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SUBJECT: Derivation of Formulas for Determining Radiation Pattern from Four-Point Sources

The formulas for determining the radiation pattern from a four-source array are derived with reference to the diagrams of Figures 1, 2, and 3. The following symbols are used:

**Definition of Symbols**

- $s_o$: Center of source array
- $s_1, s_2$: Source points
- $s_3, s_4$: Diagonal distance across array
- $s$: Distance from source plane to plane at which radiation is calculated
- $p_1', p_2'$: Points formed by extension of electron beam direction through $s_1, s_2, s_3, s_4$ to target plane
- $p_3', p_4$: Center of points $p_1, p_2, p_3, p_4$
- $\theta$: Angle of line $s_1 p_3$ with respect to line $s_1 p_1$; $\theta = \tan^{-1} \frac{s}{s}$
- $x$: Axis through $p_1, p_3$
- $p$: Point on $x$-axis for which sum of radiation from $s_1, s_2, s_3, s_4$ is calculated
- $x$: Distance from center of target plane to $p$
- $\theta_1$: Angle between $s_1 p_1$ and $s_1 p$
- $\theta_2$: Angle between $s_3 p_3$ and $s_3 p$
\[y_1\] Distance from \(p_1\) to \(p\)

\[y_2\] Distance from \(p_3\) to \(p\)

\[z_1\] Length of line \(s_o p\)

\[\alpha_1\] Angle between line \(s_o p\) and line \(s_o p_o\)

\[\zeta_1\] Distance from \(s_2\) to \(p = s_4\) to \(p\)

\[\gamma_1\] Angle between \(s_2 p\) and \(s_o p\)

\[\beta_1\] Angle between electron beam direction at \(s_2\) and line from \(s_2\) to \(p\)

\[R_{x\beta_1}^{\ell}\] Relative radiation intensity at angle \(\beta_1\) at distance \(\ell\)

\[R_{x\beta_1}^{\zeta_1}\] Relative radiation intensity at angle \(\beta_1\) at distance \(\zeta_1\)

\(x'\) - axis Axis through \(p_o\) at 45 deg to \(x\) - axis

\(p'\) Point on \(x'\) - axis at which the radiation intensity is calculated

\(x'\) Distance from \(p_o\) to \(p'\)

\[y_1'\] Distance from intersection of line \(p_1 p_4\) and \(x'\) - axis to point \(p'\)

\[y_2'\] Distance from intersection of line \(p_2 p_3\) and \(x'\) - axis to point \(p'\)

\(q_1\) Point midway between \(s_1\) and \(s_4\)

\(q_2\) Point midway between \(s_2\) and \(s_3\)

\[z_1'\] Distance from \(q_1\) to \(p'\)

\[z_2'\] Distance from \(q_2\) to \(p'\)

\[\alpha_1'\] Angle between line through \(q_1\) parallel to electron direction and line from \(q_1\) to \(p'\)

\[\alpha_2'\] Angle between line through \(q_2\) parallel to electron direction and line from \(q_2\) to \(p'\)
To calculate radiation intensity along the x-axis the procedure used was to calculate $\alpha_1$ from Equation 1, $\gamma_1$ from Equation 2, and $\beta_1$ from Equation 3. $R_{\zeta_1 \beta_1}$ was taken from the curve in Figure 4 and the value of $R_{\zeta_1 \beta_1}$ was calculated from Equation 4. If the radiation intensities at $p$ from the four-source points $s_1, s_2, s_3, s_4$, are respectively, $R_1, R_2, R_3, R_4$, then

$$R_{\zeta_1 \beta_1} = R_2 = R_4$$

For intensities $R_1$ and $R_3$ from $s_1$ and $s_3$, the following formulas apply:

$$\theta_1 = \tan^{-1} \frac{\gamma_1}{\xi}$$

$$\theta_2 = \tan^{-1} \frac{\gamma_2}{\xi}$$

Values of relative intensity were taken from Figure 4 for $\theta_1$ and $\theta_2$, and multiplied by $\cos^2 \theta_1$ and $\cos^2 \theta_2$ to give the intensities $R_1$ and $R_3$. The total radiation intensity at $p$ is the sum of $R_1, R_2, R_3, R_4$. 
The intensity along the $x'$-axis is calculated as follows:

\[
\tan \alpha_1' = \frac{y_1'}{s'}
\]

\[
\tan \gamma_1' = (1/2 \sqrt{2} (s/l)) \cos \alpha_1'
\]

\[
\cos \beta_1' = \frac{2 \sqrt{2} \sin \gamma_1'}{s/l}
\]

\[
R_{\zeta_1 \beta_1'} = R_{\xi \beta_1'} \cos^2 \beta_1'
\]  

Equation 5 gives the radiation intensity from points $s_1$ and $s_4$ at $p'$

\[
R_1' = R_4' = R_{\zeta_1 \beta_1'}
\]

\[
\tan \alpha_2' = \frac{y_2'}{s'}
\]

\[
\tan \gamma_2' = (1/2 \sqrt{2} (s/l)) \cos \alpha_2'
\]

\[
\cos \beta_2' = \frac{2 \sqrt{2} \sin \gamma_2'}{s/l}
\]

\[
R_{\zeta_2 \beta_2'} = R_{\xi \beta_2'} \cos^2 \beta_2'
\]  

Equation 6 gives the radiation intensity from points $s_2$ and $s_3$ at $p'$

\[
R_2' = R_3' = R_{\zeta_2 \beta_2'}
\]

The total radiation intensity at $p'$ is $R_1' + R_2' + R_3' + R_4'$
Figure 1. Scheme for Calculation of Four-Point Source Radiation Pattern
Figure 2. Calculation of Four-Point Source Radiation Pattern Along Two Axes (x and x')
Figure 3. Diagrams for Calculation of Radiation Patterns of Four-Point Sources
Single-Point Source

\[ I(\alpha) / I(0) \]

Relative Intensity

\[ I(\alpha) = \text{Fraction of Incident Electron Energy Radiated per Unit Solid Angle at Angle } \alpha \text{ from Electron Direction} \]

(Date from National Bureau of Standards Handbook, Series No. 55)

Thickness = 0.015 in.

\[ \alpha \text{ -- Angle with Respect to Beam Direction (deg)} \]

Figure 4. Plot of Bremsstrahlung Intensity, Single-Point Source