Measurement Notes

Note 40

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Norm Detectors for Multiple Signals

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Abstract

This paper considers some aspects of the measurement of multiple signals (currents or voltages) resulting from the excitation of a complex electronic system. For purposes of data compression this is cast into the form of an N-port norm of the interfering signals, each norm representing a bound on all N individual signals. The requirements for only small perturbation of the signals by the measurement device are discussed and also related to appropriate norms.

I. Introduction

One is often confronted with the problem of measuring a large number of electrical signals in some object (system) under test. These may be in either frequency or time domain. What is it about these signals that one can or should measure? For example, one might attempt to measure many thousands of time-domain signals, perhaps for each of several different orientations of the system in say some set of EMP simulators. This represents an enormous amount of time-sampled (or frequency-sampled) data to collect, store, and process before one attempts to draw conclusions from the test.

Is there some way that one can compress or reduce the amount of data to be taken, stored, and processed while still being able to draw valid conclusions concerning the system performance? This paper addresses this question in terms of appropriate bounds on the signals in terms of norms.

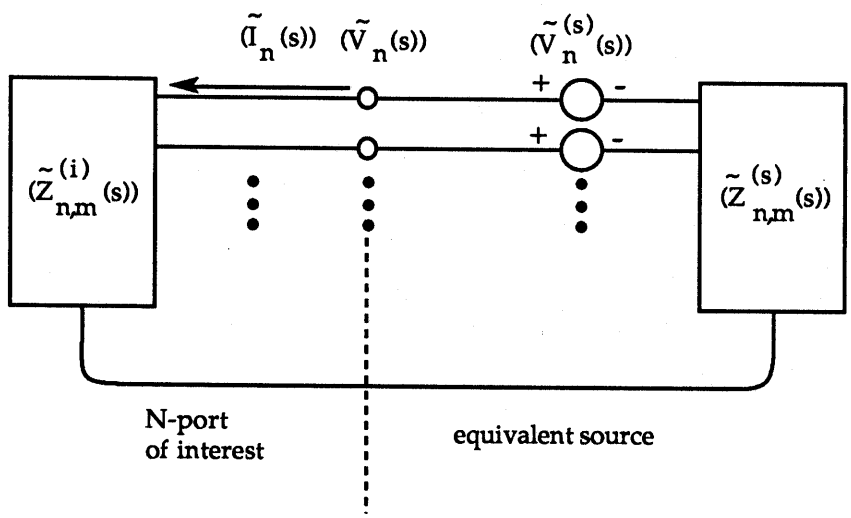
Figure 1.1 shows the general form of the problem. There is some N-port network of interest characterized by an NxN input-impedance (or admittance) matrix. This might represent the N pins (case reference) of some connector on some black box in a system where we are interested in the vector of voltages $(\tilde{V}_n(s))$ or currents $(\tilde{I}_n(s))$ related by

$$\begin{gathered}(\tilde{V}_n(s)) = (\tilde{Z}^{(i)}_{n,m}(s)) \cdot (\tilde{I}_n(s)) \\ (\tilde{Y}^{(i)}_{n,m}(s)) = (\tilde{Z}^{(i)}_{n,m}(s))^{-1}\end{gathered} \tag{1.1}$$

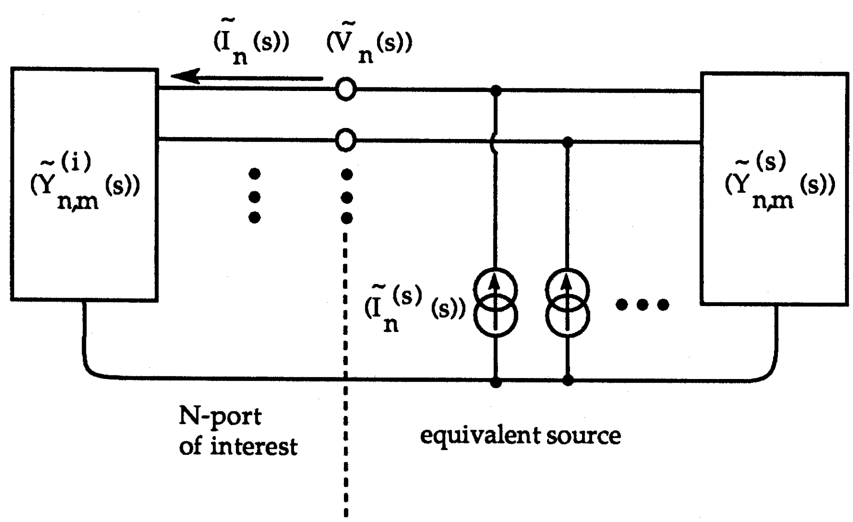
Of course, this could represent more general situations, such as the signals on all the pins into a black box, or on all the pins on some collection of such boxes in a system. Note that we have

$$\begin{gathered}\sim \equiv \text{two-sided Laplace transform} \\ s \equiv \Omega + j\omega \equiv \text{Laplace-transform variable or complex frequency}\end{gathered} \tag{1.2}$$

These N pins are connected to other parts of the system by N wires. This is presented by an NxN source impedance (or admittance) matrix, and a vector of source voltages $(\tilde{V}_n(s))$ (open-circuit or Thevenin equivalent circuit) or source currents $(\tilde{I}^{(s)}_n(s))$ (short-circuit or Norton equivalent circuit) related by



A. Thevenin (open-circuit) equivalent source



B. Norton (short-circuit) equivalent source

Fig. 1.1. Equivalent Circuits for Signals Reaching N-Port

$$(\tilde{V}_n^{(s)}(s)) = (\tilde{Z}_{n,m}^{(s)}(s)) \cdot (\tilde{I}_n^{(s)}(s))$$

$$(\tilde{Y}_{n,m}^{(s)}(s)) = (\tilde{Z}_{n,m}^{(s)}(s))^{-1} \tag{1.3}$$

The sources represent the effect of some interfering electromagnetic source inducing signals in the system. The impedance and admittance matrices are assumed to characterize in a linear, passive, receprocal, and time-invariant system in the usual senses of these technical terms for purposes of our present analysis. In time domain these matrices are also convolution operators.

Combining the various terms we have using source voltages

$$(\tilde{V}_n^{(s)}(s)) - (\tilde{V}_n(s)) = (\tilde{Z}_{n,m}^{(s)}) \cdot (\tilde{I}_n(s))$$

$$(\tilde{I}_n(s)) = (\tilde{Z}_{n,m}^{(0)}(s))^{-1} \cdot (\tilde{V}_n^{(s)}(s)) \tag{1.4}$$

$$(\tilde{Z}_{n,m}^{(0)}(s)) \equiv (\tilde{Z}_{n,m}^{(s)}(s)) + (\tilde{Z}_{n,m}^{(i)}(s))$$

and using source currents

$$(\tilde{I}_n^{(s)}(s)) - (\tilde{I}_n(s)) = (\tilde{Y}_{n,m}^{(s)}) \cdot (\tilde{V}_n(s))$$

$$(\tilde{V}_n(s)) = (\tilde{Y}_{n,m}^{(0)}(s))^{-1} \cdot (\tilde{I}_n^{(s)}(s)) \tag{1.5}$$

$$(\tilde{Y}_{n,m}^{(0)}(s)) \equiv (\tilde{Y}_{n,m}^{(s)}(s)) + (\tilde{Y}_{n,m}^{(i)}(s))$$

Note that the impedance and admittance matrices with superscript 0 are not in general mutually inverse. The use of this form will become clear later.

II. Norms of Vectors of Signals

Consider some N-component vector of signals$(f_n(t)$, or in complex-frequency domain $(\tilde{f}_n(s))$. This can represent either voltage or current as discussed in the previous section. As discussed in [6,7,9] these have norms defined (in time domain) by

$$\|(f_n(t))\| \geq 0, \quad \|(f_n(t))\| = 0 \quad \textit{iff}\,(f_n(t)) = (0_n)$$

$$\|\alpha(f_n(t))\| = \alpha\|(f_n(t))\| \tag{2.1}$$

$$\|(f_n(t)) + (g_n(t))\| \leq \|(f_n(t))\| + \|(g_n(t))\|$$

where for functions $f_n(t)$ this interpreted as excluding isolated non-zero points (also known as zero measure).

Matrix operator norms are defined as associated matrix operator norms, i.e. in terms of vector function norms. For the important operation of convolution $\circ$ with respect to time, a matrix convolution operator $(a_{n,m}(t))\circ$ is defined via

$$\|(a_{n,m}(t))\circ\| \equiv \sup_{(f_n(t))\neq(0_n)} \frac{\|(a_{n,m}(t))\circ(f_n(t))\|}{\|(f_n(t))\|} \tag{2.2}$$

Note that the norm of $(a_{n,m}(t))\circ$ is <u>not</u> the same as the norm of $(a_{n,m}(t))$. Multiplication here is in the dot-product sense as indicated. With this definition then these associated matrix operator norms have all the properties as in (2.1) with the additional product inequality

$$\|(a_{n,m}(t))\circ(b_{n,m}(t))\circ\| \leq \|(a_{n,m}(t))\circ\| \, \|(b_{n,m}(t))\circ\| \tag{2.3}$$

Note that if we are considering frequency-domain quantities, convolution is replaced by the simpler operation of multiplication (dot-product sense).

This brings us to the point: why a norm?

1. A norm reduces a set of N waveforms, each waveform being characterized by a large number (in principle infinite) of sample points, by a single, real, non-negative number.

2. A norm represents a bound on all the signals in a sense depending on a particular norm. For example, the ∞-norm is the peak of the peaks. For other p the norm of an individual signal is bounded by the norm of the vector of signals.

3. An associated matrix operator norm is a tight bound. It is a least upper bound (supremum) in the sense that for matrices and operators, if one were to assign a smaller number, there would be cases of vector waveforms for which the norm would not give a bound to the operation (or multiplication) of the matrix operator on the vector waveform.

In another paper the concept of a natural norm is introduced [8]. This is based on the symmetries of physical problem at hand. Preserving such symmetries in the norm let us require here

1. time-translation invariance

2. time-reversal invariance

3. invariance on permutation of the index n of the set of waveforms (i.e weighting all N waveforms equally).

The last point assumes no a priori knowledge of which signals are more important than others, e.g. 1 volt or 1 amp are taken as equally significant for all signals. This is less stringent than in [8] where there were only two vector components representing the polarization of an electric field and the norm was required to be coordinate-rotation invariant.

III. Examples of N-Port Norms

Applying norms to a vector of signals one might refer to such as N-port norms. Furthermore, let us restrict ourselves to natural norms and consider some examples.

A common example of a norm is the p-norm given by

$$\|(f_n(t))\|_p \equiv \left\{ \sum_{n=1}^{N} \int_{-\infty}^{\infty} |f_n(t)|^p \, dt \right\}^{\frac{1}{p}}$$

$$= \left\| \left(\|f_n(t)\|_{pf} \right) \right\|_{pv} = \left\| \|(f_n(t))\|_{pv} \right\|_{pf}$$

$$\text{for } 1 \le p < \infty \qquad (3.1)$$

$$\|(f_n(t))\|_\infty = \max_{1 \le n \le N} \left\{ \sup_t |f_n(t)| \right\}$$

$$= \sup_t \left\{ \max_{1 \le n \le N} |f_n(t)| \right\}$$

Here the subscript pv means p-norm in the vector (summation) or associated matrix sense, and the subscript pf means p-norm in the function (integration) or associated operator (convolution) sense. While the above forms are stated for time functions, they can also be applied to frequency functions. In this latter case one needs to define whether one means to have the results apply to specific (individual) frequencies, or to apply over all frequencies as a whole (integrate over frequency).

It is not necessary that the same p be used for both function and vector sense. Define a mixed p norm via

$$\|(f_n(t))\|_{p_1 f, p_2 v} \equiv \left\{ \sum_{n=1}^{N} \left\{ \int_{-\infty}^{\infty} |f_n(t)|^{p_1} dt \right\}^{\frac{p_2}{p_1}} \right\}^{\frac{1}{p_2}}$$

$$= \left\| \left(\|f_n(t)\|_{p_1 f} \right) \right\|_{p_2 v} \qquad (3.2)$$

Note that here we have first taken the p_1 function norm of the $f_n(t)$ and then taken the p_2 vector norm of the vector of N norms. (Hence one might think of these norms of vector waveforms as norms of norms, or supernorms.) The order of taking these two norms is important, since if we reverse them we have

$$\|(f_n(t))\|_{p2v,p2f} \equiv \left\{ \int_{-\infty}^{\infty} \left\{ \sum_{n=1}^{N} |f_n(t)|^{p2} \right\}^{\frac{p1}{p2}} dt \right\}^{\frac{1}{p1}}$$

$$= \left\| \|(f_n(t))\|_{p2v} \right\|_{p1f} \tag{3.3}$$

In general (3.2) and (3.3) give different norms with the summation and integration operations not commuting. In the regular p=norm in (3.1) these do commute.

An example of such a mixed p-norm is the maximum or m-norm introduced in [8] as

$$\|(f_n(t))\|_m \equiv \|(f_n(t))\|_{2v,\infty f} = \left\| \|(f_n(t))\|_{2v} \right\|_{\infty f}$$

$$= \left\| |(f_n(t))| \right\|_{\infty f}$$

$$= \sup_t |f_n(t)| \tag{3.4}$$

This as is readily seen is the supremum for all time of the vector magnitude.

Another example of such a mixed norm is based on picking the largest norm of any of the N waveforms. Let us call this the ℓ-norm given by

$$\|(f_n(t))\|_\ell \equiv \max_{1 \le n \le N} \|f_n(t)\| = \|(\|f_n(t)\|)\|_{\infty v} \tag{3.5}$$

with the interior norm of course in function sense. Specializing to the p-norm we have the $p\ell$ norm as

$$\|(f_n(t))\|_{p\ell} \equiv \|(\|f_n(t)\|)\|_{\infty v}$$

$$= \max_{1 \le n \le N} \left\{ \int_{-\infty}^{\infty} |f_n(t)|^p dt \right\}^{\frac{1}{p}} \tag{3.6}$$

This is practically interpreted by considering the individual pin voltages or currents, assigning a norm (say 2-norm (related to energy) or ∞-norm (peak)) and then taking the largest of the N norm values and assigning it to the vector waveform. Clearly the resulting number bounds the norm for each of signals and equals the norm of at least one signal.

One can also generalize these concepts to weighted norms, such as energy norms [10]. In this concept the signals are weighted, i.e. $\tilde{f}_n(j\omega)$ is replaced by some other frequency function by multiplying by some weight $\tilde{w}_n(j\omega)$ (non zero for "all" ω) which corresponds to a convolution of $\omega_n(t)$with $f_n(t)$. This weighted waveform is then inserted in the previous formulae. This might be used, for example, to correct for deficiencies in the electromagnetic environment exciting the system (if known beforehand and built in to the norm measuring device).

It should also be noted that one can combine the N signals in ways which are not norms, thereby losing the properties of norms discussed in Section II. For example, consider what is referred to as the bulk current

$$I_B(t) = \sum_{n=1}^{N} I_n(t) \tag{3.7}$$

This commonly encountered quantity is measured by passing all N wires together through a current sensor (integrating the magnetic field along a path around the wires). What should we think of such a measurement? What does it tell us about the individual $I_n(t)$? Well, basically it tells us nothing about the individual currents. $I_B(t)$ can easily be less in magnitude than any of the individual wire currents [5]. A simple example is two wires in a differential configuration

$$I_2(t) = -I_2(t)$$

$$I_B(t) = 0 \tag{3.8}$$

Clearly the bulk current bounds nothing and cannot in general be used as a norm.

IV. Effect of Loading by Measurement Device

In forming a norm of the N voltages or currents one needs to measure them somehow, combining the signals through some technique which ends up with a real, non-negative number, i.e. the norm. In making such a measurement one in general disturbs the parameter being measured, thereby introducing errors. As illustrated in fig. 4.1 one can think of the measurement device as introducing some impedances between the source and the N-port of interest. Here this is represented by a scattering supermatrix of 2x2 blocks, each NxN [2]. This accounts for the N signals on each of the two sides of the network. As indicated, the equivalent source (Thevenin or Norton form) now gives voltages $(\tilde{V}_n^{(m)}(s))$ and currents $(\tilde{I}_n^{(m)}(s))$ which are in general different from the unperturbed voltages $(\tilde{V}_n^{(m)}(s))$ and currents $(\tilde{I}_n^{(m)}(s))$.

Let us specialize the network representing the measurement device in the simpler forms in fig. 4.2. These represent series devices (low impedance) to sample current and parallel devices (low admittance) to sample voltage. These are indicated with sources in appropriate form to simply correspond with the type of loading.

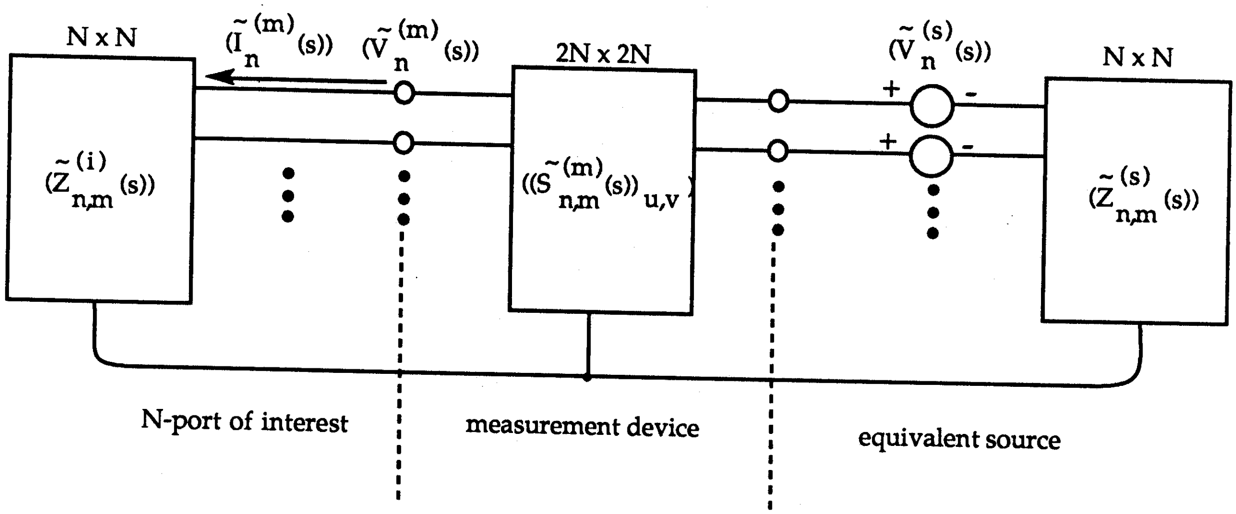
Consider the series measurement in fig. 4.2A. With no mutual impedances we have a diagonal impedance matrix

$$(\tilde{Z}_{n,m}^{(ms)}(s)) = \begin{pmatrix} \tilde{Z}_1^{(ms)}(s) & & \mathrm{O} \\ & \tilde{Z}_2^{(ms)}(s) & \\ \mathrm{O} & \ddots & \tilde{Z}_N^{(ms)}(s) \end{pmatrix} \tag{4.1}$$

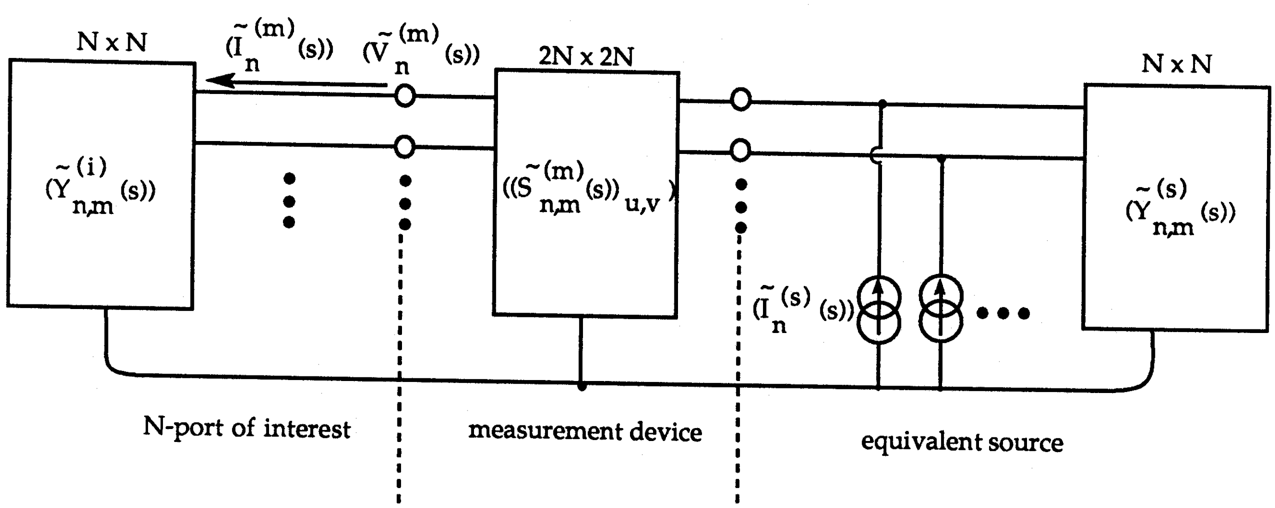
representing the loading by the measurement device. This form allows the source voltages $(\tilde{V}_n^{(s)}(s))$ to still represent the open-circuit voltages at the N-port while the effective source impedance now includes the load in (4.1) giving the measured currents

$$(\tilde{I}_n^{(ms)}(s)) = \left[(\tilde{Z}_{n,m}^{(0)}(s)) + (\tilde{Z}_{n,m}^{(ms)}(s))\right]^{-1} \cdot (\tilde{V}_n^{(s)}(s)) \tag{4.2}$$

which can be compared to the unperturbed currents in (1.4) where the superscript 0 is for the (unperturbed) source plus N-port input impedance matrix.

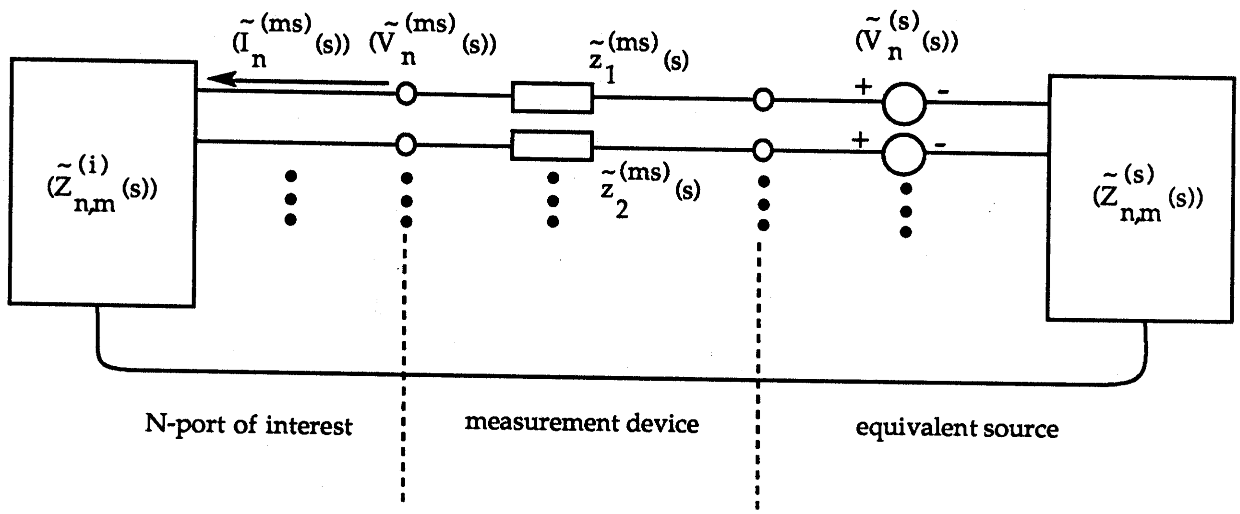


A. Thevenin form

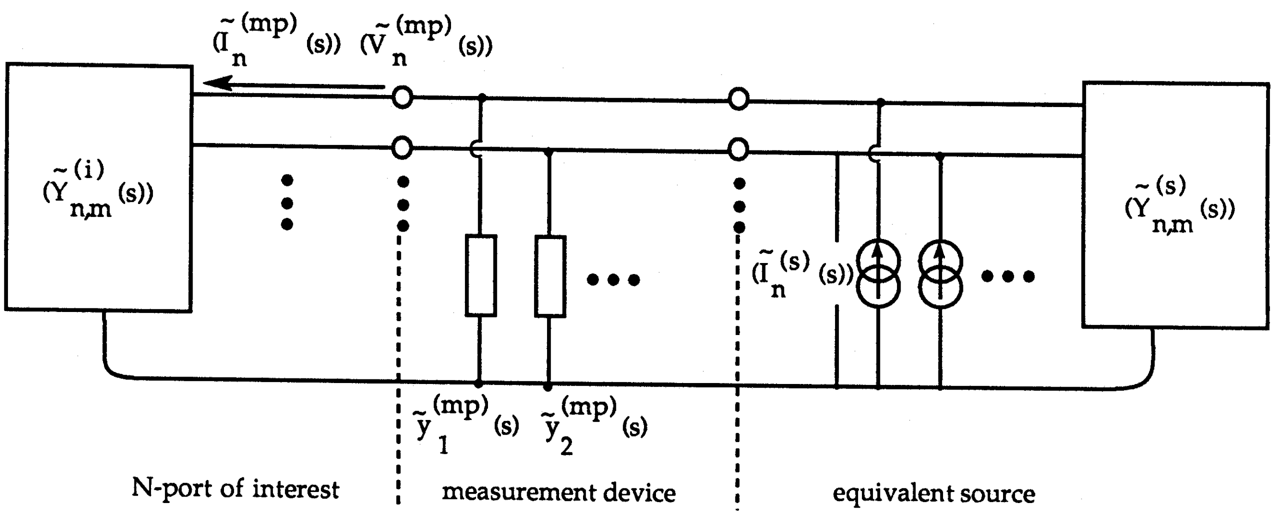


B. Norton form

Fig. 4.1. Insertion of General Measurement Device as Scattering Supermatrix



A. Series measurement (current)



B. Parallel measurement (voltage)

Fig. 4.2. Measurement Device Specialized as Diagonal Loading Elements

The perturbation (error) in the measured currents is

$$(\Delta\tilde{I}_n^{(ms)}(s) \equiv (\tilde{I}_n^{(ms)}(s)) - (\tilde{I}_n(s))$$

$$= \left\{ \left[(\tilde{Z}_{n,m}^{(0)}(s) + (\tilde{Z}_{n,m}^{(ms)}(s)) \right]^{-1} - (\tilde{Z}_{n,m}^{(0)}(s))^{-1} \right\} \cdot (\tilde{V}_n^{(s)}(s)) \tag{4.3}$$

$$\left[(\tilde{Z}_{n,m}^{(0)}(s)) + (\tilde{Z}_{n,m}^{(0)}(s)) \right]^{-1} - (\tilde{Z}_{n,m}^{(0)}(s))^{-1}$$

$$= -\left[(\tilde{Z}_{n,m}^{(0)}(s)) + (\tilde{Z}_{n,m}^{(ms)}(s)) \right]^{-1} \cdot (\tilde{Z}_{n,m}^{(ms)}(s)) \cdot (\tilde{Z}_{n,m}^{(0)}(s))^{-1}$$

$$= -(\tilde{Z}_{n,m}^{(0)}(s))^{-1} \cdot (\tilde{Z}_{n,m}^{(ms)}(s)) \cdot \left[(\tilde{Z}_{n,m}^{(0)}(s)) + (\tilde{Z}_{n,m}^{(ms)}(s)) \right]^{-1}$$

This shows that the perturbation is first order in the loading impedance which must be small (in an appropriate sense) compared to $(\tilde{Z}_{n,m}^{(0)}(s))$ for the measurement to be valid. Another useful form for this result has

$$(\Delta\tilde{I}_n^{(ms)}(s)) = -\left[(\tilde{Z}_{n,m}^{(0)}(s)) + (\tilde{Z}_{n,m}^{(ms)}(s)) \right]^{-1} \cdot (\tilde{Z}_{n,m}^{(ms)}(s)) \cdot (\tilde{I}_n(s)) \tag{4.4}$$

giving the current perturbation vector as a matrix times the unperturbed current vector. In a norm sense (frequency domain) this is

$$\frac{\left\| (\Delta\tilde{I}_n^{(ms)}(j\omega)) \right\|}{\left\| (\tilde{I}_n(j\omega)) \right\|} \le \left\| \left[(\tilde{Z}_{n,m}^{(0)}(j\omega)) + (\tilde{Z}_{n,m}^{(ms)}(j\omega)) \right]^{-1} \cdot (\tilde{Z}_{n,m}^{(ms)}(s)) \right\| \tag{4.5}$$

$$\le \left\| \left[(\tilde{Z}_{n,m}^{(0)}(j\omega)) + (\tilde{Z}_{n,m}^{(ms)}(j\omega) \right]^{-1} \right\| \left\| (\tilde{Z}_{n,m}^{(ms)}(s)) \right\|$$

Another useful form is obtained from an inequality in [3] as

$$(\Delta \tilde{I}_n^{(ms)}(s)) = -\left[(1_{n,m}) + (\tilde{Z}_{n,m}^{(0)}(s))^{-1} \cdot (\tilde{Z}_{n,m}^{(ms)}(s))\right]^{-1} \cdot (\tilde{Z}_{n,m}^{(0)}(s))^{-1} \cdot (\tilde{Z}_{n,m}^{(ms)}(s)) \cdot (\tilde{I}_n^{(ms)}(s))$$

$$\frac{\left\|(\Delta \tilde{I}_n^{(ms)}(j\omega))\right\|}{\left\|(\tilde{I}_n(j\omega))\right\|} \le \frac{\left\|(\tilde{Z}_{n,m}^{(0)}(j\omega))^{-1} \cdot (\tilde{Z}_{n,m}^{(ms)}(j\omega))\right\|}{1 - \left\|(\tilde{Z}_{n,m}^{(0)}(j\omega))^{-1} \cdot (\tilde{Z}_{n,m}^{(ms)}(j\omega))\right\|} \tag{4.6}$$

So the problem is to make the norm of $(\tilde{Z}_{n,m}^{(0)}(j\omega))^{-1} \cdot (\tilde{Z}_{n,m}^{(ms)}(j\omega))$ small enough that the norm of the error current vector is small compared to the norm of the (unperturbed) current vector. This can also be applied in time domain by making this small for all frequencies of interest or by use of norms over time.

If we further specialize the diagonal matrix in (4.1) as

$$(\tilde{Z}_{n,m}(s)) \equiv \tilde{z}^{(ms)}(s)(1_{n,m}) \tag{4.7}$$

then the results further simplify as

$$(\Delta \tilde{I}_n^{(ms)}(s)) = -\tilde{z}^{(ms)}(s)[(\tilde{Z}_{n,m}(s)) + \tilde{z}^{(ms)}(s)(1_{n,m})]^{-1}$$

$$\frac{\left\|(\Delta \tilde{I}_n^{(ms)}(j\omega))\right\|}{\left\|(\tilde{I}_n(j\omega))\right\|} \le \frac{\left|\tilde{z}^{(ms)}(j\omega)\right| \left\|(\tilde{Z}_{n,m}^{(0)}(j\omega))^{-1}\right\|}{1 - \left|\tilde{z}^{(ms)}(j\omega)\right| \left\|(\tilde{Z}_{n,m}^{(0)}(j\omega))^{-1}\right\|} \tag{4.8}$$

If one knows $\left\|(\tilde{Z}_{n,m}^{(0)}(j\omega))\right\|^{-1}$, or perhaps a bound on it, then one has some estimate of how small to make $\left|\tilde{z}^{(ms)}(j\omega)\right|$ so that the relative current error is as small as desired. Some insight is gained by writing for this symmetric matrix the assumed diagonal representation

$$(\tilde{Z}_{n,m}^{(0)}(s)) = \sum_{\beta=1}^{N} \tilde{z}_\beta^{(0)}(s)(\tilde{\zeta}_n(s))_\beta(\tilde{\zeta}_n(s))_\beta \tag{4.9}$$

$$(\tilde{\zeta}_n(s))_{\beta_1} \cdot (\tilde{\zeta}_n(s))_{\beta_2} = 1_{\beta_1,\beta_2} \quad \text{(orthonormal eigenvectors)}$$

$$\tilde{z}_\beta^{(0)}(s) \equiv \text{eigenvalues}$$

The inverse is then

$$(\tilde{Z}^{(0)}_{n,m}(s))^{-1} = \sum_{\beta=1}^{N} \tilde{z}^{(0)^{-1}}_{\beta}(s)(\tilde{\zeta}_n(s))_\beta(\tilde{\zeta}_n(s))_\beta \tag{4.10}$$

which is dominated by the <u>smallest</u> eigenvalue. So it is basically the ratio of $\left|\tilde{z}^{(ms)}(j\omega)\right|$ to this smallest eigenvalue magnitude which should be kept acceptably small for all frequencies of interest.

One can also cast these results in terms of a perturbation voltage vector as

$$\begin{aligned}(\Delta\tilde{V}^{(ms)}_n(s)) &\equiv (\tilde{V}^{(ms)}_n(s)) - (\tilde{V}_n(s)) \\ &= (\tilde{Z}^{(i)}_{n,m}(s)) \cdot (\Delta\tilde{I}^{(ms)}(s))\end{aligned} \tag{4.11}$$

Here the results for the perturbation current vector are directly applicable except they are now subject to a similarity transformation by $(\tilde{Z}^{(i)}_{n,m}(s))$. So for the case of a series measurement the current errors are more convenient to consider.

Now consider the dual situation, the parallel measurement in fig. 4.2B. With no mutual admittances we have a diagonal admittance matrix

$$(\tilde{Y}^{(mp)}_{n,m}(s)) = \begin{pmatrix} \tilde{Y}^{(mp)}_1(s) & & \mathbf{O} \\ & \tilde{Y}^{(mp)}_2(s) & \\ \mathbf{O} & \ddots & \tilde{Y}^{(mp)}_N(s) \end{pmatrix} \tag{4.12}$$

This form allows the source currents to still represent the short-circuit currents at the N-port while the effective source admittance now includes the load in (4.12) giving the measured voltages

$$\left(\tilde{V}^{(mp)}_n(s)\right) = \left[\left(\tilde{V}^{(0)}_{n,m}(s)\right) + \left(\tilde{Y}^{(mp)}_{n,m}(s)\right)\right]^{-1} \cdot (\tilde{I}^{(s)}_n(s)) \tag{4.13}$$

with the superscript 0 as in (1.5) for the (unperturbed) source plus N-port input admittance matrix.

Noting the dual nature of this parallel case to the previous series case we merely need to interchange the roles of current and voltage, and of impedance and admittance. Summarizing for the error voltage vector we have

$$(\Delta\tilde{V}_n^{(mp)}(s)) \equiv (\tilde{V}_n^{(mp)}(s)) - (\tilde{V}_n(s))$$

$$= -\left[(\tilde{Y}_{n,m}^{(0)}(s)) + (\tilde{Y}_{n,m}^{(mp)}(s))\right]^{-1} \cdot (\tilde{Y}_{n,m}^{(mp)}(s)) \cdot (\tilde{V}_n(s))$$

$$\frac{\left\|(\Delta\tilde{V}_n^{(mp)}(j\omega))\right\|}{\left\|(\tilde{V}_n(j\omega))\right\|} \le \frac{\left\|(\tilde{Y}_{n,m}^{(0)}(j\omega))^{-1} \cdot (\tilde{Y}_{n,m}^{(mp)}(j\omega))\right\|}{1 - \left\|(\tilde{Y}_{n,m}^{(0)}(j\omega))^{-1} \cdot (\tilde{Y}_{n,m}^{(mp)}(j\omega))\right\|} \tag{4.14}$$

with now $\left\|(\tilde{Y}_{n,m}^{(0)}(j\omega))^{-1} \cdot (\tilde{Y}_{n,m}^{(mp)}(j\omega))\right\|$ to be made small. For equal loading admittances we have

$$(\tilde{Y}_{n,m}^{(mp)}(s)) \equiv \tilde{y}^{(mp)}(s)(1_{n,m})$$

$$\frac{\left\|(\Delta\tilde{V}_n^{(mp)}(j\omega))\right\|}{\left\|(\tilde{V}_n(j\omega))\right\|} \le \frac{\left|(\tilde{y}^{(mp)}(j\omega))\right| \left\|(\tilde{Y}_{n,m}^{(0)}(j\omega))^{-1}\right\|}{1 - \left|\tilde{y}^{(mp)}(j\omega)\right| \left\|(\tilde{Y}_{n,m}^{(0)}(j\omega))^{-1}\right\|} \tag{4.15}$$

Assuming a diagonal form for this remaining symmetric matrix gives

$$(\tilde{Y}_{n,m}^{(0)}(s)) = \sum_{\beta=1}^{N} \tilde{y}_{\beta}^{(0)}(s)(\tilde{\xi}_n(s))_{\beta}(\tilde{\xi}_n(S))_{\beta}$$

$$(\tilde{\xi}_n(s))_{\beta_1} \cdot (\tilde{\xi}_n(s))_{\beta_2} = 1_{\beta_1,\beta_2} \quad , \quad \tilde{y}_{\beta}(s) \equiv \text{eigenvalue} \tag{4.16}$$

$$(\tilde{Y}_{n,m}^{(0)}(s))^{-1} = \sum_{\beta=1}^{N} \tilde{y}_{\beta}^{(0)^{-1}}(s)(\tilde{\xi}_n(s))_{\beta}(\tilde{\xi}_n(s))_{\beta}$$

Then it is the ratio of $\left|\tilde{y}^{(mp)}(j\omega)\right|$ to the smallest of these eigenvalue magnitudes which should be kept acceptably small for all frequencies of interest. Analogous to (4.11) one can also treat a perturbation current vector for this case, but now the voltage form is more convenient.

V. Concluding Remarks

Norms provide a powerful concept for characterizing a set of waveforms as a single, real, non-negative number. This can be used to give a bound on every waveform in the set. If one has a specification that all signals at a set of pins (in a system in some test electromagnetic environment) be less than some amount stated in such a norm sense, then a norm detector has the potential of simplifying the test.

The design of such a norm detector poses various technical problems. One of these concerns the perturbation of the voltages or currents to be measured. The actual sensor responding to each of the voltages (e.g. a parallel (to local ground reference) small admittance resistor) or currents (e.g. some small-insertion-impedance inductive coupling device) must give a negligible perturbation. For this purpose one needs to bound the appropriate eigenvalues of the source and input matrices (combined). Even the geometrical size of the devices in contact with or proximity to the pins or associated wires can be of concern because of the close packing of a large number of wires in a cable bundle or connector [1,4].

There may be some data processing desired in the norm detector. First the waveform out of the sensor will have some known relationship to the current or voltage waveform. Some kind of circuit may further modify this (integrate, differentiate, rectify, etc.). Then something about the resulting waveform may be sampled in digital or analog form. This may be as simple as some kind of peak or involve many samples over the waveform. This is then cast in some norm-appropriate form and combined for all the signals into the overall norm desired. There may be a readout at the device or the number may be telemetered to some other location for recording.

This paper has then provided an introduction to the concept of a norm detector. It has explored some of the possibilities and design problems. Such is a first step.

References

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