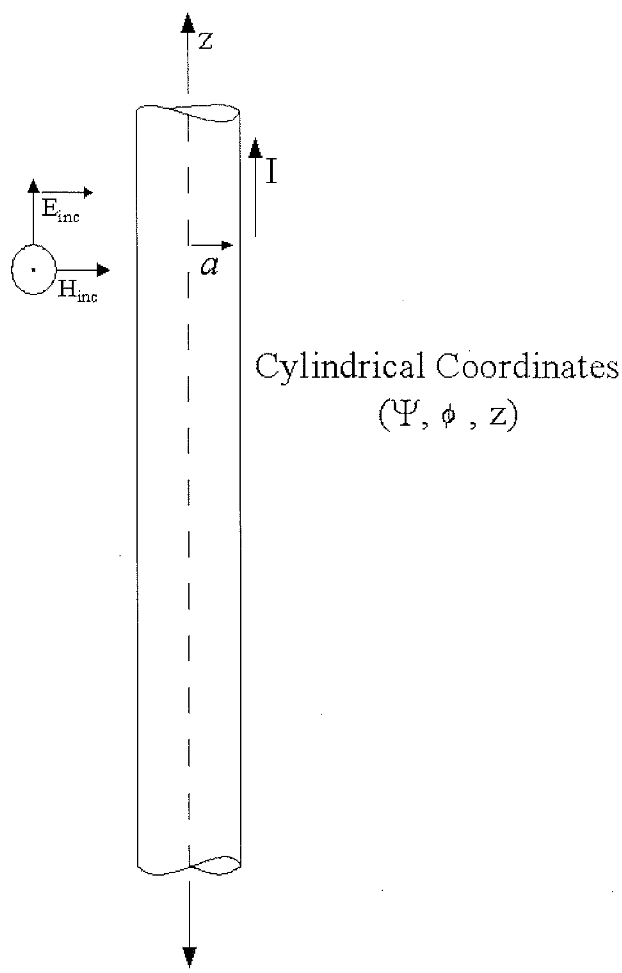
Microwave Memos
Memo 15

C.E. Baum
22 Nov. 2003

Current induced on a long wire by an electric field parallel to the wire.



Reference:

P.R. Barnes, "The Axial Current Induced on an Infinitely Long, Perfectly Conducting Circular Cylinder in Free Space by a Transient Electromagnetic Plane Wave," Interaction Note 64, March 1971

From (29)

$$I = 4\frac{E_{inc}}{Z_0\ \mathrm{k}\ H_0^{(2)}(\mathrm{k}a)} = 4\frac{E_{inc}}{\omega\mu_0 H_0^{(2)}(\mathrm{k}a)}$$

$$= \frac{2E_{inc}}{\pi \mathrm{f}\mu_0 H_0^{(2)}\left(\frac{2\pi \mathrm{f}a}{c}\right)}$$

Assume ka << 1 (or $\lambda >> a$)

Take the asymptotic expansion or $H_0^{(2)}$ for small arguments.

$$H_0^{(2)}(u) = J_0(u) - j\,Y_0(u) \cong 1 - j\frac{2}{\pi}\left[ln\left(\frac{u}{2}\right) + \gamma_e\right]$$

$\gamma_e = .577$....Euler's constant

$$I \cong j\frac{E_{inc}}{\mathrm{f}\mu_0\left[ln\left(\frac{\pi f a}{c}\right) + \gamma_e\right]}$$

Lower frequency gives more current.

Example:

$a = 1\mathrm{mm}$
$E_{inc} = 2\mathrm{kV/m}$
f = 200 MHz

$$I \cong j\frac{2\times10^3}{2\times10^8\ 4\pi\times10^{-7}\left[ln\left(\frac{\pi 2\times10^8 a}{3\times10^8}\right) + \gamma_e\right]}$$

$$ln\left(\frac{\pi \mathrm{f}a}{c}\right) = ln\left(2\times10^{-3}\right) \cong -6.17$$

$$ln + \gamma_e \cong -5.59$$

$$I \cong -j\left[4\pi 5.59\right]^{-1} 10^2 \cong 1.4\mathrm{A}\ \text{ in magnitude}$$

$$P = \frac{I^2}{2} Z_{Load} \cong 1\ \text{Watt}$$

$$\Delta t = 20ns \Rightarrow U \cong 20nJ$$

Compare to displacement current out to $\bar{\lambda}$.

$$I_d = j\omega\varepsilon_0\, \pi\bar{\lambda}^2 E_{inc} = j\varepsilon_0 \pi \frac{c^2}{\omega} E_{inc} = j\frac{\pi}{\omega\mu_0} E_{inc}$$

$$= j\frac{E_{inc}}{2\mathrm{f}\mu_0}$$

The current in the wire is minus this to cancel this displacement current.

$$I_w = -j\frac{E_{inc}}{2\mathrm{f}\mu_0}$$

In the analysis 2 is replaced by

$$-\left[ln\left(\frac{\pi \mathrm{f} a}{c}\right) + \gamma_e\right] \cong 5.59$$

So the more accurate analysis gives a slightly lower current. (The same reduction can be used for the estimate in Microwave memo 13.)

A more accurate substitution in magnitude sense includes the J_0 term and gives

$$\left[\left[\frac{\pi}{2}\right]^2 + \left[ln\left(\frac{\pi \mathrm{f} a}{c}\right) + \gamma_e\right]^2\right]^{\frac{1}{2}} \cong 5.58$$

in place of 2 (in the denominator).