Evaluation of the Integral of the Anger-Weber Function with a Complex Argument

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Abstract

A series expression for small complex arguments and an asymptotic expression for large complex arguments has been derived for evaluating the integral of Anger-Weber functions. A study of the integral has been conducted for various values of order and complex argument.
INTRODUCTION

The Anger and the Weber functions\(^1\) of the order \(\nu\) and complex argument \(K\) are defined in terms of the following integral representations

\[
J_{\nu}(K) = \frac{1}{\pi} \int_{0}^{\pi} \cos(\nu\theta - K\sin\theta) \, d\theta
\]  
\[
E_{\nu}(K) = \frac{1}{\pi} \int_{0}^{\pi} \sin(\nu\theta - K\sin\theta) \, d\theta
\]  
(1)

The Anger function \(J_{\nu}(K)\) reduces to the Bessel function \(J_{n}(K)\) when \(\nu = n\) is an integer, and the Weber function \(E_{\nu}(K)\) is same as the Lommel-Weber function \(\Omega_{\nu}(K)\) with the change of sign,\(^2\)

\[
J_{n}(K) = J_{\nu}(K), \quad \nu = n \text{ an integer}
\]  
(3)

\[
\Omega_{\nu}(K) = -E_{\nu}(K)
\]  
(4)

The integral of the particular linear combination of these two functions which has been called the Anger-Weber function,\(^3\) is given by

\[
V_{\nu}(K) = \frac{j}{2} \int_{0}^{2K} [J_{\nu}(\zeta) + jE_{\nu}(\zeta)] \, d\zeta
\]  
(5)

which can be rewritten in the following form for \(\nu\) an integer by substituting the expressions (3) and (4),

\[
V_{\nu}(K) = U_{n}(K), \quad \nu = n \text{ an integer}
\]  
(6)

\[
U_{n}(K) = \frac{1}{2} \int_{0}^{2K} [\Omega_{n}(\zeta) + jJ_{n}(\zeta)] \, d\zeta
\]  
(7)
where \( J_n(\zeta) \) and \( \Omega_n(\zeta) \) are the Bessel function and Lommel-Weber function respectively, of integer order \( n \) and complex argument \( \zeta \). The integral (7) is frequently used in the analysis of Torus type EMP simulators and loop antennas.

In the following sections, a series expansion for small complex arguments and an asymptotic expression for large complex arguments is derived for evaluating the integral expression given in (7). Using these results, the integral of the Anger-Weber function is tabulated for various values of order and complex argument. The integral is analytic in the finite complex plane \( K \) and one can introduce the general normalized transformation

\[
S = jK = \gamma a
\]

(8)

to redefine the integral (7) in a convenient form suitable for EMP work. In the expression (8), \( a \) is an appropriate characteristic dimension which makes \( S \) dimensionless. Thus the integral (7) in the \( S \) plane is

\[
W_n(S) = U(-jS) = \frac{1}{2} \int_0^{-j2S} [\Omega_n(\zeta) + jJ_n(\zeta)] \, d\zeta
\]

(9)
I. Series Expansion for Small Arguments

Combining the Bessel and the Lommel-Weber functions, we can write the bracketed term in (9) as

\[ \Omega_n(\zeta) + j J_n(\zeta) = \frac{j}{\pi} \int_0^{\pi} e^{-j[\zeta \sin \theta - n \theta]} \, d\theta \]  

(10)

Substituting the expression (10) into (9), with the transformation \( p = j \zeta \),

\[ W_n(S) = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{\pi} e^{jn\theta} e^{-p \sin \theta} \, d\theta \, dp \]

(11)

Performing the outside integration first,

\[ W_n(S) = -\frac{1}{2\pi} \int_0^{\pi} e^{jn\theta} \left[ \frac{e^{-2S \sin \theta} - 1}{\sin \theta} \right] \, d\theta \]

(12)

Next, expanding the exponential of the bracketed term in a Taylor series, we obtain

\[ W_n(S) = -\frac{1}{2\pi} \int_0^{\pi} e^{jn\theta} \left[ \sum_{\ell = 1}^{\infty} \frac{(-2S)^\ell (\sin \theta)^{\ell - 1}}{\ell!} \right] \, d\theta \]

(13)

Interchanging the summation and integration in the expression (13), one obtains,

\[ W_n(S) = -\frac{1}{2\pi} \sum_{\ell = 0}^{\infty} \frac{(-2S)^{\ell + 1}}{(\ell + 1)!} \int_0^{\pi} e^{jn\theta} (\sin \theta)^{\ell} \, d\theta \]

(14)

where the integral part in the above expression (14) can be evaluated as,

\[ \int_0^{\pi} e^{jn\theta} (\sin \theta)^{\ell - 1} \, d\theta = \frac{\pi e^{jn\pi/2}}{2^{\ell-1} \ell \beta\left[\frac{\ell+n+1}{2}, \frac{\ell-n+1}{2}\right]} \]

(15)

where \( \beta(x, y) \) is defined in terms of the gamma function.
\[ \beta(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} = \beta(y,x) \]  \hspace{1cm} (16)

\[ \Gamma(v+1) = v\Gamma(v) \]  \hspace{1cm} (17)

Substituting these results in the expression (15), we obtain

\[ \int_0^\pi e^{jn\theta}(\sin\theta)^l \, d\theta = \frac{\pi(l!) \, e^{jn\pi/2}}{2^{l+1} \, \Gamma\left(\frac{l+n}{2} + 1\right) \, \Gamma\left(\frac{l-n}{2} + 1\right)} \]  \hspace{1cm} (18)

where we have put \( \Gamma(l+1) = l! \), \( l \) an integer, and using the expression (18), the expression \( W_n(S) \) for small arguments takes the form

\[ W_n(S) = \sum_{l=0}^\infty \frac{j^n(-1)^l}{(l+1) \, \Gamma\left(\frac{l+n}{2} + 1\right) \, \Gamma\left(\frac{l-n}{2} - 1\right)} \]  \hspace{1cm} (19)

The expression \( W_n(S) \) is well behaved in the complex \( S \) plane. Also it has the property \( W_n(S^*) = [W_n(S)]^* \). In fact the expression \( W_n(S) \) is real for all values of real \( S \) and \( n \) an even integer.

The series may be computed directly, however, the accuracy is limited for large values of \( |S| \). In this study, the series (19) was summed numerically to obtain values of \( W_n(S) \). The following criterion for the series expression (19) is used for convergence check on a CDC 6600,

\[ \left| \frac{W_n^L - W_n^{L-1}}{W_n^L} \right| < 10^{-7} \]  \hspace{1cm} (20)

where the superscript \( L \) denotes the partial sum of the series (19) with \( L \) terms.

It is noted that the expression (19) is same as that obtained in reference 3 with the substitution \( S = jK \).
II. Asymptotic Expansion for Large Arguments

According to the expressions (5) and (6), we can write \( W_n(S) \), as,

\[
W_n(S) = \frac{j}{2} \int_{\zeta=0}^{-2jS} \left[ J_n(\zeta) + jE_n(\zeta) \right] d\zeta
\]  

(21)

where for the upper limit of the integration \( K = -jS \) is substituted from the definition (8). Further introducing the transformation \( p = j\zeta \) for the integration variable, the expression (21) can be rewritten as,

\[
W_n(S) = \frac{1}{2} \int_0^{2S} \left[ J_n(\zeta) + jE_n(\zeta) \right] dp \quad \zeta = -jp
\]  

(22)

The asymptotic forms of the integrand in (22) are found in reference 1, in terms of the Hankel function,

\[
J_n(\zeta) + jE_n(\zeta) \sim H_n^{(2)}(\zeta) - j \left[ \frac{1 + (-1)^n}{\pi \zeta} \right] + O\left( \frac{1}{\zeta^2} \right)
\]  

(23)

\[
\sim \left[ \frac{2}{\pi \zeta} \right]^{1/2} e^{-j(\zeta - \frac{n\pi}{2} - \frac{\pi}{4})} - j \left[ \frac{1 + (-1)^n}{\pi \zeta} \right], \quad |\arg \zeta| < \pi
\]  

(24)

Because of the presence of the term \( 1/\zeta \), the integral expression \( W_n(S) \) is not convergent for \( n \) even but is convergent for \( n \) odd which can be shown by comparing \( W_n(S) \) to the Fresnel integrals.\(^1\)

However, with the following rearrangement, where we have \( \zeta = -jp \),

\[
W_n(S) = \frac{1}{2} \int_0^{j1} \left[ J_n(\zeta) + jE_n(\zeta) \right] dp + I_1(S) + I_2(S)
\]  

(25)
where
\[
I_1(S) = \frac{1}{2} \int_{j1}^{2S} \frac{1 + (-1)^n}{\pi} dp
\]
\[
= \frac{1 + (-1)^n}{2\pi} \ln(-j2S)
\] (26)
and
\[
I_2(S) = \frac{1}{2} \int_{j1}^{2S} \left[ J_n(\xi) + jE_n(\xi) + j \frac{1 + (-1)^n}{\pi_\xi} \right] dp
\] (27)

The integral \(I_2(S)\) is convergent as \(S \to \infty\), and hence can be rewritten in the form
\[
I_2(S) = \frac{1}{2} \left[ \int_{j1}^{j\infty} + \int_{j\infty}^{2S} \right] \left[ J_n(\xi) + jE_n(\xi) + j \frac{1 + (-1)^n}{\pi_\xi} \right] dp
\] (28)

For large \(S\), the integrand in the second integral of (28) can be replaced by its asymptotic form to obtain,\(^7\)
\[
I(S) = \frac{1}{2} \int_{j\infty}^{2S} \left[ J_n(\xi) + jE_n(\xi) + j \frac{1 + (-1)^n}{\pi_\xi} \right] dp
\] (29)
\[
\sim \frac{1}{2} \int_{j\infty}^{2S} \left[ (\frac{2}{\pi_\xi})^{\frac{1}{2}} e^{-j(\xi-n\pi/2-\pi/4)} \right] dp
\] (30)
\[
|\arg(\xi = -j)\| < \pi
\]

The integral (30) simplifies to Fresnel integrals, if we now let \(\frac{\pi}{2}t^2 = \xi\), \(u = -jS\),
\[
I(S) \sim j e^{j(n\pi/2+\pi/4)} \int_{-\infty}^{2(\frac{u}{\pi})^{\frac{1}{2}}} e^{-j\frac{\pi}{2}t^2} dt
\] (31)
\[
= j e^{j(n\pi/2+\pi/4)} \left[ \int_{-\infty}^{0} + \int_{0}^{2(\frac{u}{\pi})^{\frac{1}{2}}} \right] e^{-j\frac{\pi}{2}t^2} dt
\] (32)
\[
= \left[ -\frac{1}{2} - J_{\frac{1}{2}} + Si\left[2(\frac{u}{\pi})^{\frac{1}{2}}\right] + jCl\left[2(\frac{u}{\pi})^{\frac{1}{2}}\right]\right] e^{j(n\pi/2+\pi/4)}
\] (33)
\[
|\arg u| < \pi
\]
where the Fresnel integrals

\[ \text{Ci}(u) = \int_0^u \cos\left(\frac{\pi}{2} t^2\right) \, dt \]  
(34)

\[ \text{Si}(u) = \int_0^u \sin\left(\frac{\pi}{2} t^2\right) \, dt \]  
(35)

have the following asymptotic forms, for \(|\arg u| < \pi/2|,

\[ \text{Ci}(u) \sim \frac{1}{2} + \frac{1}{\pi u} \sin\left(\frac{\pi}{2} u^2\right) + O\left(\frac{1}{u^2}\right) \]  
(36)

\[ \text{Si}(u) \sim \frac{1}{2} - \frac{1}{\pi u} \cos\left(\frac{\pi}{2} u^2\right) + O\left(\frac{1}{u^2}\right) \]  
(37)

On substituting the expressions (36) and (37) into (33), we have

\[ I(S) \sim \frac{1}{2(\pi u)^{1/2}} e^{-J[2u - n\pi/2 - \pi/4]} \]

|\arg u| < \pi/2  
(38)

Using the expressions (26), (28), and (38), for large values of \(S\), the expression (21) takes the form, \(u = -jS\),

\[ W_n(S) \sim C_n + \frac{1 + (-1)^n}{2\pi} \ln(-jS) - \frac{1}{2(\pi S)^{1/2}} e^{-2S + j(n+1)\pi/2} \]

|\arg S| < \pi  
(39)

where the coefficient

\[ C_n = \frac{1}{2} \int_0^{j1} \left[ J_n(\xi) + jE_n(\xi) \right] d\xi + \frac{1}{2} \int_{j1}^{j\infty} \left[ J_n(\xi) + jE_n(\xi) + j \frac{1 + (-1)^n}{\pi \xi} \right] d\xi + \frac{1 + (-1)^n}{2\pi} \ln 2 \]

\(40\)
To evaluate the constant $C_n$, we can make use of the already-derived asymptotic formula for $W_n(x)$ where $x$ is real argument,\footnote{Equation (41)}

$$C_n = \lim_{x \to \infty} \left[ W_n(x) - \frac{1 + (-1)^n}{2\pi} \ln x \right]$$  \hspace{1cm} (41)

Or one may follow the following method to evaluate the constant coefficient $C_n$. According to the expression (39),

$$C_n = \lim_{S \to \infty} \left[ W_n(S) - \frac{1 + (-1)^n}{2\pi} \ln(-jS) \right]$$  \hspace{1cm} (42)

Hence

$$C_{n-2} = \lim_{S \to \infty} \left[ W_{n-2}(S) - \frac{1 + (-1)^{n-2}}{2\pi} \ln(-jS) \right]$$  \hspace{1cm} (43)

Subtracting (43) from the expression (42),

$$C_n - C_{n-2} = \lim_{S \to \infty} \left[ W_n(S) - W_{n-2}(S) \right]$$  \hspace{1cm} (44)

Further we have from the expression (11),

$$W_n(S) = \frac{j}{2} \int_0^{2\pi} \left[ \frac{1}{\pi} \int_0^\pi \right. e^{jn\theta} e^{-j\zeta\sin\theta} d\zeta d\theta$$

and hence the right-hand side of (44) reduces to

$$W_{n+1}(S) - W_{n-1}(S) = -j \left[ J_n(-j2S) + jE_n(-j2S) \right] - \frac{1 - (-1)^n}{n\pi}$$  \hspace{1cm} (45)

with $n = n-1$ substituted into (45), and in the limit as $S \to \infty$,

$$C_n - C_{n-2} = -\frac{1 - (-1)^{n-1}}{(n-1)\pi}$$  \hspace{1cm} (46)

forms the recurrence relationship for $C_n$ coefficients. Rearranging the $C_n$ integral terms appropriately and noting that,\footnote{Equation (46)}
\[ \int_0^\infty J_n(\zeta) d\zeta = 1 \]  
\[ C_n = \frac{j}{2} - \lim_{S \to 0} \frac{1}{2} \int_0^{-jS} E_n(\zeta) d\zeta + \frac{1 + (-1)^n}{2\pi} \ln(-jS) \]  
which gives for \( n = 0, 8 \)
\[ C_0 = \frac{j}{2} - \frac{1}{\pi} \psi\left(\frac{1}{2}\right) \]
and for \( n = 1 \),
\[ C_1 = \frac{j}{2} \]
Using \( C_0 \) and \( C_1 \), the recurrence relation (46) yields the general term,
\[ C_n = \frac{j}{2} - \frac{1 + (-1)^n}{2\pi} \psi\left(\frac{m + 1}{2}\right) \]  
where
\[ \psi\left(\frac{1}{2}\right) = -\Gamma - 2 \ln 2 \]
\[ \psi(m+1) = \psi(m) + \frac{1}{m} \]
\[ \Gamma = 0.5772... \quad \text{Euler's constant} \]
Hence, substituting the expression (50) for constant coefficient into the expression (39), for large values of \( S \),
\[ W_n(S) \sim \frac{j}{2} + \left[ \frac{1 + (-1)^n}{2\pi} \right] \left[ \ln(-jS) - \psi\left(\frac{m+1}{2}\right) \right] \]
\[- \frac{1}{2(\pi S)^{\frac{3}{2}}} e^{-2S+j(n+1)\pi/2} \]
\[ |\arg S| < \pi \]  
(51)
III. Numerical Results and Applications

Using double precision and the series convergence criterion given in the equation (20), \( W_n(S) \) may be evaluated accurately for \(|S| < 30\) on the CDC 6600. A more detailed discussion on the range limitations for imaginary arguments may be found in Mathematics Note 25. 3

Appendix A contains tables for \( W_n(S) \) for several values of order \( n \) and complex argument \( S \), while Appendix B gives the Fortran program listing used to evaluate \( W_n(S) \).

Extensive application of the integral of the Anger-Weber function with complex argument can be found in references 6 and 9, in which the Singularity Expansion Method is applied to the study of circular loop antennas.

ACKNOWLEDGMENT

We would like to thank Dr. Carl Baum of Air Force Weapons Laboratory, Albuquerque, New Mexico, for his helpful suggestions.
REFERENCES


APPENDIX A

$W_n(S)$ for Several Values of Order $n$
and Complex Argument $S$
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APPENDIX B

Fortran Program Listing Used to Evaluate $W_n(S)$
PROGRAM TIMED(INPUT,OUTPUT)
COMPLEX SM,SNZ,Z,S,AJ
AJ=CMPLX(0.,0.*1.0)
DO 300 NNX=6,11
N=NNA-1
DO 200 IMA=1,11
SR=0.5*FLOAT(IMA-1)
SI=0.5
PRINT 151
151 FORMAT(*10,/*)
PRINT 150
150 FORMAT(18X,N,N,6X,COMPLEX ARGUMENT S*,8X,INTEGRAL SN(N,S)*,/*)
DO 100 I=1,51
S=-SR*AJ*SI*FLOAT(I-1)
SNZ=SM(N,S)
PRINT 130,N,S,SNZ
130 CONTINUE
200 CONTINUE
300 CONTINUE
STOP 10
END

COMPLEX FUNCTION SM(M,S)
SM(M,S) COMPUTES THE INTEGRAL OF THE ANGER-WEBER FUNCTION BY

SERIES FOR COMPLEX ARGUMENT

COMPLEX J*X*S
DOUBLES C,A1,ZR,ZI,SR1,S11,SNZ,S12,ZH2,ZI2,ZHL,ZIL,T11,T12,T21,
$THP=TH1,TIP=ZMAG,ZANG,Y1,Y2,PML(350),A2,A(250),B(250),SUP1,A11,ATZ,AS1
$AS2
DATA MSAVE/-1/NUM/188/J/(0.,1.)/
DATA IFLAG/-1/SP1/1.77245385045516027298167483300/
X=-J*S
XH=REAL(X)
XI=AIMAG(X)
IF(IFLAG)100,U=500,500
100 A(62)=SP1
DO 200 I=1,61
200 A(62+I)=DBLE(-2.*FLOAT(2*I-1))*A(63+I)
DO 300 I=1,1,75
300 A(62+I)=DBLE(.5*FLOAT(2*I-1))*A(61+I)
B(1)=.000
B(2)=B(1)
DO 400 I=2,175
400 B(1+I)=DBLE(FLOAT(I))*Y(I)
IFLAG=1
500 IF(MSAVE=M)1,9+1
1 C=0.000
DO 51 IL=1,NUM
L=IL-1
IF(MOD(L,M,4))2,3+2
2 A1=A((L+M+125)/2)
A2=A((L-M+125)/2)
GO TO 42
3 NAG=(L-M+2)/2
IF(NAG)4,4+41
4 PML(IL)=0.000
GO TO 51
A1=H(NAG)
A2=H((L+M+2)/2)
IF(DABS(C)GT.1.0300)GO TO 4
C=UBLE(FLOAT(IL))*A1*A2
PML(IL)=1.000/U/C
CONTINUE
MSAVE=M
NU=NUM-3
CONTINUE
IF(CABS(X)GE.35.) GO TO 117
SR1=0.000
SI1=0.000
SR2=0.000
SI2=0.000
ZHR=DUBLE(XR)
ZI=DUBLE(XI)
ZHRZ=ZHR*ZHR-ZI*ZI
ZI2=2.000*ZHR*ZI
DO 10 IL=1,NU,4
N=IL+3
ZMAG=DUBRT(ZHRZ+ZI2*ZI)
IF(ZHRNE.0.000)GO TO 30
ZANG=SQPI*SUPI*0.500
GO TO 31
30 ZANG=DATAN2(ZI,ZHR)
31 Y1=ZMAG**IL
Y2=ZANG*DUBLE(FLOAT(IL))
ZHL=Y1*DOS(Y2)
ZIL=Y1*DOSI(Y2)
TR1=PML(IL)*ZHRZ+PML(IL+2)*ZI2*ZIL-PML(IL+2)*ZHR2*ZHR
TI1=PML(IL)*ZHR2*ZIL-PML(IL+2)*ZHR2*ZIL-PML(IL+2)*ZI2*ZHR
TRP=PML(IL+1)*ZHR2*PML(IL+3)*ZI2*ZIL-PML(IL+3)*ZHR2*ZHR
TIP=PML(IL+1)*ZHR2*PML(IL+3)*ZHR2*ZIL-PML(IL+3)*ZI2*ZHR
TR2=TRP*ZR*TIP*ZI
T12=TIP*ZR*ZI1*TR
SR1=SR1+TR1
SI1=SI1+TI1
SR2=SR2+TR2
SI2=SI2+TI2
AT1=DSJRT(TR1*TR1+TI1*TI1)
AT2=DSJRT(TR2*TR2+TI2*TI2)
AS1=DSJRT(SR1*SR1+SI1*SI1)
AS2=DSJRT(SR2*SR2+SI2*SI2)
IF(AS1.0.EQ.0.000)GO TO 10
IF(AS2.0.EQ.0.000)GO TO 10
IF(DABS(AT1/AS1)GE.1.0-7)GO TO 10
IF(DABS(AT2/AS2)GE.1.0-7)GO TO 10
GO TO 11
CONTINUE
S4=SR2-SI1
SJ=SI2-SR1
SM=CMPLX(S4+S3)*J**M
RETURN
117 SM=(0.,0.,)
RETURN
END