Measurement Notes

Note 46

Considerations for Loading a Balun Using Ferrites or a Ferrite/Dielectric Sandwich

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Abstract

We consider here further properties of the ferrite/dielectric sandwich proposed earlier in Measurement Note 39. We identify an optimal ratio for the thickness of ferrite to the thickness of dielectric material in a balun, in order to achieve optimal impedance properties at high frequency. For typical sets of parameters, the optimal fill factor is half ferrite and half dielectric, as had been suggested earlier. The conditions for when this ratio is different are also identified. Some examples of the incremental impedance are plotted for real permeability constants.

I. Introduction

The purpose of ferrite in a balun is to increase the impedance of the common mode. This forces energy into the differential mode, which is the mode normally needed for radiation [1]. To achieve a high wave-impedance medium, one needs some combination of a high permeability constant (a ferrite) and a low dielectric constant (perhaps air). Normally ferrite materials have high dielectric constants, so this suggests using some combination of ferrite and a dielectric material. It was shown in [2] that alternating layers of ferrite and (low) dielectric material could provide the desired effect[2].

In this paper we first review the relevant equations for describing the phenomenon. This includes incremental impedances (impedances per unit length) for both the ferrite/dielectric sandwich, and a solid ferrite. Next, we optimize the ratio of ferrite to dielectric material, in order to achieve optimal high-frequency performance. We plot some examples for a few values of real permeability.

II. Review: Incremental Impedance of Ferrite/Dielectric Sandwich

A diagram of the configuration is shown in Figure 2.1. A solid cylindrical electric conductor is surrounded by a ferrite/dielectric sandwich. For cases where this is used in a balun, there are two conductors at the center which support a differential mode. In this case, we approximate the conductors as a single conductor, and we include a spacer layer to avoid interfering with the (desired) differential mode.

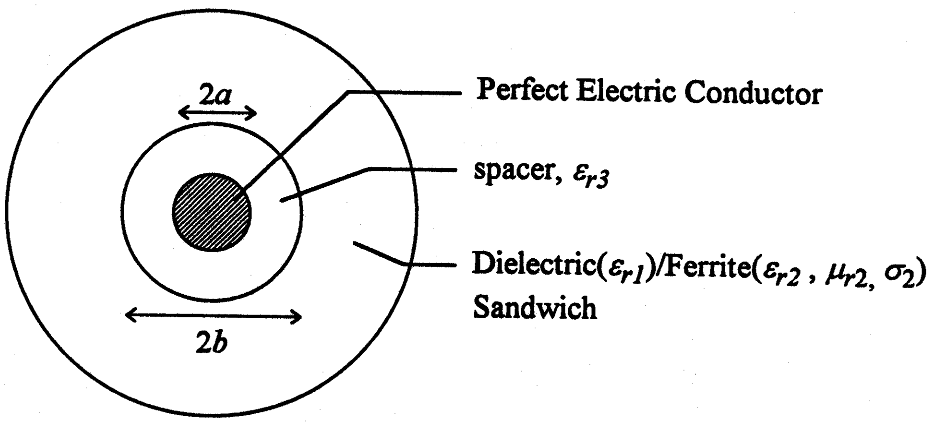

Figure 2.1. Configuration for calculating the common-mode impedance.

The details of the ferrite/dielectric sandwich are shown in Figures 2.2–2.3. This concept was first described in [2]. Electrically thin layers of dielectric and ferrite are stacked in a direction parallel to the axis of the structure. It is important to understand clearly why such a structure is expected to have an advantage over bulk ferrite in this application. At high frequencies (>1GHz) the skin depth of ferrite is quite small, on the order of a millimeter. At that thickness, very little of the ferrite has any effect. On the other hand, if thin layers of the ferrite are stacked, the wave can penetrate into the dielectric regions, thereby being exposed to a greater surface area of ferrite. By using this technique, the wave is affected by a larger volume of ferrite than would be the case if the ferrite were solid.

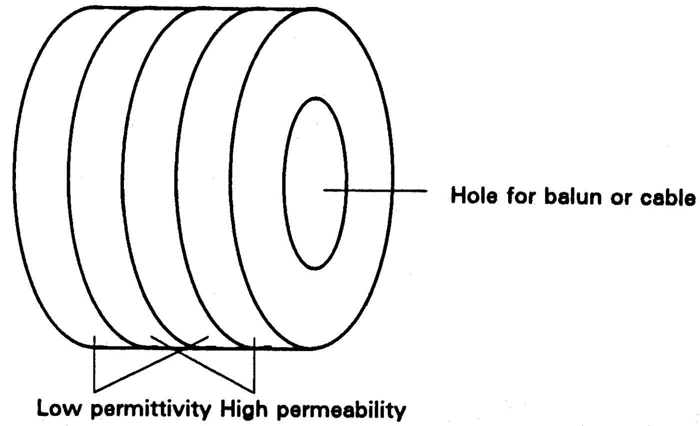

Figure 2.2. Arrangement of alternating layers of low permittivity and high permeability materials suitable for a high performance UWB choke.

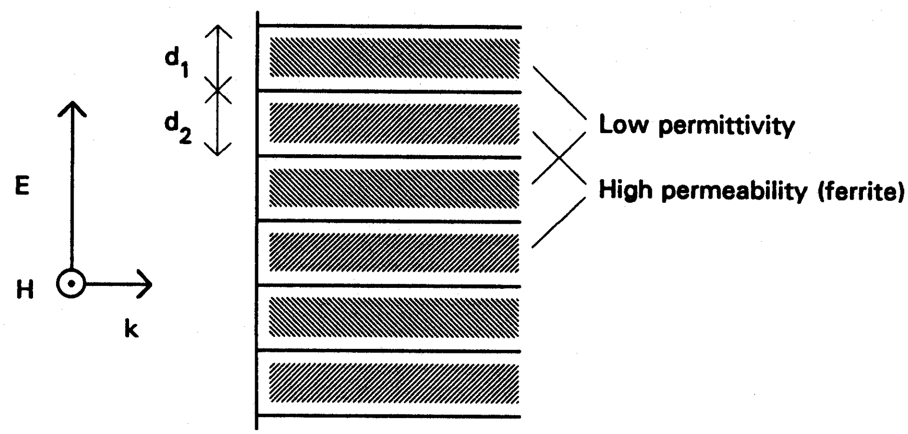

Figure 2.3. A "sandwich" of alternating materials, suitable for building a high-impedance medium for a choke. Materials of high μ_r alternate with materials of low ε_r.

We begin the analysis by determining an incremental impedance (impedance per unit length) of the ferrite/dielectric sandwich, looking from $\Psi = b$ outward. Here we use Ψ as the radial coordinate (sometimes also represented as ρ). For propagation in the radial direction, one can define relative effective permeability as [2]

$$\mu_{r\,eff} = \Delta_1 + \Delta_2 \mu_{r2} \tag{2.1}$$

where the relative permeability of the ferrite is μ_{r2}, the relative permeability of the dielectric is 1, and Δ_1 and Δ_2 are the relative fractions of the volume taken up by the dielectric and ferrite, respectively, i.e.,

$$\Delta_1 = \frac{d_1}{d_1 + d_2}\,, \qquad \Delta_2 = \frac{d_2}{d_1 + d_2} \tag{2.2}$$

Furthermore, the relative effective permittivity is expressed as

$$\varepsilon_{r\,eff} = \frac{1}{\dfrac{\Delta_1}{\varepsilon_{r1}} + \dfrac{\Delta_2}{\varepsilon_{r2} + \sigma_2/(j\omega\varepsilon_o)}} \tag{2.3}$$

where ε_{r1} and ε_{r2} are the relative dielectric constants of the dielectric and ferrite, respectively, σ_2 is the conductivity of the ferrite, and $\omega = 2\pi f$. Note that for the case of a solid ferrite with no dielectric (Δ_1=0, Δ_2=1), the above expressions reduce to the permittivity and permeability constants of the ferrite.

Under one set of simplifying assumptions, one can assume that there is an equal amount of ferrite and dielectric, and that the conductivity of the dielectric is zero. For this special case, we have for the relative permeability and permittivities and the wave impedance of the material

$$\begin{aligned} \mu_{r\,eff} &= \frac{1}{2}(1+\mu_{r2}) \\ \varepsilon_{r\,eff} &= \frac{2}{\dfrac{1}{\varepsilon_{r1}} + \dfrac{1}{\varepsilon_{r2}}} \\ \frac{\tilde{Z}}{Z_o} &= \sqrt{\frac{\mu_{r\,eff}}{\varepsilon_{r\,eff}}} = \frac{1}{2}\sqrt{(1+\mu_{r2})\left[\frac{1}{\varepsilon_{r1}} + \frac{1}{\varepsilon_{r2}}\right]} \end{aligned} \tag{2.4}$$

We will optimize the fill factors later, and achieve somewhat better performance.

Using these effective dielectric and permeability constants, we can now write an impedance and propagation constant for propagation through the sandwich in the radial direction. The impedance is just

$$\tilde{Z} = Z_o\sqrt{\mu_{r\,eff} / \varepsilon_{r\,eff}} \tag{2.5}$$

where Z_o is the impedance of free space, and the propagation constant in the radial direction is

$$\tilde{\gamma} = j\tilde{k} = \frac{j\omega}{c}\sqrt{\mu_{r\,eff}\varepsilon_{r\,eff}} \tag{2.6}$$

These expression will help to simplify expression we develop later.

Next, we can calculate the impedance per unit length looking into the ferrite/dielectric sandwich. We assume the outer radius of the sandwich is large compared to the penetration depth of the wave in the medium. In order to solve this, one must solve a boundary value problem in cylindrical coordinates. This has already been solved in [2], so we merely quote the final answer for the incremental impedance as

$$Z'_{sand} = \tilde{Z}\frac{j}{2\pi b}\frac{H_o^{(2)}(\tilde{k}\,b)}{H_1^{(2)}(\tilde{k}\,b)} \tag{2.7}$$

where $H_n^{(2)}(z) = J_n(z) - j\,Y_n(z)$ is the Hankel function of the second kind. A design goal will be to make Z'_{sand} as large as possible. It is also useful to calculate an analogous quantity for a solid ferrite. Thus we can define a Z'_{fer} to be the same as above, except that $\mu_{r\,eff}$ and $\varepsilon_{r\,eff}$ in the expression for $\tilde{k}$ are replaced by the constitutive parameters of the ferrite, μ_{r2} and ε_{r2}. The degree to which Z'_{sand} is larger than Z'_{fer} is an indicator of the improvement that the sandwich provides over the solid ferrite.

The high-frequency asymptote of the above expression leads to considerable understanding of the problem. As derived in [2], the high–frequency asymptote is just

$$Z'_{high} = \frac{\tilde{Z}}{2\pi b}\,, \qquad f \to \infty \tag{2.8}$$

where $\tilde{Z}$ is the wave impedance of either the dielectric sandwich or the medium. Since $\tilde{Z}$ is a constant at high frequencies, this asymptotic expression is also just a constant. We will see this behavior in the examples that follow

As an example, let us create some parameters to allow us to see what happens. Thus we assume the following:

$$\begin{aligned}
\text{In the dielectric:}\quad & \varepsilon_{r1} = 2.2 \\
\text{In the ferrite:}\quad & \varepsilon_{r2} = 10. \\
& \mu_{r2} = 100. \\
& \sigma_2 = 0. \\
\text{Inner radius of sandwich:}\quad & b = 2.5\text{cm} \\
\text{Radius of center conductor:}\quad & c = 1.5\text{cm} \\
\text{Fill Factors:}\quad & \Delta_1 = \Delta_2 = 0.5
\end{aligned} \tag{2.9}$$

Note that in general a number of the above constitutive parameters may be frequency dependent. This may be especially true of the permeability of the ferrite material, but for now we attempt to solve the simplest problem possible.

We plot the impedance per unit length of the sandwich and solid ferrite, Z'_{sand} and Z'_{fer}, as a function of frequency (Figure 2.4). We find that at high frequencies the sandwich has a peak near 9000 Ω/m, whereas the solid ferrite has a peak of about 7600 Ω/m. Although this represents some improvement, it may or may not be enough improvement to justify the difficulties involved with building the sandwich. Note that we can check our results by comparing our calculated high-frequency asymptotes to those specified in equation (2.8), and we get the correct values.

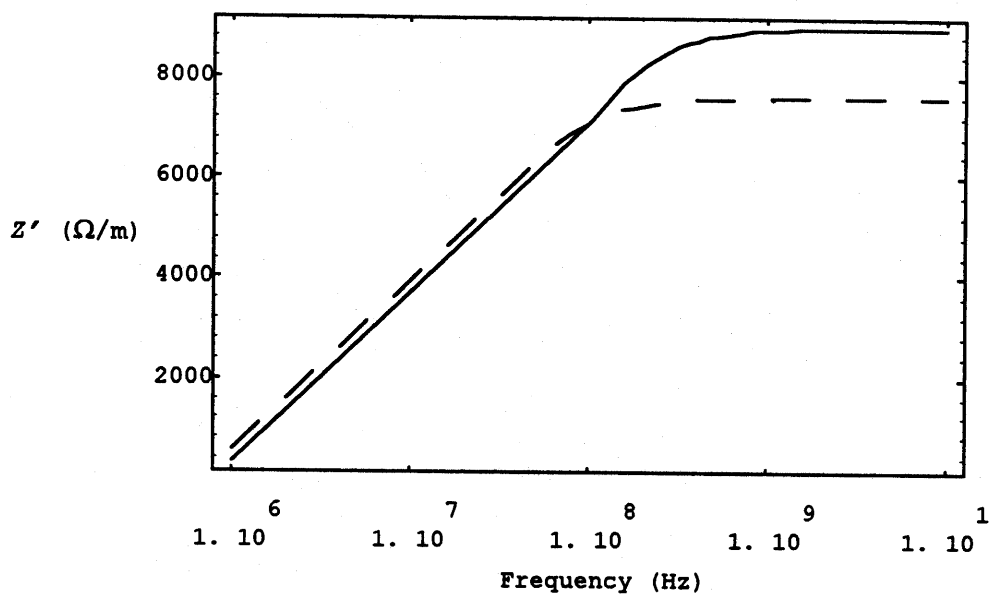

Figure 2.4. The incremental impedance of the sandwich (solid) and the solid ferrite (dashed).

What is most interesting about the above graph is that it is only the high-frequency asymptote that is of interest in the general case. This suggests that an approach for optimizing the ratio of ferrite to dielectric would involve maximizing the high-impedance asymptote in (2.8). We do so in the following section.

III. Optimization of Fill Factors for High Frequencies

Let us consider now how to optimize the fill factors in the ferrite/dielectric sandwich to obtain better high-frequency performance. The expression for high-frequency performance is

$$Z' = \frac{\tilde{Z}}{2\pi b} = \frac{Z_o}{2\pi b}\sqrt{\frac{\mu_{r\,eff}}{\varepsilon_{r\,eff}}} \tag{3.1}$$

where if $\sigma_2 = 0$, we have

$$\begin{aligned} \mu_{r\,eff} &= \Delta_1 + \Delta_2\mu_{r2} \\ \varepsilon_{r\,eff} &= \frac{1}{\dfrac{\Delta_1}{\varepsilon_{r1}} + \dfrac{\Delta_2}{\varepsilon_{r2}}} \end{aligned} \tag{3.2}$$

In order to maximize Z', one must maximize the ratio of $\mu_{r\,eff} / \varepsilon_{r\,eff}$. Let us define a function $g(\Delta_1)$, which is exactly this function. Using the relationship that $\Delta_2 = 1 - \Delta_2$, we have

$$\begin{aligned} g(\Delta_1) &= [\Delta_1 + \Delta_2\mu_{r2}]\left[\frac{\Delta_1}{\varepsilon_{r1}} + \frac{\Delta_2}{\varepsilon_{r2}}\right] \\ &= [\Delta_1 + (1-\Delta_1)\mu_{r2}]\left[\frac{\Delta_1}{\varepsilon_{r1}} + \frac{1-\Delta_1}{\varepsilon_{r2}}\right] \\ &= [\Delta_1(1-\mu_{r2}) + \mu_{r2}]\left[\Delta_1\left(\frac{1}{\varepsilon_{r1}} - \frac{1}{\varepsilon_{r2}}\right) + \frac{1}{\varepsilon_{r2}}\right] \end{aligned} \tag{3.3}$$

The optimal fill factor Δ_1 is found by differentiating $g(\Delta_1)$ and setting it equal to zero. Thus, we find

$$\begin{aligned} \frac{d\,g(\Delta_1)}{d\,\Delta_1} &= 0 \\ &= [\Delta_1(1-\mu_{r2}) + \mu_{r2}]\left(\frac{1}{\varepsilon_{r1}} - \frac{1}{\varepsilon_{r2}}\right) + \left[\Delta_1\left(\frac{1}{\varepsilon_{r1}} - \frac{1}{\varepsilon_{r2}}\right) + \frac{1}{\varepsilon_{r2}}\right](1-\mu_{r2}) \end{aligned} \tag{3.4}$$

Simplifying, we find

$$0 = \Delta_1 + \frac{\mu_{r2}}{(1-\mu_{r2})} + \Delta_1 + \frac{1}{\varepsilon_{r2}\left(\dfrac{1}{\varepsilon_{r1}} - \dfrac{1}{\varepsilon_{r2}}\right)} \tag{3.5}$$

Solving for Δ_1, and noting that $\Delta_2 = 1 - \Delta_2$, we find

$$\Delta_{1opt} = \frac{1}{2}\left[\frac{\mu_{r2}}{\mu_{r2}-1} - \frac{\varepsilon_{r1}}{\varepsilon_{r2}-\varepsilon_{r1}}\right]$$
$$\Delta_{2opt} = \frac{1}{2}\left[\frac{\varepsilon_{r2}}{\varepsilon_{r2}-\varepsilon_{r1}} - \frac{1}{\mu_{r2}-1}\right] \quad (3.6)$$

where the subscript *opt* refers to the optimal values. Note that this solution is only a maximum if the second derivative is less than zero. Thus,

$$\left.\frac{d^2 g(\Delta_1)}{d\Delta_1^2}\right|_{\Delta_1=\Delta_{1opt}} = 2(1-\mu_{r2})\left(\frac{1}{\varepsilon_{r1}} - \frac{1}{\varepsilon_{r2}}\right) < 0 \quad (3.7)$$

This is true for the normal case of $\varepsilon_{r2} > \varepsilon_{r1}$ and $\mu_{r2} > 1$.

Curiously, one can obtain a solution where values of the fill factors of less than zero or greater than one are possible. These solutions correspond to cases where it is advantageous to use either all ferrite or all dielectric material, instead of the sandwich. We must therefore specify the conditions under which this takes place. One would use all ferrite if $\Delta_1 \leq 0$ in the above expressions, or if

$$\frac{\varepsilon_{r2}}{\varepsilon_{r1}} \leq 2 - \frac{1}{\mu_{r2}} \quad (3.8)$$

Since μ_{r2} is normally a large positive number, one uses all ferrite if

$$\varepsilon_{r1} \geq \frac{\varepsilon_{r2}}{2} \quad (3.8)$$

In other words, the dielectric constant of the dielectric material must be less than half that of the ferrite, in order to achieve any benefit from the sandwich structure. Similarly, there is an unusual condition in which using all dielectric material is better. One should use all dielectric if $\Delta_2 \leq 0$ in (3.6), or if

$$\mu_{r2} \leq 2 - \frac{\varepsilon_{r1}}{\varepsilon_{r2}} \quad (3.9)$$

A ferrite with a permeability constant less than two is theoretically possible, although not very common (or good).

It is interesting now to consider when one might get a significantly different answer than the result of $\Delta_1 = \Delta_2 = 0.5$. Consider the optimized fill factors as expressed in equation (3.6). Under normal circumstances, μ_{r2}>>1 and ε_{r2}>>ε_{r1}. Under this set of circumstances, the fill factors approach 0.5 asymptotically. Thus, we see that under normal circumstances, using half of each material is a very good approximation to the optimum.

Finally, we can find the effective parameters using the above optimized fill factors. Substituting (3.6) into (3.2), we find the optimized effective constitutive parameters are

$$
\begin{aligned}
\varepsilon_{r\,eff\,opt} &= \frac{2\varepsilon_{r2}\varepsilon_{r1}(\mu_{r2}-1)}{[\mu_{r2}\varepsilon_{r2}-\varepsilon_{r1}]} \\
\mu_{r\,eff\,opt} &= \frac{1}{2}\left[\frac{\varepsilon_{r2}\mu_{r2}-\varepsilon_{r1}}{\varepsilon_{r2}-\varepsilon_{r1}}\right]
\end{aligned}
\tag{3.10}
$$

and the wave impedance of this medium is

$$
\frac{\tilde{Z}_{opt}}{Z_o} = \sqrt{\frac{\mu_{r\,eff\,opt}}{\varepsilon_{r\,eff\,opt}}} = \frac{\mu_{r2}\varepsilon_{r2}-\varepsilon_{r1}}{2\sqrt{\varepsilon_{r1}\varepsilon_{r2}(\varepsilon_{r2}-\varepsilon_{r1})(\mu_{r2}-1)}} \tag{3.11}
$$

To see whether we have made progress over a sandwich with half ferrite/half dielectric, one would compare the above results to equation (2.4). Note that the asymptotic high-frequency incremental impedance of the sandwich is given by using the above with (3.1).

Now that we have found various properties of the incremental impedance, we plot a few cases in the section that follows.

IV. Results for the Incremental Impedance

We now calculate the incremental impedance by for the following parameters.

$$
\begin{aligned}
\text{In the dielectric:} \quad \varepsilon_{r1} &= 1.,\ 2.2,\ \text{and } 4. \\
\text{In the ferrite:} \quad \varepsilon_{r2} &= 10. \\
\mu_{r2} &= 100. \\
\sigma_2 &= 0. \\
\text{Inner radius of sandwich:} \quad b &= 2.5\text{cm} \\
\text{Fill Factor of Ferrite:} \quad \Delta_2 &= 1,\ 0.5,\ \text{and optimized}
\end{aligned}
\tag{4.1}
$$

Thus, we calculate for the cases of a solid ferrite, a ferrite/dielectric sandwich with equal fills, and a sandwich with optimized fill factors. We do so for a ferrite with constitutive parameters of $\mu_{r2} = 100$ and $\varepsilon_{r2} = 10$, and dielectric materials with permittivities of $\varepsilon_{r1} = 1$, 2.2, and 4.

Note that in one case we have used a dielectric with properties the same as air. To achieve this in practice, one could either use a polystyrene foam, or one could fill the space between ferrite disks with radial spacers, as shown in Figure 4.1.

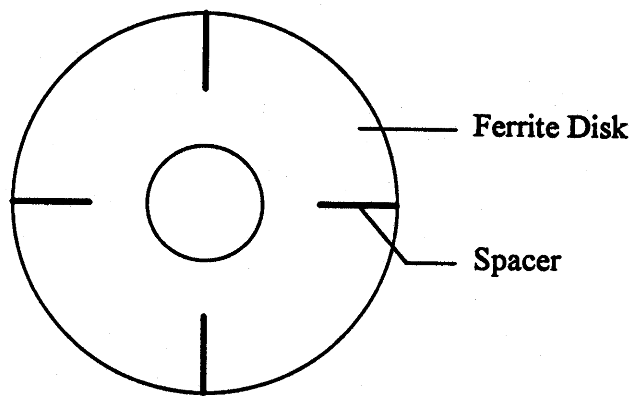

Figure 4.1. A possible arrangement for achieving air dielectric in the gap.

The incremental impedances for $\varepsilon_{r1} = 1$, 2.2, and 4 are plotted in Figures 4.2–4.4, respectively. For each of these three materials, we build the choke using three different fill factors, all ferrite, half-ferrite/half dielectric, and optimized fill factor. We can see a number of things from these results. First, it is clear that there is some advantage in using a ferrite/dielectric sandwich. There is also some advantage in using the optimized fill factors, however, the advantage of using an optimized fill factor is apparent only at the higher permittivities of dielectric material, ε_{r1}. As the permittivity of the dielectric approaches half that of the ferrite, it becomes preferable to have more and more ferrite, until one reaches the point where all ferrite is better. The point where this occurs was derived in equation (3.8).

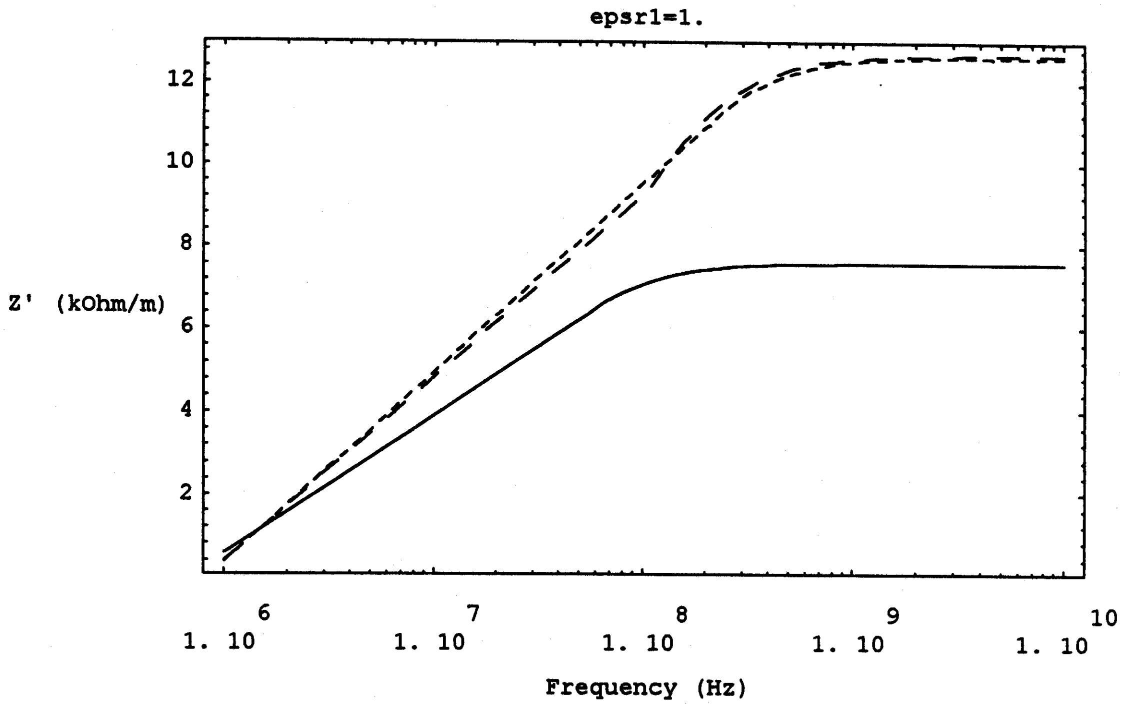

Figure 4.2 Incremental impedance for $\varepsilon_{r1} = 1$. Solid line is for all ferrite, short dashes are for half-ferrite/half-dielectric, and long dashes are for optimized ferrite/dielectric sandwich.

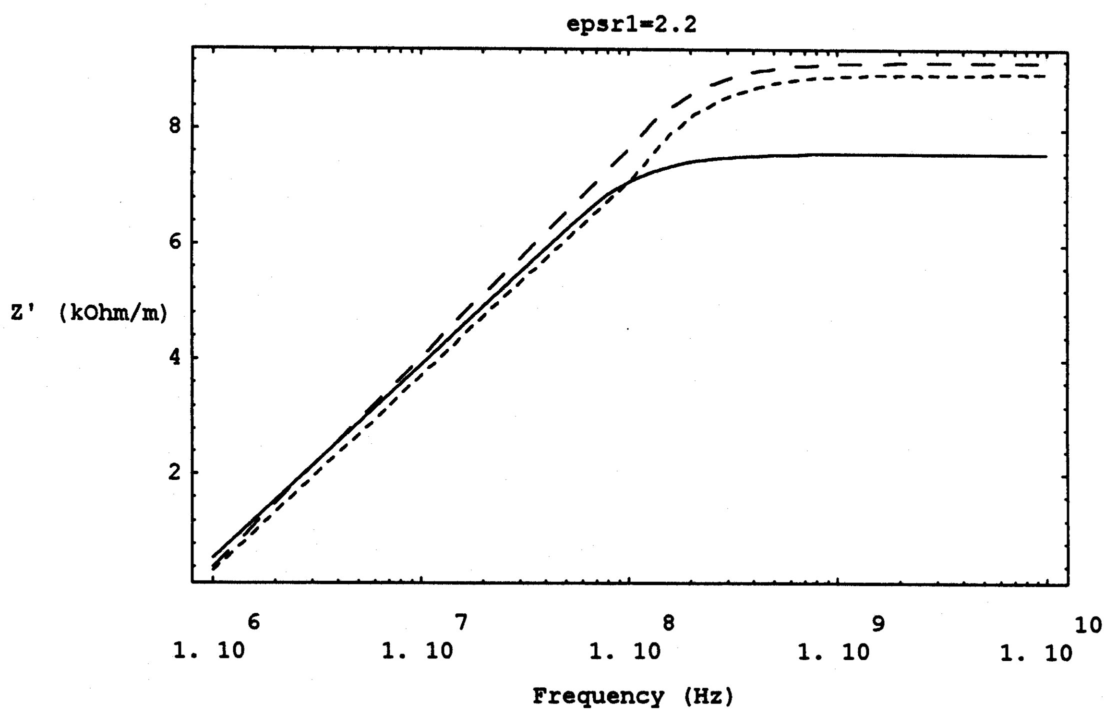

Figure 4.3 Incremental impedance for $\varepsilon_{r1} = 2.2$. Solid line is for all ferrite, short dashes are for half-ferrite/half-dielectric, and long dashes are for optimized ferrite/dielectric sandwich.

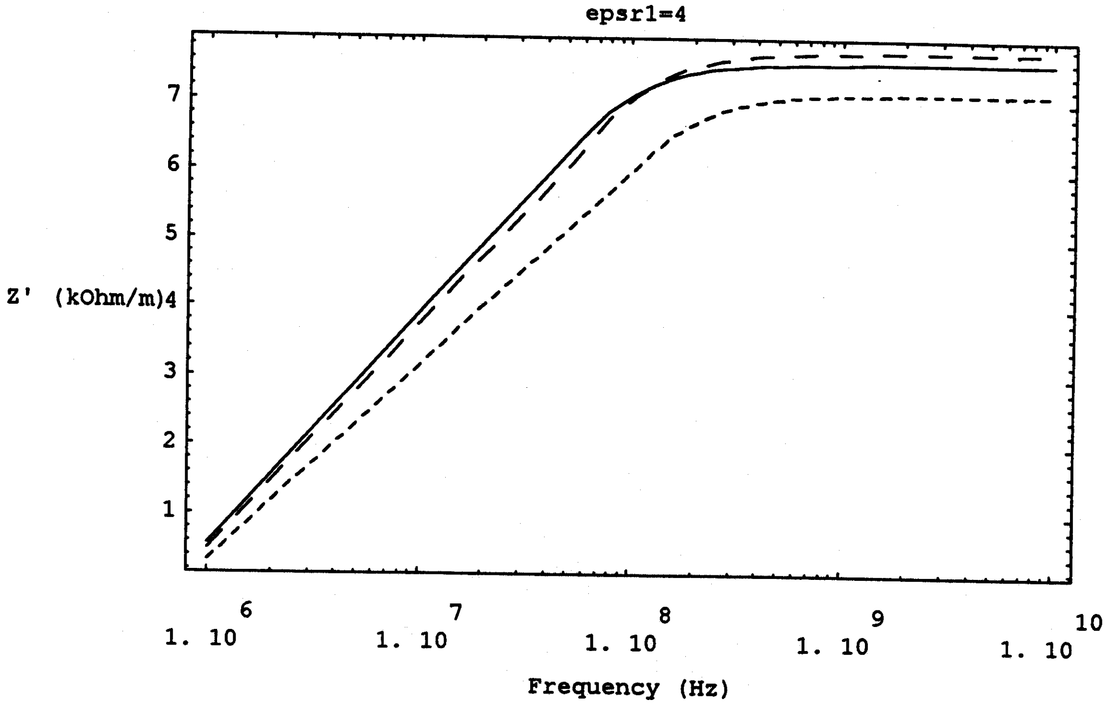

Figure 4.3 Incremental impedance for $\varepsilon_{r1} = 4$. Solid line is for all ferrite, short dashes are for half-ferrite/half-dielectric, and long dashes are for optimized ferrite/dielectric sandwich.

V. Discussion

To obtain the best possible performance, we can summarize by saying one would like the highest possible permeability in the ferrite, and the lowest possible permittivities in both the ferrite and dielectric. For most reasonable values of these parameters, the optimal fill ratio is close to half each of the ferrite and dielectric in the choke.

One would also like to keep the inner radius of the ferrite/dielectric sandwich (b in Figure 2.1) as small as possible. This must be traded off against the requirement, if in a balun, to avoid interfering with the differential mode. This points out the need for a spacer layer in a balun, however, exactly where one makes this tradeoff is not yet clear.

Finally, we point out that the general design of Figure 2.1 is optimized for high-frequency performance. However, if one is also concerned with low-frequency performance, another design worth considering would include a ferrite/dielectric sandwich close to the conductor, and a solid ferrite farther away, as shown in Figure 5.1. The rationale for this is that high frequencies can penetrate only a certain distance into the sandwich, so the sandwich is effective only out to that distance. At lower frequencies, depending upon the constitutive parameters of the various materials involved, a solid ferrite can have better performance. Thus, it may be advantageous to combine the sandwich with the solid ferrite. Note once again that the spacer may not be necessary if this is being used as a simple choke, i.e., where one does not have to worry about interfering with the differential mode in the balun.

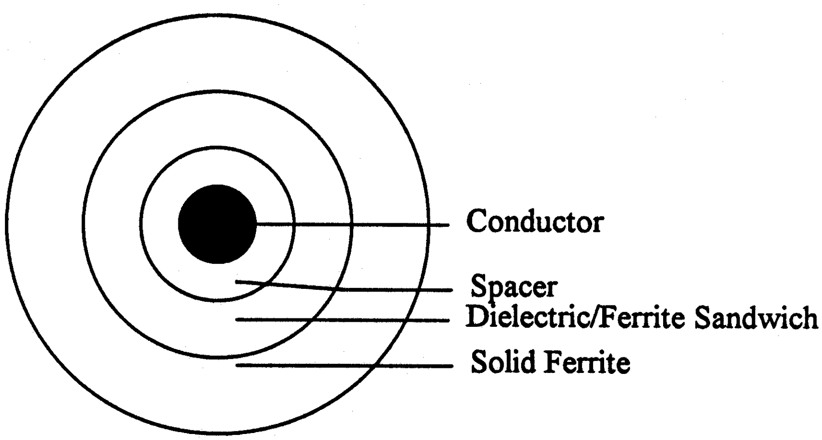

Figure 5.1. Proposed hybrid for obtaining the best characteristics of the ferrite/dielectric sandwich at high frequencies and the solid ferrite at low frequencies.

VI. Conclusions

In this paper we have provided additional details about how a ferrite/dielectric sandwich can be used to obtain a better choke than a solid ferrite. We have optimized the fill factors to achieve the highest possible incremental impedance at high frequencies. In doing so, we have proven rigorously that for most cases of interest, the previously suggested fill factors of half ferrite and half dielectric provide optimal performance. It is only for somewhat unusual sets of constitutive parameters that the optimum case differs significantly from half ferrite/half dielectric, and we have specified when that occurs. Finally, we have provided sample calculations of the incremental impedance of the ferrite/dielectric sandwich, comparing the results to the solid ferrite core, for several sets of parameters.

References

[1] E. G. Farr, *et al*, Design Considerations for Ultra-Wideband, High-Voltage Baluns, Sensor and Simulation Note 371, October 1994.

[2] C. E. Baum, An Anisotropic Medium for High Wave Impedance, Measurement Note 39, May 1991.