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ON USING A SENSE WIRE TO QUANTITATE THE MAGNETIC FLUX LEAKAGE THROUGH AN APERTURE IN AN ELECTROMAGNETIC SHIELD

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ABSTRACT

A formulation is developed for determining the sensitivity of positioning a sense wire behind an aperture in an electromagnetic shield to quantitate the flux leakage through the aperture. Both hardened and unhardened circular apertures are considered. Comparisons with measured data are used to verify the analysis.

INTRODUCTION

In order to quantitate the electromagnetic field penertration through an aperture perforated shield, a sense wire circuit may be located behind the aperture, and the voltage induced in the circuit measured [1]. A typical measurement configuration is shown in Figure 1. Here the resistor R is selected to be sufficiently large so that the aperture is not appreciably loaded. A measure of the magnetic flux penetrating the aperture is the voltage induced in the circuit.

The sense wire circuit configuration is easily implemented for apertures in flat surfaces with open regions behind the aperture. However, for apertures in curved surfaces and for apertures with restricted interior regions, some spacing between the aperture and the sense wire may be required. This paper presents an analytical study of the voltage and the current induced in the sense-wire circuit as the spacing between the aperture surface and the wire is varied.

A circular aperture in a perfectly conducting sheet is used to model an aperture perforated shield. The penetration field is obtained by using the approach presented by Jackson [2]. Analytical expressions are derived for the components of the penetrating magnetic field for a uniform magnetic field illumination. A quasi-static solution technique is used, but the results are valid for general illumination provided the aperture dimensions are small in terms of wavelength.

Faraday's law of induction is used to obtain the induced voltage in the sense-wire circuit. The equivalent circuit for the sense wire is derived, and an expression is obtained for determining the sense-wire current. Hardened apertures as well as open apertures are considered.

Comparisons of computed sense-wire currents to measured currents are made to verify the analysis.

II. ANALYSIS

In order to model a perforation in the wall of an electromagnetic shield, a circular aperture in a perfectly conducting sheet is considered (see Fig. 2). The magnetic field illuminating the aperture is considered to be uniform and directed parallel to the sheet, i.e. $H_0 a_y$. A magnetostatic solution can be obtained by solving Laplace's equation for the magnetic scalar potential while imposing the appropriate boundary conditions. Details of the procedure are given by Jackson [2]. The resulting magnetic field distribution is essentially the same that would exist if the aperture were immersed in a low frequency field (where the maximum linear dimension of the aperture is small compared to the wavelength). In this case, H_0 is simply the surface magnetic field that would exist at the aperture location if the aperture were absent.

Defining the magnetic scalar potential, Φ_M as

$$H = -\nabla\Phi_M \tag{1}$$

it is easily shown that at low frequency the potential function satisfies

$$\nabla^2\Phi_M = 0 \tag{2}$$

It is convenient to express the potential

$$\Phi_M(\vec{r}) = - H_o y + \Phi^{(1)}(\vec{r}) \quad \text{for } z > 0 \tag{3}$$

$$= - \Phi^{(1)}(\vec{r}) \quad \text{for } z < 0$$

According to Jackson [2] the solution for $\phi^{(1)}$ subject to the appropriate boundary conditions for a circular aperture in the sheet is

$$\Phi^{(1)}(\vec{r}) = \frac{2H_0a^2}{\pi} \int_0^\infty dk\ j_1(ka)\ e^{-k|z|}\ J_1(k\rho) \sin\phi \tag{4}$$

where

$$j_1(ka) = \sqrt{\frac{\pi}{2ka}}\ J_{3/2}(ka) \tag{5}$$

Here j_1 is the spherical Bessel function of the first kind; order 1 and J_1 and $J_{3/2}$ are cylindrical Bessel functions of the first kind with orders 1 and $^3/2$ respectively.

With (3) and (4) it is possible to develop an expression for $\vec{H}$ via the gradient operation, (1). An analytical evaluation of (4) is not possible in general, but results for special cases are available. In the aperture opening,

$$\vec{H}_{tan} = \frac{1}{2} H_o \hat{a}_y \tag{6}$$

$$H_z(\rho, \phi, 0) = \frac{2H_0}{\pi} \frac{\rho}{\sqrt{a^2 - \rho^2}} \sin\phi \tag{7}$$

and on the shielded side of the sheet, for $\rho \geq a$ [2]

$$H_x(\rho, \phi, 0-) = -\frac{2H_0a^3}{\pi} \frac{\cos\phi \sin\phi}{\rho^2\sqrt{\rho^2 - a^2}} \tag{8}$$

$$H_y(\rho, \phi, 0-) = -\frac{2H_0a^3}{\pi} \frac{\sin^2 \phi}{\rho^2\sqrt{\rho^2 - a^2}}$$

$$-\frac{H_0}{\pi}\left[\frac{a}{\rho}\sqrt{1 - \frac{a^2}{\rho^2}} - \sin^{-1}\left(\frac{a}{\rho}\right)\right] \tag{9}$$

Using the dipole approximation for (4) yields

$$\Phi_M^{(1)}(\vec{r}) \simeq \frac{2H_0a^3}{3\pi} \frac{\rho \sin \phi}{(\sqrt{\rho^2 + z^2})^3} \tag{10}$$

when $r^2 >> a^2$. Applying the gradient operator to (10) yields the dipole approximation to the magnetic field components. On the shielded side of the sheet the dipole approximation yields for the field components,

$$H_x(\rho, \phi, 0-) \simeq -\frac{2H_0}{\pi}\left(\frac{a}{\rho}\right)^3 \cos \phi \sin \phi \tag{11}$$

$$H_y(\rho, \phi, 0-) \simeq -\frac{2H_0}{\pi}\left(\frac{a}{\rho}\right)^3 \sin^2 \phi + \frac{2H_0}{3\pi}\left(\frac{a}{\rho}\right)^3 \tag{12}$$

It is clear from (8)-(12) that the dipole approximation is not adequate for points in the vicinity of the aperture.

For a sense wire crossing the aperture, the determination of the total flux linking the aperture requires an evaluation of the gradient of (4). The maximum flux linkage occurs when the sense wire is located in the plane of the aperture so that it bisects the aperture in the direction perpendicular to $\vec{H}_o$. In this case the magnetic flux linking the sense wire circuit is

$$\Psi_M \Big|_{max} = \mu_o \int_0^{\pi} \int_0^{a} H_z(\rho, \phi, 0)\ \rho d\rho d\phi \tag{13}$$

Using (7) in (13) yields

$$\Psi_M \Big|_{max} = \mu_o H_o a^2 \tag{14}$$

The voltage induced in the sense wire circuit is

$$V_{oc} = j\omega\Psi_M \tag{15}$$

For the sense wire configuration shown in Figure 1, the magnetic flux linking the sense-wire circuit is

$$\Psi_M = \Psi_M \Big|_{max} - 2\,\mu_o \int_{-h}^{o} \int_{o}^{\ell/2} H_y(\rho, \phi, z) \Big|_{\phi=0} d\rho dz \tag{16}$$

Substituting (3) and (4) into (1) to obtain the magnetic field and using the result in (16) along with (14) yields

$$\Psi_M = \mu_o H_o a^2 \left[1 - \frac{4}{\pi} \int_0^{\infty} dk\ j_1(ka) \int_{-h}^{o} dz\ e^{kz} \int_0^{\ell/2} d\rho\ J_1(k\rho)/\rho \right] \tag{17}$$

The foregoing may be simplified by evaluating the integral over z and applying a change of variables.

$$\Psi_M = \mu_o H_o a^2 \left[1 - \frac{4}{\pi} \int_0^{\infty} du\ j_1(u)\ \frac{1 - e^{-\ uh/a}}{u} \int_0^{(u\ell/2a)} dv\ J_1(v)/v \right] \tag{18}$$

Unfortunately, (18) can not be evaluated in closed form. However, a numerical evaluation can be accomplished as described in the Appendix A.

If the sense wire is not perpendicular to $\vec{H}_0$, the magnetic flux linking the sense wire is reduced. Suppose the angle between the sense wire and the direction normal to $\vec{H}_0$ is α. Then (13) yields

$$\Psi_M \Big|_{max,\ \alpha} = \mu_0 H_0 a^2 \cos \alpha \qquad (19)$$

It is expected that (18) would be reduced by a corresponding or greater amount.

Sense-Wire Circuit

The excitation of the sense-wire current involves both inductive coupling and capacitive coupling. An analysis of inductive coupling has been developed. A similar analysis of capacitive coupling may be developed by considering the electric flux penetrating the circular aperture while utilizing the quasistatic approximation. The magnetic coupling introduces an equivalent series voltage source driving the sense wire, and the capacitive coupling provides an equivalent shunt current source [1]. If the sense wire is thin and the aperture small then the inductive coupling should be significantly greater than the capacitive coupling.

Generally the measurement of the voltage induced in the sense wire is accomplished by inserting a large resistance in series with the wire and measuring the induced wire current. If the resistance is located midway across the aperture then the electric flux coupling provides no excitation of the resistor current. Consequently, the electric flux excitation can be ignored.

An analysis of the sense wire excitation begins with the equivalent circuit shown in Figure 3. A discussion of the development of the equivalent circuit is presented in Appendix B.

When the aperture is hardened, i.e., covered by a transparent conductive film on a glass or plastic insert or by a wire mesh bonded to the rim, the reduction in the sense-wire excitation can be modeled by an aperture load impedance, Z_ℓ [3, 4]. For the circular aperture

$$Z_\ell = \frac{3\pi}{8} [Z_s + 2 \pi R_c]$$

where Z_s is the equivalent sheet impedance of the covering and R_c is the contact resistance between the covering and the rim of the aperture [4]. For convenience in illustrating the presented analysis, only unloaded apertures are considered.

RESULTS

First, the reduction in the magnetic flux linking the sense wire circuit is studied. This also represents the reduction in the open circuit voltage V_{oc}. From (18) it is noted that

$$\Psi_M = \Psi_M(\frac{h}{a} , \frac{\ell}{a})$$

Results are presented in Table 1; here it is noted that the flux reduction is more significant for larger ℓ.

Second, the reduction in the wire current is studied for an open ($Z_\ell = \infty$) aperture and $R_W = 0$. For maximum wire current the sense wire should extend across the aperture opening ($h = 0$, $\ell = 2a$). The resulting maximum wire current is

$$I_w(h = 0) = \frac{\Psi_M(h = 0)}{L_a + L_w(h = 0)} \tag{20}$$

where the inductance for a straight wire is [5],

$$L_w(h = 0) = \frac{\mu_o \ell}{2\pi} \left[\ln \frac{2\ell}{a_w} - 0.75 \right] \tag{21}$$

Here a_w is the sense wire radius. The inductance for a small circular aperture is approximately,

$$L_a \simeq \frac{1}{2} \mu_o a \tag{22}$$

Note that in deriving (20), the image contributions of the equivalent circuit in Figure B1 are not required.

The measured data for the shorted sense wire were obtained at 3 frequencies-10, 50, and 90 MHz. Table 2 presents the average and standard deviation from the three measurements. The calculated data are obtained by considering the sense wire to be 12 gauge, $a_w = 1$ mm. These results indicate that for a small sense wire separation, $h \lesssim 0.4a$, the reduction in the sense wire current is primarily due to the reduction in flux linking the sense wire.

For $h \neq 0$ the equivalent circuit yields the sense wire current

$$I_w = \frac{2\Psi_M}{L_a + 0.5\, L_w} \tag{23}$$

where L_w is the inductance of the sense wire circuit including the image, i.e.,

$$L_w = \frac{\mu_o}{\pi} \left[\ell \ln \frac{4h\ell}{a_w(\ell+d)} + 2h \ln \frac{4h\ell}{a_w(2h+d)} + 2d - \frac{7}{4} (\ell + 2h) \right] \quad (24)$$

where

$$d = \sqrt{(2h)^2 + \ell^2} \quad (25)$$

Finally, the sense-wire current reduction can be expressed

$$\frac{I_w}{I_w(h = 0)} = \frac{\Psi_M}{\Psi_M(h = 0)} \quad \frac{L_a + L_w(h = 0)}{L_a + 0.5 L_w} \quad (26)$$

Calculated results for the sense wire current ratio are presented in Table 2. Corresponding measured results are also presented [6]. However, the measured results were obtained for a square aperture. In order to make meaningful comparisons the inductance for a square aperture is used in (26). Since $L_w >> L_a$ generally occurs, using the square aperture inductance rather than the circular aperture inductance provides only a small change in the results. For the 10" x 10" square aperture the inductance is 284 nH [7].

Table 1: Magnetic Flux Linking a Sense-Wire Circuit Behind a Circular Aperture

h/a	$\Psi_M/(\mu_o H_o a^2)$	
	$\ell/2a = 1$	$\ell/2a = 2$
0	1.0	NA
0.1	0.9088	0.8878
0.25	0.8005	0.7516
0.50	0.6761	0.5902
0.8	0.5869	0.4689
1.0	0.5493	0.4155
2.0	0.4689	0.2923

Table 2: Reduction in Sense-Wire Current for a Circular Aperture, $\ell = 2a = 10"$.

h/a	$\Psi_M/\Psi_m(h = 0)$ CALCULATED	$I_w/I_w(h = 0)$ CALCULATED	$I_w/I_w(h = 0)$ MEASURED
0	0 dB	0 dB	0 dB
0.1	- 0.83 dB		
0.25	- 1.93 dB		
0.40	- 2.87 dB	- 3.8 dB	- 4.0 ± 0.8 dB
0.50	- 3.40 dB		
0.80	- 4.63 dB	- 6.7 dB	- 7.7 ± 2.2 dB
2.0	- 6.58 dB	- 13.9 dB	- 14.7 ± 3.6 dB

CONCLUSION

A formulation is presented for determining the sensitivity of positioning a sense wire behind an aperture in an electromagnetic shield to quantitate the flux leakage through the aperture. Hardened as well as unhardened apertures are considered. A quasistatic analysis is used to determine the magnetic flux linking a sense wire configuration behind a circular aperture. Simple circuit theory is then used to obtain the sense-wire current.

Comparisons with measured data yielded excellent agreement.

REFERENCES

1. F.C. Yang and C.E. Baum, "Use of Matrix Norms of Interaction Supermatrix Blocks for Specifying Electromagnetic Peformance of Subshields," Interaction Note 427, Air Force Weapons Laboratory, April 1983.

2. J.D. Jackson, Classical Electrodynamics (2nd ed.), John Wiley, New York, 1975, section 5.13.

3. S.Y. Liao, Microwave Devices and Circuits (2nd ed.), Prentice-Hall, Englewood Cliffs, NJ, 1985, section 2.6.

4. K.F. Casey, "Low-frequency Electromagnetic Penetration of Loaded Apertures," IEEE Trans. on Electromagnetic Compatibility, Vol. EMC-23, No. 4, November 1981, pp 112-122.

5. C.D. Hodgman, R.C. Weast and S.M. Selby (ed.), Handbook of Chemistry and Physics (38th ed.), Chemical Rubber Publishing Co., Cleveland OH, 1956, p 3015.

6. D.F. Higgins, R. Wheeler and E. Wenaas, "A Comparison of Theoretical Expressions and Experimental Data for EM Penetration Through Small Apertures," IEEE Trans. on Nuclear Science, Vol. NS-32, No. 6, December 1985, pp 4340-4345.

7. L.O. Hoeft, C. Herrmann and W.D. Prather, "Measured Electromagnetic Performance of Hardening Elements for Aircraft Windows and Doors," Proceedings of the IEEE International Symposium on Electomagnetic Compatibility, Atlanta GA, August 1987.

8. M. Abramowitz and L.A. Stegun, Handbook of Mathematical Functions, National Bureau of Standards, Washington DC, 1964, p 370.

9. R.F. Harrington, Time-Harmonic Electromagnetic Fields, McGraw-Hill, New York, 1961, p 108.

APPENDIX A

The evaluation of the double integral expression, (18), for the flux linking the sense wire can be accomplished in a straightforward manner. A polynomial approximation may be used to obtain an analytical evaluation of the integral,

$$\int_0^{u\ell/2a} dv\; J_1(v)/v$$

Polynomial approximations to the integrand are available for the ranges $0 \le v \le 3$ and $3 \le v \le \infty$ with a very small error bound [8]. Note that

$$\int_0^{u\ell/2a} dv\, \frac{J_1(v)}{v} = -\,J_1\!\left(\frac{u\ell}{2a}\right) + \int_0^{u\ell/2a} J_0(v)dv \tag{A1}$$

and for $u\ell/2a \ge 10$

$$J_1\!\left(\frac{u\ell}{2a}\right) > -\,0.25 \tag{A2}$$

$$0.75 < \int_0^{u\ell/2a} J_0(v)\, dv < 1.25 \tag{A3}$$

Therefore for $u\ell/2a \ge 10$

$$\left| \int_0^{u\ell/2a} dv\, \frac{J_1(v)}{v} \right| < 1.5 \tag{A4}$$

The foregoing will be used to bound the error in truncating the integral over u.

The error in truncating the integral over u at some large u_0 is for (18)

$$\left|\epsilon\right| \le \frac{3}{2} \int_{u_o}^{\infty} du \frac{1}{u^3} = \frac{3}{4} u_o^{-2} \qquad \text{(A5)}$$

since $| j_1(u) | < u^{-2}$ for $u > u_o$. For the data that are presented $u_o = 100$ so that the truncation error is less than 10^{-4}.

APPENDIX B

In order to derive the equivalent circuit for the sense wire in proximity to the aperture, equivalent magnetic surface currents are used to represent the aperture excitation [9]. The resulting configuration with images is shown in Figure B1. Applying Faraday's law for quasistatic conditions yield

$$\oint_C \vec{E} \cdot d\vec{\ell} = - \int_S \vec{M} \cdot d\vec{s} \tag{B1}$$

where C is the contour formed by the sense wire and its image, and S is the surface with boundary C.

It is easily shown that

$$\oint_C \vec{E} \cdot d\vec{\ell} = (2R_w + j\omega L_w)\ I_w \tag{B2}$$

and

$$\int_S \vec{M} \cdot d\vec{s} = - 2V_a \tag{B3}$$

where V_a is the voltage across the aperture and L_w is the inductance of the sense wire loop with its image. The open aperture may be represented by a Thevenin equivalent circuit with a shunt impedance, Z_ℓ, representing the aperture loading as shown in Figure B2. Here L_a is the aperture inductance and V_{oc} is the open circuit voltage given by (15). The aperture inductance is studied in a separate paper [7].

Combining the equivalent circuit for the aperture voltage with the results from (B1) - (B3) yields the equivalent circuit shown in Figure 3.

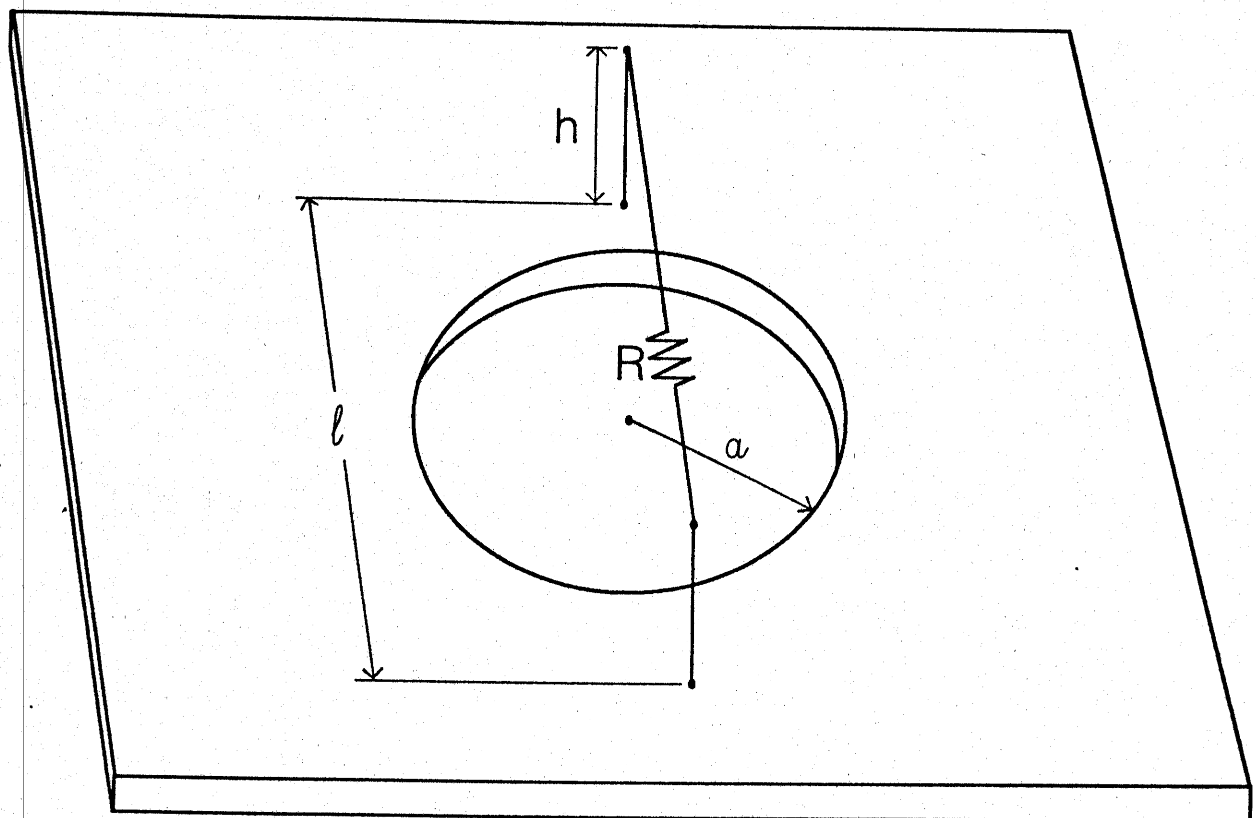

Figure 1: Sense wire behind a circular aperture to measure the magnetic flux leakage through the aperture

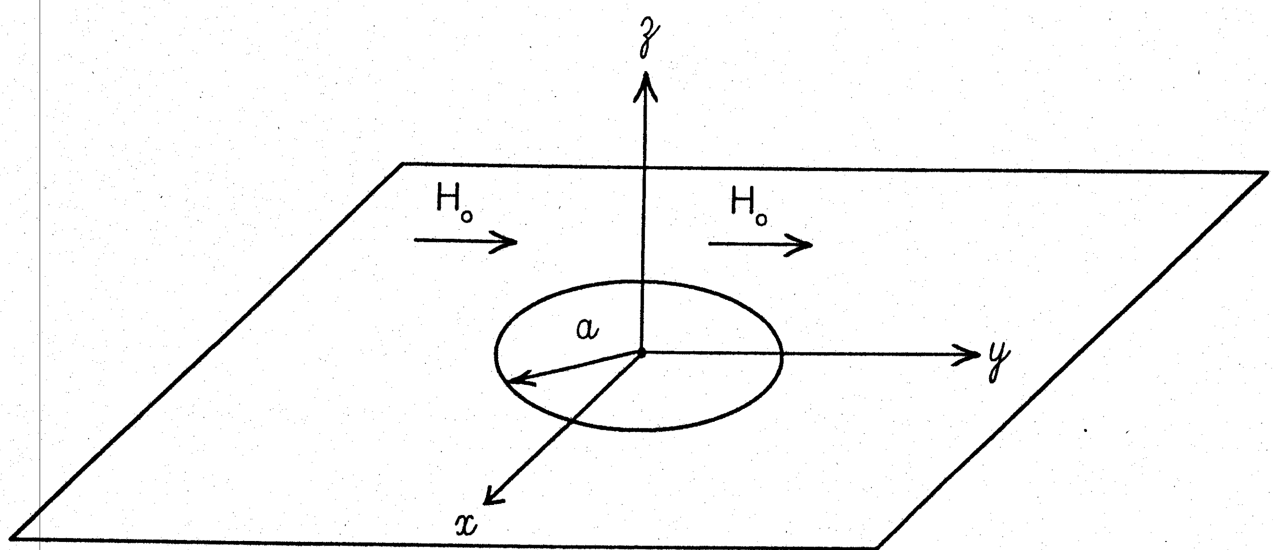

Figure 2: Circular aperture in a conducting sheet with an asymptotically uniform tangential magnetic field on one side.

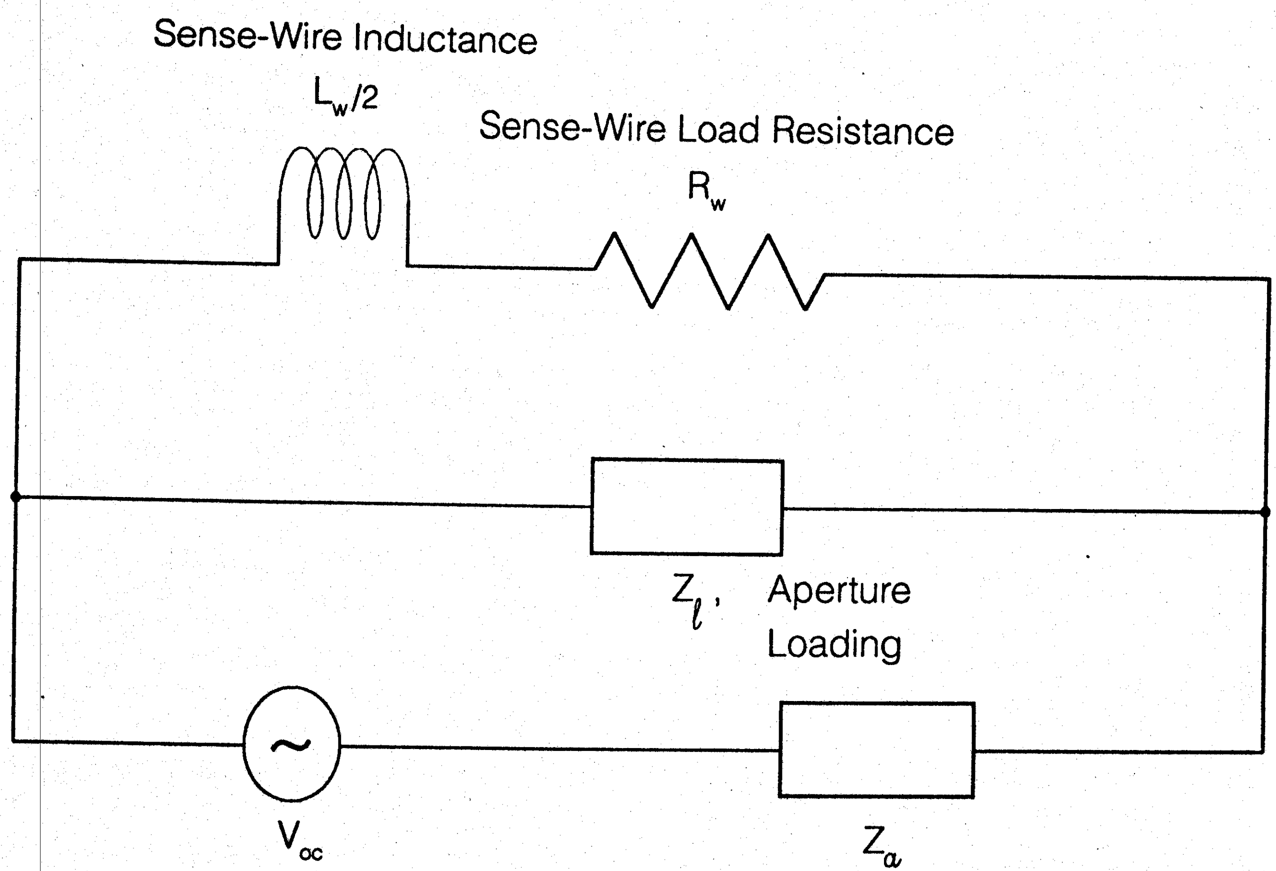

Figure 3: Equivalent circuit for the excitation of the sense wire behind an aperture.

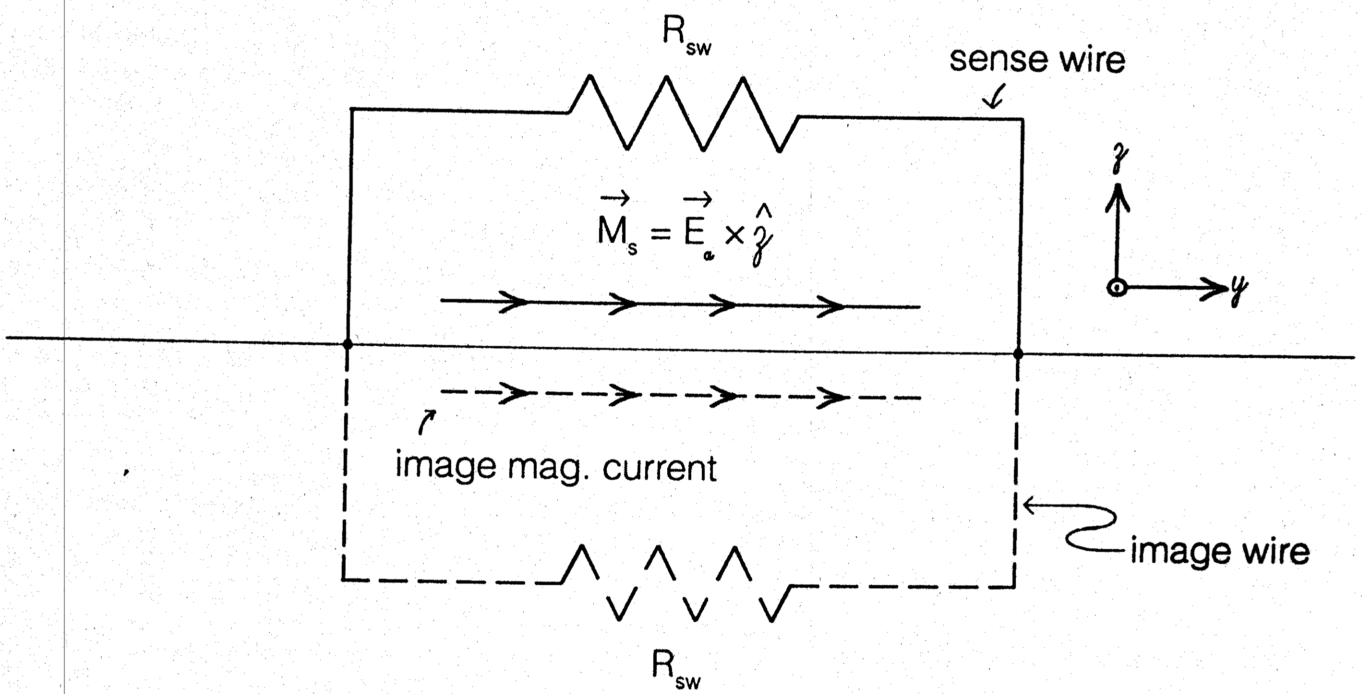

Figure B1: Sense wire configuration in proximity to an aperture.

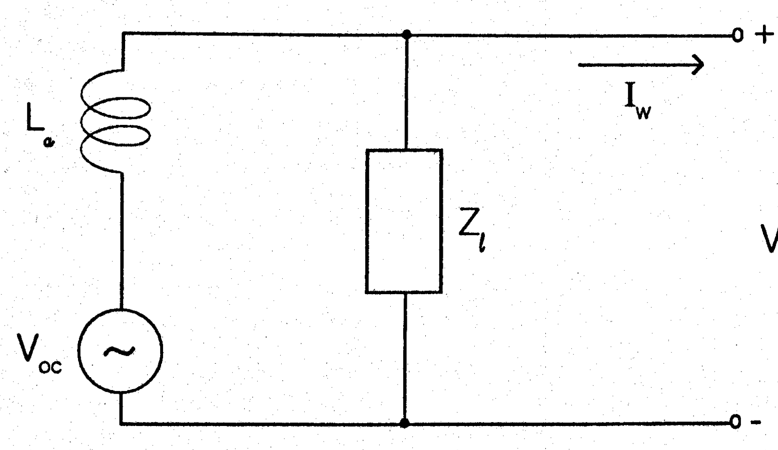

Figure B2: Equivalent circuit for the aperture voltage