Lightning Phenomenology Notes

Note 17

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Motion of Ion Clouds in Air

Carl E. Baum
Air Force Weapons Laboratory

Abstract

After the initial processes in a natural lightning event there can exist clouds of positive and/or negative ions in air. This note considers some of the processes involved in the subsequent motion of such ion clouds in spherical and cylindrical geometries. In the case of spherical geometry such a cloud can stay around for a long time. Such clouds of ions can also be moved by other electric fields in the environs.

I. Introduction

The dynamics of charge in air surrounding a natural lightning event are in general quite complex. Various papers have addressed some of the processes associated with leader pulses and return strokes. The fast times associated with these processes indicate the importance of electronic processes (avalanche and attachment) to form the requisite charge transport. This note addresses some of the late-time phenomena associated with ionic processes.

For some times (early times in general) electrons can dominate the charge transport. The important processes are avalanche [4,5] which is important for electric fields above what is referred to as a breakdown electric field (a few MV/m at STP); in the avalanche process there is an exponential increase of the electron density with time. As the electric field drops below breakdown electrons attach to neutral oxygen molecules with an exponential decay time constant for the electron density of the order of 10 ns [2-5]. At this rate the electron density decays 3 decades in about 70 ns and 4 decades in about 90 ns.

The electron mobility μ_e is about 1 $m^2/(Vs)$ at low electric fields, decreasing for larger electric fields to say .1 $m^2/(Vs)$ at fields near breakdown [2-5]. The ion mobility μ_i depends to a small degree on the ionic species present (and even the electric field) [4-5]. For present purposes let us take

$$\mu_i \simeq 2.3 \times 10^{-4} \ \frac{m^2}{Vs} \qquad (1.1)$$

as at least a representative value for low fields (say less than 1 MV/m). We shall assume this to be constant for the present analysis.

One might also consider the recombination of positive with negative ions and electrons with positive ions [1,2]. However, for simplicity in the present analysis let us assume that only one sign of ions is present after the initial breakdown and decay of the electron density. It is the subsequent motion of the resultant single-signed ions that this note addresses.

II. Uniformly Charged Sphere of Ions

Consider that by some unspecified mechanism a charge Q_0 is present in the air within some radius $0 \leq r < R_0$ in the usual (r,θ,ϕ) spherical coordinate system as indicated in fig. 2.1. Let us assume that this charge is initially uniformly distributed with a charge density ρ_0. Then we have

$$Q_0 = \frac{4}{3} \pi R_0^3 \rho_0 \tag{2.1}$$

As time evolves this charge is contained in some sphere of radius R. Outside this radius the electric field is

$$\vec{E} = \vec{1}_r E_r$$
$$E_r = \frac{Q_0}{4\pi\varepsilon_0 r^2} \quad \text{for } r > R \tag{2.2}$$

The charge within a radius r is

$$Q(r,t) = 4\pi \int_0^r r'^2 \rho(r',t)\, dr' \tag{2.3}$$

giving an electric field (radial)

$$E_r(r,t) = \frac{Q(r,t)}{4\pi\varepsilon_0 r^2} = \frac{1}{\varepsilon_0 r^2} \int_0^r r'^2 \rho(r',t)\, dr' \tag{2.4}$$

The velocity of the ions is radially outward for a unipolar (single sign) ρ as

$$v_r(r,t) = \mu_i |E_r(r,t)| = \frac{\mu_i}{\varepsilon_0 r^2} \left| \int_0^r r'^2 \rho(r',t)\, dr' \right| \tag{2.5}$$

The current density is also radial as

$$J_r(r,t) = \rho(r,t)\, v_r(r,t) \tag{2.6}$$

Now the equation of continuity gives

$$\nabla \cdot \vec{J}(r,t) = -\frac{\partial}{\partial t} \rho(r,t) = \frac{1}{r^2} \frac{\partial}{\partial r} [r^2 J_r(r,t)] \tag{2.7}$$

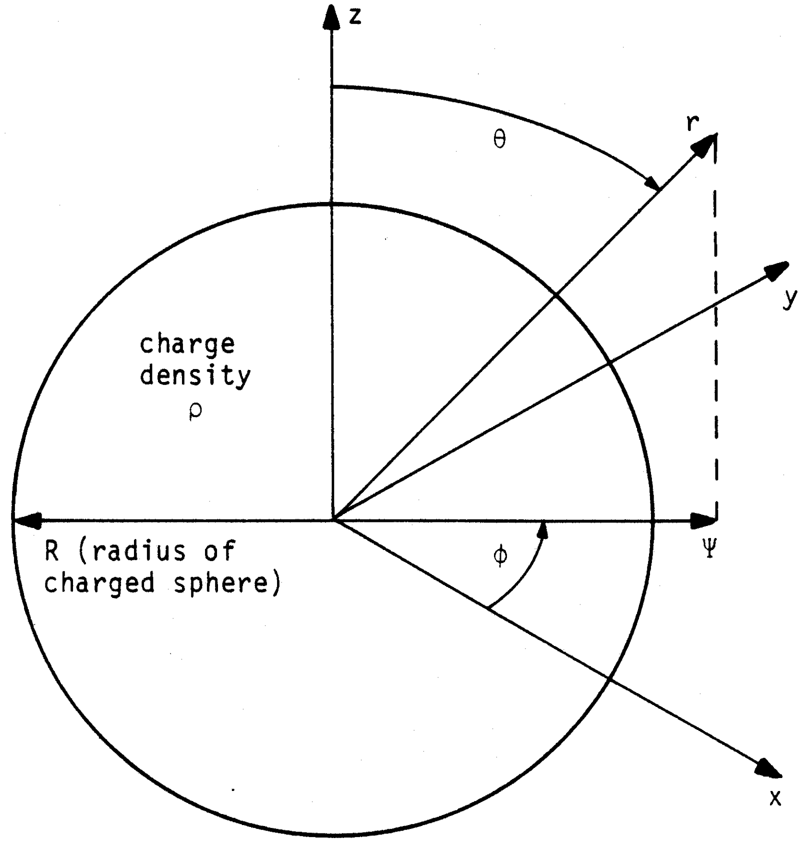

Fig. 2.1. Expansion of Uniformly Charged Sphere of Ions in Air

Thus

$$- \frac{\partial}{\partial t} \rho(r,t) = \nabla \cdot [\rho(r,t)\ v_r(r,t)]$$

$$= \frac{1}{r^2} \frac{\partial}{\partial r} [r^2 \rho(r,t)\ v_r(r,t)]$$

$$= \frac{\mu_i}{\varepsilon_0 r^2} \frac{\partial}{\partial r} [\rho(r,t) \left| \int_0^r r'^2 \rho(r',t)\ dr' \right|] \tag{2.8}$$

Since ρ is assumed to have a single sign this can be rewritten as

$$- \frac{\partial}{\partial t} |\rho(r,t)| = \frac{\mu_i}{\varepsilon_0 r^2} \frac{\partial}{\partial r} [|\rho(r,t)| \int_0^r r'^2\ |\rho(r't)|\ dr'] \tag{2.9}$$

Note that this assumption of a single sign for the charge density is important if one is to ignore recombination between positive and negative ions [1]. Such recombination is not included in these equations.

Let us look for a solution of (2.9) in the form of a uniform charge density, i.e. not a function of r for $0 \leqslant r < R$. Then

$$- \frac{\partial}{\partial t} |\rho(t)| = \frac{\mu_i}{\varepsilon_0} |\rho(t)|^2 \frac{1}{r^2} \frac{\partial}{\partial r} \int_0^r r'^2\ dr'$$

$$= \frac{\mu_i}{\varepsilon_0} |\rho(t)|^2 \tag{2.10}$$

This has a solution

$$- \int_{|\rho_0|}^{|\rho(t)|} \frac{d\rho'}{\rho'^2} = \frac{\mu_i}{\varepsilon_0} \int_{t_0}^{t} dt'$$

$$|\rho(t)|^{-1} - |\rho_0|^{-1} = \frac{\mu_i}{\varepsilon_0} (t - t_0) \tag{2.11}$$

$$|\rho(t)| = \{\frac{\mu_i}{\varepsilon_0} (t - t_0) + |\rho_0|^{-1}\}^{-1}$$

One might set $t_0 = 0$, but a more convenient choice is

$$t_0 = \frac{\varepsilon_0}{\mu_i |\rho_0|} \tag{2.12}$$

so that

$$|\rho(t)| = \frac{\varepsilon_0}{\mu_i t} = |\rho_0| \frac{t_0}{t} \text{ for } t > t_0 \tag{2.13}$$

This result indicates that an initially uniform charge density evolves as a uniform charge density. Note that (2.4) indicates that only the charge within a radius r influences the electric field at r, and hence the motion of charge at this radius. The presence (or lack of) charge at larger radii does not affect the result.

Then conserving charge within a region $0 < r < R$ with R as the outermost radius to which charge extends gives

$$Q_0 = \frac{4}{3} \pi R^3 \rho(t)$$

$$R(t) = [\frac{3}{4\pi} \frac{Q_0}{\rho(t)}]^{\frac{1}{3}} = R_0[\frac{\rho_0}{\rho(t)}]^{\frac{1}{3}} \tag{2.14}$$

$$\frac{R(t)}{R_0} = [\frac{\rho_0}{\rho(t)}]^{\frac{1}{3}} = [\frac{t}{t_0}]^{\frac{1}{3}}$$

which is quite simple in form.

The radial velocity of this outer boundary is

$$v_R(t) = \frac{dR}{dt} = \frac{1}{3} R_0 t_0^{-\frac{1}{3}} t^{-\frac{2}{3}} = \frac{R_0}{3t_0} (\frac{t_0}{t})^{\frac{2}{3}} = \frac{R_0}{3t_0} (\frac{R_0}{R(t)})^2 \tag{2.15}$$

The electric field there is

$$|E_R(t)| = \frac{1}{\mu_i} v_R(t) = \frac{R_0}{3\mu_i t_0} (\frac{t_0}{t})^{\frac{2}{3}} = \frac{R_0}{3\mu_i t_0} (\frac{R_0}{R(t)})^2 \tag{2.16}$$

Setting the initial electric field E_0 at the outer boundary as

$$E_0 = |E_R(t_0)| = \frac{R_0}{3\mu_i t_0} = \frac{R_0}{3} \frac{|\rho_0|}{\varepsilon_0} \tag{2.17}$$

gives expressions for the initial charge as

$$|\rho_o| = 3\,\frac{\varepsilon_o E_o}{R_o}$$

$$|Q_o| = \frac{4}{3}\,\pi R_o^3\,|\rho_o| = 4\pi R_o^2 \varepsilon_o E_o \tag{2.18}$$

Now using

$$E_o \simeq 2\,\frac{MV}{m} \quad \text{(around breakdown)}$$

$$\mu_i \simeq 2.3 \times 10^{-4}\;\frac{m^2}{Vs} \tag{2.19}$$

let us consider the evolution of a sphere of charge in air as indicated in table 2.1. Note here that as R increases from R_o the time increases by the cube, showing that the ionic charge in the air persists within interesting radii (in a lightning context) for quite long times.

Assumed Initial Spherical Radius R_o		$.1\ m$	$1\ m$
	$\lvert\rho_o\rvert$	$.53\ \frac{mC}{m^3}$	$53\ \frac{\mu C}{m^3}$
	$\lvert Q_o\rvert$	$2.23\ \mu C$	$.223\ mC$
	$\lvert t_o\rvert$	$73\ \mu s$	$.73\ ms$
$R = R_o$	R	$.1\ m$	$1\ m$
	$\lvert\rho\rvert$	$.53\ \frac{mC}{m^3}$	$53\ \frac{\mu C}{m^3}$
	v_R	$.46\ \frac{km}{s}$	$.46\ \frac{km}{s}$
	t	$73\ \mu s$	$.73\ ms$
$R = 10R_o$	R	$1\ m$	$10\ m$
	$\lvert\rho\rvert$	$.53\ \frac{\mu C}{m^3}$	$53\ \frac{nC}{m^3}$
	v_R	$4.6\ \frac{m}{s}$	$4.6\ \frac{m}{s}$
	t	$73\ ms$	$.73\ s$
$R = 100R_o$	R	$10\ m$	$100\ m$
	$\lvert\rho\rvert$	$.53\ \frac{nC}{m^3}$	$53\ \frac{pC}{m^3}$
	v_R	$46\ \frac{mm}{s}$	$46\ \frac{mm}{s}$
	T	$73\ s$	$.73\ ks$

Table 2.1. Evolution of a Uniform Sphere of Charge (Ions) in Air

III. Uniformly Charged Circular Cylinder of Ions

Now for comparison consider that we have some charge per unit length q_0 present in the air within some cylindrical radius $0 = \Psi < \psi_0$ in the usual (Ψ,ϕ,z) cylindrical coordinate system as indicated in fig. 3.1. Let us assume that this charge is initially uniformly distributed with a charge density ρ_0. Then we have

$$q_0 = \pi\psi_0^2\rho_0 \tag{3.1}$$

As time evolves this charge is contained in some (infinite) circular cylinder of radius Ψ. Outside this radius the electric field is

$$\vec{E} = \vec{1}_\Psi E_\Psi$$
$$E_\Psi = \frac{q_0}{2\pi\varepsilon_0\Psi} \quad \text{for } \Psi > \psi \tag{3.2}$$

The charge within a radius Ψ is

$$q(\Psi,t) = 2\pi \int_0^\Psi \Psi'\rho(\Psi',t)\, d\Psi' \tag{3.3}$$

giving an electric field (radial in cylindrical sense)

$$E_\Psi(\Psi,t) = \frac{q(\Psi,t)}{2\pi\varepsilon_0\Psi} = \frac{1}{\varepsilon_0\Psi} = \frac{1}{\varepsilon_0\Psi}\int_0^\Psi \Psi'\rho(\Psi',t)\, d\Psi' \tag{3.4}$$

The velocity of the ions is outward parallel to $\vec{1}_\Psi$ for a unipolar ρ (single sign) as

$$v_\Psi(\Psi,t) = \mu_i|E_\Psi(\Psi,t)| = \frac{\mu_i}{\varepsilon_0\Psi}\left|\int_0^\Psi \Psi'\rho(\Psi',t)\, d\Psi'\right| \tag{3.5}$$

The current density also has only a Ψ component as

$$J_\Psi(\Psi,t) = \rho(\Psi,t)\, v_\Psi(\Psi,t) \tag{3.6}$$

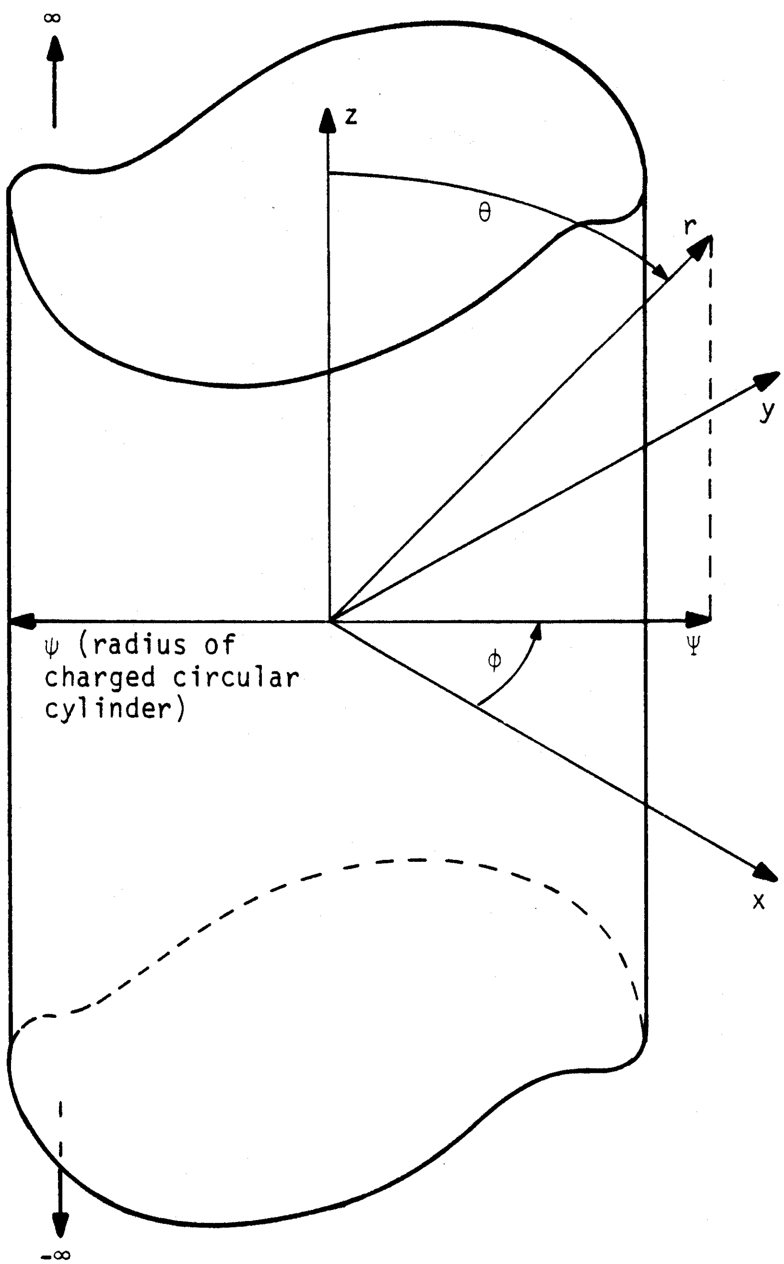

Fig. 3.1. Expansion of Uniformly Charged Circular Cylinder of Ions in Air

Now the equation of continuity gives

$$\nabla \cdot \vec{J}(\Psi,t) = -\frac{\partial}{\partial t}\rho(\Psi,t) = \frac{1}{\Psi}\frac{\partial}{\partial\Psi}[\Psi J_\Psi(\Psi,t)] \tag{3.7}$$

Thus

$$-\frac{\partial}{\partial t}\rho(\Psi,t) = \nabla \cdot [\rho(\Psi,t)\; v_\Psi(\Psi,t)]$$

$$= \frac{1}{\Psi}\frac{\partial}{\partial\Psi}[\Psi\rho(\Psi,t)\; v_\Psi(\Psi,t)]$$

$$= \frac{\mu_i}{\varepsilon_0\Psi}\frac{\partial}{\partial\Psi}[\rho(\Psi,t)\; \left|\int_0^\Psi \Psi'\rho(\Psi',t)\; d\Psi'\right|] \tag{3.8}$$

Since ρ is assumed to have a single sign this can be rewritten as

$$-\frac{\partial}{\partial t}|\rho(\Psi,t)| = \frac{\mu_i}{\varepsilon_0\Psi}\frac{\partial}{\partial\Psi}[|\rho(\Psi,t)|\int_0^\Psi \Psi'\; |\rho(\Psi',t)|\; d\Psi'] \tag{3.9}$$

Again this assumption of a single sign for the charge density is important if one is to ignore recombination between positive and negative ions.

Again look for a solution of (3.9) in the form of a charge density which is not a function of Ψ for $0 \leqslant \Psi < \psi$, giving

$$-\frac{\partial}{\partial t}|\rho(t)| = \frac{\mu_i}{\varepsilon_0}|\rho(t)|^2\frac{1}{\Psi}\frac{\partial}{\partial\Psi}\int_0^\Psi \Psi' d\Psi'$$

$$= \frac{\mu_i}{\varepsilon_0}|\rho(t)|^2 \tag{3.10}$$

As one can see this is exactly the same result as (2.10) so that (2.11) through (2.13) carry over directly from the spherical to the cylindrical case.

Then for the cylindrical case as well, an initially uniform charge density evolves as a uniform charge density. Again note that only the charge density within a cylindrical radius Ψ influences the electric field at Ψ, and hence the motion of charge at this radius.

Then conserving charge within a region $0 \leq \Psi < \psi$ with ψ as the outermost radius to which charge extends gives

$$q_0 = \pi\psi^2\rho(t)$$

$$\psi(t) = \left[\frac{q_0}{\pi\rho(t)}\right]^{\frac{1}{2}} = \psi_0\left[\frac{\rho_0}{\rho(t)}\right]^{\frac{1}{2}} \tag{3.11}$$

$$\frac{\psi(t)}{\psi_0} = \left[\frac{\rho_0}{\rho(t)}\right]^{\frac{1}{2}} = \left[\frac{t}{t_0}\right]^{\frac{1}{2}}$$

This gives the interesting result that in cylindrical geometry the charge expansion does not slow down as fast as in spherical geometry.

The cylindrical radial velocity of the outer boundary is

$$v_\psi(t) = \frac{d\psi}{dt} = \frac{1}{2}\psi_0 t_0^{-\frac{1}{2}} t^{-\frac{1}{2}} = \frac{\psi_0}{2t_0}\left(\frac{t_0}{t}\right)^{\frac{1}{2}} = \frac{\psi_0}{2t_0}\frac{\psi_0}{\psi(t)} \tag{3.12}$$

The electric field there is

$$|E_\psi(t)| = \frac{1}{\mu_i} v_\psi(t) = \frac{\psi_0}{2\mu_i t_0}\left(\frac{t_0}{t}\right)^{\frac{1}{2}} = \frac{\psi_0}{2\mu_i t_0}\frac{\psi_0}{\psi(t)} \tag{3.13}$$

Setting the initial electric E_0 at the outer boundary as

$$E_0 = |E_\psi(t_0)| = \frac{\psi_0}{2\mu_i t_0} = \frac{\psi_0}{2}\frac{|\rho_0|}{\varepsilon_0} \tag{3.14}$$

gives expressions for the initial charge as

$$|\rho_0| = \frac{2\varepsilon_0 E_0}{\psi_0}$$

$$|q_0| = \pi\psi_0^2 |\rho_0| = 2\pi\psi_0\varepsilon_0 E_0 \tag{3.15}$$

Again using the parameters in (2.19) let us consider the evolution of a circular cylinder of charge in air as indicated in table 3.1.

Assumed Initial Spherical Radius ψ_o		.1 m	1 m
	$\lvert\rho_o\rvert$	$.35\ \frac{mC}{m^3}$	$35\ \frac{\mu C}{m^3}$
	$\lvert q_o\rvert$	$1.11\ \frac{mC}{m}$	$11.1\ \frac{mC}{m}$
	$\lvert t_o\rvert$	.11 ms	1.1 ms
$\psi = \psi_o$	ψ	.1 m	1 m
	$\lvert\rho\rvert$	$.35\ \frac{mC}{m^3}$	$35\ \frac{\mu C}{m^3}$
	v_ψ	$.46\ \frac{km}{s}$	$.46\ \frac{km}{s}$
	t	.11 ms	1.1 ms
$\psi = 10\psi_o$	ψ	1 m	10 m
	$\lvert\rho\rvert$	$3.5\ \frac{\mu C}{m^3}$	$.35\ \frac{\mu C}{m^3}$
	v_ψ	$46\ \frac{m}{s}$	$46\ \frac{m}{s}$
	t	11 ms	.11 s
$\psi = 100\psi_o$	ψ	10 m	100 m
	$\lvert\rho\rvert$	$35\ \frac{nC}{m^3}$	$3.5\ \frac{nC}{m^3}$
	v_ψ	$4.6\ \frac{m}{s}$	$4.6\ \frac{m}{s}$
	t	1.1 s	11 s

Table 3.1. Evolution of a Uniform Circular Cylinder of Charge (Ions) in Air

IV. Comparison of Spherical and Cylindrical Expansion

Now let us compare the results of the previous two sections. Figure 4.1 plots the expansion of the spherical and cylindrical ion clouds for starting radii of .1 m and 1 m.

The basic result is that the spherical charge distribution expands much more slowly than the cylindrical charge distribution. In other words the spherical ion cloud "hangs around" much longer. Consider some typical numbers, say for an initial charge radius of .1 m. At 1 s the spherical charge has expanded to about 2.4 m while the cylindrical charge has expanded to about 9.5 m. At 10 s the spherical charge has expanded to about 5.2 m while the cylindrical charge has expanded to about 30 m. At 100 s the spherical charge has expanded to about 24 m while the cylindrical charge has expanded to about 950 m. Similar comparative results apply for an initial charge radius of 1 m.

What these results indicate is that for times of a few seconds or more the spherical ion distribution is only slowly expanding as far as a human observer can determine without special instruments provided some phenomenon (such as visual or acoustic) indicates the presence of this charge. The associated dimensions are a few meters. On the other hand a cylindrical ionic charge distribution is still quite rapidly expanding so that a nearby human observer might not so readily identify its "quasi stationary" presence on such time scales.

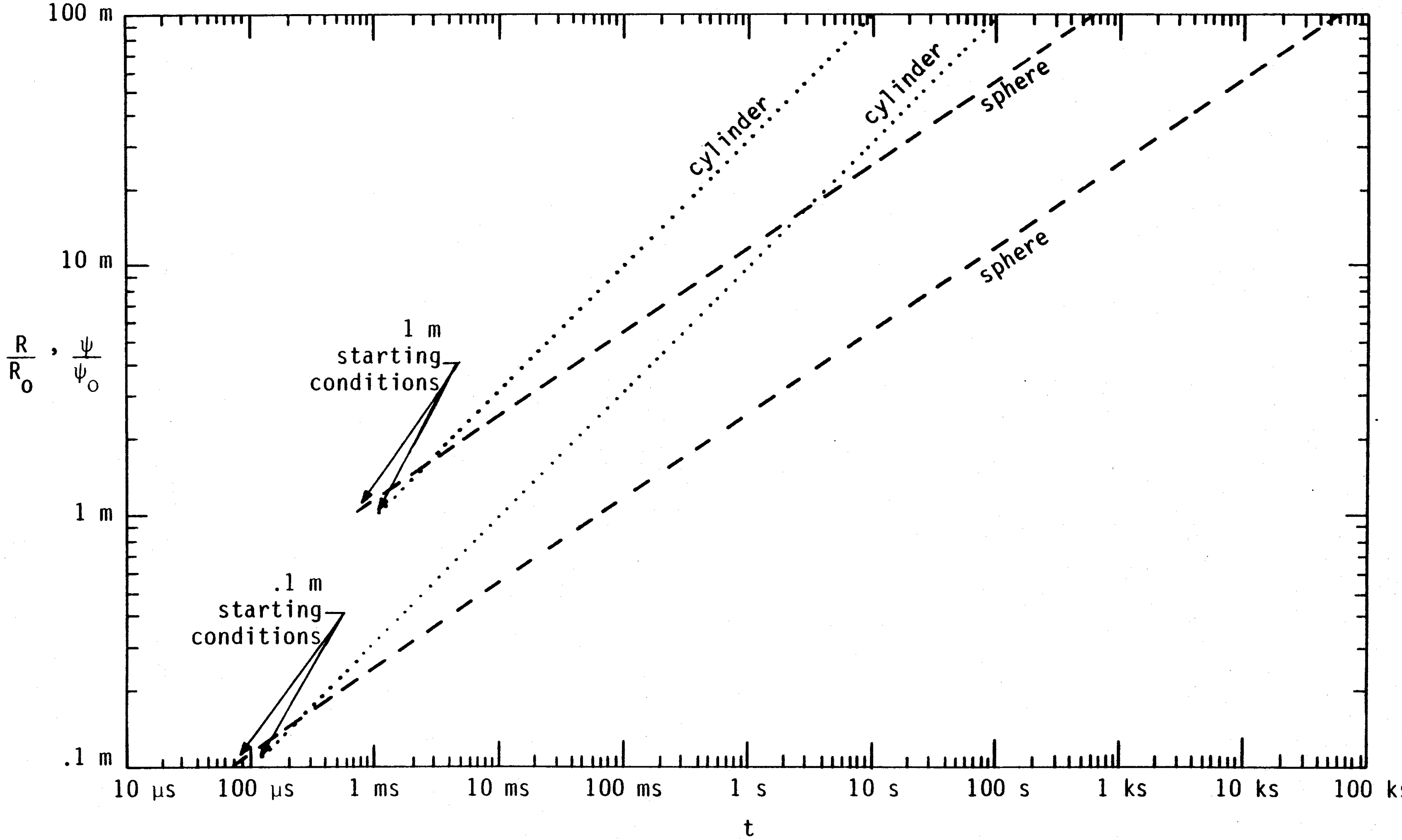

Fig. 4.1. Comparison of Spherical and Cylindrical Expansion of Ions

V. Movement of Ions in Uniform Electric Field

While the previously discussed charge expansion is occurring there are other factors to consider. In a lightning environment there is a background vertical electric field near the ground associated with the charge buildup in the clouds. This field can in turn move such a spherical distribution of ions through the air.

Let us assume for present purposes such an ambient electric field E_1 of about

$$E_1 \approx 10^4 \ \text{V/m} \tag{5.1}$$

This gives a drift velocity of the ions of about

$$v_d \approx \mu_i E \approx 2.3 \ \frac{m}{s} \tag{5.2}$$

Comparing this velocity to the previous results we can see that for a spherical ion cloud this mechanism dominates the spherical expansion velocity after about .7 s for an initial spherical radius of .1 m. So for a human observer this drift velocity dominates very quickly.

The "spherical" charge may be formed around some sharp protrusion near the ground surface. After the above time the "spherical ion cloud" may drift free of its formation location.

VI. Movement of Charged Sphere of Air in Air

Concerning the movement of a spherical ionic charge in air, one might ask whether a background vertical electric field would exert enough force on the charge to move the sphere of air containing the charge along with the charge. For present purposes then consider the idealized problem of a solid charged sphere moving through the air.

Using standard aerodynamic formulas

$$F_D = C_D(\pi a^2)\ \rho_m \frac{v_s^2}{2}$$

$$F_D \equiv \text{drag force (Newtons)}$$

$$C_D \equiv \text{coefficient of drag (dimensionless)}$$

$$a \equiv \text{sphere radius} \qquad (6.1)$$

$$\rho_m \equiv \text{mass density of fluid (kg/m}^3\text{)}$$

$$v_s \equiv \text{sphere speed}$$

For our problem let us use

$$C_D \simeq 0.5$$

$$\rho_m \simeq 1.29 \text{ kg/m}^3 \quad \text{(STP air)} \qquad (6.2)$$

This drag coefficient is appropriate for small speeds below the transition speed corresponding to a Reynolds number less than 385,000 [6]. For STP air this corresponds to about 3 m/s for a 1-m radius sphere; this critical speed is proportional to the reciprocal of the sphere radius. Then we have (in MKS units)

$$F_D \simeq a^2 v_s^2$$

$$v_s \simeq \frac{1}{a}\sqrt{F_D} \qquad (6.3)$$

This drag force is balanced by an electrostatic force

$$F_Q = |Q_o|\ E_1 \qquad (6.4)$$

where as before Q_0 is the total charge. Equating a to R as the radius of the spherical ion cloud gives

$$v_s \simeq \frac{1}{R}\sqrt{E_1\ |Q_0|} \tag{6.5}$$

Choosing

$$E_1 \simeq 10^4 \quad \text{V/m} \tag{6.6}$$

gives

$$v_s \simeq \frac{10^2}{R}\sqrt{|Q_0|} \tag{6.7}$$

To see the relative importance of the motion of a charged sphere of air through air refer to table 6.1. As the ion sphere expands, the speed v_s of the charged air sphere through the air falls below 1 m/s. Comparing these results to those of section 5, the drift of the ions through the air dominates this effect.

Assumed Initial Spherical Radius R_o		.1 m	1 m
Total Charge $\lvert Q_o \rvert$		2.23 μC	.223 mC
$R = R_o$	R	.1 m	1 m
	v_s	1.5 $\frac{m}{s}$	1.5 $\frac{m}{s}$
$R = 10R_o$	R	1 m	10 m
	v_s	.15 $\frac{m}{s}$	.15 $\frac{m}{s}$
$R = 100R_o$	R	10 m	100 m
	v_s	15 $\frac{mm}{s}$	15 $\frac{mm}{s}$

Table 6.1. Motion of Charged Sphere Through Air

VII. Summary

This note has considered the late-time phenomena associated with the movement of ions in air such as might be associated with natural lightning. It has been shown that a spherical cloud of ions expands at late time much more slowly than a cylindrical ion cloud such as one might associate with the region surrounding a lightning arc. The numerical results show that spherical ion clouds can stay around at a few meters radius for times that are sufficiently long (seconds) to be observed by human observers provided there are accompanying optical, acoustic, etc., phenomena on similar time scales.

In comparison to these results we have considered the motion of the spherical ion cloud in some background electric field, such as associated with thunderstorms. The ions can drift through the air at a few meters per second; this process can dominate the ion cloud expansion after a few seconds. Another process is the motion of the sphere of air associated with the ion cloud. However, the speed of this sphere of air seems to be less than that of the ion drift through the air.

Not addressed here is the mechanism for launching a spherical (or roughly spherical) ion cloud. Near various structures one can have a strongly enhanced background electric field creating some charge distribution in the surrounding air by breakdown processes. Here the electric field is not simply radial, but much more complex. For example, a spherical charge near a ground plane has an image charge of opposite sign giving significant forces if the spherical charge is sufficiently close to the ground plane. The model discussed in this paper then applies after the charge cloud has been effectively launched.

There are perhaps other effects of significance not discussed here. Let us mention the wind which can move an ion cloud with speeds greater than some of those discussed here. The air associated with the ion cloud may also be heated in the process of formation. Such air may be of lower density giving it a bouyancy in the presence of the surrounding air. These and other processes as well as the formation of such ion clouds need further consideration.

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