\exp(i\omega t)$. What this means is the present context is that $M(s)$ is the Laplace transform of the time-dependent current moment $m(t)$. That is

$$M(s) = \int_0^\infty m(t)e^{-st}dt \tag{1}$$

with the variable s identified with $i\omega$. The generality of the transform is extended if we allow $\mathrm{Re}\, s > 0$. Symbolically we write Equation 1 as

$$M(s) = L\, m(t) \tag{2}$$

where L is the Laplace transform operator.

For the moment we define $m(t)$ the instantaneous current moment as follows:

$$m(t) = \int_0^\ell i(z,t)\, dz \tag{3}$$

where ℓ is a fixed height beyond which (i.e., $z > \ell$) the current $i(z,t)$ is always zero. The situation is illustrated in Figure 1 when with reference to cylindrical coordinates (ρ,ϕ,z) the earth's surface is at $z = 0$ and the current element is located at $\rho = 0$ but just above $z = 0$. The region $z > 0$ is assumed to be free space with permittivity ε_0 and permeability μ.

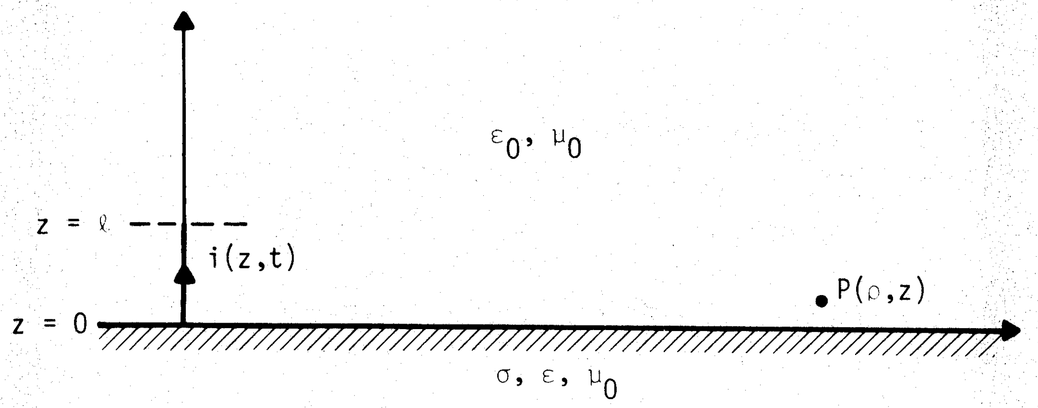

Figure 1. The basic prototype model for looking at pulse propagation over the earth.

The primary objective of this prototype problem is to determine the electromagnetic field at a distance point $P(\rho,z)$ bearing in mind that symmetry in ϕ prevails (i.e., $\partial/\partial\phi = 0$).

A word might be said here about what we mean by the specification of a filamental current $i(z,t)$ on the axis $\rho = 0$. The link is Ampere's Law which says that

$$2\pi\rho_0 H_p(\rho_0,z) = i(z,t) \tag{4}$$

where $H_\rho(\rho_0,z,t)$ is the azimuthal magnetic field as a function of time at some radius ρ_0 that just exceeds the radius of the current filament. Thus, at this stage, we do not need to describe the discharge mechanism explicitly.

With reference to the model depicted in Figure 1 we make some further idealizations that are later relaxed. Specifically we assume that the fields are to be observed in the insulator at $z = 0$ just above the earth's surface. Also we assume that the horizontal range is much greater than the maximum height ℓ (chosen arbitrarily) of the source dipole. That is $\rho \gg \ell$. This inequality is a necessary but not a sufficient condition for the validity of the "moment approximation" (Refs. 1-3).

In the transform s domain we can now write down an expression for the vertical electric field $E_z(\rho,0,s)$, denoted $E_z(s)$, at $P(\rho,0)$ as follows

$$E_z(s) = E_0(s)\ F(p) \qquad (5)$$

where F(p) is an attenuation function usually attributed to Sommerfeld and Norton. The normalization is such that F(p) = 1 if the ground conductivity were infinite (i.e., $\sigma = \infty$). Thus $E_0(s)$ is really twice the free-space field of the current element at a distance ρ in the broadside direction. Thus we may write, without further approximation (Ref. 3), that

$$E_0(s) = -\frac{\mu_0 s}{2\pi\rho}\ e^{-s\rho/c}\left[1 + \frac{c}{s\rho} + \left(\frac{c}{s\rho}\right)^2\right]M(s) \qquad (6)$$

where

$$c = (\varepsilon_0\ \mu_0)^{-1/2} \simeq 3 \times 10^8$$

The attenuation function mentioned above can be well approximated as follows:

$$F(p) \simeq 1 - i(\pi p)^{1/2}\ e^{-p}\ \ \text{erfc}\ (ip^{1/2}) \qquad (7)$$

or

$$F(p) \simeq 1 - i2p^{1/2}e^{-p} \int_{ip^{1/2}}^{\infty} e^{-x^2}\ dx \qquad (8)$$

when p the numerical distance is defined here by

$$p = p(s) \simeq -\frac{s^2\rho}{2\sigma\mu_0 c^3(1 + \delta)}\left(1 - \frac{s}{\sigma\mu_0 c^2(1 + \delta)}\right) \qquad (9)$$

where, in turn, $\delta = \varepsilon s/\sigma$. A self contained derivation of Equation 7, or Equation 8, is available in a recent textbook (Ref. 4). As indicated then some key approximations are made in the process. For example, in performing the saddle point method of integration, we have assumed that, in the present context, $1\ s\rho/c \gg 1$. This far-field approximation is clearly violated in the near field but in that case $|p| \ll 1$ and $F(p) \simeq 1$. Nevertheless, as we indicated below, this point requires further study. Another key approximation is that $|(\sigma + \varepsilon s)/(\varepsilon_0 s)|^2 \gg 1$. However, this restriction is not particularly stringent if the form for the numerical distance given by (Eq. 9) above is employed. Of course some additional simplifications are possible if p is further simplified to

$$p \simeq - Ks^2\rho \tag{10}$$

where

$$K = (2\sigma\ \mu_0 c^3)^{-1} = (240\pi\sigma c)^{-1}$$

This form for p is valid if $|\varepsilon s/\sigma|^2$ is sufficiently small. It is important to note here that the form given by (Eq. 10) neglects displacement currents in the ground but not in the air.

To obtain results in the time domain we need to invert (Eq. 5). In their words, if $E_z(\rho,0,t)$ or $E_z(t)$ is the vertical electric field at $P(\rho,0)$ for a source dipole moment $M(t)$, then we must deal with

$$E_z(s) = \int_0^\infty E_z(t)\ e^{-st}\ dt = L\ E_z(t) \tag{11}$$

where $E_z(s)$ is known. The inverse operation is

$$E_z(t) = \frac{i}{2\pi i} \int_{b-i\infty}^{b+i\infty} E_z(s)\ e^{st}\ ds = L^{-1}\ E_z(s) \tag{12}$$

where $b(>0)$ is some small real constant.

Note: An equivalent statement of (Eq. 12) is:

$$E_z(t) = \frac{1}{2\pi} \int_{-\infty+ib}^{+\infty+ib} E_z(i\omega)e^{i\omega t}\ d\omega$$

The evaluation of the inverse transform given by (Eq. 12) is not a simple task even when various simplifications in the s plane are made. Furthermore the validity in the time domain is not easily established. To cope with this problem we describe the time domain results for the simplest case first and then progressively increase the complexity by relaxing the various approximations.

To provide a comparison between various approaches it is also convenient to adopt a standard source. A desirable form is the ramp current source chosen such that

$$\frac{dm(t)}{dt} = A\ u(t) \tag{13}$$

where

$$u(t) = 1 \text{ for } t > 0$$
$$= 0 \text{ for } t < 0$$

is the unit (Heaviside) step function. Then $m(t) = Atu(t)$ which rises linearly with time for all $t > 0$. Now clearly

$$M(s) = L\ m(t) = A/s^2 \tag{14}$$

Thus, our task is to evaluate

$$E_z(t) = -\frac{\mu_0}{2\pi\rho} AR\left(t - \frac{\rho}{c}\right) \tag{15}$$

where

$$R(t) = L^{-1}\frac{1}{s}\left[1 + \frac{c}{s\rho} + \left(\frac{c}{s\rho}\right)^2\right]F(p)$$

or

$$R(t) \approx L^{-1} \frac{1}{s} \left[F(p) + \frac{c}{s\rho} + \left(\frac{c}{s\rho}\right)^2 \right] \qquad (16)$$

In the simplest case, where $\sigma = \infty$, $F(p) = 1$ and we easily deduce that

$$R(t) = R_0(t) = \left[1 + \frac{ct}{\rho} + \left(\frac{ct}{\rho}\right)^2 \frac{1}{2} \right] u(t) \qquad (17)$$

For the "radiation zone", $\rho \gg ct$. The response has the expected step-function form. The function $R_0(t - \rho/x)$ is shown plotted in Figure 2 for various ranges; ρ = 20, 50 and 100 km. The rise of these curves above 1.0 exemplifies the influence of the induction and static field components.

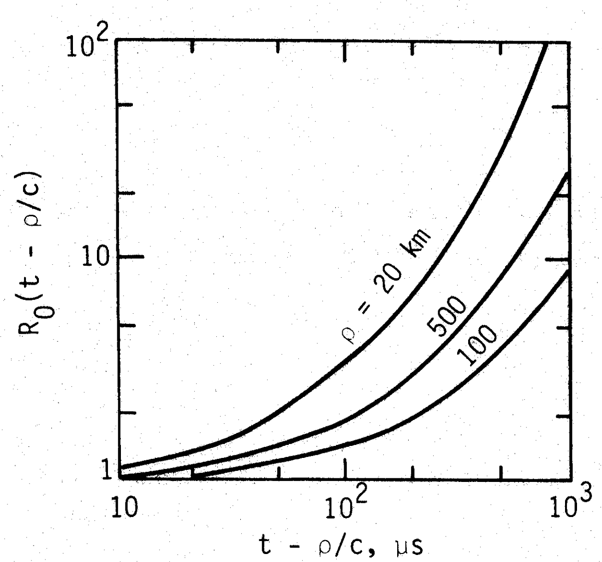

Figure 2. The transient response $R_0(t - \rho/c)$ at distance ρ from a source dipole energized by a ramp current at $t = 0$ computed for a flat perfectly conducting earth. (Curves are actually applicable for finitely conducting earth if $\sigma > 10^{-3}$.)

In the case of a finitely conducting earth we find that

$$R(t) \approx \left[R_r(t) + \frac{ct}{\rho} + \frac{1}{2}\left(\frac{ct}{\rho}\right)^2\right]u(t) \tag{18}$$

where

$$R_r(t) = L^{-1}\frac{1}{s} F(p) \tag{19}$$

is the "radiation field" that is primarily affected by the finite conductivity of the earth. In the simplest case, where we neglect the displacement currents in the ground, we may use (Eq. 10) for p in (Eq. 7) to yield the remarkably simple form (Refs. 5 and 6):

$$R_r(t) = \left[1 - \exp\left(\frac{t^2}{4\ K\rho}\right)\right] u(t) \tag{20}$$

where $K = (240\pi\sigma c)^{-1}$. As indicated this rises from zero at $t = 0$ and reaches a maximum of 1.0 for $t \to \infty$.

The function $R(t - \rho/c)$, as defined by (Eq. 18) with $R_r(t)$ given by (Eq. 20), is shown plotted by the solid curve in Figure 3 as a function of the retarded time $t - \rho/c$ is microseconds. The range ρ is taken to be 50 km and the conductivity σ is 10^{-1} mhos/m. The slight rise of the function $R(t - \rho/c)$ above unity for the later times is the influence of the static and induction fields.

We now need to deal with the displacement currents in the ground. An approximate approach (Ref. 5) is based on the evaluation of the inverse transform

$$R_r(t) = L^{-1}\frac{1}{s} F(p) \tag{21}$$

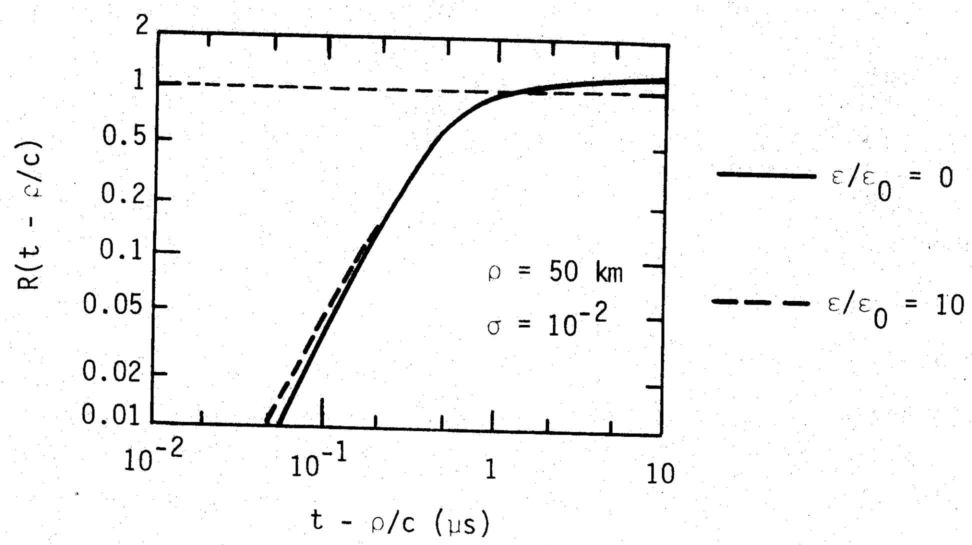

Figure 3. The transient response $R(t - \rho/c)$ at distance ρ from a source dipole for a finitely conducting earth showing the effect of displacement currents at small times.

where

$$p = p(s) = -\frac{s^2 K\rho}{1 + \delta}\left(1 - \frac{sc}{K(1 + \delta)}\right) \tag{22}$$

where $\delta = \varepsilon s/\sigma$. The validity of the asymptotic expansion

$$F(p) \simeq -\frac{1}{2p} - \frac{1.3}{(2p)^2} - \frac{1\cdot 3\cdot 5}{(2p)^3} - \cdots \tag{23}$$

is tacitly assumed bearing in mind that p is large for small times. Then we find that

$$R_r(t) \simeq \left[1 - \exp\left(-\frac{t^2}{4K\rho}\right)\right] + \Delta(t) \tag{24}$$

where $\Delta(t)$ is a first order correction (Refs. 5 and 7) given by

$$\Delta(t) \simeq \frac{2\varepsilon_0}{\sigma t}\left(\frac{\varepsilon}{\varepsilon_0} + 1\right) J(x) \tag{25}$$

and

$$J(x) = x^2(1 - x^2)e^{-x^2} \tag{26}$$

Calculations of the response $R(t-\rho/c)$ based on (Eqs. 18 and 24) are shown in Figure 3 by the dashed curve. The rather small enhancement of the transient response at short times, when displacement currents are considered, is typical. Actually the inverse transform of (Eq. 21) can be performed with the more accurate form of $p(s)$ given by (Eq. 22). Such an evaluation was carried out by Walsh and Rahman (Ref. 8). They compared their calculations with those based on our (Eq. 24) and found the results were virtually identical in the region of interest (i.e. 10^{-2} to 1 μs for ranges from 1 to 100 km and $\sigma \simeq 10^{-2}$ to 10^{-3} mhos/m). Walsh (Private Communication) has also extended these transient calculations for the flat earth model to raised antennas where again the Sommerfeld-Norton form of the steady state attenuation function is employed at the outset.

In a recent interesting paper Cooray and Lundquist (Ref. 9) studied the changes in the rise times and the attenuation of the initial peaks of the radiation fields from lightning constructed according to the most recent observational data on lightning return stroke radiation fields. Their results showed the importance of taking into account the propagation effects when trying to estimate rise time, rate of rise, and peak current from measured fields. The predictions based on the formulation of Wait

(Ref. 5) showed good agreement with their data. As pointed out by Cooray and Lundquist (Ref. 9) the distance dependence of the risetime is critically dependent on the effective earth conductivity.

In dealing with pulse propagation over the earth various approximations are made to facilitate both the spatial and temporal integrations. Various checks are desirable in addition to observational tests that may not be convincing. When there is an exact solution to a prototype problem, one should confirm that the approximated solution reduces to such a form under appropriate conditions. For example van der Pol (Ref. 10) has provided us with an exact solution of the problem formulated above in the extreme limiting situation when the ground conductivity σ was set equal to zero. A careful comparison (Ref. 11) of our approximated solution outlined here concluded that the Sommerfeld-Norton form of the attenuation function is applicable to pulse propagation over a flat earth provided the dielectric constant (i.e., $\varepsilon/\varepsilon_0$) is reasonably large compared with unity.

Some further questions and concerns have arisen more recently about the validity of the attenuation function used in the transient calculations for the flat earth model. For example, Haddad and Chang (Ref. 12) developed an accurate procedure to deal with the inverse frequency transform of the exact integral formula for the dipole source excited by impulsive current. They showed an interesting comparison of the early time solution based on (Refs. 5 and 13), and their admittedly more accurate (but more complicated form). As they conclude the results for a range of 10 km, for $\sigma = 10^{-2}$ and 10^{-3} mhos/m agreed in their "overall pattern" for all times from 0.01 to 10 μs. However, there were differences in the magnitudes and the precise locations of the nulls and peaks. It would have been useful if Chang and Haddad (Ref. 12) had integrated their responses twice with respect to time so that their results could be compared directly with our results for a ramp current source. It appears that such a step would tend to submerge the differences in the various theoretical approaches (not meaning we wish to sweep them under the rug!).

Using a surface impedance model Wait (Ref. 14) also reformulated the pulse problem for the dipole on the boundary and showed that the traditional approximations in the frequency domain could omit a slow tail in the transient response. However, the presence of the static and induction fields, not explicitly considered by Chang and Haddad (Ref. 12), would tend to swamp the influence of the slow tail not to mention ionospheric influences. Nevertheless, further work on these more accurate representations would be well worthwhile. A promising approach is to make use of the known properties of the integral representations of the incomplete Hankel functions (Refs. 15, 16). In this manner one may avoid the asymptotic approximations that are apparently violated when the Sommerfeld-Norton form is used for low frequencies (or large times).

Finally it might be mentioned that integral equation approaches to deal with the influence of finite conductivity are fruitful. In fact Hufford (Ref. 17) shows many years ago that the Sommerfeld-Norton form could be developed by a direct application of Green's theorem using essentially a surface impedance model of the homogeneous half-space. This approach has been followed up by others (Refs. 18, 19) where more complicated geometries are treated. A contemporary review appears in (Ref. 3) (see Chapter VII).

Footnote: In Theoretical Note 311, June 1980, K.S.H. Lee, derives the Sommerfeld-Norton form of the attenuation function from an integral equation whose use is attributed to C. L. Longmire that has the same form as in References 17, 18, and 19. Lee also obtains the expected time-domain response as in Reference 5.

III. SPHERICAL EARTH MODEL

In discussing pulse propagation over the earth's surface one should really consider earth curvature. Or at least one should ascertain if the assumed flat earth model is adequate. The most direct approach is to formulate the problem in spherical geometry which leads to spherical Bessel functions and Legendre polynomials. In some cases this intricate approach is required and the numerical results in both the frequency and time domain are useful. However if the range is short it is desirable to examine the spherical model in such a fashion that the corrective terms to the flat earth model are clearly evident.

The curvature corrected flat-earth attenuation function was developed by Bremmer and Wait in 1955 and the results appeared in two papers (Refs. 20, 21). The method is based on an asymptotic development of the field (Spatial) transform that permitted a term by term inversion to yield the desired time-harmonic response. Thus, for example we were able to say that corrected attenuation function $\bar{F}(p)$, in the s plane, is

$$F(p) = F(p) - \frac{1}{2}\Omega\left[1 - i(\pi p)^{1/2} - (1 + 2p)F(p)\right]$$

$$+ \Omega^2\left[1 - i(\pi p)^{1/2}(1 - p) - 2p + \frac{5}{6}\ p^2 + \left(\frac{1}{2}\ p^2 - 1\right)F(p)\right]$$

$$+ \text{terms in } \Omega^3,\ \Omega^4,\ \text{etc.} \qquad (27)$$

where

$$\Omega = \frac{[\mu\omega\sigma(1 + \delta)]^{3/2}}{(s/c)^4 a} \quad \text{and} \quad \delta = \frac{\varepsilon s}{\sigma}$$

and a is the earth's radius. The flat earth attenuation function F(p) and the numerical distance p are as defined by Equations 7 and 9 above. In spite of the complexity of Equation 27 the inverse Laplace transform can be obtained without undue difficulty (Refs. 5, 13). We do not need to exhibit the analytical details here. But the transient response for the standard ramp current source dipole is shown in Figure 4 where corrective terms for earth curvature are indicated. For the range indicated, the correction terms are quite small and in fact the third order correction would not produce a further modification at least to the graphical accuracy of the figure.

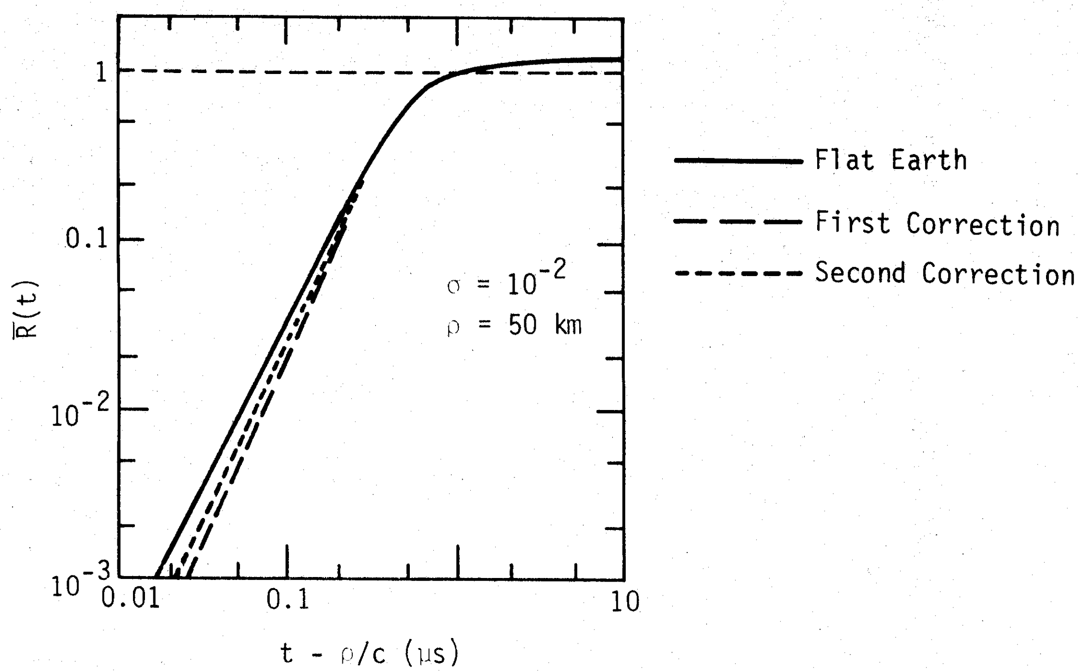

Figure 4. The transient response R(t-ρ/c) at (great circle) distance ρ from a source dipole for a finitely conducting spherical earth. The flat earth model is shown along with first and second order corrections. When the third order correction is added, there is no perceptible change to the second order corrected curve.

Using the curvature corrected formulae, the transient responses, for the ramp current source dipole, are plotted in Figures 5a and 5b for ranges of 20 and 50 km, respectively. Both the effects of changing ground conductivity and displacement currents are shown on the figure. Not surprisingly the initial rise of the waveforms are delayed significantly when the ground conductivity is reduced from 10^{-2} to 10^{-3} mho/m.

Additional short time transient responses are given in Reference 13 for both ramp- and step-function current sources. Some of these results were privately communicated by Dr. John Malik and J. R. Johler.

At greater ranges (e.g., >300 km or so) the curvature corrected flat earth formulae become poorly convergent and many higher order corrections are needed. The mathematical convergence is conjectured to be absolute in spite of the asymptotic nature of the attenuation function in transform space. A related question was addressed by Wait (Ref. 22). There it was shown that the asymptotic expansion of F(p), indeed, could be inverted term by term to lead to a convergent time-domain response.

As a practical matter, it is more convincing if somewhat inelegant to work directly with the residue series representations for the spherical earth attenuation function. Such a series is highly convergent deep in the shadow zone well (beyond the optical horizon). This approach is appropriate for ranges beyond 300 km.

In accordance with conventional notation, we now designate the attenuation function, for time harmonic variation, by $W(\omega)$. Actually it is normalized so that it would reduce to F(p) for the flat earth model. Again we are dealing with pure ground wave transmission so atmospheric effects are being ignored--at least for the moment. Thus, with reference to a general treatment of the problem (Ref. 23), we may write for a time factor $\exp(i\omega t)$:

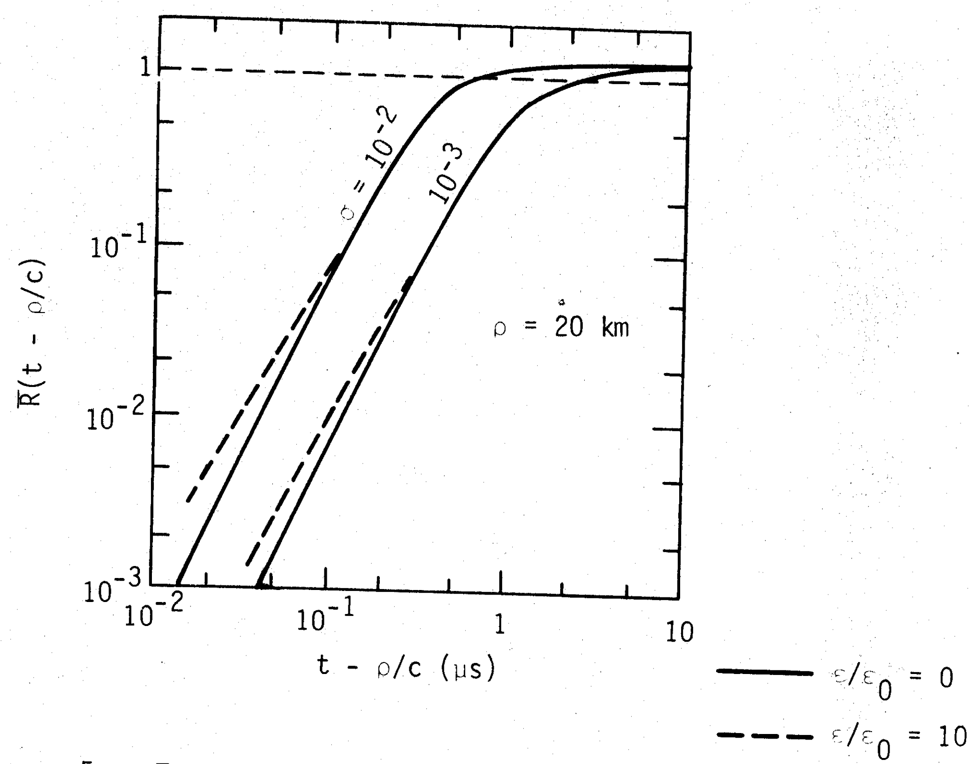

Figure 5a. Transient response at distance ρ on spherical earth showing dependence on ground conductivity and displacement currents (ρ = 20 km).

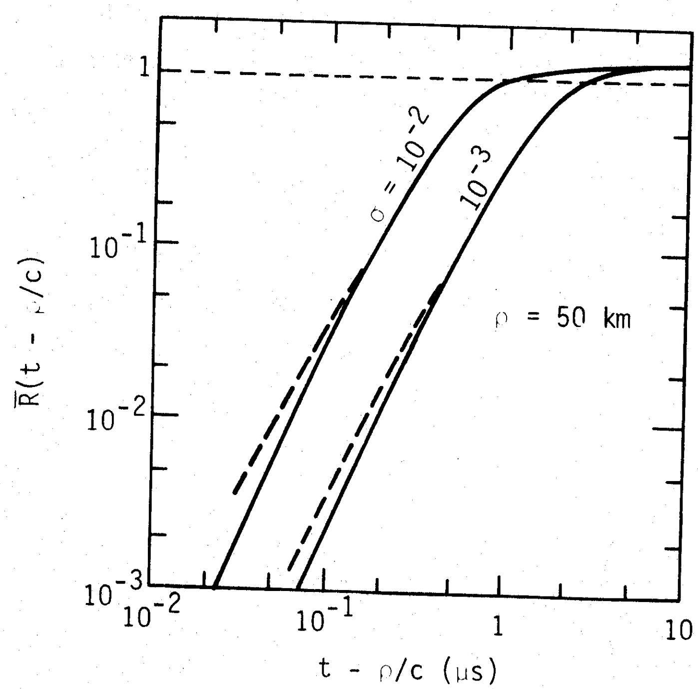

Figure 5b. Same as Figure 5a for ρ = 50 km.

$$W(\omega) = (\pi x)^{1/2} e^{-i\pi/4} \sum_{s=1,2,3...}^{\infty} \frac{e^{-ixt_s}}{t_s - q^2} \cdot \frac{W_1(t_s - y)}{W_1(t_s)} \cdot \frac{W_1(t_s - y')}{W_1(t_s)} \tag{28}$$

where

$$y = \left(\frac{2c}{\omega a}\right)^{1/3} \frac{\omega}{c} z \quad , \quad x = \left(\frac{\omega a}{2c}\right)^{1/3} \frac{d}{a}$$

$$y' = \left(\frac{2c}{\omega a}\right)^{1/3} \frac{\omega}{c} z' \quad , \quad iq = \left(\frac{\omega a}{2c}\right)^{1/3} Z_g(\omega)/120\pi$$

and where Z_g is the effective surface impedance at the ground (i.e., at $r = a$ in spherical coordinates). The function $W_1(t)$, of argument t not to be confused with time, is defined by

$$W_1(t) = \pi^{1/2}[Bi(t) - iAi(t)] \tag{29}$$

where Ai(t) and Bi(t) are conventional Airy functions as defined for example in the Handbook of Mathematical Functions. The summation in Equation 28 extends over the discrete roots t_s which are solutions of

$$[dW_1(t)/dt] - qW_1(t) = 0 \tag{30}$$

This "mode equation" assures that the surface impedance boundary condition is satisfied at $r = a$. In fact, the surface impedance for a homogeneous ground with properties σ, ε and μ_0 takes the form

$$Z_g = \left(\frac{i\mu_0\omega}{\sigma + i\varepsilon\omega}\right)^{1/2} \left(1 - \frac{i\varepsilon_0\omega}{\sigma + i\varepsilon\omega}\right)^{1/2}$$

which is, in fact, exactly true for a vertically polarized plane wave incident at grazing angles on a homogeneous half-space model of the earth. Actually, stratified earth models can be handled easily by using more general forms (Refs. 20, 23) of Equation 31.

The geometry of the situation is shown in Figure 6 where key parameters are indicated. Over the years, a great deal of effort has gone into the evaluation of the time-harmonic form $W(\omega)$ such as the residue series given by Equation 28 although the notation may be different (e.g., Hankel functions of order 1/3 are sometimes used in place of the less confusing Airy

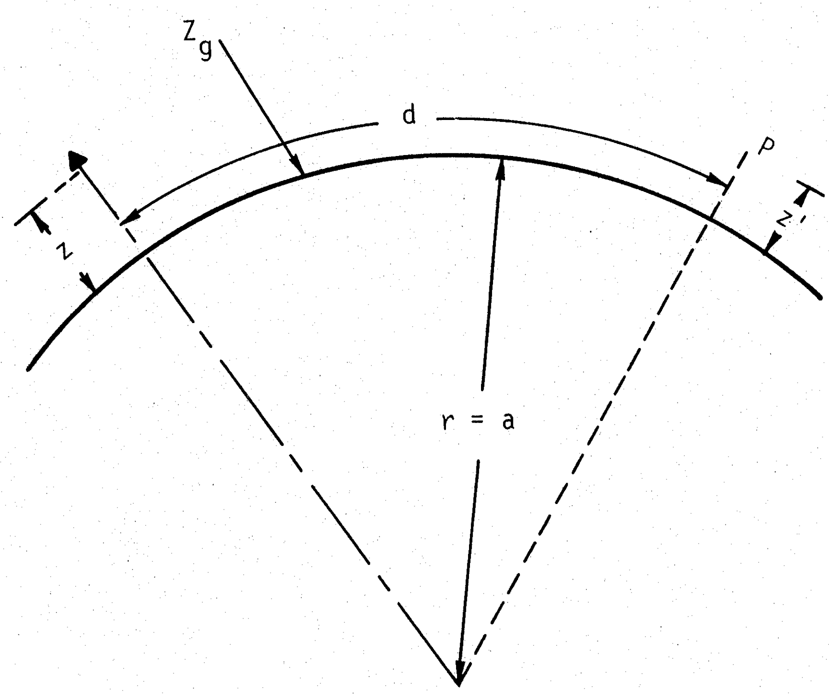

Figure 6. Geometry for spherical earth model for ground wave transmission from vertical (i.e., radial) current element at P at height z'.

functions). Less effort has been made to determine the transient solutions. Our approach (Ref. 24) was to perform a straightforward inverse Fourier transform of the time harmonic data. A similar approach was employed by the Soviets (Ref. 25) and also by J. R. Johler (private communications in 1965). Many of these results are summarized and reviewed in Reference 13 which is readily accessible.

To obtain the desired transient response for a ramp current source, we may proceed formally by using the inverse Fourier integration over real ω:

$$R(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{W(\omega)}{i\omega} e^{i\omega t} d\omega$$

$$= 1 - \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{1 - W(\omega)}{i\omega} e^{i\omega t} d\omega$$

$$= 1 + \frac{2}{\pi} \int_{0}^{\infty} \frac{I_m W(\omega)}{\omega} \cos(\omega t) d\omega \qquad (31)$$

To illustrate our procedures, we will assume here that the earth is perfectly conducting (essentially the same as sea water). Then to perform the integration in Equation 31 we employ the simplified form of Equation 28 which, for z = z' = 0, is given by (Ref. 26):

$$W(\omega) \simeq W^{\infty}(\omega) = (2\pi x)^{1/2} e^{-i\pi/4} \sum_{s=0}^{\infty} \frac{e^{-ix\tau_s^{\infty}}}{2\tau_s^{\infty}} \qquad (32)$$

where

$$\tau_0^\infty = 0.808\ e^{-i\pi/3}$$

$$\tau_1^\infty = 2.577\ e^{-i\pi/3}$$

$$\tau_2^\infty = 3.824\ e^{-i\pi/3}$$

$$\tau_3^\infty = 4.892\ e^{-i\pi/3}$$

$$\tau_s^\infty \simeq \frac{1}{2}\left[3\pi\left(s + \frac{1}{4}\right)\right]^{2/3} e^{-i\pi/3} \quad , \quad \text{for } s \geq 5$$

An alternative form of Equation 32, usable for small x and/or low frequencies, is adapted from (Refs. 5, 26) or specialized from Equation 27 above:

$$W^\infty(\omega) = 1 - \frac{\pi^{1/2}}{8}(1 + i)g^{3/2} + \frac{7i}{120}g^3 + \frac{\pi^{1/2}7(1 - i)}{2048}g^{3/2}$$

$$\pm \text{ terms in } g^6,\ g^{15/2},\ \text{etc.} \qquad (33)$$

where

$$g = (\omega D/c)(\omega a/c)^{-2/3}$$

The analytical form of Equation 33 allows us to perform the integration in Equation 3 to yield the expansion

$$R(t) \simeq 1 - \frac{1}{4\sqrt{2}\ T^{1/2}} + \frac{7}{2048\sqrt{2}\ T^{3/2}} \pm \text{terms in } \frac{1}{T^{5/2}},\ \frac{1}{T^{7/2}},\ \text{etc.} \qquad (34)$$

where

$$T = tca^2/d$$

This representation (Ref. 27) is obviously most suitable for "large" times in the transient response.

Another approach (Ref. 27) is to evaluate the infinite integral in Equation 31 by the saddle point method (Ref. 28). This leads to

$$R(t) \simeq \frac{3}{2^{3/2}} \sum_{s=0}^{\infty} \exp\left(- \frac{2}{3^{3/2}} \left|\tau_s^{\infty}\right|^{3/2} \frac{1}{T^{1/2}}\right) / \left|\tau_s^{\infty}\right|^{3/2} \tag{35}$$

which is valid only for "small" times (i.e., T small).

To cope with the difficult intermediate region where T is neither large or small, we must resort to numerical integration using the latter form of Equation 31 that involves integration over real frequencies from 0 to ∞. Actually, this task is not that difficult, if good data for $W(\omega)$ is available (Ref. 26), because $W(\omega)$ is heavily damped at higher frequencies.

The overlapping validity of the three approaches to the transient response for the spherical earth model is illustrated in Figure 7. Such hydrid numerical-analytical approaches are highly recommended in dealing with such transient waveform predictions. The numerical method involving double integration over wave number and frequency is particularly prone to error at small times in the transient response. This point of view has been also expounded in an elegant fashion by Felsen (Ref. 29).

Another important subject that we call attention to is the influence of land/sea boundaries and other mixed path conditions (Ref. 30). Some analytical work on this topic has been carried in the context of pulsed ground waves (Refs. 22, 31, 32).

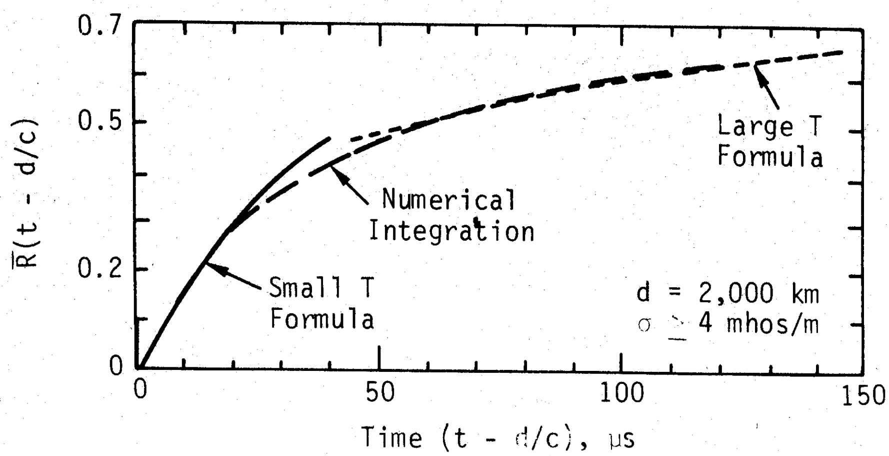

Figure 7. The transient response for a ramp current source on a spherical earth model showing comparisons between three different methods (Ref. 27) of calculation.

To illustrate the mixed path problem we consider a two-section ground. As indicated in Figure 8, the total range is d, the land portion of conductivity σ_1 has length d_1 and the sea portion of conductivity σ_1 is of length $d - d_1$. All displacement currents are neglected. The attenuation function for an observer at P on the land portion can be derived from an integral equation formulation (Refs. 19, 30) and In the s (=iω) plane we obtain the convolution integral solution:

$$F'(s) = F(-Ks^2 d) - s\left(\frac{\varepsilon_0 d}{2\pi c}\right)^{1/2}\left(\frac{1}{\sigma_1^{1/2}} - \frac{1}{\sigma^{1/2}}\right)$$

$$\times \int_0^{d_1} \frac{F(-K(d-\alpha)s^2)F(-K_1\alpha s^2)}{[\alpha(d-\alpha)]^{1/2}}\, d\alpha \qquad (36)$$

where

$$K = (2\sigma\mu_0 c^3)^{-1} = (240\pi\sigma c)^{-1}$$

$$K_1 = (2\sigma_1\mu_0 c^3)^{-1} = (240\pi\sigma_1 c)^{-1}$$

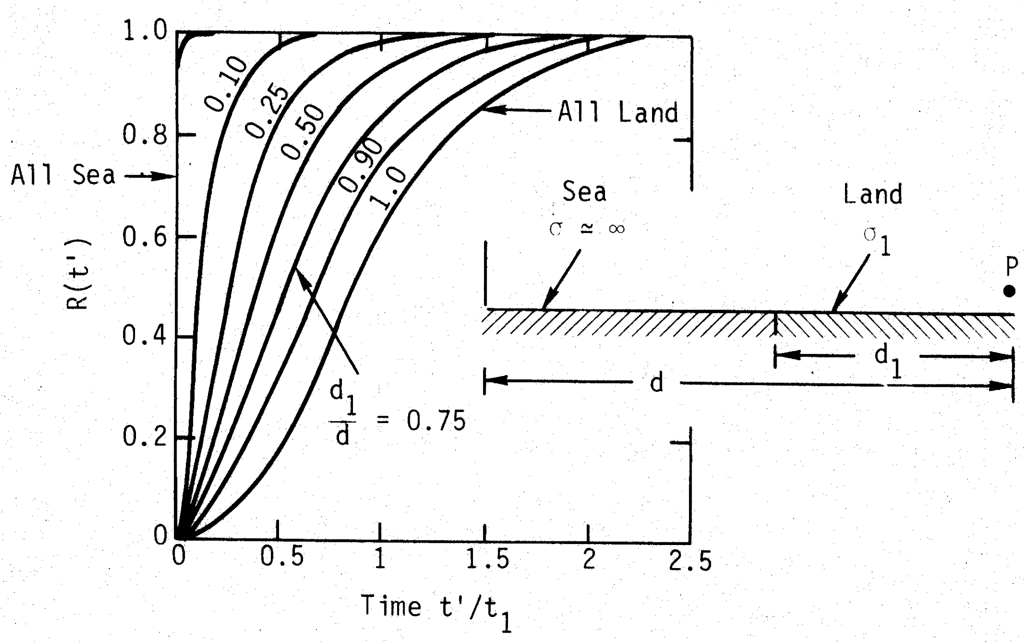

Figure 8. Transient response at P at distance d from the source dipole energized with a ramp current at t = 0. the path has two sections; the first section is perfectly conducting (e.g., sea water) and the second section of length d_1 is finitely conducting (i.e., land). The retarded time $t' = t - d/c$ and the time parameter $t_1 = 2\sqrt{K_1 d_1}$ where $K_1 = (2\sigma_1\mu_0 c^3)^{-1}$.

and the (unprimed) F's are the attenuation functions appropriate for propagation over homogeneous paths. Thus the mixed path attenuation function F'(s) can be obtained by suitable convolving of the homogeneous attenuation functions. The next step of course is to perform the inverse transform of F'(s)/s to get the ramp current response at P. This operation leads in general to a double convolution integral representation for the desired time domain function (Refs. 22, 31, 32). In the case where $\sigma \gg \sigma_1$ (i.e., sea water conductivity ≫ land conductivity) the general result reduces to the remarkable simple form

$$R'(t') = \left\{1 - \exp\left(-\left(\frac{t}{t_1}\right)^2\right) \operatorname{erfc}\left[\frac{t'}{t_1}\left(\frac{d}{d_1} - 1\right)^{1/2}\right]\right\} u(t') \qquad (37)$$

where erfc is the complement of the error function of the indicated argument. Of course if $d_1/d \to 0$, $R'(t') \simeq u(t')$ within this approximation of a perfectly conducting all sea path. On the other hand, if $d_1 \to d$, corresponding to all land the response reduces to

$$R'(t') \simeq R(t') = \left[1 - \exp\left(-(t'/t_1)^2\right)\right] u(t') \tag{38}$$

which is the same as Equation 20.

The mixed path transient response function is sketched in Figure 8 for various values of d_1/d from 0 (i.e., all sea) to 1.0 (i.e., all land).

One of the major simplifications to ground wave propagation theory was the introduction of the surface impedance concept. Thus, for example, layered models (Refs. 13, 19, 20) and mixed path geometries could be considered in a straightforward manner (Refs. 30, 33). With certain limitations the surface impedance can also be applied to the description of lossy hemispherical bosses on an otherwise perfectly conducting plane (Ref. 34). This idea has been followed up in recent years to more complicated situations (Refs. 35, 36). Including the case of wide band signals from lightning propagating over a rough sea (Refs. 37, 38). Malaga, in particular, has considered the attenuation function in the form given by Equation 28 here for a spherical earth model. He introduces a spectral model for the effective (normalized) surface impedance $Z_{eff}/120\pi = \Delta(\omega)$ in the form

$$\Delta(\omega) = \Delta_0(\omega) + \frac{i\omega}{4c} \int_0^\infty dK \int_0^{2\pi} d\theta \, \frac{K^3 \cos^2\theta S(K,\theta)}{\sqrt{K^2 + (2\omega K)c)\cos\theta}} \tag{39}$$

where the first term $\Delta_0(\omega)$ is the normalized impedance for a smooth surface and the second term represents the effective increase which depends on the two-dimensional wave number spectrum $S(K,\theta)$.

Various forms of the wave number spectrum for the rough ocean surface have been proposed (Ref. 39). For a fully developed (steady state) sea they have the form

$$S(K,\theta) = \frac{CM(\theta,\alpha)}{K^p} e^{-\beta(K_m/K)^n} \tag{40}$$

where K_m is the wave number at the peak of the spectrum, p is the spectrum power law dependence, $M(\theta,\alpha)$ is the wind directionality dependence, α is the wind direction relative to the direction of propagation ($\theta = 0$) and C is a proportionality constant. An example from Malaga's (Ref. 38) paper is shown in Figure 9 where the response for a step-current excited dipole is plotted as a function of the retarded time for d = 150 km, wind 0, 20 and 30 knots (see state 0, 4 and 6, respectively) using Phillips isotropic spectrum. Not surprisingly the roughness tends to increase the dispersion somewhat. Further studies at this important effect are certainly warranted. A recent thesis written by Srivastava (Ref. 39) includes a rigorous formulation of time-harmonic scattering from a three-dimensional (moving) surface. This could be the starting point for further studies of the transient problem.

In propagating to long distances over a spherical earth, it is necessary to consider ionospheric influences. This is a vast subject in its own right and the history extends back to the early days of radio when Appleton, Watson-Watt, and others studies the waveforms of radio atmospherics or "sferics" as they are often called. Probably the most convenient approach here is to treat the earth-ionosphere space as a waveguide (Refs. 40, 1). For the time harmonic problem the vertical electric field $E(\omega)$ at (great-circle) distances d as a sum of waveguide modes is

$$E(\omega) \simeq \left[\frac{d/a}{\sin d/a}\right]^{1/2} \frac{(2\pi cd)^{1/2}}{h} \frac{\bar{e}_0(\omega)}{(i\omega)^{1/2}} \sum_{n=0}^{\infty} s_n^{3/2} \delta_n e^{(-2\pi d/h)u_n} e^{-iF(\omega)} \tag{41}$$

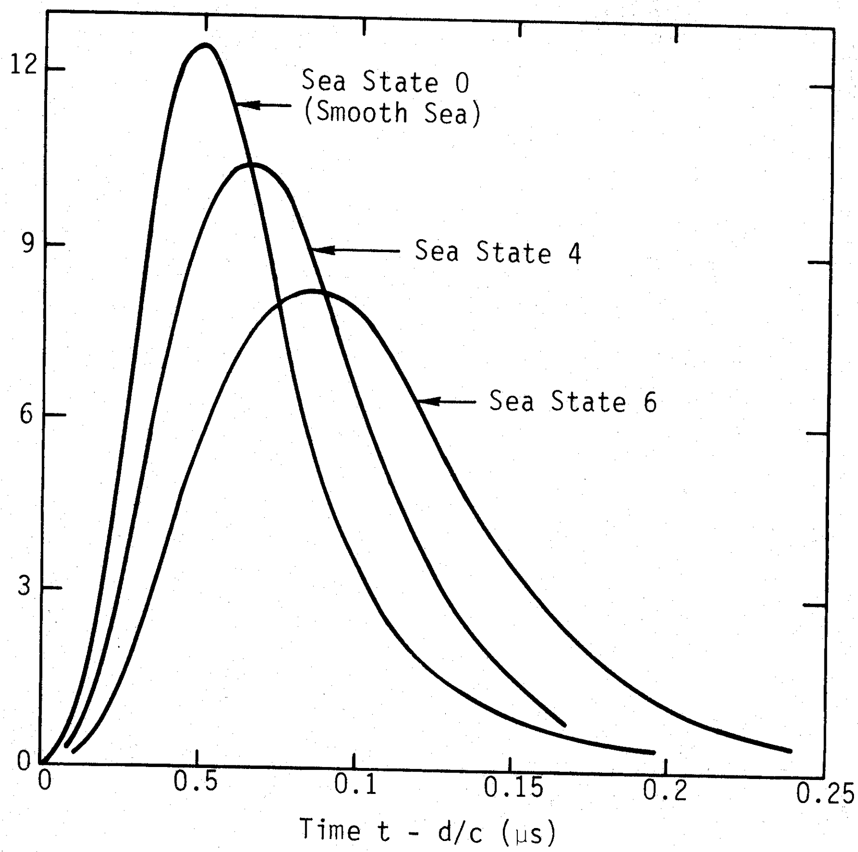

Figure 9. Normalized response for a step function current source at distance 150 km over a fully developed rough sea (downwind) (adapted from Malaga).

where u_n and s_n are the attenuation and phase constants of the waveguide mode of order n, δ_n is an excitation coefficient and roughly $\delta_0 \approx 1/2$ and $\delta_n = 1$ for $n \neq 0$, h is the effective height of the waveguide, a is the earth's radius and $F(\omega) = (\omega/c)s_n d - \omega t$. Here $\overline{e}_0(\omega)$ is the transform of the equivalent radiation field $E_0(\omega)$ at distance d for a perfectly conducting ground plane, i.e.,

$$\overline{e}_0(\omega) = \int_0^\infty \overline{E}_0(t)e^{-i\omega t}dt \tag{42}$$

Now, of course, the desired transient response to the time domain is

$$E(t) = \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} E(\omega) e^{i\omega t} d(i\omega) \tag{43}$$

This turns out to be a rather complicated problem because $u_n(\omega)$ and $s_n(\omega)$ must be obtained as a solution of a mode equation for each frequency. In our early work we simplified the process by using the approximate stationary phase method to evaluate the inverse transform over $i\omega$ or s. Also for very great distances one needs only consider several terms in the waveguide mode expansion.

An example at the transient response is shown in Figure 10 which is adapted from Reference 40. The oscillatory nature of the waveform is typical of transmission of pulses to great distances. In fact, the variation of the quasi-half periods also varies with range in a predictable manner that is only weakly dependent on source characteristics. Such a behavior has been observed by Hepburn (Ref. 41) and there is a qualitative agreement with theory at least for daytime propagation paths. We also call attention to related investigations (Refs. 42-44).

At intermediate ranges it is probably more convenient to use a geometrical optical representation for the total field. This is the approach in a monumental effort by Gardner (Ref. 45). His general formulation and computer codes should lay the foundation for further quantitative analyses.

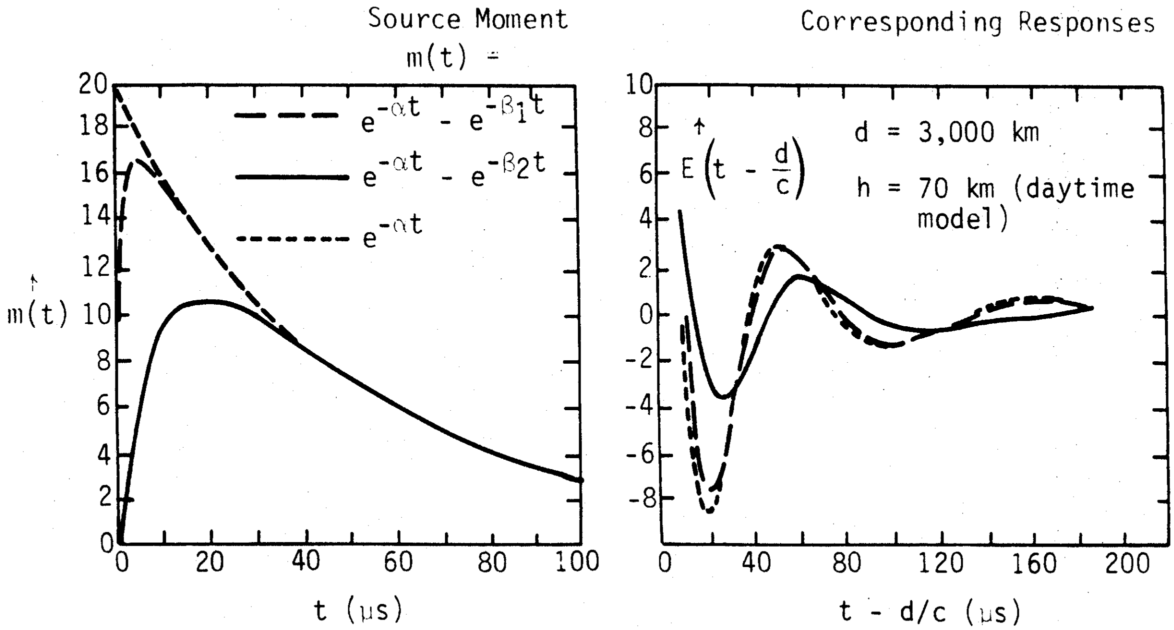

Figure 10. Transient response of then n = 1 dominant mode at a distance of 3000 km from a source dipole with a current moment as shown.

IV. CONCLUDING REMARK

In this review of the propagation phenomenology, we have tried to summarize in a critical fashion the basic theoretical work on the subject. The writer will admit a certain bias to referencing his own work. But the diligent reader will discover many additional references to other investigators in the bibliographies to these papers.

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