This note is appendix II of RPN 35 and is not included here.

RPN 34

PHYSICS INTERNATIONAL INTERNAL REPORT NO. 55

AUTHOR: John Creedon July 16, 1965

SUBJECT: Derivation of Formulas for Determining Radiation Pattern from Four-Point Sources

The formulas for determining the radiation pattern from a four-source array are derived with reference to the diagrams of Figures 1, 2, and 3. The following symbols are used:

Definition of Symbols

s_o	Center of source array
s_1, s_2, s_3, s_4	Source points
s	Diagonal distance across array
ℓ	Distance from source plane to plane at which radiation is calculated
p_1, p_2, p_3, p_4	Points formed by extension of electron beam direction through s_1, s_2, s_3, s_4 to target plane
p_o	Center of points p_1, p_2, p_3, p_4
θ	Angle of line s_1p_3 with respect to line s_1p_1; $\theta = \tan^{-1} \frac{s}{\ell}$
x-axis	Axis through p_1, p_3
p	Point on x-axis for which sum of radiation from s_1, s_2, s_3, s_4 is calculated
x	Distance from center of target, plane to p
θ_1	Angle between s_1p_1 and s_1p
θ_2	Angle between s_3p_3 and s_3p

y_1	Distance from p_1 to p
y_2	Distance from p_3 to p
Z_1	Length of line $s_o p$
α_1	Angle between line $s_o p$ and line $s_o p_o$
ζ_1	Distance from s_2 to p = s_4 to p
γ_1	Angle between $s_2 p$ and $s_o p$
β_1	Angle between electron beam direction at s_2 and line from s_2 to p
$R_{\ell\beta_1}$	Relative radiation intensity at angle β_1 at distance ℓ
$R_{\zeta_1\beta_1}$	Relative radiation intensity at angle β_1 at distance ζ_1
x'-axis	Axis through p_o at 45 deg to x-axis
p'	Point on x'-axis at which the radiation intensity is calculated
x'	Distance from p_o to p'
y_1'	Distance from intersection of line $p_1 p_4$ and x'-axis to point p'
y_2'	Distance from intersection of line $p_2 p_3$ and x'-axis to point p'
q_1	Point midway between s_1 and s_4
q_2	Point midway between s_2 and s_3
Z_1'	Distance from q_1 to p'
Z_2'	Distance from q_2 to p'
α_1'	Angle between line through q_1 parallel to electron direction and line from q_1 to p'
α_2'	Angle between line through q_2 parallel to electron direction and line from q_2 to p'

To calculate radiation intensity along the x-axis the procedure used was to calculate α_1 from Equation 1, γ_1 from Equation 2, and β_1 from Equation 3. $R_{\ell\beta_1}$ was taken from the curve in Figure 4 and the value of $R_{\zeta_1\beta_1}$ was calculated from Equation 4. If the radiation intensities at p from the four-source points s_1, s_2, s_3, s_4, are respectively, R_1, R_2, R_3, R_4, then

$$R_{\zeta_1\beta_1} = R_2 = R_4$$

For intensities R_1 and R_3 from s_1 and s_3, the following formulas apply:

$$\theta_1 = \tan^{-1} \frac{y_1}{\ell}$$

$$\theta_2 = \tan^{-1} \frac{y_2}{\ell}$$

Values of relative intensity were taken from Figure 4 for θ_1 and θ_2, and multipled by $\cos^2 \theta_1$ and $\cos^2 \theta_2$ to give the intensities R_1 and R_3. The total radiation intensity at p is the sum of R_1, R_2, R_3, and R_4.

The intensity along the x'-axis is calculated as follows:

$$\tan \alpha_1' = y_1'/\ell$$

$$\tan \gamma_1' = (1/2\sqrt{2})(s/\ell) \cos \alpha_1'$$

$$\cos \beta_1' = \frac{2\sqrt{2} \sin \gamma_1'}{s/\ell}$$

$$R_{\zeta_1 \beta_1}' = R_{\ell \beta_1}' \cos^2 \beta_1' \qquad (5)$$

Equation 5 gives the radiation intensity from points s_1 and s_4 at p'

$$R_1' = R_4' = R_{\zeta_1 \beta_1}'$$

$$\tan \alpha_2' = y_2'/\ell$$

$$\tan \gamma_2' = (1/2\sqrt{2})(s/\ell) \cos \alpha_2'$$

$$\cos \beta_2' = \frac{2\sqrt{2} \sin \gamma_2'}{s/\ell}$$

$$R_{\zeta_2 \beta_2}' = R_{\ell \beta_2}' \cos^2 \beta_2' \qquad (6)$$

Equation 6 gives the radiation intensity from points s_2 and s_3 at p'

$$R_2' = R_3' = R_{\zeta_2 \beta_2}'$$

The total radiation intensity at p' is $R_1' + R_2' + R_3' + R_4'$

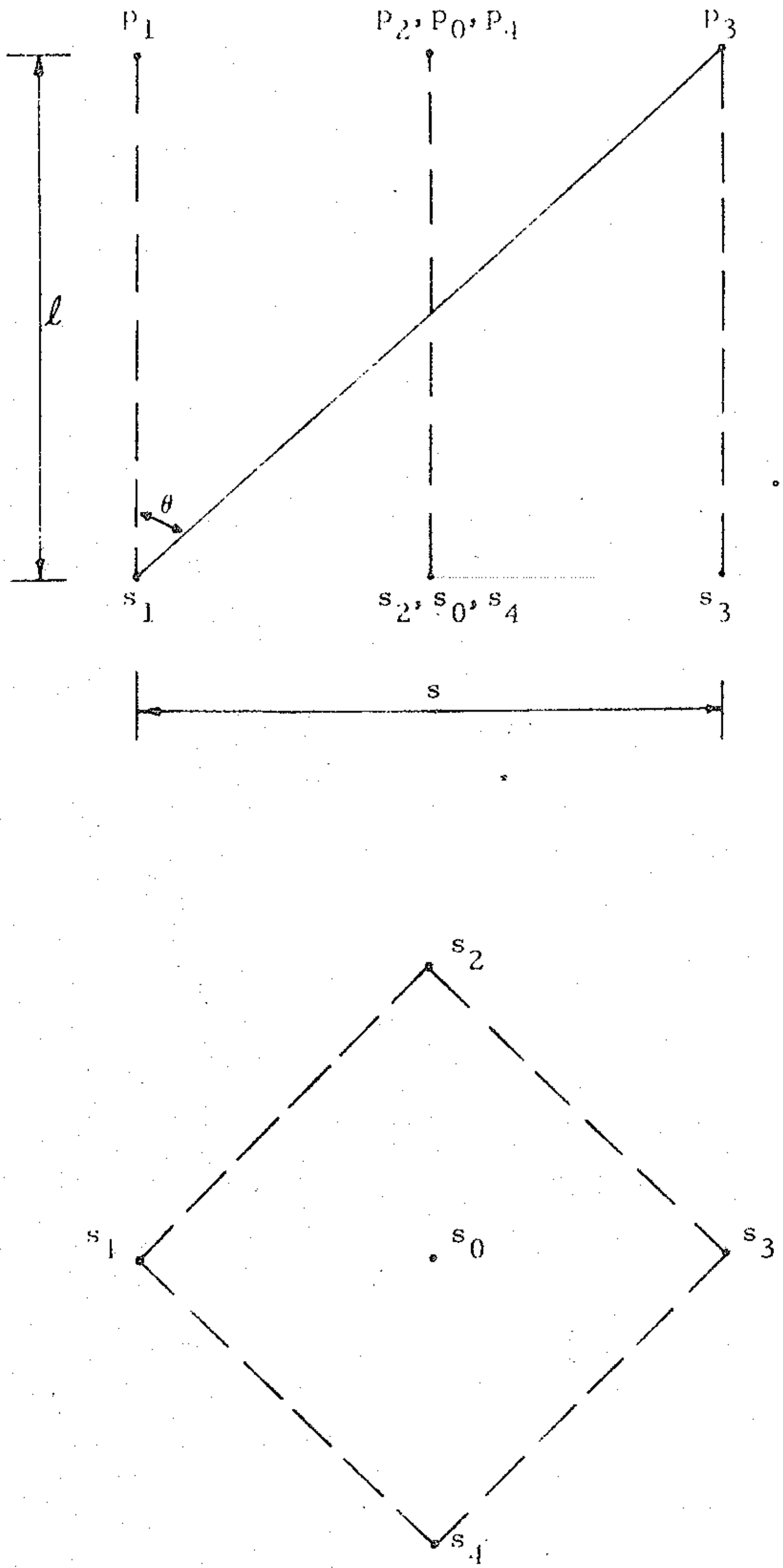

Figure 1. Scheme for Calculation of Four-Point Source Radiation Pattern

1252

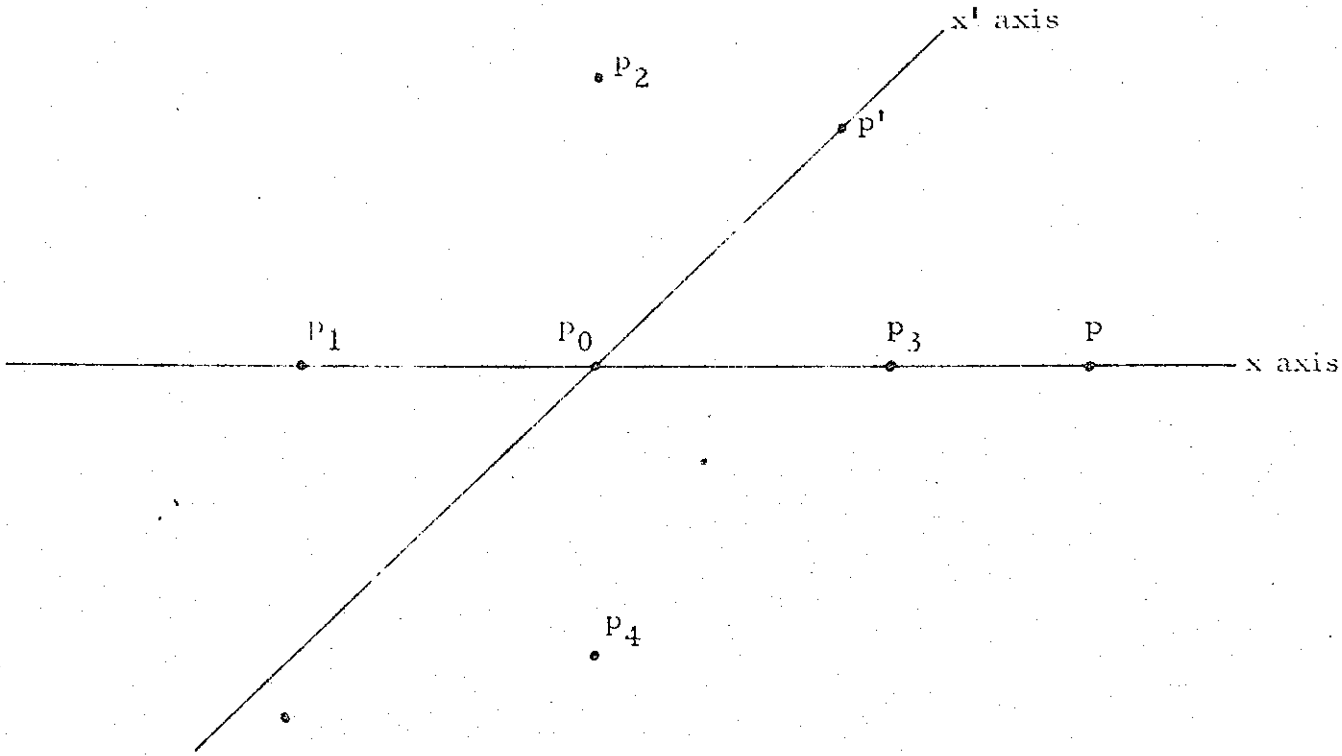

Figure 2. Calculation of Four-Point Source Radiation Pattern Along Two Axes (x and x')

1253

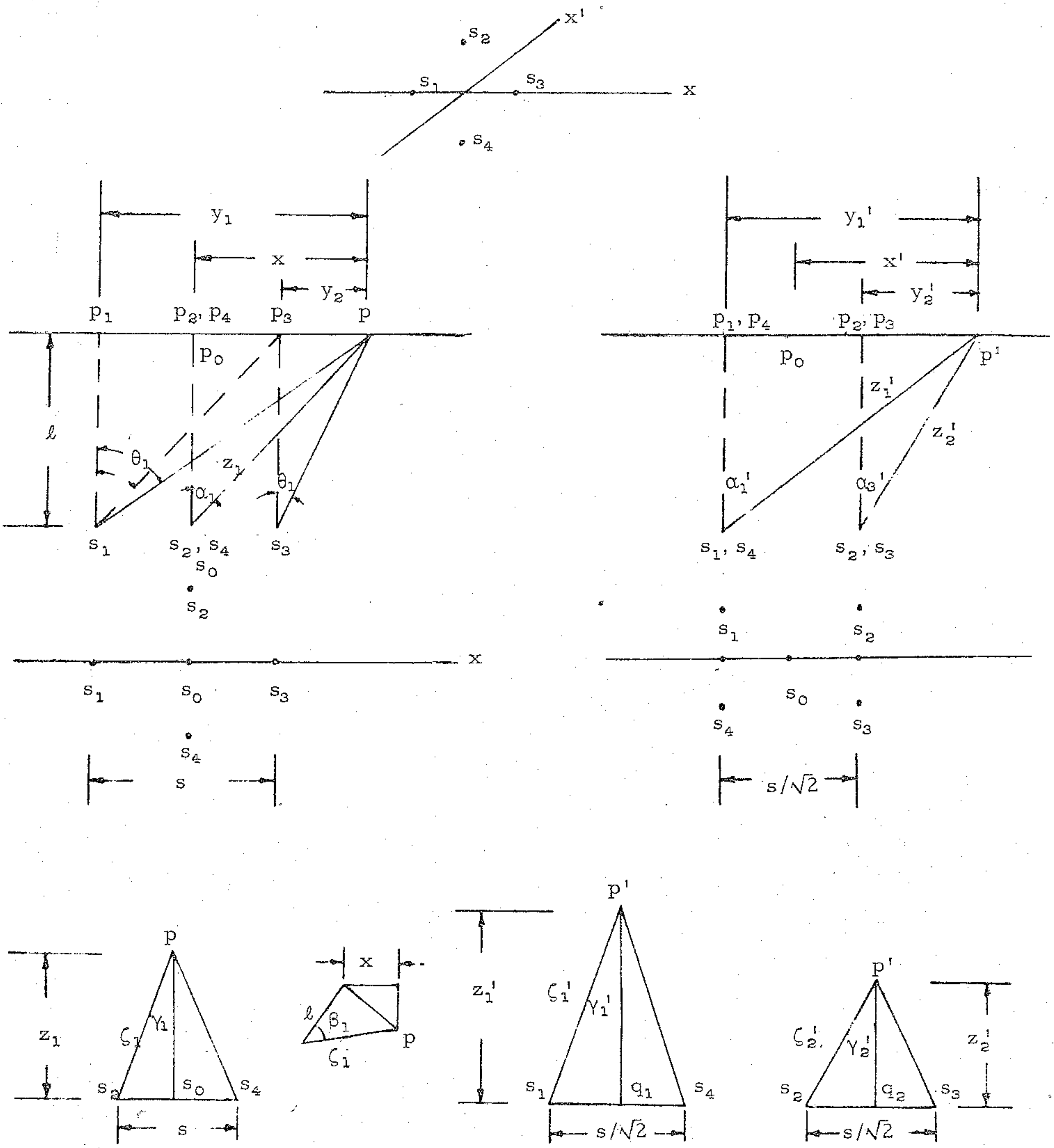

Figure 3. Diagrams for Calculation of Radiation Patterns of of Four-Point Sources

1250

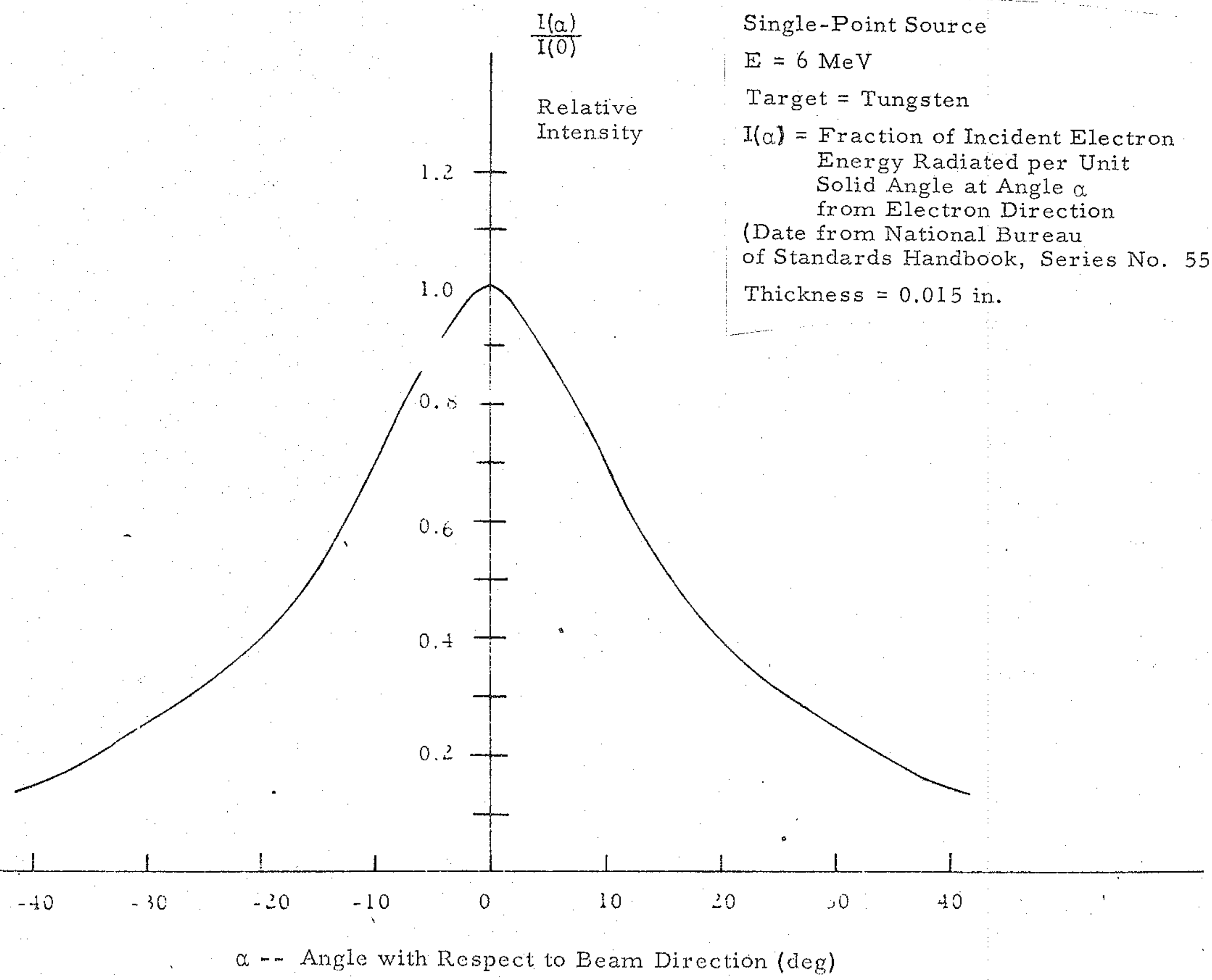

Figure 4. Plot of Bremsstrahlung Intensity, Single-Point Source