Microwave Memos
Memo 13

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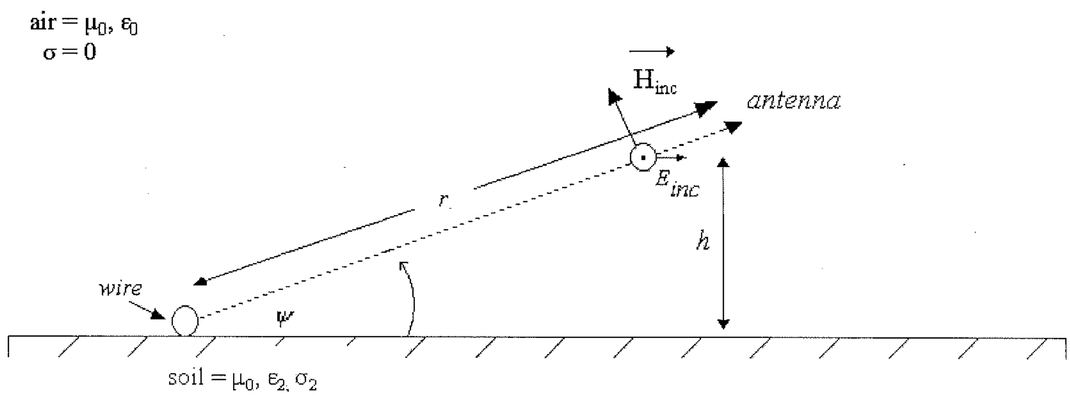
Backscattering of Horizontally Polarized Electromagnetic Wave from a Wire on or near the Ground Surface



Approximate the fields from the antenna as a plane wave of amplitude E_{inc} incident broadside to the wire on or near the ground surface.

Let:

$$\varepsilon_2 \equiv \varepsilon_r \ \varepsilon_0, \quad \varepsilon_r \cong 10$$
$$\sigma_2 \cong 10^{-2} \text{ to } 10^{-3}$$

Above $\omega = \dfrac{\sigma_2}{\varepsilon_2}$ or $f \cong 2$ to 20 MHz

Then ε_2 dominates σ_2.

Reference:
C.E. Baum, "The Reflection of Pulsed Waves from the Surface of a Conducting Dielectric", Theoretical Note 25, February 1967.

Field resultant on a ground surface in absence of wire is $T_e E_{inc}$

$$T_e = 1 + R_e = \frac{2\sin(\psi)}{\sin(\psi) + \left[\varepsilon_r - 1 + \sin^2(\psi)\right]^{\frac{1}{2}}}$$

If the ground were free space ($\varepsilon_r = 1$) then $T_e \cong 1$. But for $\varepsilon_r >> 1$ we have:

$$T_e \cong \frac{2\sin(\psi)}{\sqrt{\varepsilon_r}} \quad , \quad E_{\tan} = T_e E_{inc}$$

A more accurate model might include a ground wave component since the incident field is not truly a plane wave coming from ∞ (for small ψ).

At the wire:

$$E_{inc} = T_V V_a$$

$$V_a = Z_a I_a = \text{voltage into antenna}$$

The current on the wire can be crudely thought to collect displacement current out to a radian wavelength in the soil

$$\bar{\lambda} = \frac{\bar{\lambda}_0}{\sqrt{\varepsilon_r}} = \frac{\lambda_0}{2\pi\sqrt{\varepsilon_r}} = \frac{c}{2\pi \, \mathrm{f} \sqrt{\varepsilon_r}} = \frac{c}{\omega\sqrt{\varepsilon_r}}$$

$$I_{sc} = F \frac{\pi \bar{\lambda}^2}{2} j\omega\varepsilon_2 E_{\tan} = \mathrm{f} \frac{\pi}{2} \frac{c^2}{\omega} \mathrm{j}\varepsilon_0 E_{\tan}$$

F = fudge factor to allow for more accurate scattering analysis
The same analysis applies to the common mode for two closely spaced wires.

Open-circuit voltage at terminal pair (port) introduced into wire (common mode if two wires)

$$V_{oc} = \frac{Z_c I_{sc}}{2}$$

$Z_e \equiv$ characteristic impedance of wire near earth as an approximate transmission line (typically 50 or 100 Ω or so)

The factor of 2 accounts for two waves propagating away from the port.

Now we have:

$$V_{oc} = X\, I_a$$

$$X = j\frac{Z_c}{2} F \frac{\pi}{2} \frac{1}{\omega\mu_0} T_e T_V Z_a$$

$$\frac{j}{8}\frac{F}{\mathrm{f}\mu_0} T_e T_V Z_c Z_a \quad \text{(dimension } \Omega\text{)}$$

By reciprocity a current into the wire port produces an open circuit voltage at the antenna as:

$$V_{aoc} = XI_{sc} = X\frac{2}{Z_c}V_{oc}$$

Now we can construct a transfer function from volts into the antenna to volts received by the antenna as:

$$T \equiv \frac{V_{aoc}}{V_a} = X\frac{I_{sc}}{V_a} = X\frac{2}{Z_c}\frac{V_{oc}}{V_a} = X^2\frac{2}{Z_c}\frac{I_a}{V_a}$$

$$= X^2 \frac{2}{Z_c Z_a}$$

$$= -\frac{F^2}{32\mathrm{f}^2 \mu_0^2} T_e^2 T_V^2 Z_c Z_a$$

Examining these factors we find:

$$T_e^{\,2} \propto \sin^2(\psi) \cong \psi^2 \quad \text{for small } \psi$$

$$T_V^{\,2} \propto r^{-2}$$

$$T \propto \frac{\sin^2(\psi)}{r^2} \cong \frac{h^2}{r^2} \quad \text{for small } \psi$$

pointing out the need for small r and large h.

For an estimate of T_V we have reference:
C.E. Baum, "General Properties of Antennas", Sensor and Simulation Note 330, 1991; IEEE Trans. EMC, 2002, pp. 18-24.

$$T_V = \frac{e^{-\gamma r}}{r} F_V = \frac{1}{r} F_V \text{ in magnitude sense}$$

$$\text{Power density radiated} = \frac{E^2}{2Z_0} = \frac{G}{4\pi^2} \frac{V^2}{2\,\mathrm{Re}[Z_a]}$$

G ≡ gain of antenna

Z_0 ≡ wave impedance of free space

≅ 377Ω

Assume Z_a real

In magnitude sense

$$T_V = \frac{E_{inc}}{V_a} = \left[\frac{Z_0}{Z_a}\right]^{\frac{1}{2}} \frac{G^{\frac{1}{2}}}{\sqrt{2\pi Z_a}} = \left[\frac{Z_0}{4\pi Z_a}\right]^{\frac{1}{2}} \frac{G^{\frac{1}{2}}}{r}$$

$$\cong \left[\frac{30\Omega}{Z_a}\right]^{\frac{1}{2}} \frac{G^{\frac{1}{2}}}{r}$$

$$\frac{30\Omega}{Z_a} \text{ is of general order 1}$$

G is perhaps 10, depending on antenna design.

For example let:

$F \cong 1$, $G \cong 10$
$f = 100\text{MHz}$
$\varepsilon_r \cong 10$
$Z_c = Z_a\, 100\Omega$
$r = 100\text{m}$
$h = 10\text{m}$

In magnitude sense:

$$T \cong \frac{1}{32 f^2 \mu_0^{\ 2}} \frac{4}{\varepsilon_r} \left[\frac{h}{r}\right]^2 \frac{G}{r^2} Z_c Z_a$$

$$\cong \frac{1}{32 \times 10^{16} [4\pi]^2 10^{-14}} .4 \frac{10^2}{10^8} 10^4$$

$$\cong \frac{10^{-5}}{8[4\pi]^2} \cong .8 \cdot 10^{-8}$$

If one changed r to 10m this would be about 10^{-4}.

Consider the current in the wire (common mode for a pair of wires).

$$I_{sc} \cong F\frac{\pi}{2}\frac{1}{\omega\mu_0}T_e E_{inc} \qquad \text{(magnitude)}$$

with say $E_{inc} = 1\,\text{kV/m}$

$$I_{sc} \cong \frac{1}{4\text{f}\mu_0}\frac{\sin(\psi)}{\sqrt{\varepsilon_r}}E_{inc}$$

$$\cong \frac{1}{4\times10^8\times4\pi\times10^{-7}}\frac{.1}{\sqrt{10}}10^3$$

$$\cong 6.3\times10^{-2} \cong 63\text{mA}$$