Backscattering of Horizontally Polarized Electromagnetic Wave from a Wire on or near the Ground Surface

Approximate the fields from the antenna as a plane wave of amplitude $E_{inc}$ incident broadside to the wire on or near the ground surface.

Let:

$$\varepsilon_2 \equiv \varepsilon_r \varepsilon_0, \quad \varepsilon_r \approx 10$$

$$\sigma_2 \approx 10^{-2} \text{ to } 10^{-3}$$

Above

$$\omega = \frac{\sigma_2}{\varepsilon_2} \quad \text{or} \quad f \approx 2 \text{ to } 20 \text{ MHz}$$

Then $\varepsilon_2$ dominates $\sigma_2$. 
Reference:

Field resultant on a ground surface in absence of wire is \( T_e E_{inc} \)

\[
T_e = 1 + R_e = \frac{2 \sin(\psi)}{\sin(\psi) + [\varepsilon_r - 1 + \sin^2(\psi)]^{1/2}}
\]

If the ground were free space (\( \varepsilon_r = 1 \)) then \( T_e \approx 1 \). But for \( \varepsilon_r >> 1 \) we have:

\[
T_e \approx \frac{2 \sin(\psi)}{\sqrt{\varepsilon_r}}, \quad E_{tan} = T_e E_{inc}
\]

A more accurate model might include a ground wave component since the incident field is not truly a plane wave coming from \( \infty \) (for small \( \psi \)).

At the wire:

\[
E_{inc} = T_e V_a
\]

\[
V_a = Z_a I_a = \text{voltage into antenna}
\]

The current on the wire can be crudely thought to collect displacement current out to a radian wavelength in the soil

\[
\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_r}} = \frac{\lambda_0}{\frac{1}{2\pi} \frac{1}{\sqrt{\varepsilon_r}}} = \frac{c}{\frac{1}{2\pi} \frac{1}{\sqrt{\varepsilon_r}}} = \frac{c}{\omega \sqrt{\varepsilon_r}}
\]

\[
I_{se} = F \frac{\pi \lambda^2}{2} j \omega \varepsilon_2 E_{tan} = \frac{\pi}{2} \frac{c^2}{\omega} j \varepsilon_0 E_{tan}
\]
F = fudge factor to allow for more accurate scattering analysis. The same analysis applies to the common mode for two closely spaced wires.

Open-circuit voltage at terminal pair (port) introduced into wire (common mode if two wires)

\[ V_{oc} = \frac{Z_c I_{sc}}{2} \]

\[ Z_c \equiv \text{characteristic impedance of wire near earth as an approximate transmission line (typically 50 or 100 } \Omega \text{ or so)} \]

The factor of 2 accounts for two waves propagating away from the port.

Now we have:

\[ V_{oc} = X I_a \]

\[ X = j \frac{Z_c}{2} F \frac{\pi}{2} \frac{1}{\omega \mu_0} T_e T_y Z_a \]

\[ = j \frac{F}{8 f \mu_0} T_e T_y Z_c Z_a \quad \text{(dimension } \Omega \text{)} \]

By reciprocity a current into the wire port produces an open circuit voltage at the antenna as:

\[ V_{aoc} = X I_{sc} = X \frac{2}{Z_c} V_{oc} \]

Now we can construct a transfer function from volts into the antenna to volts received by the antenna as:

\[ T \equiv \frac{V_{aoc}}{V_a} = X \frac{I_{sc}}{V_a} = X \frac{2}{Z_c} \frac{V_{oc}}{V_a} = X^2 \frac{2}{Z_c} \frac{I_a}{V_a} \]

\[ = X^2 \frac{2}{Z_c Z_a} \]

\[ = -\frac{F^2}{32 f^2 \mu_0} T_e^2 T_y^2 Z_c Z_a \]
Examining these factors we find:

\[ T_e^2 \propto \sin^2 (\psi) \approx \psi^2 \quad \text{for small } \psi \]
\[ T_v^2 \propto r^{-2} \]
\[ T \propto \frac{\sin^2 (\psi)}{r^2} \approx \frac{h^2}{r^2} \quad \text{for small } \psi \]

pointing out the need for small \( r \) and large \( h \).

For an estimate of \( T_v \) we have reference:

\[ T_v = \frac{e^{-\gamma}}{r} F_v = \frac{1}{r} F_v \quad \text{in magnitude sense} \]

Power density radiated:
\[ \frac{E^2}{2Z_0} = \frac{G}{4\pi^2} \frac{V^2}{2 \text{Re}[Z_a]} \]

\( G \) = gain of antenna
\( Z_0 \) = wave impedance of free space
\( \equiv 377\Omega \)

Assume \( Z_a \) real
In magnitude sense

\[ T_\nu = \frac{E_{inc}}{V_a} = \left[ \frac{Z_0}{Z_a} \right]^{\frac{1}{2}} \frac{G^2}{\sqrt{2\pi Z_a}} = \left[ \frac{Z_0}{4\pi Z_a} \right]^{\frac{1}{2}} \frac{G^2}{r} \]

\[ \approx \left[ \frac{30\Omega}{Z_a} \right]^{\frac{1}{2}} \frac{G^2}{r} \]

\[ \frac{30\Omega}{Z_a} \] is of general order 1

G is perhaps 10, depending on antenna design.

For example let:

\[ F \equiv 1, \quad G \equiv 10 \]
\[ f = 100\text{MHz} \]
\[ \varepsilon_r \approx 10 \]
\[ Z_e = Z_a, 100\Omega \]
\[ r = 100\text{m} \]
\[ h = 10\text{m} \]

In magnitude sense:

\[ T \approx \frac{1}{32\pi^2 \mu_0^2 \varepsilon_r} \left[ \frac{h}{r} \right]^2 \frac{G}{r^2} Z_e Z_a \]

\[ \approx \frac{1}{32 \times 10^{16}} \left[ \frac{4\pi}{2} \right]^2 \times 10^{-14} \times 4 \times 10^2 \times 10^8 \times 10^4 \]

\[ \approx \frac{10^{-5}}{8} \approx 0.10^{-8} \]

If one changed \( r \) to 10m this would be about \( 10^{-4} \).
Consider the current in the wire (common mode for a pair of wires).

\[ I_{sc} \approx F \frac{\pi}{2} \frac{1}{\omega \mu_0} T_e E_{inc} \quad \text{(magnitude)} \]

with say \( E_{inc} = 1 \text{kV/m} \)

\[ I_{sc} \approx \frac{1}{4f \mu_0} \sin(\psi) E_{inc} \]

\[ \approx \frac{1}{4 \times 10^8 \times 4\pi \times 10^{-7}} \frac{1}{\sqrt{10}} 10^3 \]

\[ \approx 6.3 \times 10^{-2} \approx 63 \text{mA} \]