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Consequences of the Electron Equation of Motion

Pt II

A Theory of the Electron

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ABSTRACT

The failure of classical physics in the atomic domain as documented is remarkable on two counts. Firstly that such a powerful theory with such wide application to the universe could be so wrong, and secondly that the failures should be displayed as actually inconsistent with the assumptions of classical physics. The equation of motion derived in Physics Note No 5 was found to have solutions that removed some of these failures and further, was shown to allow discontinuous motion. These observations led to the idea that perhaps classical physics is not wrong as such, but merely incomplete. The adoption of one extra hypothesis taken from quantum mechanics leads to a model of the electron that is both visualizable and consistent with quantum mechanics.

I. INTRODUCTION

The failure of classical physics in the atomic domain as documented is remarkable on two counts. Firstly that such a powerful theory with such wide application to the universe could be so wrong, and secondly that the failures should be displayed as actually inconsistent with the assumptions of classical physics[1]. It has been this latter point that has been the objective of the current research, to see if some of the inconsistencies could be removed. The development of a relativistic equation of motion in Physics Note No 5[2] that retained causality and conservation of energy would appear to have solved this problem.

In the previous paper[3] a number of results were obtained for various forces acting on an electron, and as is to be expected the radiation term leads to decay of the motion. However it was shown that only a finite amount of energy can be imparted to an electron either impulsively or by sinusoidal fields. This suggests that the 'ultraviolet catastrophe'[4] predicted by the Rayleigh-Jeans derivation of the thermal spectrum would not have occurred had the above result been taken into account as all radiation must have originated as the result of accelerating charges. The unexpected result of discontinuous motion[3] in the case of electron-electron bremsstrahlung suggested that this equation might be closer to describing the quantum world than previous formulations of the electron equation of motion. These observations led to the idea that perhaps classical physics is not wrong as such, but merely incomplete. The problem then becomes what minimum assumption needs to be added to classical physics to obtain results consistent with quantum mechanics. A possible solution comes from a study of the equations of quantum mechanics themselves. The differential equations describing the stationary states via wave functions make use of imaginary operators, id/dt and 1/id/dx[5]. *The hypothesis is now introduced that the complex conjugate of these same operators should replace the real operators in the equation of motion to determine stationary states of charged particles..*

This paper is concerned with applying this hypothesis to an assumed point electron in zero external field. In so doing a model of the electron emerges that can not only be visualised, but is also consistent with the known properties of the electron. In addition a limiting process leads to the essential properties of the neutrino.

II. THE STATIONARY STATE EQUATION

We consider first rectilinear motion. The non-relativistic equation for linear motion is[2]

$$\dot{v} + \tau \frac{\dot{v}^2}{v} = \frac{f}{m}$$

Making the replacement

$$\frac{d}{dt} \rightarrow \frac{1}{i}\frac{d}{dt}$$

the equation becomes, on setting to zero and cancelling out the acceleration now assumed not to be zero

$$\frac{\dot{v}}{v} = -\frac{i}{\tau}$$

and this has solution

$$v = v_0 e^{-\frac{it}{\tau}}$$

We assumed linear motion and have obtained a complex solution. The imaginary part of the solution is not to be discarded- it means that motion occurs in a direction orthogonal to the one chosen, that is we have two dimensional motion and in particular we have motion in a circle. We have arrived at a model of a spinning electron! To confirm that this really is the case we now consider two dimensional motion, and we write

$$z = x + iy$$

and the equation for stationary states becomes

$$-\ddot{z} + \tau \frac{[-\ddot{z}.-\ddot{z}^*]}{i\dot{z}.-i\dot{z}^*}(-i\dot{z}) = 0$$

Reducing to its simplest terms

$$\frac{\ddot{z}^*}{\dot{z}^*} = -\frac{1}{i\tau}$$

Taking the complex conjugate

$$\frac{\ddot{z}}{\dot{z}} = -\frac{i}{\tau}$$

Integrating, the solution becomes

$$\dot{z} = \dot{z}_0 e^{-\frac{it}{\tau}}$$

Integrating again

$$z = i\tau v_0 e^{-\frac{it}{\tau}}$$

The motion is circular with radius $v_0\tau$ and constant speed v_0, and we observe that this motion is consistent with the 'zitterbewegun'[5] of quantum mechanics. If this spinning charge is to represent the observed electron, the associated parameters must agree, and so we attempt to match the angular momentum

$$m_0 v_0^2 \tau = \frac{\hbar}{2}$$

Assuming m_0 is the observed rest mass of the electron, we obtain for the velocity

$$v_0 = \sqrt{\frac{\hbar}{2m_0\tau}} \sim 3.10^9 \mathrm{m/s}$$

and so we see that a relativistic treatment is necessary.

III. THE RELATIVISTIC SPINNING ELECTRON

The relativistic equation of motion derived in [2] is, for zero applied force,

$$\frac{d}{dt}\frac{mv}{[1-\frac{v^2}{c^2}]^{1/2}} + \frac{m\tau}{[1-\frac{v^2}{c^2}]^2}\left\{\dot{v}^2 + \frac{(v.\dot{v})^2}{c^2[1-\frac{v^2}{c^2}]}\right\} = 0$$

We have previously considered the electron to have a rest mass m_0, but on this model the electron is taken to be moving at speeds close to c, and it will be the spinning electron that will have an apparent rest mass m_0. The postulated particle will have an intrinsic mass m_i such that the relativistic speed gives the observed rest mass. We observe that the radiation time constant depends on m, and so we write

$$m_i\tau_i = m_0\tau_0$$

We now make the assumption of circular motion at constant speed as indicated in the non-relativistic solution in section IV. We then have

$$\frac{d}{dt}v^2 = 0$$

$$v.\dot{v} = 0$$

and the equation reduces to

$$\dot{v} + \frac{\tau\dot{v}^2}{[1-\frac{v^2}{c^2}]^{3/2}}\frac{v}{v^2} = 0$$

Making the replacements

$$r \to z \qquad \frac{d}{dt} \to \frac{1}{i}\frac{d}{dt}$$

the equation becomes

$$-\dot{z} + \frac{\tau_i}{[1-\frac{v^2}{c^2}]^{3/2}}\frac{\ddot{z}\ddot{z}^*}{i\dot{z}^*} = 0$$

Simplifying

$$\frac{\ddot{z}^*}{\dot{z}^*} = \frac{i}{\tau_i}[1-\frac{v^2}{c^2}]^{3/2}$$

Taking the complex conjugate

$$\frac{\ddot{z}}{\dot{z}} = \frac{-i}{\tau_i}[1 - \frac{v^2}{c^2}]^{3/2}$$

and the solution is

$$\dot{z} = v_0 \exp\{-i[1 - \frac{v^2}{c^2}]^{3/2}\frac{t}{\tau}\}$$

Integrating again

$$z = \frac{i v_0 \tau_i}{[1 - \frac{v^2}{c^2}]^{3/2}} \exp\{-i[1 - \frac{v^2}{c^2}]^{3/2}\frac{t}{\tau}\}$$

and the radius of the circle is

$$r \quad \frac{v\tau_i}{[1 - \frac{v^2}{c^2}]^{3/2}}$$

IV. THE ELECTRON SPIN

The angular momentum is

$$\mathbf{\Omega} = m_0 \mathbf{v} \times \mathbf{r} = \frac{m_0 v^2 \tau_i}{[1 - \frac{v^2}{c^2}]^{3/2}}\hat{\mathbf{n}} = \frac{m_0 v^2 \tau}{[1 - \frac{v^2}{c^2}]^2}\hat{\mathbf{n}}$$

Equating the magnitude to the known value of the electron spin

$$m_0 \tau \frac{v^2}{[1 - \frac{v^2}{c^2}]^2} = \frac{\hbar}{2}$$

or

$$\frac{v}{[1 - \frac{v^2}{c^2}]} = \pm\sqrt{\frac{\hbar}{2m_0\tau}}$$

Introducing α, the fine structure constant

$$\alpha = \frac{3}{2}\frac{m_0 c^2 \tau}{\hbar}$$

and we have

$$\frac{v}{[1 - \frac{v^2}{c^2}]} = \pm c\sqrt{\frac{3}{4\alpha}}$$

Rearranging we obtain a quadratic for v/c

$$\pm\sqrt{\frac{3}{4}\alpha}\left[\frac{v}{c}\right]^2 + \frac{v}{c} \mp \frac{\sqrt{3}}{4\alpha} = 0$$

Taking the upper sign

$$\frac{v}{c} = \frac{-1 \pm \sqrt{1 + \frac{3}{\alpha}}}{\sqrt{\frac{3}{\alpha}}}$$

We must take the positive sign for v<c, with the result that

$$\frac{v}{c} = \frac{\sqrt{1 + \frac{3}{\alpha}} - 1}{\sqrt{\frac{3}{\alpha}}} = 0.9518956$$

The angular velocity of the charge is

$$\omega = \frac{[1 - \frac{v^2}{c^2}]^{3/2}}{\tau_i} = \frac{[1 - \frac{v^2}{c^2}]^2}{\tau}$$

From the solution for the spin velocity

$$[1 - \frac{v^2}{c^2}]^2 = [\frac{v}{c}]^2 . \frac{4\alpha}{3}$$

and so the angular velocity on the present theory is

$$\omega = [\frac{v}{c}]^2 \frac{4\alpha}{3\tau}$$

The quantum mechanical result for the 'zitterbewegun' gives the frequency as

$$\omega = \frac{2E}{\hbar} = \frac{2m_0c^2}{\hbar} = \frac{4\alpha}{3\tau}$$

Comparison of the two results suggests that the standard result treats the electron as having zero intrinsic mass, that is, all the observed mass of the electron is due to its spin. On the present model the intrinsic mass is

$$m_i = m_0[1 - \frac{v^2}{c^2}]^{1/2} = 0.0938947\,m_0$$

The radius of the circle is

$$r_e = \frac{v\tau}{[1 - \frac{v^2}{c^2}]^2}$$

This reduces to

$$r_e = \frac{\left(\frac{3}{\alpha}\right)^{3/2} c\tau}{4\left\{\sqrt{\frac{3}{\alpha} + 1} - 1\right\}}$$

Introducing the Compton wavelength via the relation

$$\lambda\!\!\!^{-} = \frac{3c\tau}{2\alpha}$$

the expression becomes

$$2r_e = \left(\frac{v}{c}\right)\lambda\!\!\!^{-} \quad = 0.9518956\,\lambda\!\!\!^{-}$$

ie the spin circle diameter is just under a Compton wavelength.

V. THE ELECTRON MAGNETIC MOMENT

The magnetic moment is the turning force per unit magnetic flux density,

$$\boldsymbol{\mu} = q\mathbf{v}\times\mathbf{r} = q\frac{v^2\tau}{[1 - \frac{v^2}{c^2}]^2}\hat{\mathbf{n}} = \frac{q\hbar}{2m_0}\hat{\mathbf{n}} = \boldsymbol{\mu}_B$$

that is the magnetic moment is the Bohr Magneton.

VI. THE NEUTRINO

The equation of motion is not restricted to electrons and we are at liberty to consider other parameters. We write for an arbitrary spin half particle

$$m_i \tau_i \frac{v^2}{[1 - \frac{v^2}{c^2}]^2} = \frac{\hbar}{2}$$

If we consider the possibility of mi→0 we must expect v→c in such a way that

$$\mathop{Lt}_{\substack{m_i \to 0 \\ v \to c}} \frac{m_i}{[1 - \frac{v^2}{c^2}]^{1/2}} = \frac{E}{c^2}$$

where E is the energy of the particle. Inserting this result

$$\mathop{Lt}_{\substack{m_i \to 0 \\ v \to c}} \frac{E}{c^2} \cdot \frac{\tau_i c^2}{[1 - \frac{v^2}{c^2}]^{3/2}} = \frac{\hbar}{2}$$

For this limit to exist, we must have $\tau_i \to 0$ as v→c in such a way that

$$\mathop{Lt}_{\substack{\tau_i \to 0 \\ v \to c}} \frac{\tau_i}{[1 - \frac{v^2}{c^2}]^{3/2}} = \frac{\hbar}{2E}$$

Note that as $\tau_i = 0$, we have $q = 0$ and so we observe that charge and mass are tied together, suggesting that they are aspects of the same reality. The radius of rotation is given by

$$2r = 2 \mathop{Lt}_{\substack{\tau_i \to 0 \\ v \to c}} \frac{c\tau_i}{[1 - \frac{v^2}{c^2}]^{3/2}} = \frac{\hbar c}{E_v} = \lambda$$

The stationary state equation is then consistent with the existence of neutrinos.

VII. SUMMARY AND DISCUSSION

The relativistic equation of motion supplemented by the stationary state hypothesis has resulted in a visualizable model of a spinning electron, that is a point charge rotating around a centre with a velocity close to c, the motion being such that the observed mass, m_o, is given by the relativistic increase in an intrinsic mass $m_i \sim m_0 /10$. The rotation diameter is just less than the Compton wavelength of the electron and the angular frequency is slightly less than the quantum mechanical result for 'zitterbewegun', this being attributable to a non-zero intrinsic mass on this model. The magnetic moment of the electron on this model is one Bohr magneton. A mathematical limiting process leads to a model of the neutrino. It appears to follow from the eqations that mass and charge vanish together, suggesting that they are different aspects of the same reality.

The results appear to be deterministic, but this is illusory as we cannot know the initial conditions. The results simply tell us *how* the electron moves, not what its speed or position is at any particular time. The results of any measurement of position will be uncertain by $\Delta x = \sqrt{2}r$, while the uncertainty in momentum will be $\Delta p = \sqrt{2}\, m_0 v$, and from the appropriate equations

$$\Delta x \Delta p = \sqrt{2}\,\frac{v\tau}{[1 - \frac{v^2}{c^2}]^2} \times \sqrt{2} m_0 v = \hbar$$

and the result is seen to be consistent with the uncertainty principle.

An objection to the use of point charge models is the belief that point charges infer infinite self energy of the electrostatic field. It is argued here that this is not so. The way in which the field energy is calculated is to assume the incremental assembly of the charge, and in the limit of forcing all the charge into a sphere of zero volume, an infinite amount of work is required. However, a point charge cannot be divided into increments either for assembly or disintegration, and the mathematical process does not mirror any physical process. The point charge just exists. If it is considered to be in an otherwise empty universe, the field will spread out isotropically, and the concept of doing work is meaningless. If we now bring in another like charge the fields will interact and work will be required to bring them close together, in fact an infinite amount of work will be required to make them occupy the same position in space. It is only *distorted* electric fields that can be regarded as storing energy. In contrast magnetic fields always represent stored energy due to the apparent non-existence of magnetic monopoles.

It remains to be seen how well this approach applies to other particles and systems such as atoms, some preliminary results for the hydrogen atom being encouraging, but whether or not further progress is possible, the essential point that has been made is that classical physics is not as far removed from quantum mechanics as the bulk of publications would have us believe.

VIII. REFERENCES

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