Current induced on a long wire by an electric field parallel to the wire.

Cylindrical Coordinates
$(\Psi, \phi, z)$

Reference:
P.R. Barnes, "The Axial Current Induced on an Infinitely Long, Perfectly Conducting Circular Cylinder in Free Space by a Transient Electromagnetic Plane Wave," Interaction Note 64, March 1971
From (29)
\[ I = 4 \frac{E_{inc}}{Z_0 k H_0^{(2)}(ka)} = 4 \frac{E_{inc}}{\omega \mu_0 H_0^{(2)}(ka)} \]

\[ = \frac{2E_{inc}}{\pi f \mu_0 H_0^{(2)} \left( \frac{2\pi f a}{c} \right)} \]

Assume \( ka \ll 1 \) (or \( \lambda >> a \))

Take the asymptotic expansion or \( H_0^{(2)} \) for small arguments.

\[ H_0^{(2)}(u) = J_0(u) - j Y_0(u) \equiv 1 - j \frac{2}{\pi} \left[ \ln \left( \frac{u}{2} \right) + \gamma_e \right] \]

\[ \gamma_e = 0.577... \text{ Euler's constant} \]

\[ I \approx j \frac{E_{inc}}{\pi f \mu_0 \left[ \ln \left( \frac{\pi f a}{c} \right) + \gamma_e \right]} \]

Lower frequency gives more current.

Example:

\( a = 1 \text{mm} \)
\( E_{inc} = 2 \text{kV/m} \)
\( f = 200 \text{ MHz} \)

\[ I \approx j \frac{2 \times 10^3}{2 \times 10^8 4\pi \times 10^{-7} \left[ \ln \left( \frac{\pi 2 \times 10^5 a}{3 \times 10^8} \right) + \gamma_e \right]} \]

\[ \ln \left( \frac{\pi f a}{c} \right) = \ln \left( 2 \times 10^{-3} \right) \approx -6.17 \]

\[ \ln + \gamma_e \approx -5.59 \]

\[ I \approx -j \left[ 4\pi 5.59 \right] 10^2 \approx 1.4 \text{A in magnitude} \]

\[ P = \frac{I^2}{2} Z_{load} \approx 1 \text{ Watt} \]

\[ \Delta t = 20 \text{ns} \Rightarrow U \approx 20nJ \]
Compare to displacement current out to $\bar{\lambda}$.

\[ I_d = j\omega\varepsilon_0\pi\bar{\lambda}^2E_{inc} = j\varepsilon_0\pi\frac{c^2}{\omega}E_{inc} = j\frac{\pi}{\omega\mu_0}E_{inc} \]

\[ = j\frac{E_{inc}}{2f\mu_0} \]

The current in the wire is minus this to cancel this displacement current.

\[ I_w = -j\frac{E_{inc}}{2f\mu_0} \]

In the analysis 2 is replaced by

\[ -\left[ \ln\left(\frac{\pi\lambda}{c}\right) + \gamma_e \right] \approx 5.59 \]

So the more accurate analysis gives a slightly lower current. (The same reduction can be used for the estimate in Microwave memo 13.)

A more accurate substitution in magnitude sense includes the $J_0$ term and gives

\[ \left[ \frac{\pi}{2}^2 + \left( \ln\left(\frac{\pi\lambda}{c}\right) + \gamma_e \right)^2 \right]^{\frac{1}{2}} \approx 5.58 \]

in place of 2 (in the denominator).