AN EXPERIMENTAL INVESTIGATION OF THE MICROWAVE RADIATION EMITTED BY A HIGH CURRENT RELATIVISTIC ELECTRON BEAM

BY

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LPS 41                    APRIL 1970

This work was supported by the Office of Naval Research Contract No. N00014-67-A-0077-0002.
ABSTRACT

The development of a high frequency instability in a relativistic electron beam, propagating through a drift space, has been studied from measurements of the microwave emission from the beam. The results show that a part of the radiation is emitted from the region close to the beam front and that the intensity of this emission grows in time as it is convected along with the beam. The growth rate is of the order of $10^8 \text{ sec}^{-1}$ and depends on the macroscopic flow parameters. Experiments in different gases show that the growth rates are approximately proportional to the ionization probability of the gas. Precursor radiation is also monitored at comparable power levels to those found at the beam front.

Possible explanations of the radiation are proffered and the physical importance of the emission ($\sim 100 \text{ kw/m. length of drift tube}$) on beam neutralization and structure are indicated.
I. Introduction

During the past few years considerable interest has developed in the properties and transport of intense beams of relativistic electrons propagating through drift regions filled with various gases at low pressures (0. 1-10 Torr.).\textsuperscript{1, 2, 3, 4} The measurements reported, to date, in the literature relate mainly to beam transport processes, although recent work has discussed possible applications of these beams to such widely diverse fields as thermonuclear containment devices\textsuperscript{5} and ion accelerators.\textsuperscript{6} This paper presents measurements on the microwave radiation properties of such a beam (55 kA, 300 kV) propagating through air, hydrogen, and helium. These measurements complement the transport studies referred to earlier and suggest possible potential of the beam as an intense pulsed radiation source.\textsuperscript{7}

The experimental investigation describes the temporal and spatial development of microwave radiation from the Cornell relativistic electron beam. Observations, principally in the range 8 to 12 gigahertz, show an exponential growth of the radiation level over four orders of magnitude with growth rates of about $10^8$ sec$^{-1}$ (approximately equal to the orbital period of the electron in the self-magnetic field to the electron beam). The radiation is broad band in the frequency range indicated above and shows evidence of complex structure. The radiation intensity considerably exceeds levels anticipated on the basis of single particle incoherent radiation. The growth of the radiation has been followed through
to saturation when the radiation power was of the order of 20 Watt/cm²/steradian transverse to the beam. Precursor signals, of comparable magnitude, were observed up to fifty nanoseconds in advance of the beam front. The phenomenology of these observations is discussed and a qualitative explanation of the observations is proffered.

2. Beam Equilibria and Electron Orbits

We review here and apply, as relevant to the radiation studies reported, beam transport theory. The earliest studies by Alfvén in 1939 and later Lawson demonstrated the existence of a critical current, above which a fully neutralized beam should not propagate. This critical current arises from considerations of the effect of the self-magnetic field of the beam on the motion of the electrons. When the current is sufficiently large, the electrons describe re-entrant orbits; and the axial motion is in the limiting case reduced to zero. The value of this critical current $I_A'$ is readily shown to be

$$I_A = 17,000 \beta \gamma \text{ amps} \quad (1)$$

where $\beta$ is the ratio of the axial velocity of the electrons to the speed of light and $\gamma = (1 - \beta^2)^{-1/2}$. For a 300 KeV electron beam the critical current has a value of 21.2 K. Amps. The ratio of the actual beam current $I$ to this critical current defines the electron orbital motion. In the experiments performed the injection current was typically 55 K. Amps.
and counterstreaming of secondary electrons reduced this value to a net current of about 10 to 15 K. Amps.

More recently, Rostocker and Hammer\(^{(4)}\) derived a self-consistent equilibrium starting from the relativistic Vlasov equation. Their work demonstrated that it is possible to have a propagating electron beam with a net current in excess of the Alfvén limit provided that the current distribution is nonuniform across the drift tube radius. The majority of the current flows in a shell at the edge of the beam, the thickness of this shell being about \(c/\omega_p\), the collisionless skin depth.

We now examine the results derived by Rostocker and Hammer as applied to the case of a fully neutralized beam with \(I/I_A < 1\). An examination of the approximations made shows that the expansions involved probably yield results good to about 10\(^{\circ}/0\) for the experimental conditions. It is readily shown that the radial motion of the electrons is described by the equation

\[
\frac{\vec{r}}{b} = \frac{1}{\sqrt{2}} \left[ \cos (\omega_o t + \psi x) - \sin (\omega_o t - \psi) \vec{y} \right] 
\]

where \(b\) is the beam radius

\[
\omega_o = \frac{2v}{\gamma} \frac{V_z^2}{c^2}
\]

\(v = N \rho_o\) (\(N = \) line density of electrons and \(\rho_o = \) classical electron radius)
In summary we conclude that the effect of the self-magnetic field on the beam propagation is not inconsistent with existing theory. We have, though, no guarantee that the equilibrium studied can be approached in the time scale of the experiment. However, it seems likely that the qualitative features of this analysis are probably correct. It is also clear that the electron acceleration produced by the beam self-magnetic field is capable of producing 'synchrotron' like radiation. The power levels monitored are substantially higher than those calculated on the basis of single particle radiation using the model given.

3. Experimental Facility and Measurement Techniques

3.1 The Experimental Facility

The experimental configuration used to produce the beam of relativistic electrons has been described elsewhere \(^{(10)}\) and will only be outlined in the present article. The fast pulse section of the system consists of a planar 2 Ω Blumlein transmission line with an electrical length of 40 n. secs. The line is reasonably charged from a Marx generator erecting in about 600 n sec. The line feeds a vacuum diode with a thin foil (1 mil) Titanium anode. The diode spacing is adjusted to match the impedance of the line. The electron beam emerges from the foil and enters a cylindrical drift region with an aluminum mesh lining the inside of the drift tube. The liner serves the purpose of providing a return path for the beam current. Measurements on the propagation characteristics
of this beam have been reported elsewhere. The majority of experiments described here were carried out in a 6.4 m, long, fourteen cm. internal diameter, drift tube.

3.2 Microwave Measurement Techniques

These measurements fall into two categories - (a) Measurements with a standard gain x band horn mounted in a side arm. (b) Measurements of the radiation picked up by magnetic field search coils, approximately 6 mm x 4 mm, mounted just inside the aluminum liner.

In both cases the detection system was identical, with the exception of the antenna, and was calibrated using a swept frequency backward wave oscillator of known(measured)power output. In the latter case the output was fed from the loop antenna through 18 inches of 50 ohm cable to a type N coaxial to RG52 waveguide adaptor. This was followed by an isolator, with greater than 20 db isolation and an attenuator to the detector. The detector was a balanced mixer (Microwave Associates MA 1113), modified for use as a simple crystal detector and fed a matched 50 ohm cable to the video detector. The response of this system, excluding the antenna, was flat over the range 8.5 to 12.4 GHz to within 3 db and fell off rapidly outside this range. Six such detectors were used in the experiments to be described. The average difference between detectors was calibrated and found to be less than 3 db. The local output (at a given frequency) did not differ from one detector to the next by more
than 3 db at any frequency. The same systems, including the loop
antennas were calibrated against each other in the experimental conditions
(i.e., at one fixed axial location and under nanosecond pulse conditions)
and showed on average the same responses integrated over the frequency
band studied, as had been obtained on the calibration set up using the
B. W. O.

It should be noted that the calibrations were performed at the
signal levels used in the experiment and were checked subsequent to the
experiment. No evidence of significant change was detected.

The loops were placed at different axial locations along the
drift tube with the loop antennas oriented in the r-z plane. Using the
calibrations made we consider that absolute signal levels recorded were
good to approximately a factor of two (3 db).

In addition to recording the microwave emission in the x band,
streak records, x ray scintillator photo diodes and magnetic probes were
used to diagnose the beam behavior. Time correlated records of the
microwave emission and wall magnetic field were obtained and absolute
times recorded, with reference to the onset of the beam current, through
the use of the multiple display of the records obtained from the wall
magnetic field probes. Overall absolute time accuracy is of the order of
or less than ±2.5 n secs. The video output was displayed on Tektronix
555 and 556 oscilloscopes. The rise time of the recorded pulses was
about 10 n secs., consequently, some loss of signal was probably present
in the 555 records. Under no circumstances do we believe that the loss of signal in the 555 record, compared to that on the 556, was greater than 3 db.

The experiments to be described were carried out in Air (200 m. Torr to 1.6 Torr), Helium (900 m. Torr to 6.8 Torr) and Hydrogen (690 m. Torr to 5.2 Torr) in the pressure ranges indicated. In each case the microwave emission had fallen off substantially by the upper pressure limit indicated. The mean operating level of the beam was 300 kV and 55 kA with an r.m.s. scatter of 8%o. A reproduction of typical current and voltage oscillograms and a streak camera record of the beam propagation are shown in Figs. 1a and 1b.

4. Experimental Results

4.1 Growth of Radiation

The microwave emission from the relativistic electron beam, monitored using the multiple detector system described in the last section, was examined in three gases, air, hydrogen, and helium in the pressure ranges indicated. The loop antennas were mounted in the side wall of the drift space at axial locations 1.0, 1.92, 3.16, 4.07, 5.0, and 6.23 metres, measured from the diode anode foil. A second set of measurements with the detectors located 0.23, 0.39, 0.69, 1.9, 1.31, and 1.62 metres from the diode were also taken to study the development of the radiation pulse on a finer space and time scale. All oscilloscope traces were time
correlated with each other through multiple displays of the magnetic field at the tube wall. Ports at 1, 3.1, 2.54, 3, 16, and 6.23 metres were used for this purpose. The zero of time was taken as the onset of beam current.

A typical display of the data is shown in Fig. 2. In this figure the oscilloscope waveforms are reproduced and arranged vertically to correspond with the axial (z) location of the detector. The luminous front of the beam, as recorded using an S.T.L. image converter operated in the streak mode is also shown on the same figure. The writing speed of the image converter as recorded on the film corresponded to a 'beam writing speed' of 7.3 cm/n. sec. and a time resolution capability of about 2 n. sec. limited by the diameter of the drift tube. Thin sheets of plastic scintillator, shielded from a beam by a 5 mil. strip of aluminum responded to bombardment by energetic electrons and indicated that the visible light emission occurred promptly (within about 2 n. sec. over the pressure range studied) with the arrival of the primary electrons. Peak levels of the received microwave signal are indicated beside each trace (OdB = 1 m watt). Three features of these records are evident:

1) A microwave signal is recorded closely time coincident with the rise in the magnetic field at the tube wall and the onset in luminosity as recorded by the streak camera. This particular feature is especially evident in the data taken in close to the diode.
power growth rate by a factor of two. To a good approximation the growth
depend on the macroscopic flow properties of the beam. (i.e.,
A beam propagates with similar characteristics at different ambient
pressures in different gases.). The propagation (see Fig. 5) and signal
growth rates scale fairly closely with the ionization probability of the gas.
A more detailed study of beam propagation in different gases substantiating
this statement has been presented elsewhere. 11, 12

A low drift tube pressures (e.g., 150 m. Torr in Air) dis-
continuities are evident in the beam propagation. This phenomenon has
been described elsewhere. 13 Fig. 6 shows a plot, typical of those found
under these conditions. It will be observed that the emission level of
the beam plasma system is substantially reduced at this point and that the
precursor signal seems to originate at the discontinuity. This process
has been noted consistently for operation under these conditions. Similar
data has been obtained in hydrogen at 690 m. Torr and 900 m. Torr
in helium. Careful observation has demonstrated that the signal dis-
continuity in the low pressure regime is an extreme case of the periodic
structure observed in the streak pictures at higher pressures. In the
'high' pressure cases, the discontinuities are not so pronounced and do
not lead to gross reductions in the microwave emission. It may well be,
however, that the precursor signals seen in Fig. 2 and 6 both have a
common origin. The 'periodicity' of the discontinuity structure is inversely
proportional to the pressure and at the highest pressures utilized (1.6 Torr
in Air) occurs with a spatial wavelength of about 0.3 metres (~2 n. sec. time interval) and hence would not be resolvable in the emission patterns observed oscillographically.

4.2 Observations with Horn Antennas Mounted in Side Arms

Measurements of the beam radiation were also made using standard gain x-band horns mounted in a side arm approximately 2 metres from the diode. A summary of the results obtained is given below:

1) The time from beam initiation to the onset of radiation varied inversely with pressure asymptoting to a constant value, at high pressure, of approximately twice the delay expected on the basis of rectilinear motion of the electrons. The delay cannot be explained simply on the basis of non-neutralization of the beam head. 9

2) The onset of the radiation occurred simultaneously with the arrival of primary electrons as indicated by scintillator diode x-ray measurements and magnetic field probes mounted in the tube walls. The precursor signals referred to earlier had not developed in the first two metres of the drift tube.

3) The duration of the radiation pulse varied with ambient pressure, decreasing from 70 n. sec. at 50 m. Torr. (in Air) to 15 n. sec. at 600 m. Torr. At 250 m. Torr., the lower limit reported using the loop antennas, the pulse width was 30 n. sec. These measurements are consistent with the present data.
4) Measurements with the E plane of the detector along and transverse to the direction of beam propagation suggested that the radiation was primarily polarized in the direction of beam propagation. This is consistent with propagation in the extraordinary wave mode in the plasma.

5) Measurements with different frequency band detectors indicate that the radiation existed at the levels described, only for frequencies below 13 GHz. These measurements were carried out, up to 80 GHz. No measurements were made below 7.8 GHz. Above 13 GHz the radiation level fell off rapidly with increasing frequency.

The use of low Q tuned cavities showed that the signal was broad band. This result is in agreement with the data obtained from dispersing the incident pulse to obtain a measure of the frequency spectrum of the microwave signal. This measurement is described in more detail below.

The cavity measurements suggested that all the frequencies monitored were radiated simultaneously. This was, however, on a shot to shot basis. The use of a tapered transition piece to a KU band guide and detector, monitored simultaneously with the complete X band signal, showed clearly that signals received above 10 GHz had an identical duration to those from 8 to 12 GHz, confirming the broad band nature of the signal and illustrating that all frequencies were radiated simultaneously.
4.3 Spectrum Analysis of the Microwave Emission

The short duration of the radiation pulse (∼30 n. sec.) and the rapid fall off in emission at frequencies greater than 13 GHz enables one to utilize the dispersive property of a wave guide, where propagation is known to be restricted to the T. E. 10 mode, to measure the spectral content of the emitted signal. At 95 m. length of x band waveguide was used for this purpose. Details of its performance and limitations will be published separately. Under the conditions prevailing in the experiment we were able to obtain reliable information for frequencies from about 7.5 to 10.5 GHz. These measurements gave power received integrated over the radiation pulse width (∼30 n. sec.), as a function of frequency on a single shot basis. The shot to shot repeatability of the dispersed signal was poor and the results only yielded a trend evidenced in all of the records obtained.

Fig. 7 shows an example of the dispersed pulse taken, in this case in Helium at a pressure of 3.4 Torr, with operating voltage and current 320 kV and 57.5 kA, respectively. It will be noted that the radiation extends over the band 7.8 to 12.0 GHz and indeed to frequencies substantially below the lower limit shown. The radiation level at frequencies above 12.5 GHz was at least 20 db down on the level at 12.0 GHz. The monitored output, at a port about 1 1/2 m from the diode, is observed to be 'spikey', consisting of a series of discreet frequencies rather than a
continuum. The limits of resolution set by the dispersive nature of the guide are indicated by the horizontal bars above the spectrum. The rise time of the oscilloscope (Tektronik 454) played no role in this limitation. The frequency separation of the peaks in the radiation pattern varied from about 100 MHz at the lowest frequencies to about 400 MHz at the highest frequency. It is not clear if this increase in separation is real or simply a function of the decreased sensitivity at high frequency. It should be noted that the envelope of the spectrum is flat over the complete range shown and that the signal intensity falls off rapidly with increasing frequency above 12.0 GHz. This feature is consistent with any bunching mechanism, which could cause coherent effects, failing at short wavelengths. That is we interpret this as indicating bunching dimensions as being no shorter than about 1 cm. Furthermore, it should be noted that the separation in frequency of these peaks is about that expected on the basis of a synchrotron radiation mechanism. A similar spread in frequencies could also be obtained from the eigenmodes of the conducting wall drift tube.

5. Discussion of Results

It is clear that the radiation phenomenon observed is at a level substantially in excess of the single particle radiation expected on the basis of electron motion in the self-magnetic field of the beam. Evidence
substantiating the nonrectilinear motion of the electrons rests primarily in the beam transit time through a drift region exceeding the times calculated for rectilinear motion. Such orbital motion would lead to 'synchrotron' radiation. To account for the radiation levels monitored it would be necessary to have bunching of the electron beam. The growth of the radiation is clearly capable of providing such a mechanism. It is not possible to confirm the nature of the instability from the measurements made. It should be noted, however, that the measured growth rate is close to the relativistic cyclotron frequency based on the value of the magnetic field at the tube wall. A second mechanism capable of producing bunching of the electrons relies on the finite rise time of the current pulse. The Fourier components of the signal in the frequency band of interest are sufficiently large to account for the radiated signal. In addition the microwave signal is usually largest at the time of maximum rate of change of magnetic field. There is also some evidence of steepening of the current front as the beam propagates along the drift tube. The evidence is however marginal as the rise time of the self-magnetic field is close to the rise time of the 556 oscilloscopes used. Measurements at the diode also show two radiated pulses—one on the leading and one on the trailing edge of the current pulse, suggesting the finite rise time of the pulse plays at least a role in the development of the radiation.

If we associate the radiation with the orbital motion of the electrons then we must note that at the lowest frequency monitored
we would be observing radiation at close to the fortieth harmonic.
This is high, but possible since similar observations have been made
in lower energy systems.\textsuperscript{16} The rapid decrease in signal at high
frequencies (\(-12\ \text{GHz}\)) would be consistent with the failure to produce
the requisite bunching density of electrons in a half wavelength at the
highest frequencies. An additional feature based on the synchrotron
radiation model is that low frequency radiation (less than the plasma
frequency of the beam) would be heavily absorbed in the background
plasma. Since the power monitored corresponds to radiation levels
of a few hundred kilowatts per meter, we expect substantial micro-
wave heating and ionization of the background gas from the radiation.
Saturation of the radiated signal could result from such a process.
The r.f. ionization can be estimated as about comparable to the
primary ionization rate. In addition, it should be noted that the
precursor signals monitored are close to the threshold for r.f.
breakdown of the gas ahead of the beam.\textsuperscript{17} No measurements, however,
have been made of electron density ahead of the beam front. The
effect of the conducting wall in the drift space is that of an oversize
waveguide channelling the r.f. The cut-off frequency of the lowest
order mode in the guide is about\(1.5\ \text{GHz}\), consequently, the precursor
can propagate ahead of the beam with a velocity extremely close to the
speed of light.

An alternative mechanism for the radiation growth, initially
suggested by Eastlund\textsuperscript{18}, is the Čerenkov effect. Since observations
on the plasma conditions. This result holds even in the absence of a magnetic field. The r.f. power then causes further ionization and enhances the Čerenkov radiation. In our case initial ionization is provided by the electron beam and initial bunching is produced by the pulse rise time. Čerenkov or any other r.f. radiation is then capable of enhancing the ionization thus leading to a growth of the radiation in space and time. A mechanism of this type is qualitatively capable of accounting for the observed discontinuity structures in the beam propagation. The power levels monitored are, however, insufficient to account for the discontinuities but the conversion efficiency to radiated power and the fractional radiated power to dielectric and ionization losses are not readily assessed. A more detailed account of these processes has been given by Eastlund. 26

Experiments by Kornilov et. al. 27 using a steady state beam-plasma experiment at energies of 2-5 keV and beam currents of 10-50 mA in a guide field of up to 2k gauss have demonstrated a rapid transition in the radiation properties of the beam plasma system. In their configuration oscillations at the electron cyclotron frequency were observed to grow with increasing discharge chamber length. For sufficiently long drift space and high ambient gas pressure (~2m Torr) the radiation built up to a sufficiently large value to cause additional ionization resulting in a change in plasma density of three orders of magnitude. Accompanying the change the emission spectrum switched to radiation close to the plasma frequency. The change in nature is attributed to the Čerenkov effect in the frequency zone $\omega_p$ to $(\omega_p^2 + \omega_c^{21/2})$. An energy analyser showed complete stopping of the primary electron beam. The experimental conditions in
this experiment are very different to those used in our experiment. The beam propagation discontinuities in our own case, however, bear a strong qualitative similarity to those found by Kornilov. The recommencement of the beam motion would then be attributed to changes in the effectiveness of the Čerenkov radiation mechanism. It seems possible that both mechanisms, 'synchrotron' and Čerenkov, could well play important roles in both experimental regimes. At present no information exists on the nature of the radiation spectrum in the vicinity of beam discontinuities.
Conclusions

The temporal and spatial development of microwave emissions from a drifting high current (net $v/\gamma \sim 0.5$) relativistic electron beam has been studied experimentally. It is found that the radiated power grows exponentially in time and is convected along with the beam. The growth rate scales with the ambient gas such that the behavior is essentially related through the ionization probability of that gas. The macroscopic properties, i.e., flow velocity, discontinuity structure periodicity, and instability growth, all scale proportionally to the rate of ionization of the gas.

It is not possible to establish unambiguously the source of the instability although two stream instability (the beam and the secondary electrons) must be regarded as a strong possibility. The effect of the instability is to enhance the radiated energy to a level where r.f. ionization and heating is significant. The radiation probably arises from either a Čerenkov or 'synchrotron' mechanism. In conclusion precursor radiation is observed at levels marginal for the preionization of the gas ahead of the beam. This may be an important effect in the propagation of high current beams.
ACKNOWLEDGMENTS

We wish to express our gratitude to the members of the Cornell Relativistic Electron Beam Group for their constant assistance and constructive criticism of this work. Helpful discussions with Dr. Bernard Eastlund of the U.S.A.E.C. regarding Čerenkov radiation mechanisms are especially appreciated.

This work was supported by O.N.R. contract number N00014-67A-0077-0002.
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7. J. A. Nation - to be published.

26. B.J. Eastlund, to be published (Private communication).

Figure 1a  Upper Trace - Diode Current - 55 KA peak  Sweep Time 400 nsec.
         Lower Trace - Diode Voltage - 300 KV peak at 50 nsec/divn.

Figure 1b  Streak Record at 350m Torr in Air
         Horizontal Axis 200 nsec sweep
         Vertical Axis 6.4m length of drift tube
         Diode is located at upper part of the picture.
Figure 2. Plot showing distance-time-amplitude characteristics of the microwave emission (Air 700 m, Torr.) and their time correlation with the streak camera record and wall magnetic field probes.
Figure 6. Distance-time-amplitude characteristics of the microwave emission and their time correlation with the streak camera record and wall magnetic field probes. This record is typical of 'low' pressure data (Air 150 m. Torr.) where large steplike discontinuities are evident in beam propagation.
Figure 7: Frequency Spectrum of Microwave Emission in Helium at 3.4 Torr.