

Measurement Notes

Note 30

March, 1984

AN INVESTIGATION INTO THE USE OF
DETONATING FUSES TO CREATE CONDUCTING
PATHS IN THE ATMOSPHERE

by

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Abstract

Detonating fuses, of the types used in blasting operations, can create conducting paths in the atmosphere behind a shock wave that propagates at about 6000 m/second. Measurements of some of these paths behind a detonated fuse indicate electrical resistances of a few kilohms per meter with conductive durations of up to 25 ms.

Attempts to prolong this duration by electrical heating showed that currents in excess of 1 A would be required to maintain an arc. Estimates of the displacement currents that will flow when a long length of fuse is detonated in the presence of a strong, external electric field suggest that the resulting currents may be sufficient to aid in the triggering of lightning by detonating fuses.

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Figure 13 Plot of displacement currents in an ellipsoidal conductor with the semi-major axis growing at 6000 m/sec an electric field of 20 kV/m at the earth's surface under various point discharge current densities.

Introduction

Brook et al (1961) proposed that lightning could be initiated artificially by the sudden injection of a conductor into a region with strong electric fields. Later, Newman (1965) and his associates utilized the technique to trigger lightning by towing a grounded wire upward behind a rocket launched beneath a thundercloud. The lightning initiated in this fashion is similar to the upper portion of a cloud-to-ground discharge where positive streamers propagate outward into an undisturbed cloud from the top of the return-stroke channel from the earth.

Some interest now has developed in the artificial initiation of discharges aloft so as to simulate natural lightning more closely with a downward coming leader and an upward propagating return-stroke, uncontaminated by the remains of the grounded wire. One means of achieving this, suggested by Baum and others, is the sudden creation of an elevated, conducting channel by the use of a long length of a detonating fuse, suspended vertically from a captive balloon.

This report describes a preliminary investigation of some detonating fuses and assesses the utility of these fuses for lightning initiation experiments.

Properties of Detonating Fuses

Detonating fuses typically consist of a penta erythritol tetra nitrate (PETN) explosive powder inserted into long fabric

tubes in amounts ranging from 20 grains/foot [4.25 gm/m] to 400 grains/foot [85 gm/m].

When initiated by a blasting cap at one end of the tube, a detonation wave-front propagates down the tube with speeds of up to 6000 m/sec. This fuse is commonly used to initiate widely separated blasts nearly simultaneously, and is therefore available in long sections, 300 m or greater in length. One readily available, PETN detonating fuse is known under the trade name of "Primacord" which is furnished by:

Ensign Bickford Company
660 Hopmeadow Street
Simsburg, Conn 06070

PETN has an explosive energy of about 6300 J/gm which is equivalent to about 1340 J/m per grain/foot in Primacord.

Experiments with Detonating Fuses

The first questions to be asked about the use of detonating fuses for the creation of conducting channels are "what is the channel's electrical resistance and how long does conduction persist?" To answer these, we carried out several tests in the TERA explosives facility here at New Mexico Tech using various lengths and loadings of Primacord between two, steel armor plate electrodes about 45 cm long and 4 cm thick. The surfaces of these electrodes had been exposed to the weather and were therefore covered with a coat of rust. We cleaned these by use of a steel wire brush until the underlying metal could be seen. Each electrode was isolated from the earth by placing it on a platform

of dry wood. A section of Primacord was stretched between the electrodes and taped in place by use of duct tape. At one end, the Primacord extended several meters beyond the electrode to a blasting cap that was used to initiate a detonation.

A view of the electrode arrangement is given in Figure 1. A potential difference of 6.28 volts was applied to the two electrodes and any current between them, after the detonation of the Primacord, was sensed by measurement of the voltage drop developed across the input resistance of a digital volt meter that was connected in series with the conducting channel and the battery. The voltmeter used here was a Biomation 8100 waveform analyzer with a 50 ohm input impedance. It sampled at 20 microsecond intervals and stored the results in a 2 kilobyte shift register. A sketch of the current measuring circuit is shown in Figure 2.

A light-sensing silicon diode (EG&G Light Mike, Model 560B) was used to monitor the duration of any luminosity associated with the explosion. The output from the diode was recorded on a second Biomation 8100 waveform analyzer and the data from both Biomation units were recorded on magnetic tape for analysis.

The results of typical tests are shown in Figures 3 and 4 with plots of the current carried between the two electrodes and of the channel luminosity detected by the photo diode. These data indicate two separate regimes in each explosion, one associated with the initial detonation of the PETN and a later one,

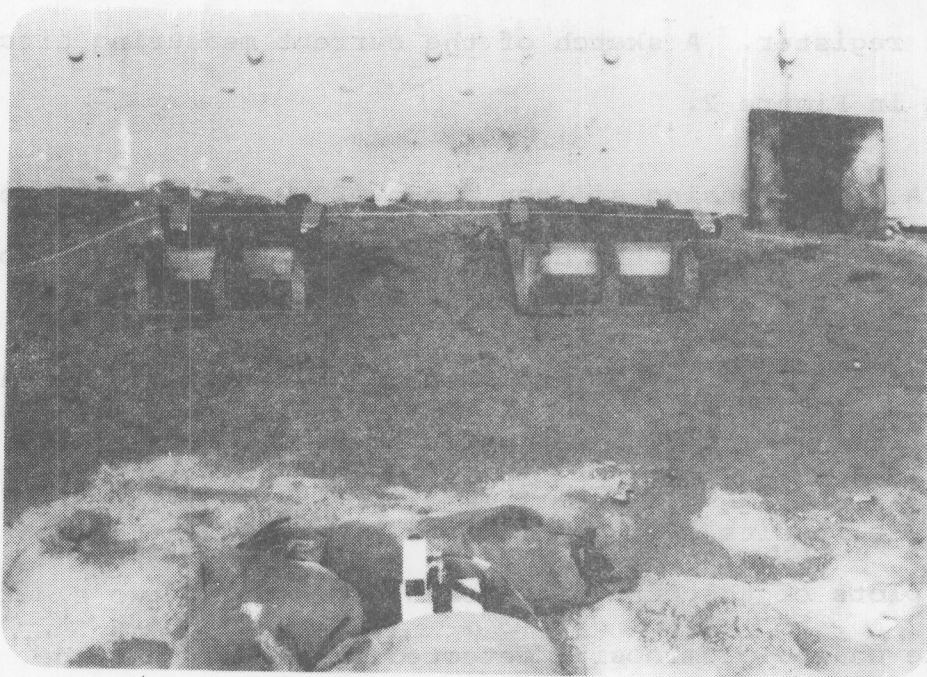
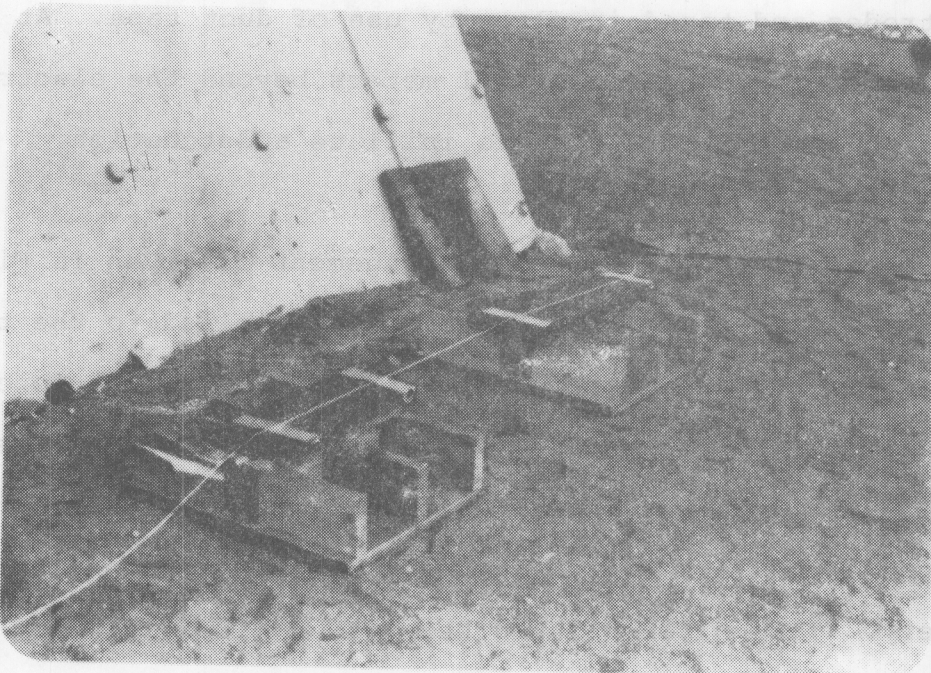


Figure 1 Views of the Primacord experiment with armor plate electrodes supported on wooden platforms and separated by 0.5 m. The light string-like line shows a length of 10.6 gm/m (50 grain/foot) Primacord. The white instrument in the foreground of the lower picture is the EG&G Light Mike.

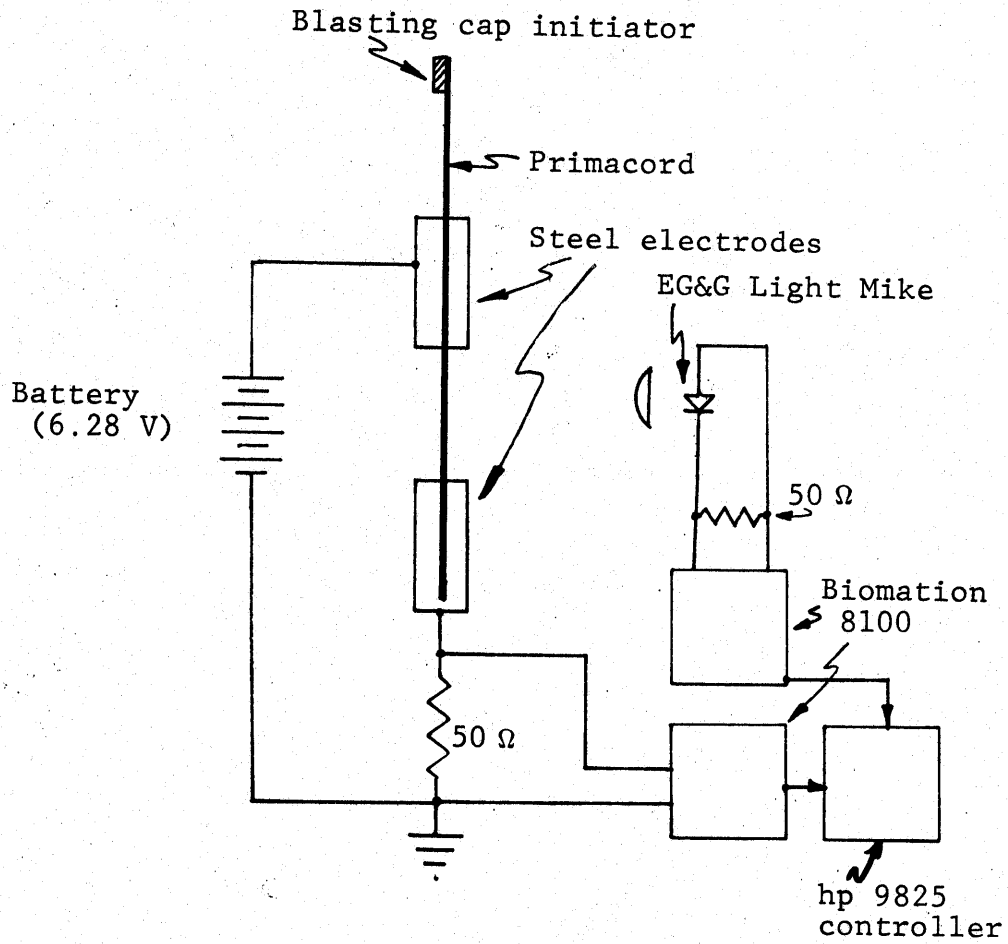


Figure 2 Circuit diagram for the measurement of conduction current and luminosity associated with a Primacord detonation.

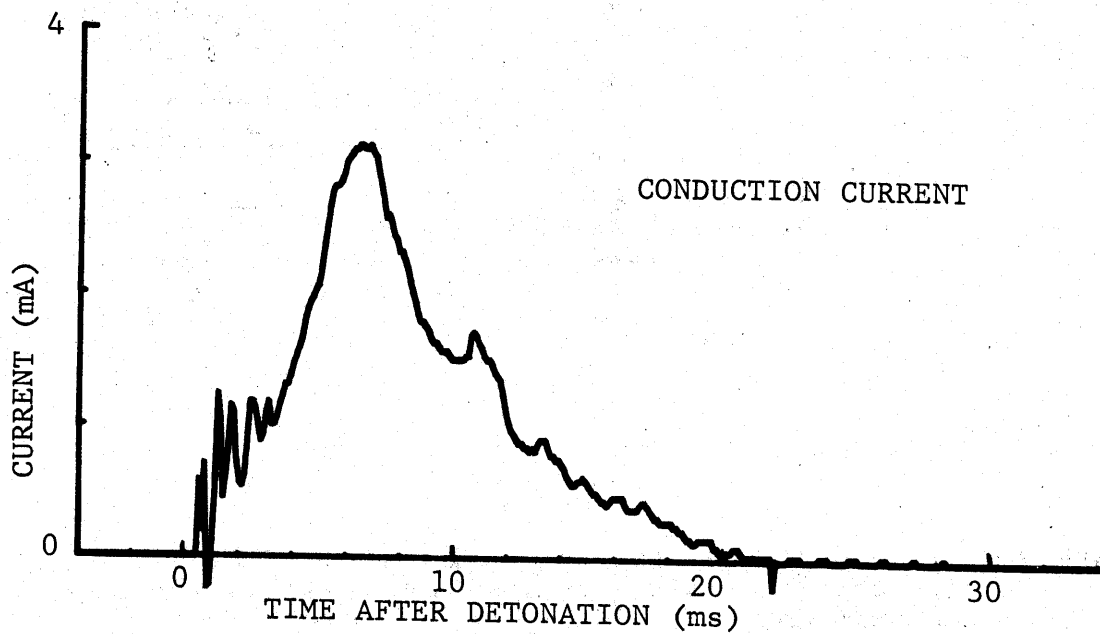
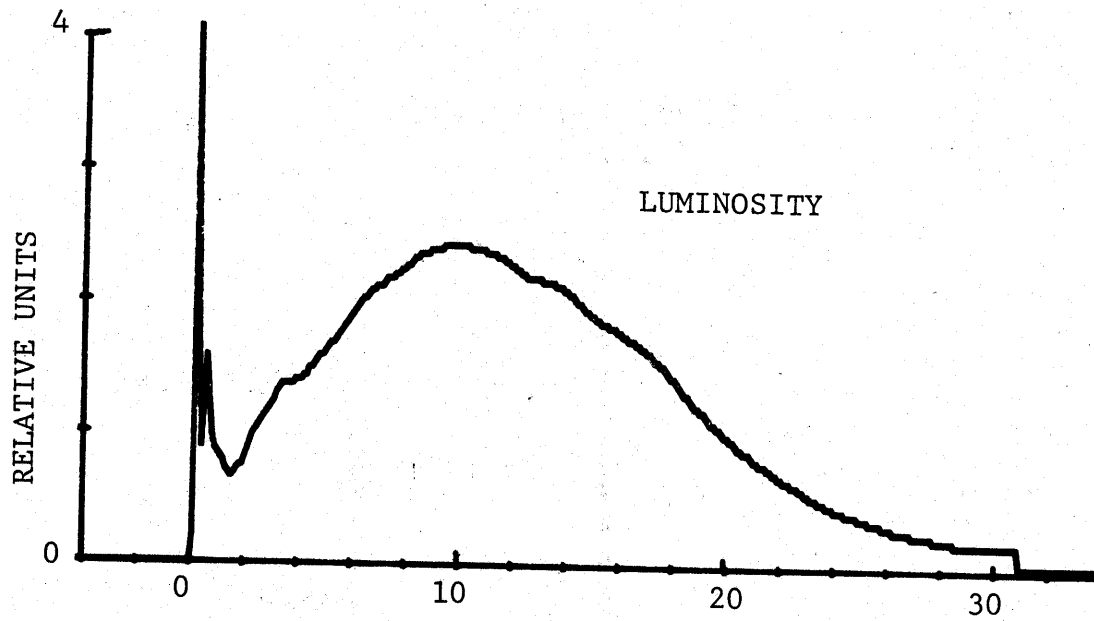


Figure 3 Luminosity and current associated with the detonation of a 0.5 m length of 10.6 gm/m Primacord.

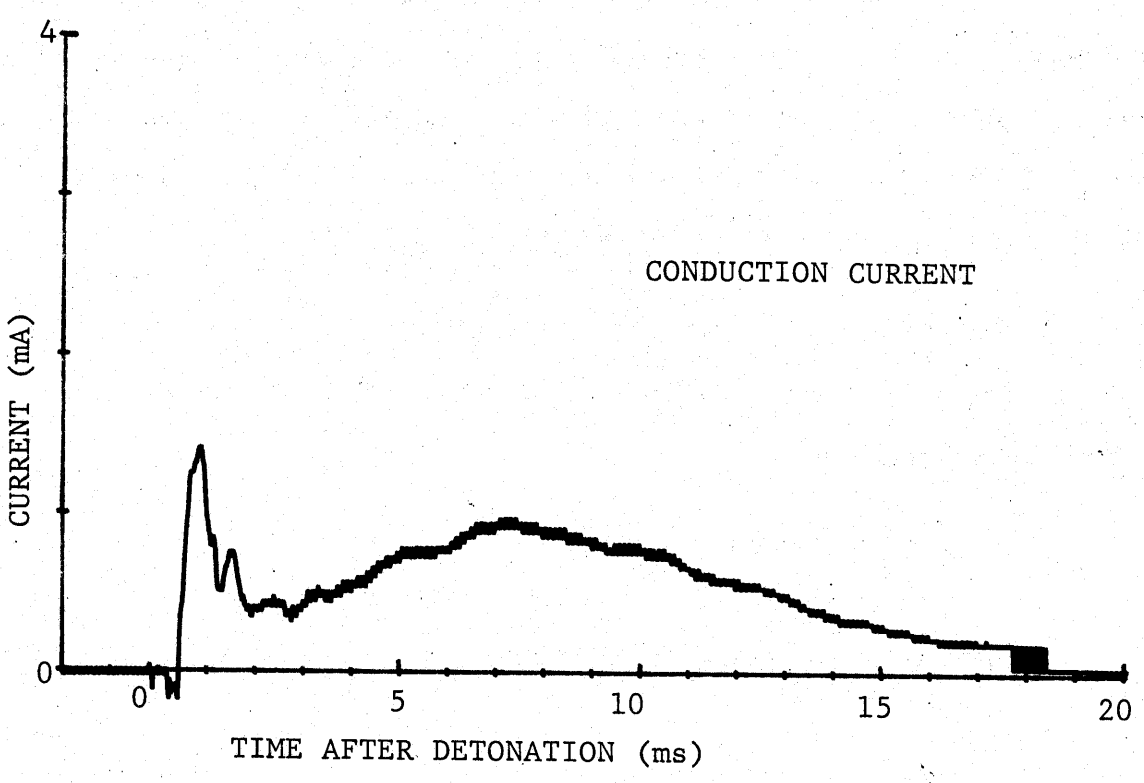
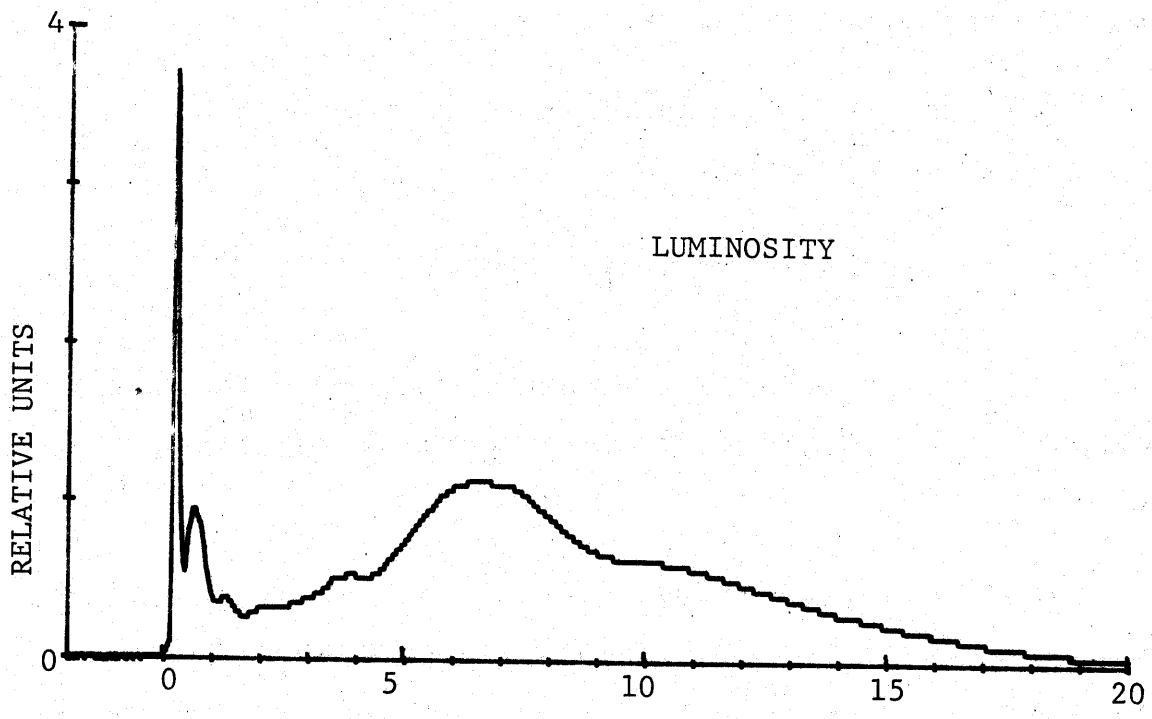


Figure 4 Luminosity and current associated with the detonation of a 3 m length of 10.6 gm/m Primacord.

presumably associated with combustion of the fabric fuse casing and other unburned components.

The total resistance, R_t , between the two electrodes for each detonation was calculated from:

$$R_t = \left[\frac{6.28 \text{ volts}}{V_{\text{Biomation}}} - 1 \right] 50 \text{ ohms}$$

Plots of the channel resistance versus time are shown in Figures 5 and 6 for the current measurements displayed in Figure 3 and 4.

In an effort to separate the resistance of the explosion channel from that at the electrodes, two different lengths of Primacord were detonated in each set of tests. In the first detonation of each set, the electrodes were separated by 0.50 m and were connected by a straight run of Primacord. In the other half of each test, the electrodes were placed 3.0 m apart and again connected by a straight section of fuse. The results of these tests are shown in Table I.

Estimates for the electrode resistance, R_e , and channel resistance/unit length, dR/dl , were then obtained by a solution of the two equations:

$$R_e + 0.5 \, dR/dl = R_t$$

$$R_e + 3.0 \, dR/dl = R_t$$

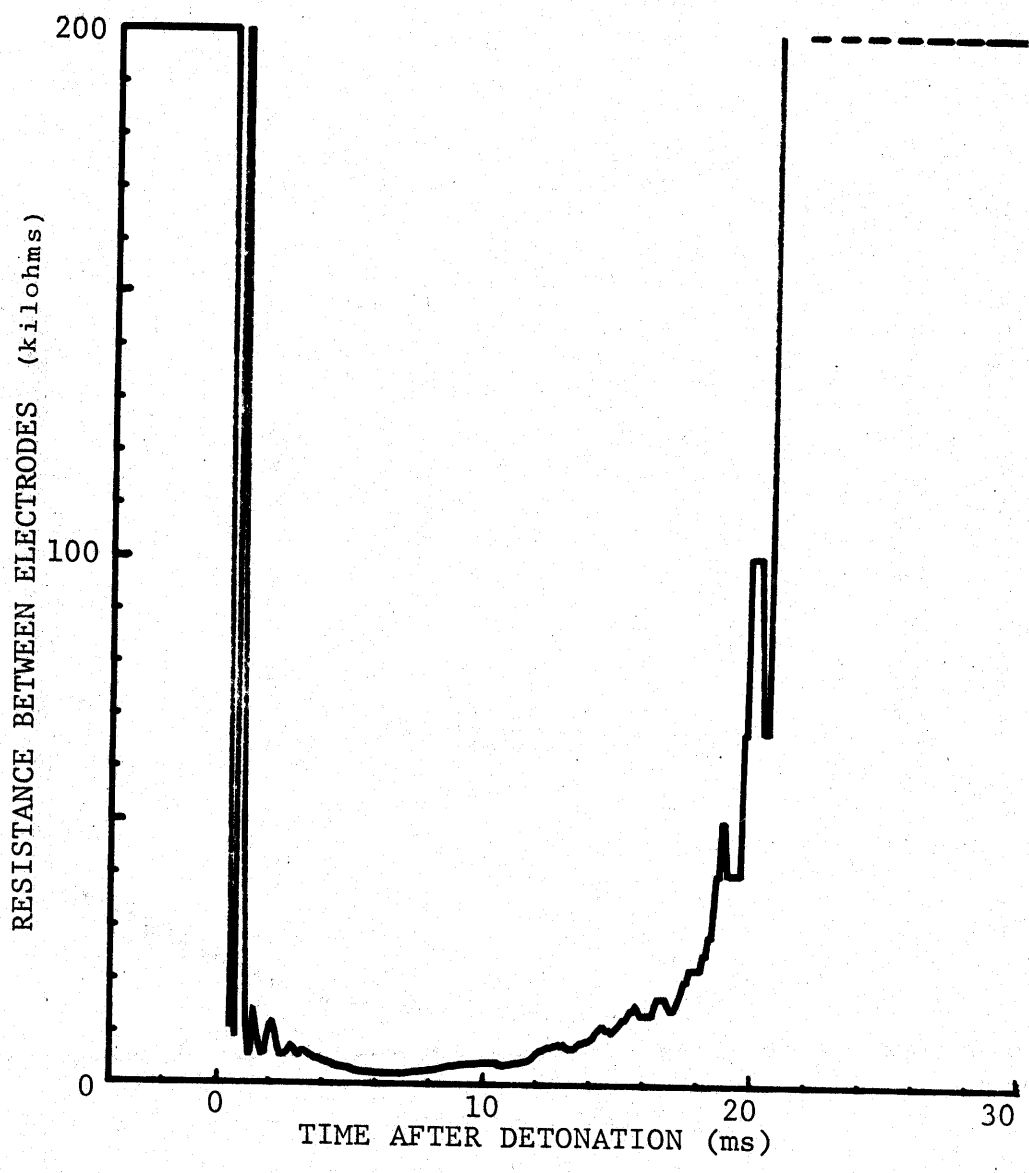


Figure 5 The electrical resistance between electrodes of the 0.5 m length of 10.6 gm/m Primacord shown in Figure 3.

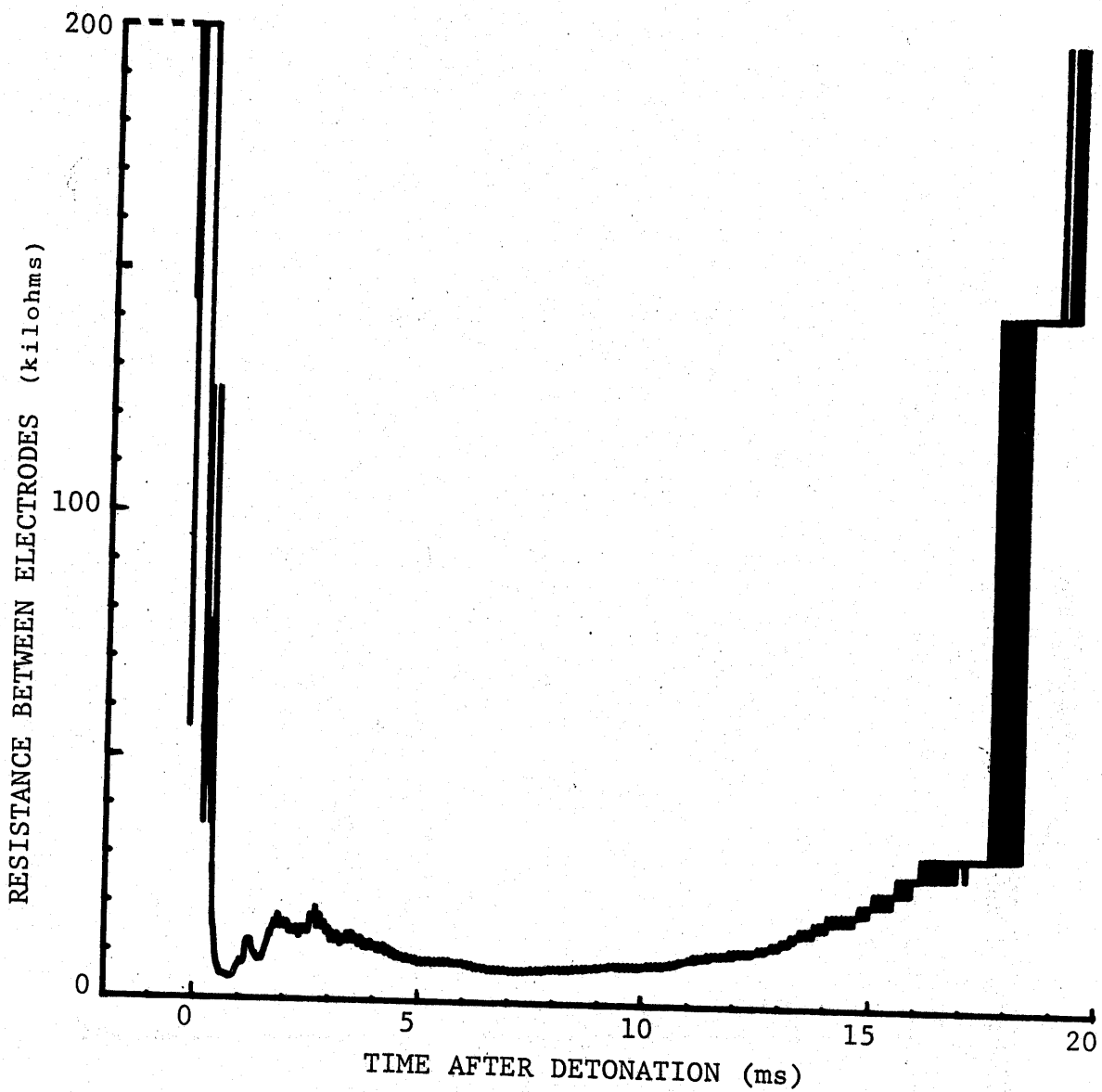


Figure 6 The electrical resistance between electrodes of the 3 m length of 10.6 gm/m Primacord shown in Figure 4.

TABLE I

Measurements Obtained from First Primacord Experiments

Explosive Loading (gm/m)	Primacord Length (m)	Electrode Type	Minimum Resistance Observed (ohms)	Current Duration (ms)	Luminosity Duration (ms)
5.3	0.5	armor plate	50,000	11.8	18.2
5.3	0.5	armor plate	22,000	7.9	12.3
5.3	0.5	stainless steel	<3,165	18.4	-
5.3	0.5	stainless steel	<1,560	18.1	-
5.3	0.5	stainless steel	1,440	9.0	-
5.3	0.5	stainless steel	<1,560	14.1	-
5.3	3	stainless steel	5,300	6.9	-
5.3	3	armor plate	5,000	11.7	13.2
10.6	0.5	armor plate	1,960	20.4	27.6
10.6	0.5	armor plate	1,900	21.4	-
10.6	0.5	double armor plates	7,100	7.4	22.6
10.6	0.5	stainless steel	<1,560	16.7	-
10.6	3	stainless steel	3,740	21.8	-
10.6	3	double armor plates	2,950	9.6	22.9
10.6	3	armor plate	4,400	17.9	18.7
10.6	10	armor plate	6,900	21	-
85	0.5	armor plate	5,100	14.7	24.1
85	3	armor plate	3,140	8.0	19.2
black powder fuse surrounded by 6 lengths of 5.32 gm/m Primacord	0.5	stainless steel	7,100	14.8	-
	3	stainless steel	5,000	16.2	-

Results of these calculations are shown in Table II. This exercise suggested that a large fraction of the resistance occurred at the electrodes which were still somewhat rusty, despite the wire brushing.

In an effort to improve the electrode coupling, we substituted two stainless- steel slabs for the armor plate electrodes but these did not dramatically decrease the electrode resistance nor prolong the duration.

In another effort to improve the electrode coupling, the ends of the Primacord passing over armor plate electrodes were sandwiched beneath additional slabs of steel that were connected electrically to the underlying support electrodes. When this arrangement was tested, the electrode resistance increased and the duration of conduction decreased, apparently as a result of more rapid separation of the plasma from the confining electrodes.

We noted in these experiments that the duration of the luminosity was significantly longer than that of the current conduction. One possible reason is that the impulse supplied by the explosion propelled the plasma channel away from the electrode, soon breaking any electrical connection. The duration of the luminosity therefore may be a better measure of the duration of the conducting phase in the atmosphere than is the measured current flow between fixed electrodes.

The maximum duration of channel luminosity was found to be about 25 milliseconds during which, an advancing propagation

TABLE II

Resistances Inferred from Primacord Measurements

Explosives Configuration	Total Electrode Resistance (ohms)	Plasma Resistance (ohm/m)
5.32 gm/m Primacord, stainless steel electrodes	670	1,550
10.63 gm/m Primacord, stainless steel electrodes	<1,100	<870
10.63 gm/m Primacord, armor plate electrodes	1,900	670
10.63 gm/m Primacord, double armor plate electrodes	6,800	340
black powder fuse surrounded by 6 lengths of 5.32 gm/m Primacord, stainless steel electrodes	7,300	6,500

wave-front would have traveled about 150 m from its source. To trigger lightning, we need to have our conductor cut across a potential difference of more than 2 MV and this could require a conductor length in excess of 200 m for ambient electric field strengths of the order of 10^4 V/m. Accordingly, the observed plasma duration produced by the detonation may be marginal or insufficient for the lightning triggering applications. We, therefore, considered means of prolonging the duration of conduction. An increase in the amount of explosive by the use of 85 gm/m (400 grain/foot) fuses did not prolong the conductive regime, so that we tried the seeding of the plasma with potassium ions and then by heating of the channel with electrical discharges.

The seeding was attempted by surrounding lengths of "slow fuse" containing a potassium-based, black powder in the center of six strands of 25 grain Primacord taped together to form a cylindrical rope. This composite arrangement with a central core of potassium was placed between the electrodes and detonated as before. In this first test we found no significant change in the duration of the channel conduction but the test was preliminary and not conclusive. Other configurations of alkali metal seeding agents are possible: The Ensign Bickford people have indicated that, with suitable support, they could make specially formulated fuses containing cesium or potassium salts. These have not yet been investigated.

Channel Heating by Electrical Discharges

In another attempt to prolong the duration of a conducting plasma, we discharged high voltage capacitors through the explosively - produced plasma channels to simulate possible heating by incipient lightning. The experimental arrangement is shown in Figure 7. In this experiment, a capacitor was charged by a 15 kV high voltage supply. One side of the capacitor was connected to earth and the other was connected to one of the steel electrodes that was isolated from earth by plastic insulators. The other steel electrode, attached to the first by a 0.5 m length of 10.6 gm/m (50 grain/foot) Primacord was connected to earth through a 1.2 ohm resistor. The current flow was sensed by measuring the voltage drop across this resistor by a Biomation 8100 waveform analyzer through a protective voltage divider. When the Primacord was detonated, charge from the capacitor flowed through the resulting plasma channel and energies of up to 1600 J were supplied to augment the 33,500 J released by the detonating fuse. After each test, the capacitor was found to be partially discharged by the plasma channel which usually extinguished at a capacitor potential of about 3000 to 4000 volts.

The luminosity detector was not used in these tests because the second Biomation unit was employed in measuring the voltage drop across the plasma channel. The results from these first tests with a capacitor are shown in Table III. Currents of up to 10 amperes were detected in the discharge channels but the

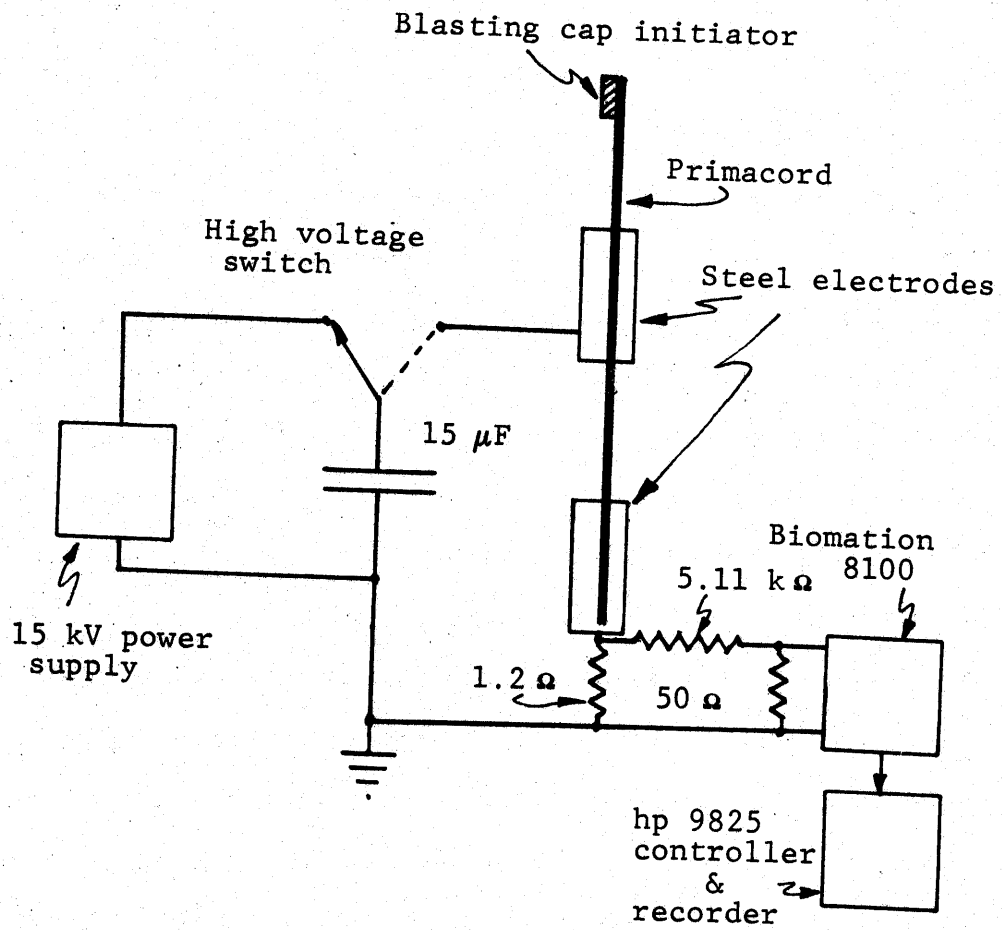


Figure 7 Circuit diagram for experiments aimed at electrically heating the Primacord detonation channel by discharge of a high voltage capacitor.

TABLE III

Measurements Obtained from Primacord Plasma Channels,
Heated Electrically

Capacitor (μ F)	Initial Potential (V)	Final Potential (V)	Resistance (Channel & Electrodes) Mean Minimum	Current Duration (ms)	Maximum Current (A)	Electrical Energy Release (J)	
100	3,690	3,075	4,780	835	18.6	4.0	310
15.3	10,410	3,530	5,900	830	16.0	8.4	720
15.3	14,950	3,910	4,900	930	14.8	10.1	1,560

Armor Plate Electrodes
0.5 m lengths of 10.6 gm/m Primacord

durations of the conductive period were not prolonged over those for the unheated channels.

These results suggested that the electrical discharges through the channel were ineffectual in extending the plasma duration due to the relatively small amount of electrical energy and to the negative resistance effect: The hotter the plasma, the more it was ionized and the lower was its resistance which discharged the capacitor more rapidly.

To reduce this effect, a current limiting resistor was used in the next set of experiments with the electrical discharges. A 1686 ohm, 500 watt resistor was inserted in series with the Primacord channel in the circuits shown schematically by Figure 8. The results from electrical discharges through 0.5 m lengths of 10.6 gm/m Primacord on stainless-steel electrodes are shown in Figures 9, 10 and 11 and are listed in Table IV. In this arrangement, the capacitor potential at arc extinction could not be read directly and the granularity of the Biomatron reading gave an uncertainty in the voltage drop readings. As can be seen, however, this technique kept some of the the arc discharge in a conducting state for significantly longer durations than achieved earlier. After the analysis described below, we concluded that the load resistance was too large causing the arc to be starved, which led to its extinction. With a smaller load resistance, the duration of the arc could have been extended to about 150 ms before the capacitor voltage dropped to the extinction level.

To explain our experimental results, W. P. Winn plotted Grotrian's (1915) relation between the current through an electric arc in air between iron electrodes and the potential difference across the arc:

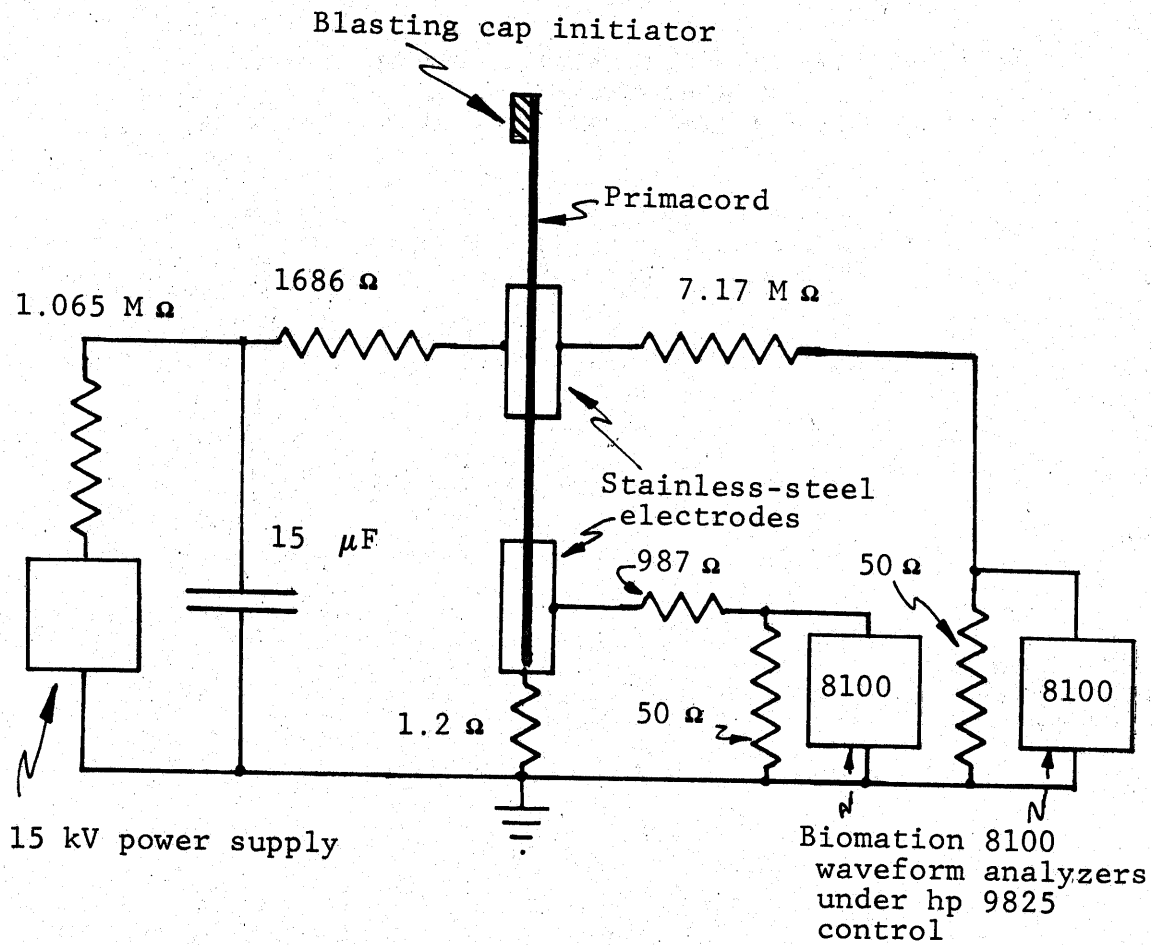


Figure 8 Circuit diagram for experiments aimed at electrically heating the Primacord detonation channel by discharge of a high voltage capacitor through a 1686 ohm load resistor. The voltage drop across the arc was measured in these experiments.

TABLE IV

Measurement Obtained from Primacord Channels,
Heated Electrically
Through a 1686 Ohm Current Limiting Resistor

Initial Potential (V)	Resistance (Channel & Electrodes)		Current Duration (ms)	Maximum Current (A)
	Mean	Minimum		
10,850	2,900	550	53	2.0
15,060	11,760	4,100	51.5	3.6
15,250	20,140	4,300	32.6	3.4
14,890	16,100	2,800	15.9	3.3

15.3 μ F Capacitor
Stainless steel electrodes separated by 0.5 m
10.6 gm/m Primacord

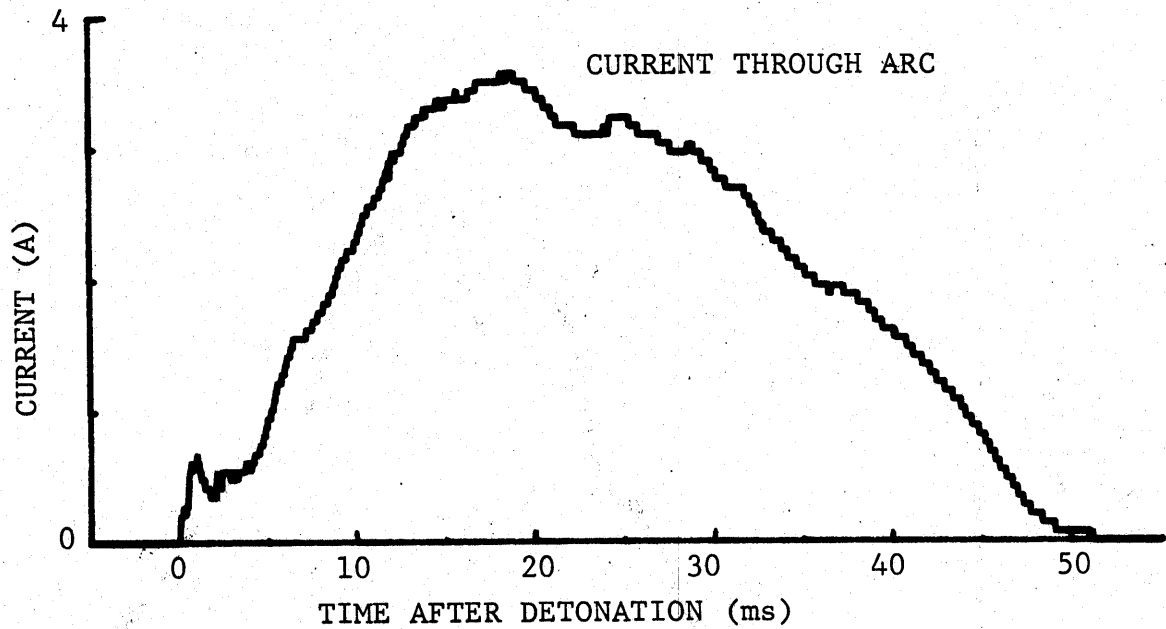
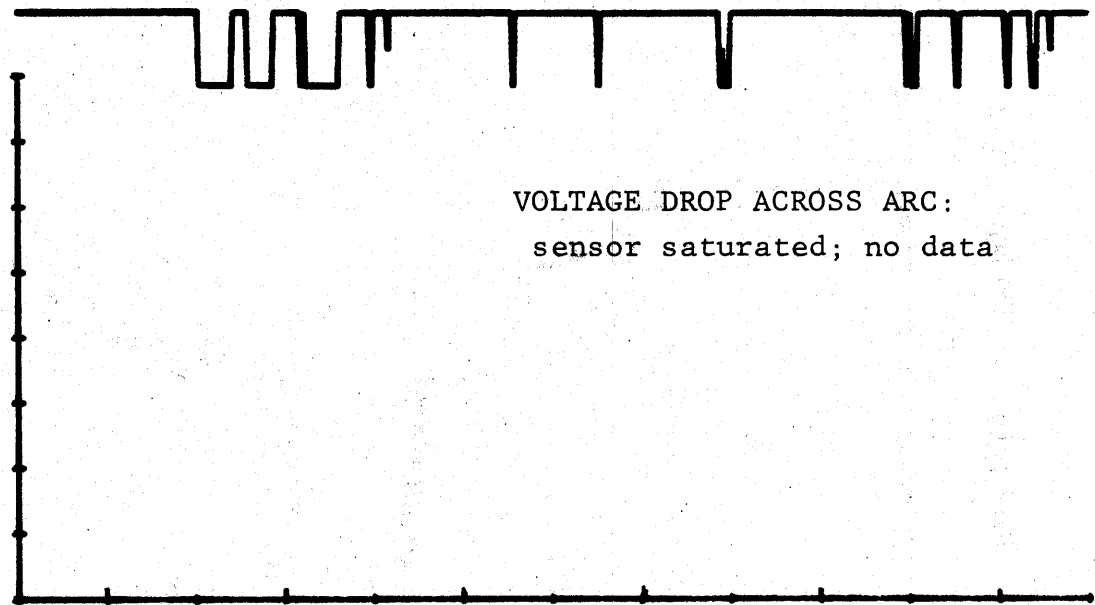


Figure 9 Plot of the electric current through the arc created by the detonation of a 0.5 m length of 10.6 gm/m Primacord connected across a 15 μ F capacitor initially charged to 15.1 kV.

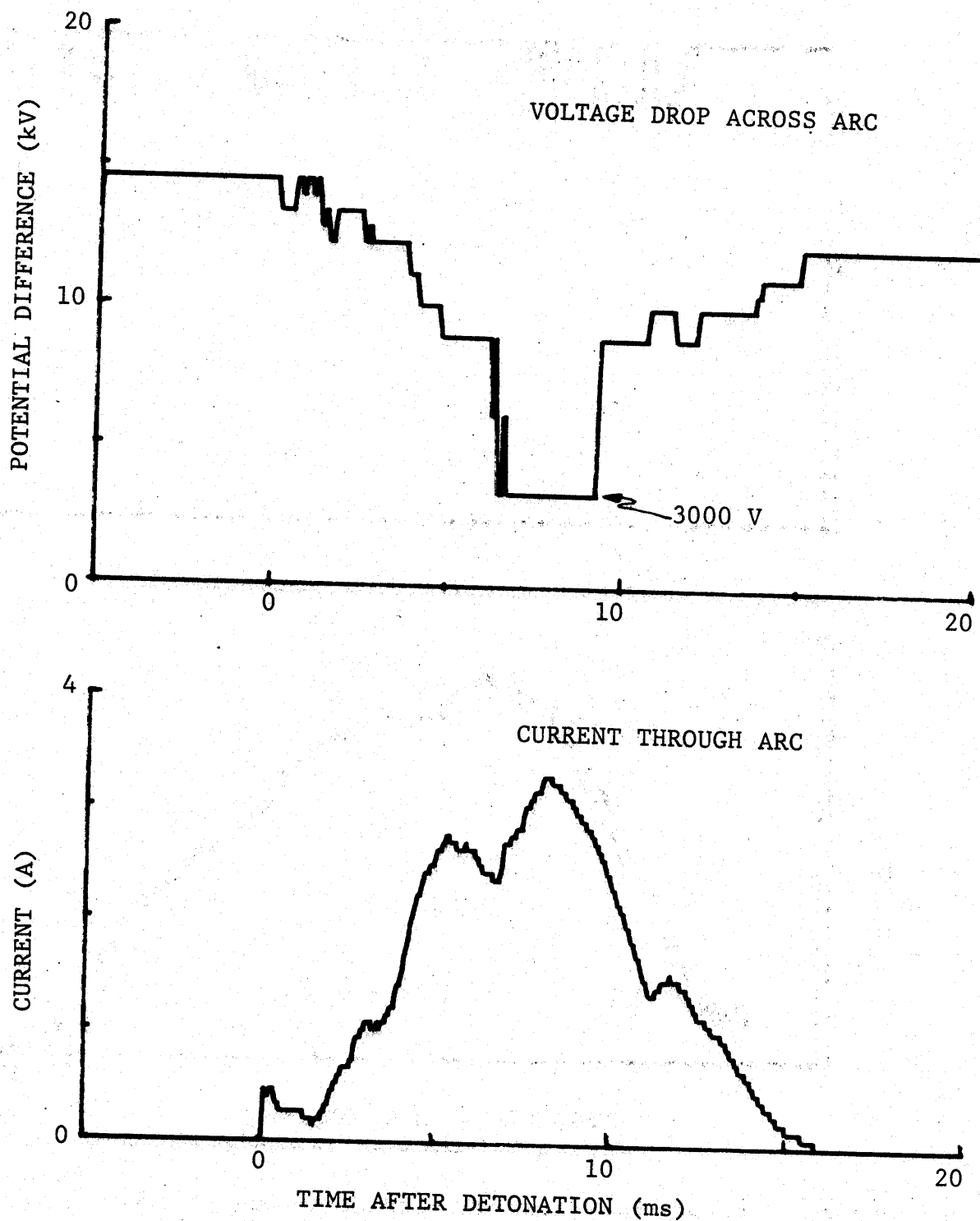


Figure 10 Plot of the voltage drop and electric current through an arc created by the detonation of a 0.5 m length of 10.6 gm/m Primacord connected across a 15 μ F capacitor initially charged to 14.9 kV. The arc apparently extinguished after its voltage drop decreased somewhere below 3000 V.

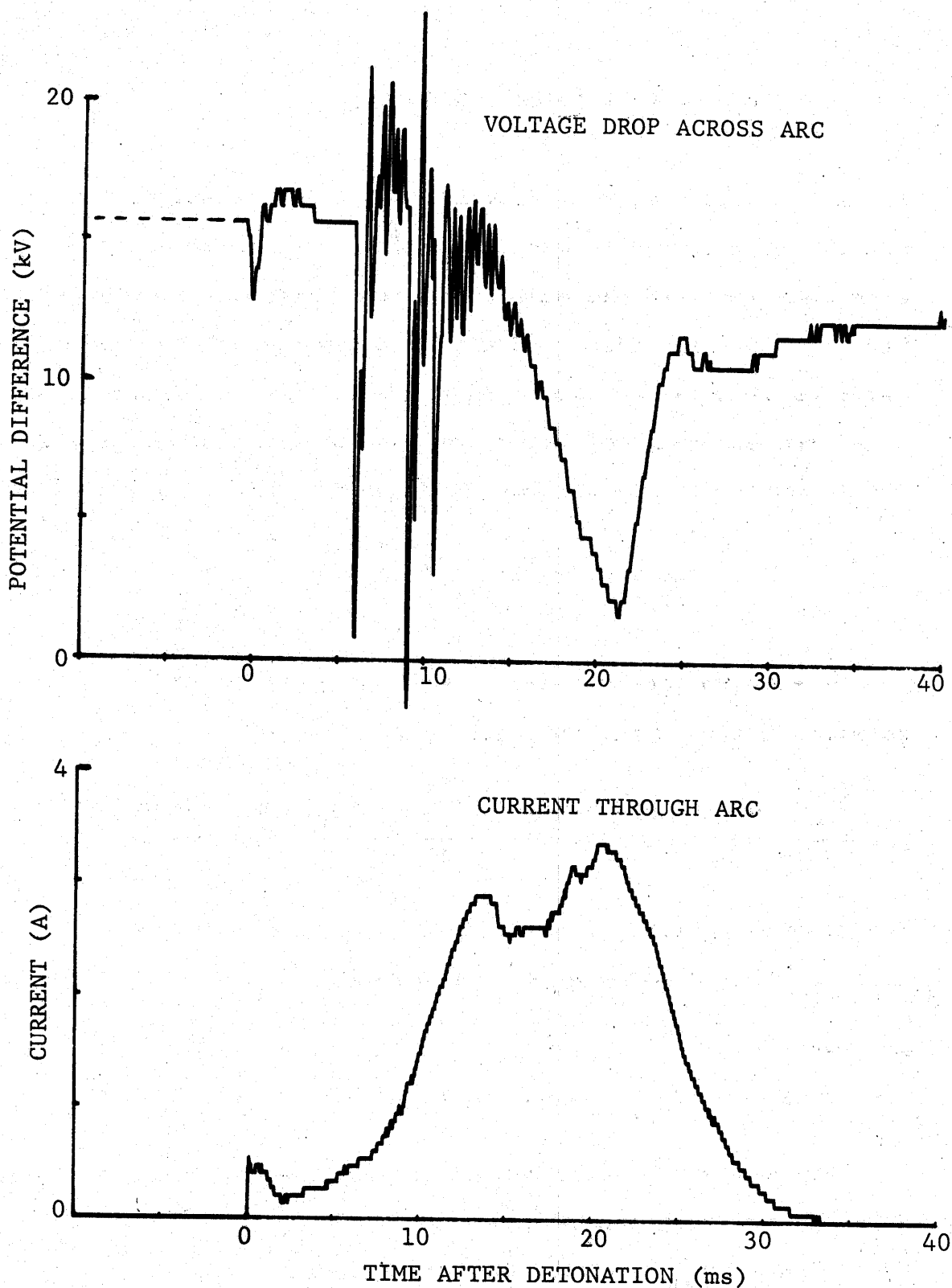


Figure 11 Plot of the voltage drop and electric current through an arc created by the detonation of a 0.5 m length of 10.6 gm/m Primacord connected across a 15 μ F capacitor initially charged to 15.25 kV. The arc apparently extinguished after its voltage drop decreased to about 1600 volts.

$$V_{\text{arc}} = 62 + (1140 + 3260/I)L$$

where I is the current in amperes and L is the arc length in meters. The 62 V term reflects a voltage drop across the electrode vapor at the ends of the arc while the length dependent term is a characteristic of the long gas column. A plot of this relation is shown in Figure 12 on which has been drawn a "load line" representing the effect of the current limiting resistor R. The equation for the voltage difference is given by

$$V_{\text{power supply}} = V_{\text{arc}} + IR.$$

A solution for the arc current from Grotrian's relation intersecting the load line gives

$$I = \frac{V_{\text{ps}} - 62 - 1140L + [(62 + 1140L - V_{\text{ps}})^2 - 4 \times 3260RL]^{1/2}}{2R}$$

The minimum current is given when

$$(62 + 1140L - V_{\text{ps}})^2 = 4 \times 3260RL$$

so that

$$I_{\text{min}} = (V_{\text{ps}} - 62 - 1140L)/2R.$$

The maximum R value for an intersection of its load line with the arc relation is given by

$$R = (V_{\text{ps}} - 1140L - 62)^2 / (4 \times 3260L)$$

or about 1262 ohms for our final potentials.

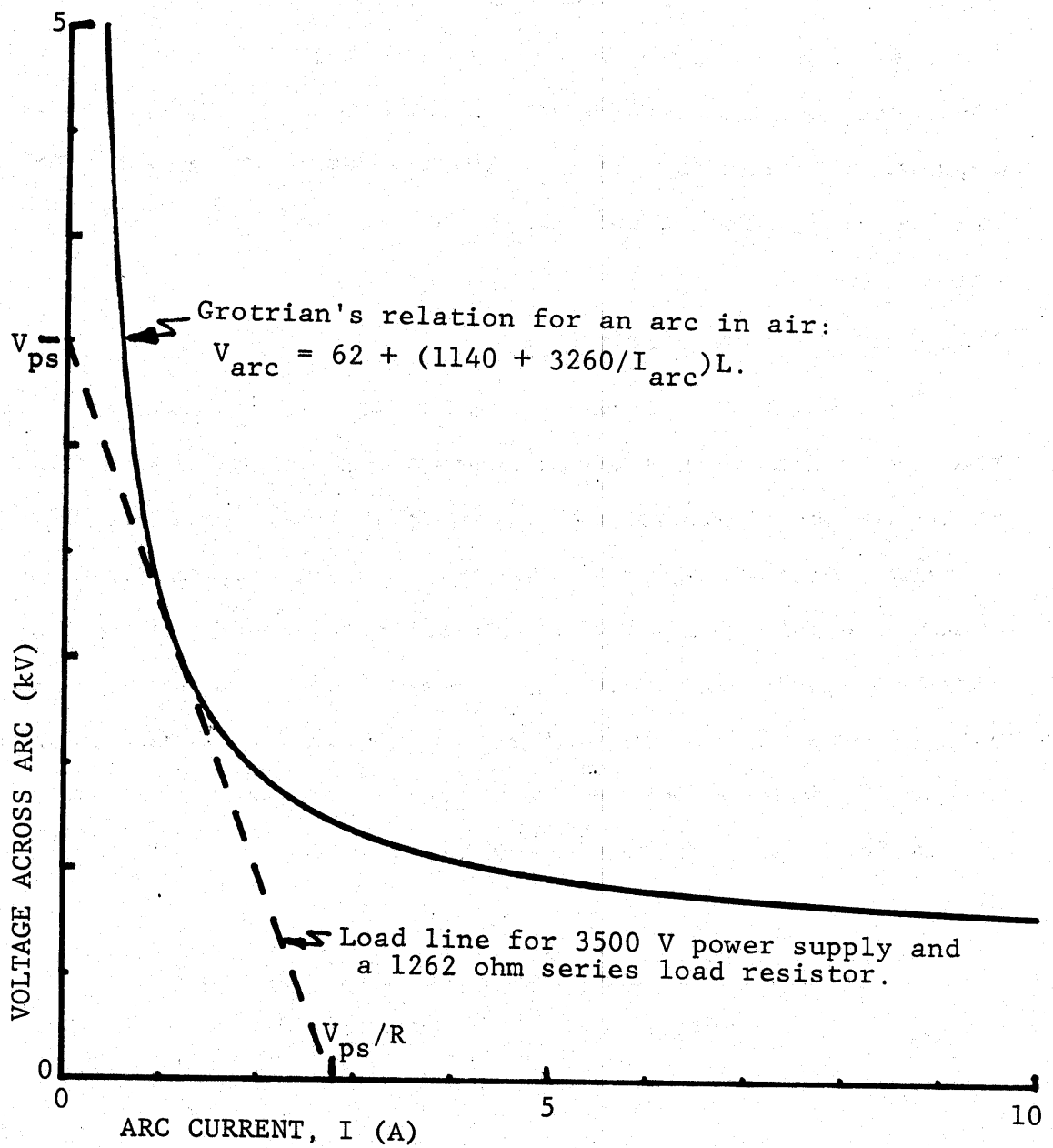


Figure 12 Grotrian's (1915) plot of arc current versus voltage with a 1262 ohm load line showing an arc current minimum of about 1.1 A.

For our experimental conditions, I_{\min} is about 1.14 A for an arc extinguishing at an arc voltage drop of 2100 V. This value is compatible with our experimental results and is of interest as an estimate of the minimum required for the "long continuing currents" associated with lightning. It further suggests that a power input of greater than 4800 W/m may be required to sustain an arc in air at a pressure of 0.85 atmospheres.

Displacement Current Estimates

It is also of interest to calculate the displacement current that would flow in the conducting channel behind a detonating wave through a length of suspended Primacord under the influence of an external, vertical electric field, E_0 . For this calculation, we assume the conducting channel to be approximately represented by a vertical, prolate ellipsoid with a semi-major axis length of c and a tip radius of curvature of a . From Gauss' law, the induced charge, Q , on the upper half of the conducting surface of an ellipsoid is given by Moore (1983) as

$$Q = \frac{\pi \epsilon E_0 (c^2 - ac)}{0.5(1 - a/c)^{-\frac{1}{2}} \ln \left[\frac{1 + (1 - a/c)^{\frac{1}{2}}}{1 - (1 - a/c)^{\frac{1}{2}}} \right] - 1}$$

The time derivative of this relation gives the displacement current I through the midsection of the ellipsoid. For constant E_0 , and an ellipsoidal conductor growing at dc/dt in the direction of E :

$$\frac{dQ}{dt} = \pi \epsilon E (dc/dt) \left\{ \frac{2c}{D} + \frac{(c^2 - ac)}{2D^2} \left[\frac{a \ln \left[\frac{1 + (1-a/c)^{1/2}}{1 - (1-a/c)^{1/2}} \right]}{2c^2 (1-a/c)^{3/2}} + \frac{1}{2(c-a)} \right] \right\}$$

where

$$D = 0.5(1-a/c)^{-1/2} \ln \left[\frac{1 + (1-a/c)^{1/2}}{1 - (1-a/c)^{1/2}} \right] - 1$$

At our area of operations on South Baldy Peak, the value of E_0 at the surface is frequently in excess of 20 kV/m. For this field strength and for $c = 150$ m, $dc/dt = 6000$ m/sec and $a = 1$ cm, the calculated value for I is about 0.25 A. Under these strong electric fields, the earth emits point discharge ions with current densities of up to 10 nA/m^2 . The emission of these ions limits E_0 , the strength of the field at the earth's surface. The fields aloft, above many of these point discharge ions, however are much stronger. According to Wilson (1925) the strength $E(z)$ of the field aloft is given by

$$E(z) = [E_0^2 + 2jz/E_0k]^{1/2}$$

where j is the point discharge ion current density

z is height

K is the point discharge ion mobility.

Since the field lines are concentrated near the upper tip of a conducting ellipsoid, the use of the field strength at this level is appropriate for a calculation of an upper limit on the displacement current at the center of a full, prolate ellipsoid.

Plots are given in Figure 12 of the displacement currents versus the length c of the semi-major axis growing at 6000 m/sec above a surface E_0 of 20,000 V/m with various point discharge current densities. We may expect these currents to increase significantly whenever a plasma streamer develops for its propagation velocity may be much greater than that of the Primacord.

A plasma streamer may develop whenever the strength of the electric field at the tip of the growing channel exceeds some critical value. In stronger fields, the energy required for the ionization of a new volume is provided by the energy released by the exclusion of the field from this volume.

Phelps (1974) has found that positive streamers, once initiated, will grow exponentially in electric fields stronger than 4×10^5 V/m at one atmosphere.

The equilibrium strength, E_t , of the electric field at the tip of a conducting prolate ellipsoid in an external field of E_0 is given by Moore (1983) as

$$E_t = E_0 \frac{(1 - a/c)^{3/2}}{(0.5a/c) \ln \left[\frac{1 + (1 - a/c)^{1/2}}{1 - (1 - a/c)^{1/2}} \right] - (1 - a/c)^2}$$

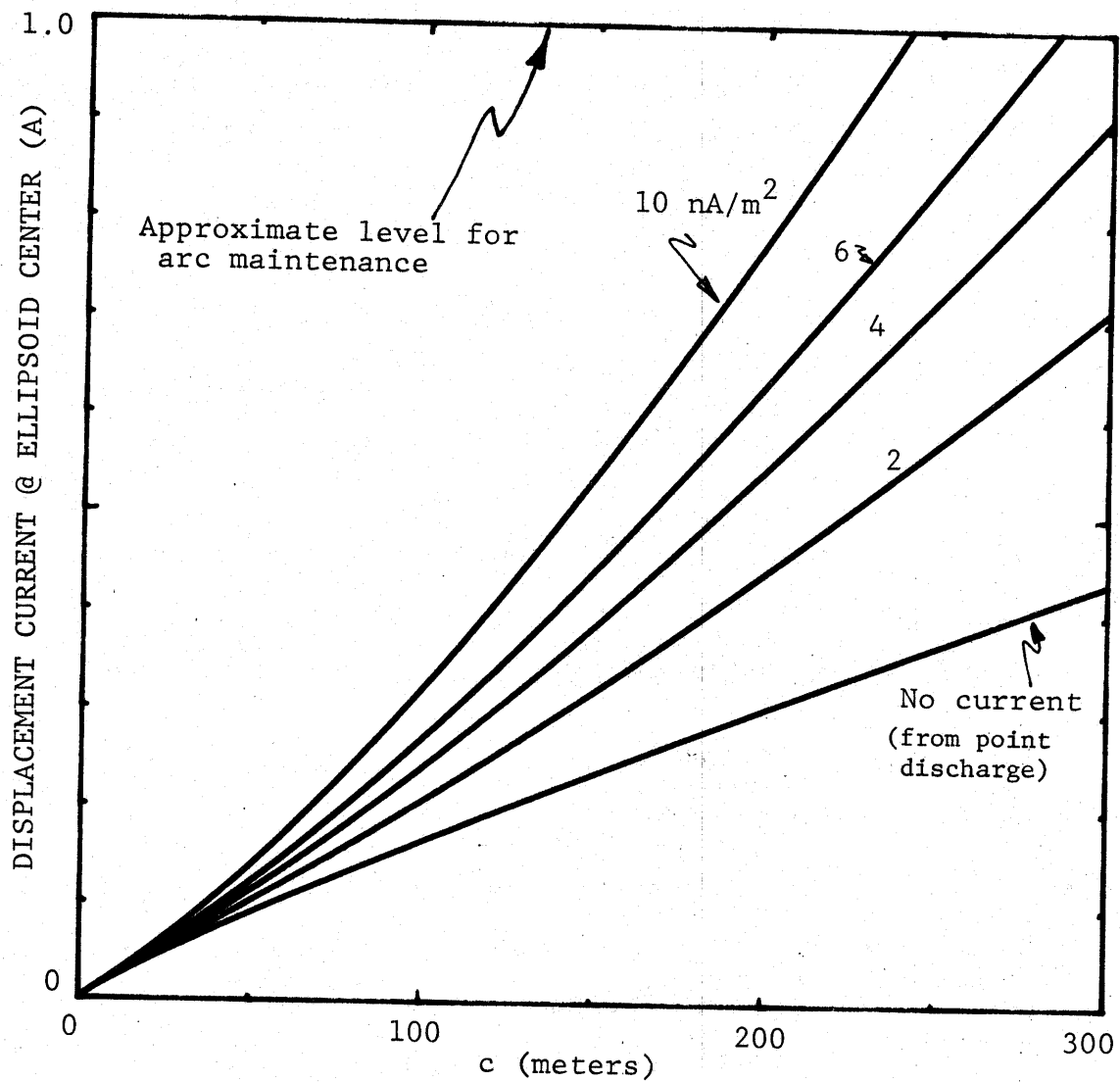


Figure 13 Plot of displacement currents in an ellipsoidal conductor with the semi-major axis growing at 6000 m/sec an electric field of 20 kV/m at the earth's surface under various point discharge current densities.

For $c > 50$ m, $a = 1$ cm and $E_o > 10$ kV/m, $E_t > 10$ MV/m. It seems likely, therefore, that the vertical, conducting channel produced by the detonation of a 50 m or greater length of Primacord in strong electric fields could grow and develop into a lightning discharge.

From this exercise, it appears that detonating fuses may have some utility in the triggering of lightning.

Conclusions

From the preceding results, we expect that conducting channels can be produced in the atmosphere by the detonation of vertically hanging lengths of Primacord. The maximum conductive duration of these channels is about 25 ms so that the resulting maximum conductor length probably extends approximately 150 m from the detonating wave front back to where the plasma is extinguishing and is no longer a conductor.

As Carl Baum has suggested, initiation of the detonation at the center of a vertical length of Primacord could cause two separating wave fronts that could diverge by 300 m before the center section becomes non conductive. If this approach were still marginal, simultaneous firing of detonating caps at the $1/4$ and at the $3/4$ points on a 600 m length of Primacord could initiate 4 wave fronts, briefly making the entire length into a conductor.

The next step clearly is to suspend vertical lengths of Primacord from a captive balloon beneath a thundercloud and to determine if lightning triggering can be achieved.

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