Sensor and Simulation Notes

Note 437

20 June 1999

Unipolarized Currents for Antenna Polarization Control

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Abstract

For polarimetric radar it is important to be able to control, or at least accurately know, the polarization of the fields radiated to the target from the radar antennas. This paper considers the properties of antennas with currents all parallel to a fixed axis (hence, unipolarized). This results in a frequency-independent polarization at each point in the far field. Formulae are developed to relate the antenna far fields (and reception by reciprocity) to the usual h,v polarizations at the target. These formulas further simplify if the far-field patterns are those of electrically small antennas characterized by unipolarized electric dipole moments. In this latter case the antenna patterns are frequency independent with simple formulae for the angular dependence applying to both frequency and time domains. Implications for loading such electrically small antennas are discussed.

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1. Introduction

For certain types of radars, specifically polarimetric SAR (synthetic aperture radar), control of the polarization of the incident wave on the target and the polarization of the receive antennas is important. This relates to certain signatures in the target scattering which can be used for target identification [13-16].

Symmetry is an important concept in controlling antenna patterns, including polarization [6, 18]. This can be thought of as complementary to the symmetry in the target which strongly influences the scattering pattern, including polarization [8, 9, 11-15, 17]. The antenna symmetry can be used for polarization control, but with limitations depending on the relative orientations of the antenna and target (assumed in the antenna's far field).

In the present paper we consider another technique for antenna polarization control: the restriction of the antenna currents to flow only parallel/antiparallel to a given frequency/time-independent direction designated $\vec{1}_a$. This gives a particular real frequency/time-independent polarization at each point in the far field. However, this polarization in general differs from point to point. Taking two choices for $\vec{1}_a$ (conveniently orthogonal) for two such antennas gives two independent polarizations at each point in the far field which can be used to mathematically construct the usual h,v radar polarizations scattering dyadic to the transmit/receive properties of the two antennas. Furthermore, by use of two orthogonal symmetry planes for the two antennas (with currents on thin wires) the two antennas can be made mutually noninteracting so as not to disturb each other's pattern/polarization. Making the antennas electrically small further simplifies the analysis by making the pattern simply that of an electric dipole.

2. Antenna With Unipolarized Currents

Consider, as in Fig. 2.1, a coordinate system for an antenna with a preferred axis $\vec{1}_a$ which is taken as the z_a axis (subscript a for these coordinates). These are Cartesian (x_a, y_a, z_a), cylindrical (Ψ_a, ϕ_a, z_a), and spherical (r_a, θ_a, ϕ_a) coordinates related as

$$\begin{aligned} x_a &= \Psi_a \cos(\phi_a) \quad , \quad y_a = \Psi_a \sin(\phi_a) \\ z_a &= r_a \cos(\theta_a) \quad , \quad \Psi_a = r_a \sin(\theta_a) \end{aligned} \tag{2.1}$$

The associated unit (direction) vectors are

$$\begin{aligned} \vec{1}_{\Psi_a} &= \vec{1}_{x_a} \cos(\phi_a) + \vec{1}_{y_a} \sin(\phi_a) \quad , \quad \vec{1}_{\phi_a} = -\vec{1}_{x_a} \sin(\phi_a) + \vec{1}_{y_a} \cos(\phi_a) \\ \vec{1}_{r_a} &= \vec{1}_{\Psi_a} \sin(\theta_a) + \vec{1}_{z_a} \cos(\theta_a) = \left[\vec{1}_{x_a} \cos(\phi_a) + \vec{1}_{y_a} \sin(\phi_a)\right] \sin(\theta_a) + \vec{1}_{z_a} \cos(\theta_a) \\ \vec{1}_{\theta_a} &= \vec{1}_{\Psi_a} \cos(\theta_a) - \vec{1}_{z_a} \sin(\theta_a) = \left[\vec{1}_{x_a} \cos(\phi_a) + \vec{1}_{y_a} \sin(\phi_a)\right] \cos(\theta_a) - \vec{1}_{z_a} \sin(\theta_a) \\ \vec{1}_{z_a} &= \vec{1}_a \end{aligned} \tag{2.2}$$

The far field radiated by a current distribution limited to a region of space with finite linear dimensions (say within a sphere of radius b with $b \ll r_a$ and centered on $\vec{r}_a = 0$) is [10]

$$\vec{E}_f(\vec{r}_a, t) = -\frac{\mu_0}{4\pi r_a} \overset{\leftrightarrow}{1}_{r_a} \cdot \frac{\partial}{\partial t} \int_{V'_a} \vec{J}\left(\vec{r}'_a,\, t - \frac{r_a}{c} + \frac{\vec{1}_{r_a} \cdot \vec{r}'_a}{c}\right) dV'_a$$

$$\tilde{\vec{E}}_f(\vec{r}_a, s) = -\frac{s\mu_0 e^{-\gamma r_a}}{4\pi r_a} \overset{\leftrightarrow}{1}_{r_a} \cdot \int_{V'_a} e^{\gamma \vec{1}_{r_a} \cdot \vec{r}'_a}\, \tilde{\vec{J}}(\vec{r}'_a, s)\, dV'_a$$

$c = [\mu_0 \varepsilon_0]^{-\frac{1}{2}}$ = speed of light

$\sim$ ≡ two-sided Laplace transform over time t

$s \equiv \Omega + j\omega$ ≡ Laplace-transform variable or complex frequency

$\gamma \equiv \frac{s}{c}$ ≡ propagation constant

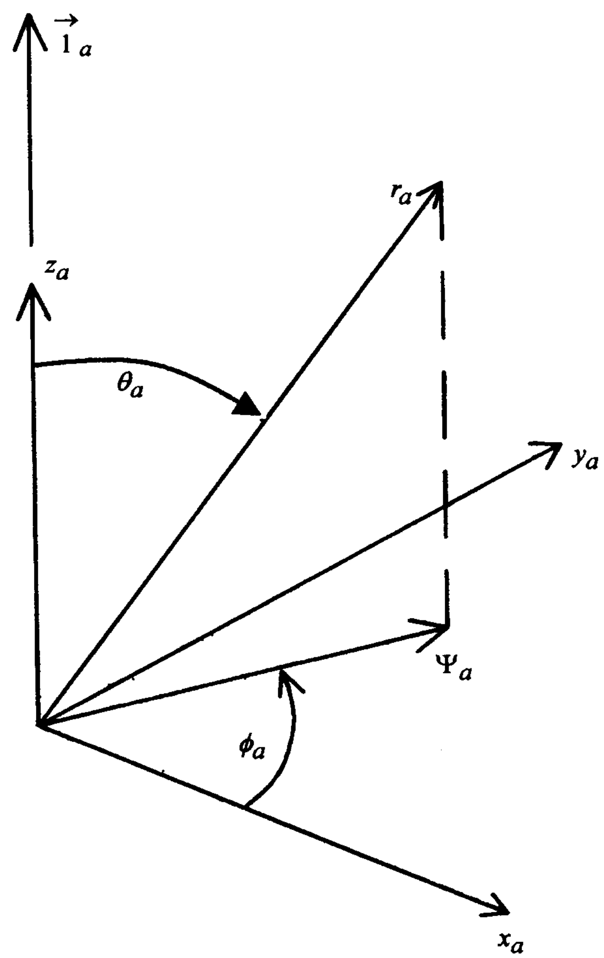


Fig. 2.1 Antenna Coordinates.

$$\overleftrightarrow{1}_{r_a} \equiv \overleftrightarrow{1} - \vec{1}_{r_a}\vec{1}_{r_a} \tag{2.3}$$

$$\overleftrightarrow{1} = \vec{1}_{x_a}\vec{1}_{x_a} + \vec{1}_{y_a}\vec{1}_{y_a} + \vec{1}_{z_a}\vec{1}_{z_a} = \vec{1}_{r_a}\vec{1}_{r_a} + \vec{1}_{\theta_a}\vec{1}_{\theta_a} + \vec{1}_{\phi_a}\vec{1}_{\phi_a}$$

$\equiv$ identity

Here the integration is over the primed coordinates over the domain V'_a of the antenna. Note the weighting function $e^{\gamma \vec{1}_{r_a} \cdot \vec{r}'_a}$ giving phase from the various positions on the antenna (a complicating factor).

The basic idea here is to constrain all the antenna currents to be parallel (including antiparallel) to $\vec{1}_a$, specifically,

$$\vec{J}(\vec{r}'_a, t) = J(\vec{r}'_a, t)\,\vec{1}_a \tag{2.4}$$

Then noting that

$$\overleftrightarrow{1}_{r_a} \cdot \vec{1}_a = \overleftrightarrow{1}_{r_a} \cdot \vec{1}_{z_a} = \left[\vec{1}_{\theta_a}\vec{1}_{\theta_a} + \vec{1}_{\theta_a}\vec{1}_{\theta_a}\right] \cdot \vec{1}_{z_a} = -\sin(\theta_a)\,\vec{1}_{\theta_a} \tag{2.5}$$

the far electric field takes the form

$$\begin{aligned} \vec{E}_f(\vec{r}_a, t) &= \frac{\mu_0 \sin(\theta_a)}{4\pi r_a}\,\vec{1}_{\theta_a} \frac{\partial}{\partial t} \int_{V'_a} J\left(\vec{r}'_a,\, t - \frac{r_a}{c} + \frac{\vec{1}_{r_a} \cdot \vec{r}'_a}{c}\right) dV'_a \\ \tilde{\vec{E}}_f(\vec{r}_a, s) &= \frac{s\mu_0 e^{-\gamma r_a} \sin(\theta_a)}{4\pi r_a}\,\vec{1}_{\theta_a} \int_{V'_a} e^{\gamma \vec{1}_{r_a} \cdot \vec{r}'_a}\, \tilde{J}(\vec{r}_a, s)\, dV'_a \end{aligned} \tag{2.6}$$

and, *voila*, the far-field polarization is given by $\vec{1}_{\theta_a}$ which is independent of time and frequency. It is a real direction in space, perpendicular to $\vec{1}_{r_a}$ (direction from the antenna), and in a plane containing the z_a axis and the observer. However, as θ_a and ϕ_a are varied there is still the complicated variation associated with $e^{\gamma \vec{1}_{r_a} \cdot \vec{r}'_a}$.

As illustrated in Fig. 2.2A one way to realize this condition is to constrain all (net) currents to thin wires, all of which are parallel to the z_a axis, and hence to $\vec{1}_a$. Note also that we have a reference plane designated S_a and

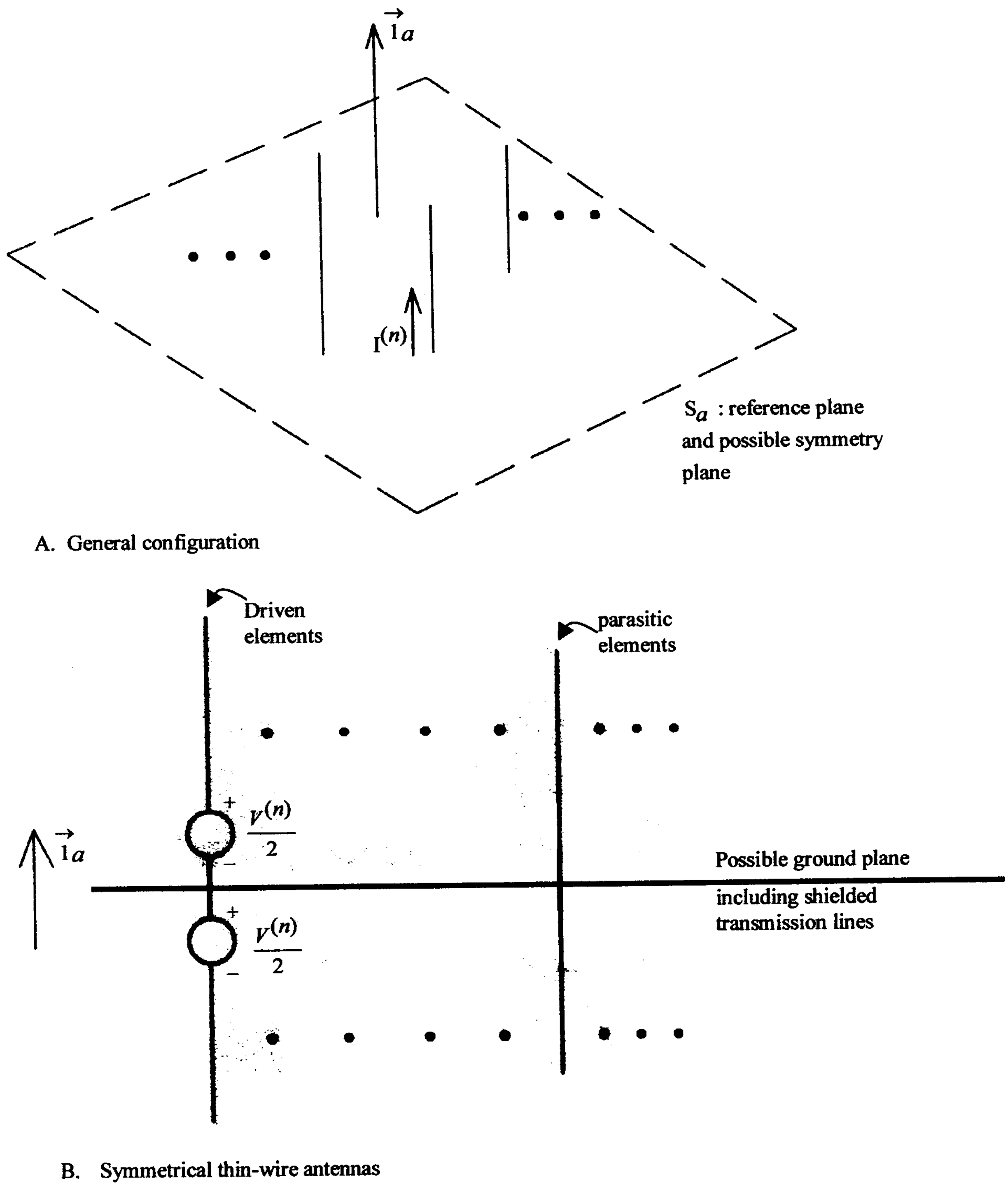


Fig. 2.2 Antenna With Unipolarized Currents.

which is one of the planes perpendicular to $\vec{1}_a$ (say $z_a = 0$). Now there can be many thin wires (say N), all perpendicular to S_a with various currents $I^{(n)}$ which can vary along the length of the wires. With various lengths and various coordinates on S_a (i.e., (x_a, y_a)) this configuration is quite general, constrained only by (2.4) and the limited size of the source (antenna) region.

Going a step further, now let S_a be a symmetry plane as illustrated in Fig. 2.2B. In particular let the currents be antisymmetric [7, 18], i.e.,

$$I^{(n)}(z_a) = I^{(n)}(-z_a) \tag{2.7}$$

where S_a is taken as the $z_a = 0$ plane, and positive current is taken in the $+z_a$ (or $+\vec{1}_a$) direction. The wires, of course, have equal extent in the $+z_a$ and $-z_a$ directions but the various wires need not have the same lengths. This is a kind of array which can be driven (with antisymmetric sources, such as illustrated), or parasitic (undriven). These wires can be impedance loaded as well as long as the symmetry with respect to S_a is maintained. The pattern of this antenna is then also antisymmetric with respect to S_a. Note that the symmetry plane can now be in part a conducting sheet and no net surface current density (sum from $z_a = 0_+$ (above) and $z_a = 0_-$ (below)) will flow on it. Furthermore, letting the conducting sheet have some small thickness this can be used to route (hide) various transmission lines connected to the antenna elements and source(s) at the antena terminal(s).

For the special (but important) case that the antenna is electrically small the antenna is characterized in transmission by its electric dipole moment

$$\begin{aligned} \tilde{\vec{p}}(s) &= \frac{1}{s} \int_{V_a'} \tilde{\vec{J}}(\vec{r}_a', s)\, dV_a' \\ \vec{p}(t) &= \frac{1}{s} \int_{-\infty}^{t} \vec{J}(\vec{r}_a', t')\, dV_a'\, dt' \end{aligned} \tag{2.8}$$

which in (2.3) gives

$$\begin{aligned} \vec{E}_f(\vec{r}_a, t) &= -\frac{\mu_0}{4\pi r_a} \overleftrightarrow{1}_{r_a} \cdot \frac{\partial^2}{\partial t^2} \vec{p}\left(t - \frac{r_a}{c}\right) \\ \tilde{\vec{E}}_f(\vec{r}_a, s) &= -\frac{s^2 \mu_0 e^{-\gamma r_a}}{4\pi r_a} \overleftrightarrow{1}_{r_a} \cdot \tilde{\vec{p}}(s) \end{aligned} \tag{2.9}$$

This, of course, assumes that $\vec{p} \neq \vec{0}$ so that we need not go to other moments of the current distribution. Specializing to the case of unipolarized currents gives

$$
\begin{aligned}
\vec{p}(t) &= p(t)\,\vec{1}_a = p(t)\,\vec{1}_{z_a} \\
\vec{E}_f(\vec{r}_a, t) &= \frac{\mu_0 \sin(\theta_a)}{4\pi r_a}\,\vec{1}_{\theta_a}\,\frac{\partial^2}{\partial t^2}\,p\!\left(t - \frac{r_a}{c}\right) \\
\tilde{\vec{E}}_f(\vec{r}_a, s) &= \frac{s^2 \mu_0 e^{-\gamma r_a} \sin(\theta_a)}{4\pi r_a}\,\vec{1}_{\theta_a}\,\tilde{p}(s)
\end{aligned}
\tag{2.10}
$$

This is a very simple pattern function $\sin(\theta_a)\,\vec{1}_{\theta_a}$ in addition to the simple polarization. (Note that a unipolarized electric dipole moment does not in general require unipolarized currents.)

Assuming that the antenna has a single port for transmission/reception we can characterize its performance in various ways, including voltage, current, and wave variables, including reciprocity between transmission and reception [3, 5]. The various forms of these transmission and reception parameters are relatable to each other. For convenience, let us use the wave variables for which we have in transmission

$$
\begin{aligned}
\tilde{\vec{E}}_f(\vec{r}_a, s) &= \frac{e^{-\gamma r_a}}{r_a}\,\tilde{\vec{F}}_t(\vec{1}_{r_a}, s)\,\tilde{V}_t(s) \\
\tilde{V}_t(s) &= \frac{\tilde{Z}_{in}(s)}{\tilde{Z}_L(s) + \tilde{Z}_{in}(s)}\,\tilde{V}_s(s) \equiv \text{transmitted voltage} \\
\tilde{V}_s(s) &\equiv \text{source voltage} \\
\tilde{Z}_L(s) &\equiv \text{source impedance in transmission} \\
&= \text{load voltage (termination) in reception} \\
\tilde{Z}_{in}(s) &= \frac{\tilde{V}_t(s)}{\tilde{I}_t(s)} \equiv \text{antenna input impedance}
\end{aligned}
\tag{2.11}
$$

In reception we have

$$\tilde{\vec{E}}^{(inc)}(\vec{r}_a, s) = \tilde{\vec{E}}_0(s)\,e^{-\gamma \vec{1}_i \cdot \vec{r}_a} \equiv \text{incident plane wave}$$

$$\vec{1}_i \equiv \text{direction of incidence (scattering from target)}$$

$$= -\vec{1}_{r_a}$$

$$\tilde{V}_r(s) = \vec{\tilde{h}}_t(\vec{1}_i,s) \cdot \vec{\tilde{E}}^{(inc)}(0,s) = \vec{\tilde{h}}_t(\vec{1}_i,s) \cdot \vec{\tilde{E}}_0(s) = \text{voltage across } \tilde{Z}_L(s) \tag{2.12}$$

$\vec{\tilde{h}}_t(\vec{1}_i,s) =$ effective height

$\vec{\tilde{h}}_t(\vec{1}_i,s) \cdot \vec{1}_i = 0$

$\tilde{Z}_L(s) = \dfrac{\tilde{V}_r(s)}{\tilde{I}_r(s)}$ noting opposite direction conventions for I_t and I_r

Applying reciprocity we have

$$\vec{\tilde{F}}_t(\vec{1}_{r_a},s) = \frac{s\mu_0}{4\pi}\left[\tilde{Y}_{in}(s) + \tilde{Y}_L(s)\right] \vec{\tilde{h}}_t(-\vec{1}_{r_a},s) \tag{2.13}$$

which for the simple but useful case of

$$\tilde{Z}_{in}(s) = \tilde{Z}_L(s) = R \tag{2.14}$$

gives the result

$$\vec{\tilde{F}}_t(\vec{1}_{r_a},s) = \frac{s\mu_0}{2\pi R} \vec{\tilde{h}}_t(-\vec{1}_{r_a},s) \tag{2.15}$$

The signal into the antenna results in the currents in (2.3) from which we can identify

$$\vec{\tilde{F}}_t(\vec{1}_{r_a},s) = -\frac{s\mu_0}{4\pi\tilde{V}_t(s)} \vec{1}_{r_a} \cdot \int_{V_a'} e^{\gamma \vec{1}_{r_a} \cdot \vec{r}_a'} \vec{\tilde{J}}(\vec{r}_a', s)\, dV_a' \tag{2.16}$$

Specializing to unipolarized currents as in (2.4) gives

$$\vec{\tilde{F}}_t(\vec{1}_{r_a},s) = \frac{s\mu_0 \sin(\theta_a)}{4\pi\tilde{V}_t(s)} \vec{1}_{\theta_a} \int_{V_a'} e^{\gamma \vec{1}_{r_a} \cdot \vec{r}_a'} \tilde{J}(\vec{r}_a', s)\, dV_a' \tag{2.17}$$

showing the $\vec{1}_{\theta_a}$ constant polarization. Writing this in the form

$$\vec{\tilde{F}}_t(\vec{1}_{r_a},s) = \vec{\tilde{F}}_t(\theta_a,\phi_a;s) = \tilde{F}_t(\theta_a,\phi_a;s)\,\vec{1}_{\theta_a} \tag{2.18}$$

the assumption of antisymetric currents as in (2.7) gives an antisymmetric radiation pattern as

$$\tilde{F}_t(\theta_a,\phi_a;s) = \tilde{F}_t(\pi-\theta_a,\phi_a;s) \tag{2.19}$$

Note similarly in reception that (2.18) implies

$$\begin{aligned} \vec{\tilde{h}}_t(\vec{1}_i,s) &= \vec{\tilde{h}}_t(-\vec{1}_{r_a},s) = \vec{\tilde{h}}_t(\pi-\theta_a,\phi_a-\pi;s) = \tilde{h}_t(\pi-\theta_a,\phi_a-\pi;s)\,\vec{1}_{\theta_a} \\ \tilde{F}_t(\vec{1}_{r_a},s) &= \frac{s\mu_0}{2\pi R}\,\tilde{h}_t(-\vec{1}_{r_a},s) \end{aligned} \tag{2.20}$$

and the antisymmetric currents further imply

$$\tilde{h}_t(\pi-\theta_a,\phi_a-\pi;s) = \tilde{h}_t(\theta_a,\phi_a-\pi;s) \tag{2.21}$$

For an electrically small antenna in transmission (2.9) gives

$$\vec{\tilde{F}}_t(\vec{1}_{r_a},s) = -\frac{s^2\mu_0}{4\pi\tilde{V}_t(s)}\,\overleftrightarrow{1}_{r_a}\cdot\vec{\tilde{p}}(s) \tag{2.22}$$

For currents polarized in the $\vec{1}_a$ direction (2.10) applies and gives

$$\vec{\tilde{F}}_t(\vec{1}_{r_a},s) = -\frac{s^2\mu_0}{4\pi\tilde{V}_t(s)}\sin(\theta_a)\,\vec{1}_{\theta_a}\,\tilde{p}(s) = \tilde{F}_t(\vec{1}_{r_a},s)\,\vec{1}_{\theta_a} \tag{2.23}$$

with the same result as for antisymmetric currents. For convenience we can define

$$\tilde{T}_p(s) = \frac{\tilde{p}(s)}{\tilde{I}_t(s)} \tag{2.24}$$

as a characteristic of the antenna (an electric-dipole transfer function). Then we have

$$\vec{\tilde{F}}_t(\vec{1}_{r_a},s) = -\frac{s^2\mu_0}{4\pi}\tilde{T}_p(s)\sin(\theta_a)\,\vec{1}_{\theta_a} \tag{2.25}$$

so that the angular dependence and frequency dependence are completely separated as distinct factors.

Note that in general $\vec{F}_t$ (and hence $\vec{h}_t$) can be different for different antennas. In such a case a superscript a (later taken as $1, 2, \cdots$) can be used. For convenience these functions are assumed to be the same for the various antennas, as expressed in appropriate antenna-based coordinates.

3. Two Colocated Orthogonal Antennas, Each With Unipolarized Currents

For measuring the target scattering dyadic two polarizations are required. This in turn implies at least two antennas for transmission and two for reception (which may be the transmitter antennas as well). Retaining the unipolarized currents in each antenna it is important that the two antennas do not significantly interact. Note that the two antennas need to have different polarizations for the scattering-dyadic measurement.

One approach to this design is as illustrated in Fig. 3.1. Here we take two antennas such as discussed in Section 2. Denoting these as 1 and 2, they are assumed to have symmetry planes S_1 and S_2 (mutually perpendicular). The polarization direction $\vec{1}_a$ for the currents now takes on two values $\vec{1}_1$ and $\vec{1}_2$ with
\

$$\vec{1}_1 \cdot \vec{1}_2 = 0 \quad , \quad S_1 \perp \vec{1}_1 \quad , \quad S_2 \perp \vec{1}_2 \tag{3.1}$$

Furthermore, let the 1 antenna elements lie on S_2 so that the 2 currents do not excite currents on the 1 elements (by symmetry), and conversely. This does not imply that the two antennas are identical except for a rotation by $\pi / 2$ about the axis designated by $\vec{1}_f$ as the line of intersection of S_1 and S_2, with

$$\vec{1}_f = -\vec{1}_1 \times \vec{1}_2 \tag{3.2}$$

This axis can be used to define the location of a (thin) conducting tube, inside of which transmission lines can be placed to feed both sets of antenna elements from two ports also located on or near this axis.

Various designs are still possible for the two antennas, including a single element pair (thin-wire antenna) or an array of elements such as a log-periodic antenna. As mentioned previously, symmetrically positioned pairs of identical impedances can also be included in the elements.

Of course, a convenient choice for the two antennas is to have them identical so as to have the same radiation and reception characteristics except for a coordinate rotation ($\vec{r}_a$ now becoming two sets of coordinates, $\vec{r}_1$and $\vec{r}_2$). If desired, one antenna can be shifted (translated) along the axis with respect to the other, but this introduces a phase shift (dependent on angles to the target) between the two antennas.

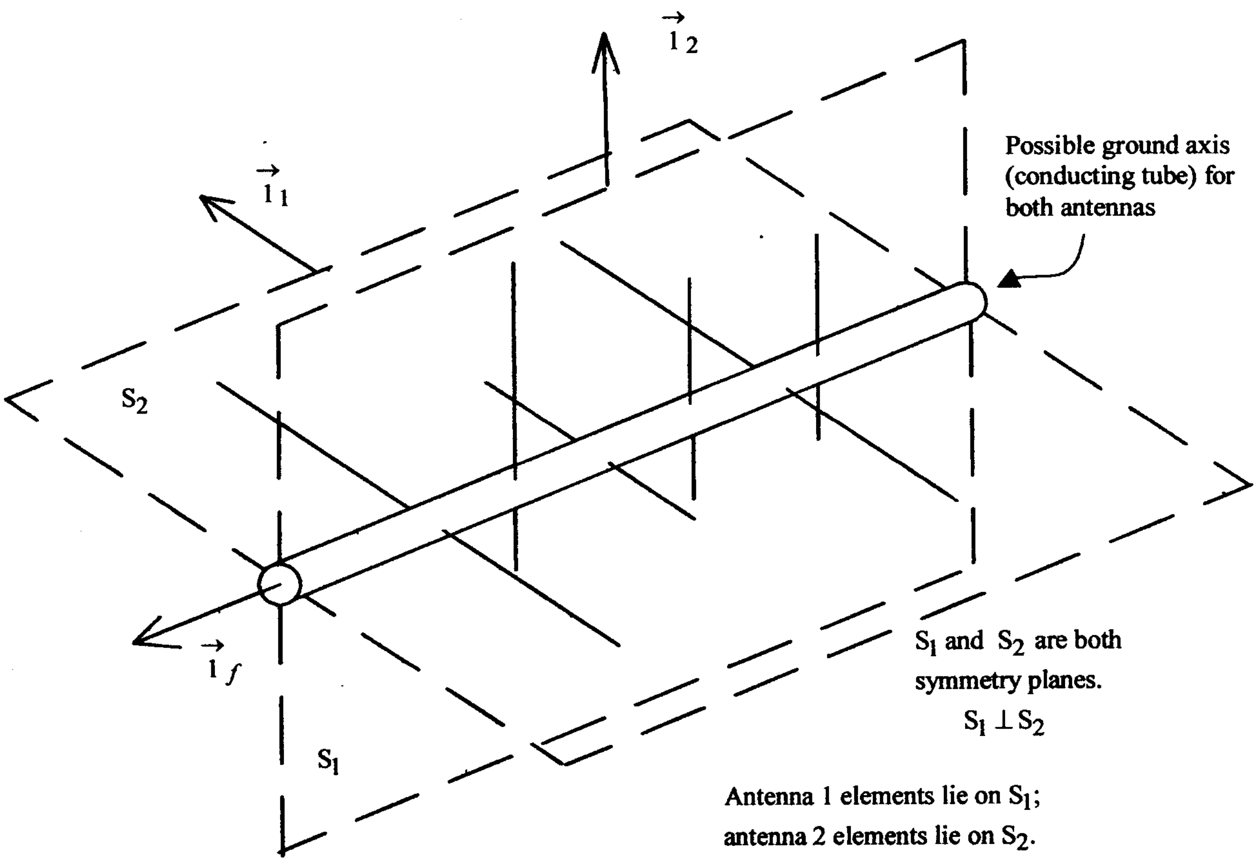


Fig. 3.1 Two Colocated Orthogonal Antennas, Each With Unipolarized Currents.

4. Antenna Positions and Orientations With Respect to Target Site

The antennas are now positioned and oriented as illustrated in Fig. 4.1. First, establish site coordinates as in Fig. 4.1A. These are the standard Cartesian, cylindrical, and spherical coordinates as in (2.1) and (2.2), except with no subscript. The coordinate origin, $\vec{r} = \vec{0}$, is taken at a height h above the ground surface S_e which is assumed flat. The z axis is taken as perpendicular to S_e. The x axis is assumed extended over the target site, an orientation appropriate for a side-looking SAR with antenna motion in the y direction.

Note that horizontal polarization on the target site is given by

$$\vec{1}_h = -\vec{1}_\phi \tag{4.1}$$

and vertical polarization is given by

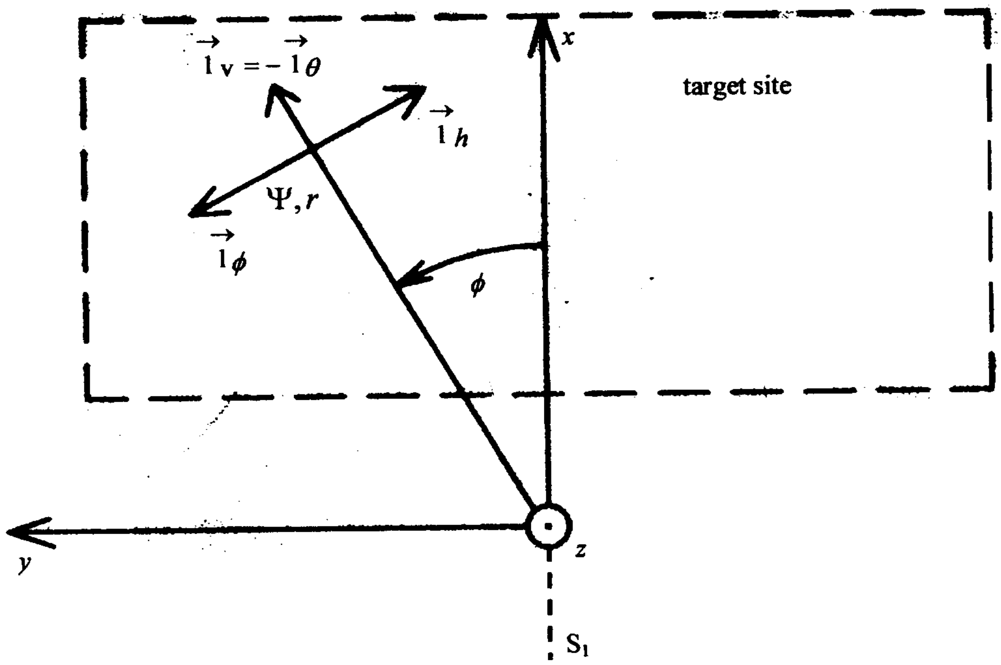
$$\vec{1}_v = -\vec{1}_\theta \tag{4.2}$$

These vary over the target site, but they are appropriate to characterizing the target scattering, including the lack of a crosspol component for targets with O_2 symmetry, including the ground presence (the vampire signature) [13-16]. In general, these polarizations are not the same as those radiated by the antennas to the target location. For later use we have the transverse dyadic with respect to (direction to the target) as

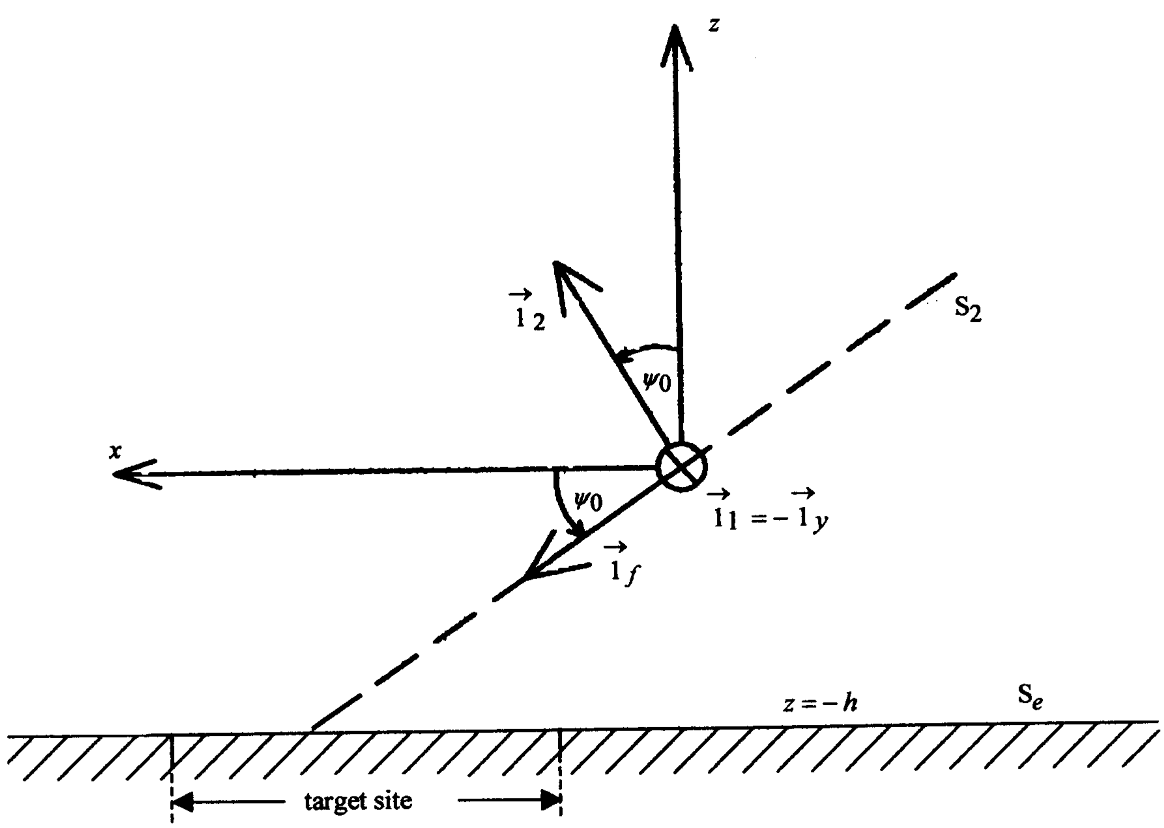
$$\overleftrightarrow{1}_r \equiv \overleftrightarrow{1} - \vec{1}_r \vec{1}_r = \vec{1}_\theta \vec{1}_\theta + \vec{1}_\phi \vec{1}_\phi = \vec{1}_h \vec{1}_h + \vec{1}_v \vec{1}_v \tag{4.3}$$

Now take antenna 1 with $\vec{1}_1$ in the y direction so that S_1, the symmetry plane perpendicular to the antenna elements, is the $y = 0$ plane and is perpendicular to S_e. This symmetry then includes the ground as well. Antenna 1 has horizontally oriented elements, but this is not the same as horizontal polarization $\vec{1}_h$ on the target site, except on S_1. As indicated in Fig. 4.1B the antenna may be canted (rotated) downward toward the target site by an angle ψ_0 with respect to the x axis. This gives the orientation of the other symmetry plane S_2 of the antenna.

Antenna 2 now has $\vec{1}_2$ in the $y=0$ plane and oriented at an angle of ψ_0 from the vertical axis. Regarding $\vec{1}_f$ as the nominal forward direction from both antennas to the target site, this is also in the y plane and depressed by the angle ψ_0 from the x axis. This antenna has the same symmetry planes S_2 and S_1 as antenna 1. While we might think of antenna 2 as approximately vertical, this does not in general give vertical polarization $\vec{1}_v$ on the target site, except on S_1.



A. Top view: site coordinates



B. Side view: antenna orientations

Fig. 4.1 Site Coordinates and Antenna Orientations.

For the special case that $\psi_0 = 0$ we have the interesting result that antenna 2 produces pure vertical polarization since $\vec{1}_{\theta_2} = \vec{1}_\theta$ all over the target site. However, antenna 2 still does not give pure horizontal polarization, except in the limit of small h so that the incident fields are nearly in the $z = 0$ plane and $\vec{1}_{\theta_1} \simeq \vec{1}_\phi$.

Antennas 1 and 2 can be designed for both transmit and receive. If one desires greater isolation between transmission and reception, one can supplement this antenna pair by a second pair: antennas 3 and 4 as indicated in Fig. 4.2. In this case, if we locate the centers of both pairs on the $y = 0$ plane then we can have $S_1 = S_3$ as a common symmetry plane for both pairs. S_4 is then different from S_2, but can be made parallel to it if desired. Furthermore, neither S_2 nor S_4 is a symmetry plane for the entire array, but a plane centered between S_2 and S_4 can be.

In order to maintain the lack of coupling between the various antennas it is now necessary to make the separation $D >> b$. Not only does this reduce the coupling from one antenna pair (say 1 and 2 in transmission) to the second pair (say 3 and 4 in reception). It also reduces the scattering from one pair off the second pair back to the first. This reduces the effect of symmetry breaking (S_2 distinct from S_4) in allowing say antenna 2 to couple to the conductors (thin wires) in antennas 1 and 3, and thereby distort the polarization purity implied by (2.4). Note that the common $S_1 = S_3$ symmetry plane implies that antennas 1 and 3 can mutually couple, as can 2 and 4. However, this symmetry also means that in terms of signals at the antenna ports neither can antenna 1 mutually couple with antenna 4, nor 2 with 3.

One may also wish to constrain $D << r$ so that the various angles of both antenna pairs with respect to a target are nearly the same, simplifying the analysis, and thereby the data reduction.

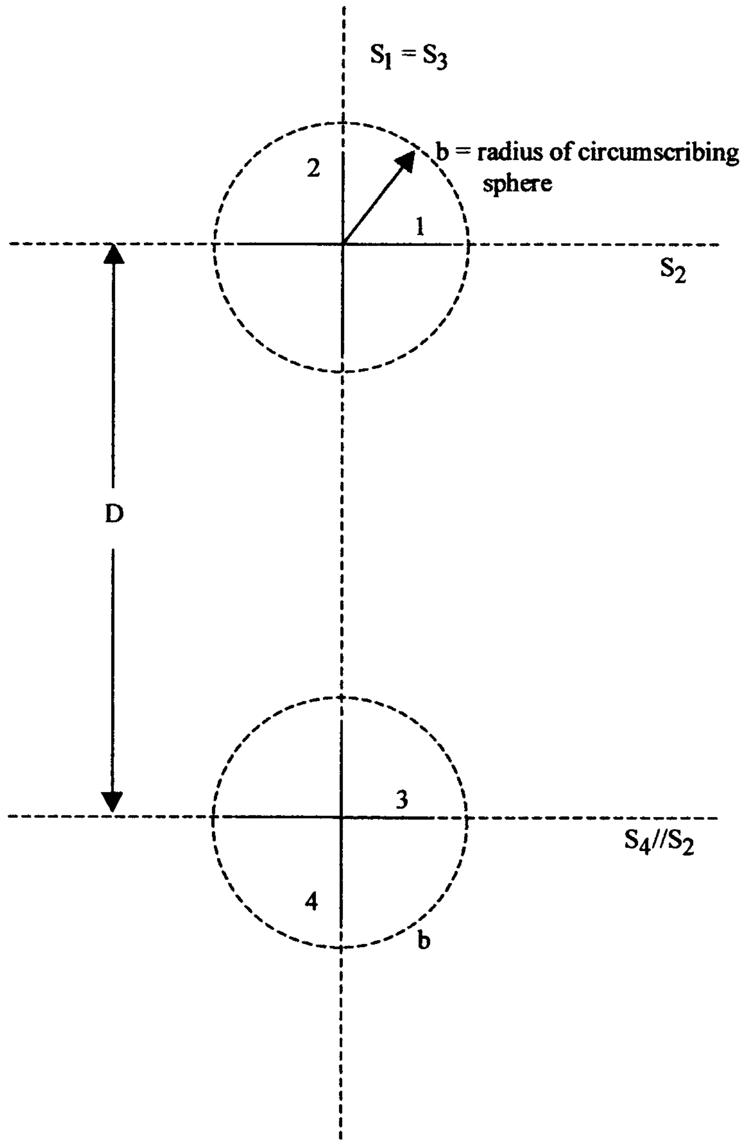


Fig. 4.2 Separate Transmit and Receive Antenna Pairs.

5. Fields of Antenna 1 in Site Coordinates

Having the fields of antenna 1 described in $\vec{r}_1$ coordinates in Section 2, these coordinates can now be related to the site coordinates discussed in Section 4. In Cartesian form, this relation is indicated in Fig. 5.1 and given by

$$
\begin{aligned}
x_1 &= x\cos(\psi_0) - z\sin(\psi_0) \\
y_1 &= x\sin(\psi_0) + z\cos(\psi_0) \\
z_1 &= -y \\
\vec{1}_{x_1} &= \vec{1}_x\cos(\psi_0) - \vec{1}_z\sin(\psi_0) \\
\vec{1}_{y_1} &= \vec{1}_x\sin(\psi_0) + \vec{1}_z\cos(\psi_0) \\
\vec{1}_{z_1} &= -\vec{1}_y
\end{aligned}
\tag{5.1}
$$

The electric field incident on the target is polarized in the $\vec{1}_{\theta_1}$ direction which we can think of as quasi-horizontal (with a minus sign). With horizontal polarization as in (4.1) then we can form

$$
\begin{aligned}
f_{1,h} &= \vec{1}_h \cdot \left[-\vec{1}_{\theta_1}\right] = \vec{1}_\phi \cdot \vec{1}_{\theta_1} \\
&= \left[-\vec{1}_x\sin(\phi) + \vec{1}\cos(\phi)\right] \cdot \left[\left[\vec{1}_{x_1}\cos(\phi_1) + \vec{1}_{y_1}\sin(\phi_1)\right]\cos(\theta_1) - \vec{1}_{z_1}\sin(\theta_1)\right] \\
&= -\cos(\psi_0)\cos(\phi_1)\cos(\theta_1)\sin(\phi) - \sin(\psi_0)\sin(\phi_1)\cos(\theta_1)\sin(\phi) + \sin(\theta_1)\cos(\phi) \\
&= -\cos(\phi_1 - \psi_0)\cos(\theta_1)\sin(\phi) + \sin(\theta_1)\cos(\phi)
\end{aligned}
\tag{5.2}
$$

showing the rotation of ϕ_1 by $-\psi_0$. This gives the portion of the field with horizontal polarization.

Simiarly with vertical polarization as in (4.2) we can form

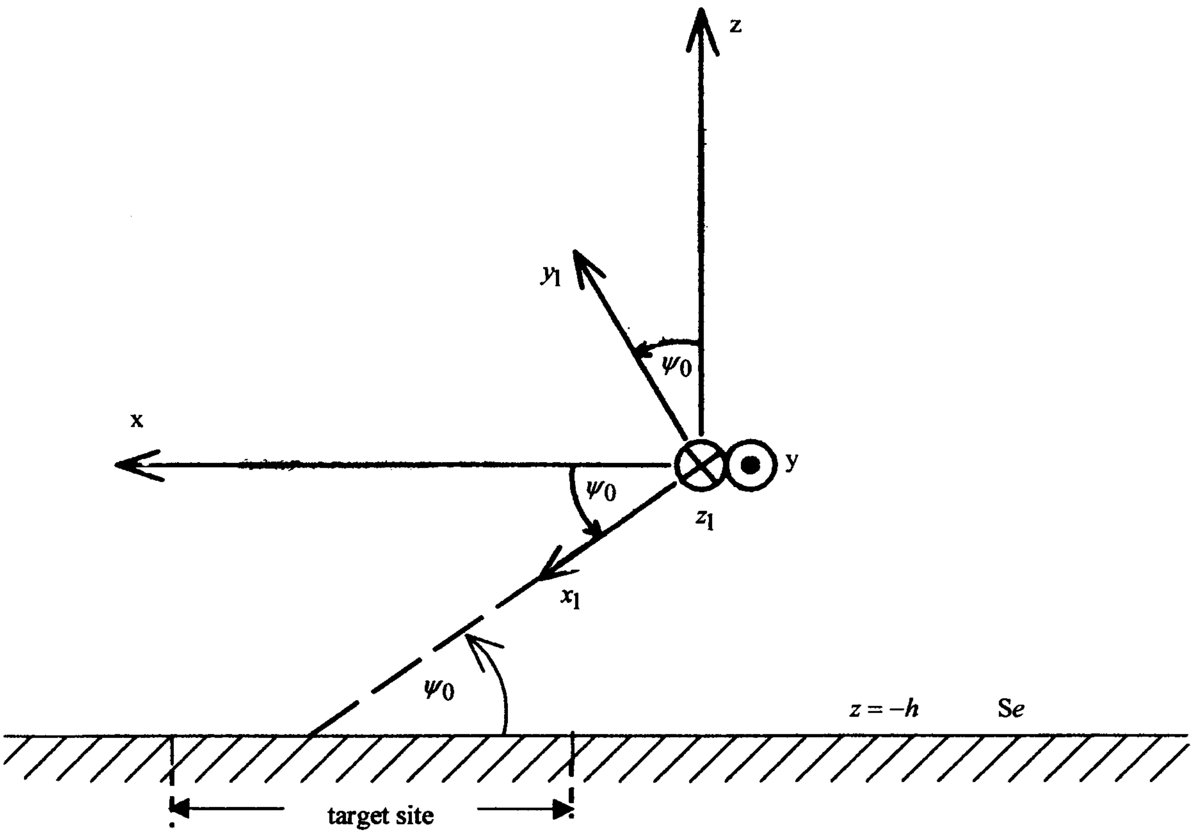


Fig. 5.1 Coordinates for Antenna 1.

$$
\begin{aligned}
f_{1,\mathrm{v}} &= \vec{1}_{\mathrm{v}} \cdot \left[-\vec{1}_{\theta_1} \right] = \vec{1}_{\theta} \cdot \vec{1}_{\theta_1} \\
&= \left[\left[\vec{1}_x \cos(\phi) + \vec{1}_y \sin(\phi) \right] \cos(\theta) - \vec{1}_z \sin(\theta) \right] \\
&\quad \cdot \left[\left[\vec{1}_{x_1} \cos(\phi_1) + \vec{1}_{y_1} \sin(\phi_1) \right] \cos(\theta_1) - \vec{1}_{z_1} \sin(\theta_1) \right] \\
&= \cos(\psi_0)\cos(\phi_1)\cos(\theta_1)\cos(\phi)\cos(\theta) \\
&\quad + \sin(\psi_0)\sin(\phi_1)\cos(\theta_1)\cos(\phi)\cos(\theta) + \sin(\theta_1)\sin(\phi)\cos(\theta) \\
&\quad - \sin(\psi_0)\cos(\phi_1)\cos(\theta_1)\sin(\theta) + \cos(\psi_0)\sin(\phi_1)\cos(\theta_1)\sin(\theta) \\
&= \cos(\phi_1 - \psi_0)\cos(\theta_1)\cos(\phi)\cos(\theta) + \sin(\theta_1)\sin(\phi)\cos(\theta) \\
&\quad - \sin(\phi_1 - \psi_0)\cos(\theta_1)\sin(\theta)
\end{aligned}
\tag{5.3}
$$

This gives the portion of the field with vertical polarization. For computing $f_{h,h}$ and $f_{1,\mathrm{v}}$ for a given location, $\vec{r} = (x, y, z)$ on the target site, one can use (5.1) to compute $\vec{r}_1(x_1, y_1, z_1)$. These in turn via (2.1) can be used to compute the various angles and/or the appropriate trigonometric functions of these angles which are in turn substituted into (5.2) and (5.3).

Note that on the S_1 symmetry plane we have over the target site

$$
\begin{aligned}
&\theta_1 = \frac{\pi}{2} \quad , \quad \phi = 0 \\
&f_{1,h} = 1 \quad , \quad f_{1,\mathrm{v}} = 0
\end{aligned}
\tag{5.4}
$$

The fields radiated to the target are

$$
\begin{aligned}
\vec{E}_f^{(1)}(\vec{r}_1, s) &= \frac{e^{-\gamma r}}{r} \vec{F}_t^{(1)}(\vec{1}_{r_1}, s) \tilde{V}_t^{(1)}(s) \\
&= \frac{e^{-\gamma r}}{r} \tilde{F}_t(\theta_1, \phi_1; s) \vec{1}_{\theta_1} \tilde{V}_t^{(1)}(s) \\
r_1 &= r \quad , \quad \vec{1}_r = \vec{1}_{r_1}
\end{aligned}
\tag{5.5}
$$

For present purposes we assume that the two antennas are identical except for the rotation previously discussed. The above incident field can now be decomposed into h and v components as

$$\vec{1}_h \cdot \vec{E}_f^{(1)}(\vec{r}_1,s) = -\frac{e^{-\gamma r}}{r}\tilde{F}_t(\theta_1,\phi_1;s)\, f_{1,h}\, \tilde{V}_t^{(1)}(s)$$
$$\vec{1}_v \cdot \vec{E}_f^{(1)}(\vec{r}_1,s) = -\frac{e^{-\gamma r}}{r}\tilde{F}_t(\theta_1,\phi_1;s)\, f_{1,v}\, \tilde{V}_t^{(1)}(s) \qquad (5.6)$$

For an electrically small antenna this further reduces as

$$\vec{1}_h \cdot \vec{E}_f^{(1)}(\vec{r}_1,s) = -\frac{\mu_0 s^2 e^{-\gamma r}}{4\pi r}\sin(\theta_1)\, f_{1,h}\, \tilde{T}_p(s)\, \tilde{V}_t^{(1)}(s)$$
$$\vec{1}_v \cdot \vec{E}_f^{(1)}(\vec{r}_1,s) = \frac{\mu_0 s^2 e^{-\gamma r}}{4\pi r}\sin(\theta_1)\, f_{1,v}\, \tilde{T}_p(s)\, \tilde{V}_t^{(1)}(s) \qquad (5.7)$$

In this last form we see one advantage of electrically small antennas in the factorization of the dependences on frequency and angles, simplifying the analysis of experimental data.

6. Fields of Antenna 2 in Site Coordinates

Having the fields of antenna 2 described in $\vec{r}_2$ coordinates, these also need to be related to site coordinates. In Cartesian form this relation is indicated in Fig. 6.1 and given by

$$
\begin{aligned}
x_2 &= x\cos(\psi_0) - z\sin(\psi_0) \\
y_2 &= y \\
z_2 &= x\sin(\psi_0) + z\cos(\psi_0) \\
\vec{1}_{x_2} &= \vec{1}_x \cos(\psi_0) - \vec{1}_z \sin(\psi_0) \\
\vec{1}_{y_2} &= \vec{1}_y \\
\vec{1}_{z_2} &= \vec{1}_x \sin(\psi_0) + \vec{1}_z \cos(\psi_0)
\end{aligned}
\tag{6.1}
$$

The electric field incident on the target is polarized in the $\vec{1}_{\theta_2}$ direction which we can think of as quasi-vertical (with a minus sign). With vertical polarization as in (4.2) then we can form

$$
\begin{aligned}
f_{2,h} &= \vec{1}_h \cdot \left[-\vec{1}_{\theta_2} \right] = \vec{1}_\phi \cdot \vec{1}_{\theta_2} \\
&= \left[-\vec{1}_x \sin(\phi) + \vec{1}_y \cos(\phi) \right] \\
&\quad \cdot \left[[\vec{1}_{x_2} \cos(\phi_2) + \vec{1}_{y_2} \sin(\phi_2)] \cos(\theta_2) - \vec{1}_{z_2} \sin(\theta_2) \right] \\
&= -\cos(\psi_0)\cos(\phi_2)\cos(\theta_2)\sin(\phi) + \sin(\psi_0)\sin(\theta_2)\sin(\phi) + \sin(\phi_2)\cos(\theta_2)\cos(\phi)
\end{aligned}
\tag{6.2}
$$

This gives the portion of the field with horizontal polarization.

With the vertical polarization as in (4.2) we can form

$$
\begin{aligned}
f_{2,v} &= \vec{1}_v \cdot \left[-\vec{1}_{\theta_2} \right] = \vec{1}_\theta \cdot \vec{1}_{\theta_2} \\
&= \left[[\vec{1}_x \cos(\phi) + \vec{1}_y \sin(\phi)] \cos(\theta) - \vec{1}_z \sin(\theta) \right] \\
&\quad \cdot \left[[\vec{1}_{x_2} \cos(\phi_2) + \vec{1}_{y_2} \sin(\phi_2)] \cos(\theta_2) - \vec{1}_{z_2} \sin(\theta_2) \right]
\end{aligned}
$$

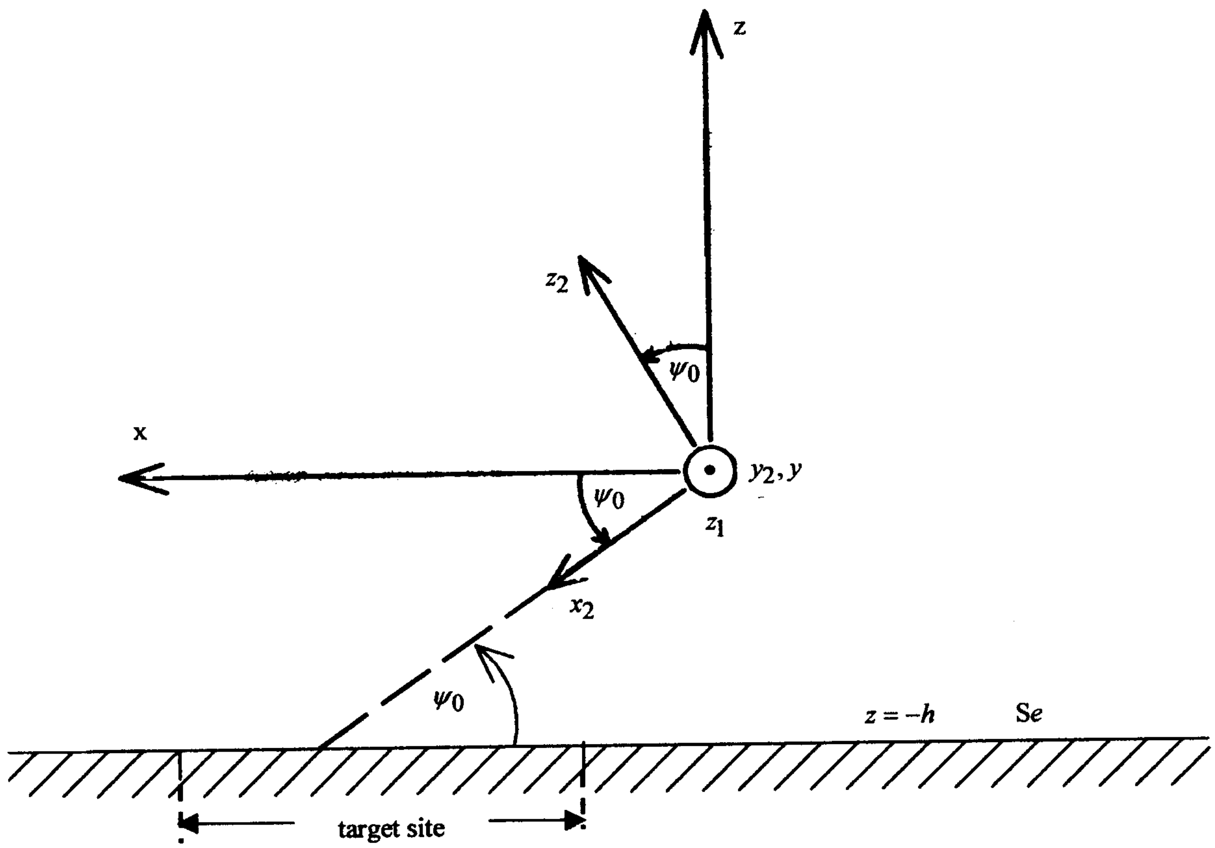


Fig. 6.1 Coordinates for Antenna 2.

$$
\begin{aligned}
&= \cos(\psi_0)\cos(\phi_2)\cos(\theta_2)\cos(\phi)\sin(\phi) \\
&\quad - \sin(\psi_0)\sin(\theta_2)\cos(\phi)\cos(\theta) + \sin(\phi_2)\cos(\theta_2)\sin(\phi)\cos(\theta) \\
&\quad + \sin(\psi_0)\cos(\phi_2)\cos(\theta_2)\sin(\theta) + \cos(\psi_0)\sin(\theta_2)\sin(\theta)
\end{aligned}
\tag{6.3}
$$

This gives the portion of the field with vertical polarization. As in the previous section one can now compute the various angles from the two sets of Cartesian coordinates for the target.

On the S_1 symmetry plane we have over the target site

$$
\begin{aligned}
&\theta_2 = \theta - \psi_0 \quad , \quad \phi = 0 \quad , \quad \phi_2 = 0 \\
&f_{2,h} = 0 \\
&f_{2,v} = \cos(\psi_0)\cos(\theta_0)\cos(\theta) - \sin(\psi_0)\sin(\theta_0)\cos(\theta) \\
&\qquad + \sin(\psi_0)\cos(\theta_2)\sin(\theta) + \cos(\psi_0)\sin(\theta_0)\sin(\theta) \\
&\quad = \cos(\psi_0)\cos(\theta - \theta_0) + \sin(\psi_0)\sin(\theta - \theta_0) \\
&\quad = 1
\end{aligned}
\tag{6.4}
$$

Also we have the special case for $\psi_0 = 0$ which gives everywhere over the target site

$$
\begin{aligned}
&\theta = \theta_2 \ , \ \phi = \phi_2 \\
&\quad f_{2,h} \\
&f_{2,v} = \cos^2(\phi)\cos^2(\theta) + \sin^2(\phi)\cos^2(\theta) + \sin^2(\theta) \\
&\quad = 1
\end{aligned}
\tag{6.5}
$$

which is perfect vertical polarization.

The fields radiated to the targets are just like in the previous section ((5.5) – (5.7)) except with the index 1 replaced by 2.

7. Fields Scattered from the Target to the Antennas and Inference of the Backscattering Dyadic

The target in turn scatters the incident fields back to the two antennas. These fields are naturally represented in the usual h,v radar coordinates. The far scattered electric field back at the antennas ($\vec{r} = \vec{0}$) is just [11, 13]

$$
\begin{aligned}
\vec{\tilde{E}}_f^{(sc,a)}(\vec{0},s) &= \frac{e^{-\gamma r}}{4\pi r}\overleftrightarrow{\Lambda}_b(\vec{1}_r,s)\cdot \vec{\tilde{E}}_f^{(a)}(\vec{r},s) \\
\overleftrightarrow{\Lambda}_b(\vec{1}_r,s) &= \overleftrightarrow{\Lambda}_b^T(\vec{1}_r,s) \quad \text{(reciprocity)} \\
&\equiv \quad \text{backscattering dyadic}
\end{aligned}
\tag{7.1}
$$

For present purposes we can regard this dyadic as 2 x 2 relating to the transverse fields in *the h*, v coordinates. Note the superscript "a" since the above relation applies to fields incident from both the 1 and 2 antennas. This also applies to the combination of fields from both antennas.

At the antennas the reception is described by [3]

$$
\begin{aligned}
\tilde{V}_r^{(a)}(s) &= \vec{h}_t^{(a)}(-\vec{1}_{r_a},s)\cdot \vec{\tilde{E}}_f^{(sc)}(\vec{0},s) \\
\vec{h}_t^{(a)}(-\vec{1}_{r_a},s) &= \frac{4\pi}{s\mu_0}\left[\tilde{Y}_{in}(s)+\tilde{Y}_L(s)\right]^{-1}\vec{F}_t^{(a)}(\vec{1}_{r_a},s) \\
\tilde{V}_r^{(a)}(s) &= \frac{4\pi}{s\mu_0}\left[\tilde{Y}_{in}(s)+\tilde{Y}_L(s)\right]^{-1}\vec{F}_t^{(a)}(\vec{1}_{r_a},s)\cdot \vec{\tilde{E}}_f^{(sc)}(\vec{0},s)
\end{aligned}
\tag{7.2}
$$

With the case of a resistive antenna and load impedance matched as in (2.14) this reduces to

$$
\tilde{V}_r^{(a)}(s) = \frac{2\pi R}{s\mu_0}\vec{F}_t^{(a)}(\vec{1}_{r_a},s)\cdot \vec{\tilde{E}}_f^{(sc)}(\vec{0},s) \tag{7.3}
$$

Again the 1 and 2 antennas are assumed identical except for a rotation. Here the far scattered field is a linear combination of the fields scattered from the incident fields from both antennas 1 and 2.

The previous two sections have considered the properties of $\vec{F}_t^{(a)}$ for the two antennas, as well as its projection onto h,v components. What we need is a 2 x 2 matrix equation relating transmission and reception of the scattering in the 1,2 channels to obtain some effective scattering matrix which can be related to the scattering dyadic in h,v coordinates.

Collecting the various terms we have

$$\begin{pmatrix} \tilde{V}_r^{(1)}(\vec{r},s) \\ \tilde{V}_r^{(2)}(\vec{r},s) \end{pmatrix} = \frac{e^{-2\gamma r}}{s\mu_0}\left[\tilde{Y}_{in}(s)+\tilde{Y}_L(s)\right]^{-1}(\tilde{X}_{n,m}^{(1)}(\vec{1}_r,s))\cdot\begin{pmatrix} \tilde{V}_t^{(1)}(s) \\ \tilde{V}_t^{(2)}(s) \end{pmatrix}$$

$$(\tilde{X}_{n,m}^{(1)}(\vec{1}_r,s)) = \tag{7.4}$$

$$\begin{pmatrix} \vec{F}_t^{(1)}(\theta_1,\phi_1;s)\cdot\overleftrightarrow{\Lambda}_b(\vec{1}_r,s)\cdot\vec{F}_t^{(1)}(\theta_1,\phi_1;s) & \vec{F}_t^{(1)}(\theta_1,\phi_1;s)\cdot\overleftrightarrow{\Lambda}_b(\vec{1}_r,s)\cdot\vec{F}_t^{(2)}(\theta_2,\phi_2;s) \\ \vec{F}_t^{(2)}(\theta_2,\phi_2;s)\cdot\overleftrightarrow{\Lambda}_b(\vec{1}_r,s)\cdot\vec{F}_t^{(1)}(\theta_1,\phi_1;s) & \vec{F}_t^{(2)}(\theta_2,\phi_2;s)\cdot\overleftrightarrow{\Lambda}_b(\vec{1}_r,s)\cdot\vec{F}_t^{(2)}(\theta_2,\phi_2;s) \end{pmatrix}$$

As one can readily see this matrix is symmetric as is required by reciprocity. The problem is now to calculate $\overleftrightarrow{\Lambda}_b$ from $(\tilde{X}_{n,m}^{(1)})$ which we can obtain by measurement. We have four matrix components, presumably from measurements from which we can infer the four components of the backscattering dyadic in h,v coordinates. Of course, by reciprocity only one of the off-diagonal components needs to be computed.

We can expand the backscattering dyadic as

$$\begin{aligned} \overleftrightarrow{\Lambda}_b(\vec{1}_r,s) &= \tilde{\Lambda}_{b_{h,h}}(\vec{1}_r,s)\,\vec{1}_h\,\vec{1}_h+\tilde{\Lambda}_{b_{h,\mathrm{v}}}(\vec{1}_r,s)\,\vec{1}_h\,\vec{1}_\mathrm{v}+\tilde{\Lambda}_{b_{\mathrm{v},h}}(\vec{1}_r,s)\,\vec{1}_\mathrm{v}\,\vec{1}_h+\tilde{\Lambda}_{b_{\mathrm{v},\mathrm{v}}}(\vec{1}_r,s)\,\vec{1}_\mathrm{v}\,\vec{1}_\mathrm{v} \\ &= \tilde{\Lambda}_{b_{h,h}}(\vec{1}_r,s)\,\vec{1}_h\,\vec{1}_h+\tilde{\Lambda}_{b_{h,\mathrm{v}}}(\vec{1}_r,s)\left[\vec{1}_h\,\vec{1}_\mathrm{v}+\vec{1}_\mathrm{v}\,\vec{1}_h\right]+\tilde{\Lambda}_{b_{\mathrm{v},\mathrm{v}}}(\vec{1}_r,s)\,\vec{1}_\mathrm{v}\,\vec{1}_\mathrm{v} \end{aligned} \tag{7.5}$$

$$\tilde{\Lambda}_{b_{\mathrm{v},h}}(\vec{1}_r,s)=\tilde{\Lambda}_{b_{h,\mathrm{v}}}(\vec{1}_r,s)$$

This can be used to write the matrix elements in the form

$$\begin{aligned} &\vec{F}_t^{(n)}(\theta_n,\phi_n;s)\cdot\overleftrightarrow{\Lambda}_b(\vec{1}_r,s)\cdot\vec{F}_t^{(m)}(\vec{1}_r,s) \\ &= \tilde{X}_{n,m}(\vec{1}_r,s)\left[\vec{F}_t^{(n)}(\theta_n,\phi_n;s)\cdot\vec{1}_h\right]\left[\vec{F}_t^{(m)}(\theta_m,\phi_m;s)\cdot\vec{1}_h\right]\tilde{\Lambda}_{b_{h,h}}(\vec{1}_r,s) \end{aligned}$$

$$+\left[\vec{F}_t^{(n)}(\theta_n,\phi_n;s)\cdot\vec{1}_h\right]\left[\vec{F}_t^{(m)}(\theta_m,\phi_m;s)\cdot\vec{1}_v\right]\tilde{\Lambda}_{b_{h,v}}(\vec{1}_r,s)$$

$$+\left[\vec{F}_t^{(n)}(\theta_n,\phi_n;s)\cdot\vec{1}_v\right]\left[\vec{F}_t^{(m)}(\theta_m,\phi_m;s)\cdot\vec{1}_h\right]\tilde{\Lambda}_{b_{v,h}}(\vec{1}_r,s) \tag{7.6}$$

$$+\left[\vec{F}_t^{(n)}(\theta_n,\phi_m;s)\cdot\vec{1}_v\right]\left[\vec{F}_t^{(m)}(\theta_m,\phi_m;s)\cdot\vec{1}_v\right]\tilde{\Lambda}_{b_{v,v}}(\vec{1}_r,s)$$

In a shorthand form we can define

$$F_{n',h'} \equiv \vec{F}_t^{(n')}(\vec{1}_r,s)\cdot\vec{1}_{h'} \tag{7.7}$$

giving four factors based on the choices of

$$n = n', m' \quad , \quad h' = h, v \tag{7.8}$$

All four of these factors are available from measurement (or calculation) of the antenna transmission function as in (5.5), together with its decomposition into h,v coordinates as discussed in Sections 5 and 6. With the four choices of the n, m combination we can form the matrix equation

$$\begin{pmatrix}\tilde{X}_{1,1}^{(2)}\\ \tilde{X}_{1,2}^{(1)}\\ \tilde{X}_{2,1}^{(1)}\\ \tilde{X}_{2,2}^{(1)}\end{pmatrix} = \begin{pmatrix} F_{1,h}^2 & F_{1,h}F_{1,v} & F_{1,v}F_{1,h} & F_{1,v}^2\\ F_{1,h}F_{2,h} & F_{1,h}F_{2,v} & F_{1,v}F_{2,h} & F_{1,v}F_{2,v}\\ F_{2,h}F_{1,h} & F_{2,h}F_{1,v} & F_{2,v}F_{1,h} & F_{2,v}F_{1,v}\\ F_{2,h}^2 & F_{2,h}F_{2,v} & F_{2,v}F_{2,h} & F_{2,v}^2\end{pmatrix}\cdot\begin{pmatrix}\tilde{\Lambda}_{b_{h,h}}\\ \tilde{\Lambda}_{b_{h,v}}\\ \tilde{\Lambda}_{b_{v,h}}\\ \tilde{\Lambda}_{b_{v,v}}\end{pmatrix} \tag{7.9}$$

Inverting this 4 x 4 matrix with the $X_{n,m}$ known from measurement we have the four components of $\tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_r,s)$ in h,v coordinates.

After solving (7.9) we should have equality of the off-diagonal components of the backscattering dyadic. Of course, this is only approximate in real measurements due to noise. At this stage one might average the off-diagonal components for a hopefully more accurate estimate. An alternate procedure is to impose this (7.5) from the start as well as

$$\tilde{X}_{2,1}^{(1)} = \tilde{X}_{1,2}^{(1)} \tag{7.10}$$

which we can impose by averaging the measurements of these two latter parameters, or by measuring only one of them and using this measurement for both. We can then set up a matrix equation in the form

$$\begin{pmatrix} \tilde{X}_{1,1}^{(1)} \\ \tilde{X}_{1,2}^{(1)} \\ \tilde{X}_{2,2}^{(1)} \end{pmatrix} = \begin{pmatrix} F_{1,h}^2 & 2\, F_{1,h} F_{1,v} & F_{1,v}^2 \\ F_{1,h} F_{2,h} & F_{1,h} F_{2,v} + F_{1,v} F_{2,h} & F_{1,v} F_{2,v} \\ F_{2,h}^2 & 2\, F_{2,h} F_{2,v} & F_{2,v}^2 \end{pmatrix} \cdot \begin{pmatrix} \tilde{\Lambda}_{b_{h,h}} \\ \tilde{\Lambda}_{b_{h,v}} \\ \tilde{\Lambda}_{b_{v,v}} \end{pmatrix} \tag{7.11}$$

which reduces the problem to the inversion of a 3 x 3 matrix.

With the antenna polarizations controlled to be in the $\vec{1}_{\theta_n}$ directions we have as discussed in Sections 5 and 6

$$\begin{aligned} \vec{F}^{(n)}(\theta_n, \phi_n; s) &= \tilde{F}^{(n)}(\theta_n, \phi_n; s)\, \vec{1}_{\theta_n} \\ F_{n',h'} &\equiv \tilde{F}_t^{(n')}(\theta_n, \phi_n; s)\, \vec{1}_{\theta_{n'}} \cdot \vec{1}_{h'} \end{aligned} \tag{7.12}$$

Here the frequency dependence is separated in a scalar factor giving frequency-inependent polarizations. Writing out the four terms we have

$$\begin{aligned} F_{1,h} &= -\tilde{F}_t^{(1)}(\theta_1, \phi_1; s)\, f_{1,h} \quad , \quad F_{1,v} = -\tilde{F}^{(1)}(\theta_1, \phi_1; s)\, f_{1,v} \\ F_{2,h} &= -\tilde{F}_t^{(2)}(\theta_2, \phi_2; s)\, f_{2,h} \quad , \quad F_{2,v} = -\tilde{F}^{(2)}(\theta_2, \phi_2; s)\, f_{2,v} \end{aligned} \tag{7.13}$$

where the angular functions $f_{n',h'}$ are tabulated in Sections 5 and 6. Substituting these in (7.9) and (7.11) we can see that the scalar pattern functions $\tilde{F}_t^{(n')}(\theta_{n'}, \phi_{n'}; s)$ still appear in the matrix in a mixed way such that they cannot be factored out of the matrix which is then still frequency dependent in a complicated way. However, we do have the advantage of effectively having scalar antenna patterns times an easily calculable frequency-independent polarization.

Going further, let us further simplify the problem by assuming that the antennas are electrically small. Recalling (2.25) we then have

$$\tilde{F}_t^{(n')}(\theta_{n'},\phi_{n'};s) = -\frac{s^2\mu_0}{4\pi}\tilde{T}_p(s)\sin(\theta_{n'}) \tag{7.14}$$

Noting that the two antennas are identical except for the angular rotation, we can now rewrite (7.4) in the form

$$\begin{pmatrix} \tilde{V}_r^{(1)}(\vec{r},s) \\ \tilde{V}_r^{(2)}(\vec{r},s) \end{pmatrix} = -\frac{\mu_0}{16\pi^2} s^3 \tilde{T}_p^2(s)\left[\tilde{Y}_{in}(s)+\tilde{Y}_L(s)\right]^{-1}(\tilde{X}_{n,m}^{(2)}(\vec{1}_r,s))\cdot\begin{pmatrix} \tilde{V}_t^{(1)}(s) \\ \tilde{V}_t^{(2)}(s) \end{pmatrix}$$

$$\begin{aligned} \tilde{X}_{n,m}^{(2)}(\vec{1}_r,s) = \sin(\theta_n)\sin(\theta_m)\Big[& f_{n,h}f_{m,h}\tilde{\Lambda}_{b_{h,h}} + f_{n,h}f_{m,\mathrm{v}}\tilde{\Lambda}_{b_{h,\mathrm{v}}} \\ & + f_{n,\mathrm{v}}f_{m,h}\tilde{\Lambda}_{b_{\mathrm{v},h}} + f_{n,\mathrm{v}}f_{m,\mathrm{v}}\tilde{\Lambda}_{b_{h,\mathrm{v}}}\Big] \end{aligned} \tag{7.15}$$

In this case we know these matrix elements from measurement and the $\theta_{n'}$ and $f_{h',h'}$ are purely geometrical and need not be recomputed for each frequency, but only for different target locations in the data processing.

The matrix equation corresponding to (7.9) can now be written as

$$\begin{pmatrix} \tilde{X}_{1,1}^{(2)} \\ \tilde{X}_{1,2}^{(2)} \\ \tilde{X}_{2,1}^{(2)} \\ \tilde{X}_{2,2}^{(2)} \end{pmatrix} = \begin{pmatrix} \sin^2(\theta_1)f_{1,h}^2 & \sin^2(\theta_1)f_{1,h}f_{1,\mathrm{v}} & \sin^2(\theta_1)f_{1,\mathrm{v}}f_{1,h} & \sin^2(\theta_1)f_{1,\mathrm{v}}^2 \\ \sin(\theta_2)\sin(\theta_2)f_{1,h}f_{2,h} & \sin(\theta_2)\sin(\theta_2)f_{1,h}f_{2,\mathrm{v}} & \sin(\theta_1)\sin(\theta_2)f_{1,\mathrm{v}}f_{2,h} & \sin(\theta_1)\sin(\theta_2)f_{1,\mathrm{v}}f_{2,\mathrm{v}} \\ \sin(\theta_2)\sin(\theta_1)f_{2,h}f_{1,h} & \sin(\theta_2)\sin(\theta_1)f_{2,h}f_{1,\mathrm{v}} & \sin(\theta_2)\sin(\theta_1)f_{2,\mathrm{v}}f_{1,h} & \sin(\theta_2)\sin(\theta_1)f_{2,\mathrm{v}}f_{1,\mathrm{v}} \\ \sin^2(\theta_2)f_{2,h}^2 & \sin^2(\theta_2)f_{2,h}f_{2,\mathrm{v}} & \sin^2(\theta_2)f_{2,\mathrm{v}}f_{2,h} & \sin^2(\theta_2)f_{2,\mathrm{v}}^2 \end{pmatrix} \cdot \begin{pmatrix} \tilde{\Lambda}_{b_{h,h}} \\ \tilde{\Lambda}_{b_{h,\mathrm{v}}} \\ \tilde{\Lambda}_{b_{\mathrm{v},h}} \\ \tilde{\Lambda}_{b_{\mathrm{v},\mathrm{v}}} \end{pmatrix} \tag{7.16}$$

An alternate form can be found by moving the sine functions to the left side as

$$
\begin{pmatrix} \csc^2(\theta_1)\tilde{X}_{1,1}^{(2)} \\ \csc(\theta_1)\csc(\theta_2)\tilde{X}_{2,1}^{(2)} \\ \csc(\theta_2)\csc(\theta_1)\tilde{X}_{2,1}^{(2)} \\ \csc^2(\theta_2)\tilde{X}_{2,2}^{(2)} \end{pmatrix} = \begin{pmatrix} f_{1,h}^2 & f_{1,h}f_{1,\text{v}} & f_{1,\text{v}}f_{1,h} & f_{1,\text{v}}^2 \\ f_{1,h}f_{2,h} & f_{1,h}f_{2,\text{v}} & f_{1,\text{v}}f_{2,h} & f_{1,\text{v}}f_{2,\text{v}} \\ f_{2,h}f_{1,h} & f_{2,h}f_{1,\text{v}} & f_{2,\text{v}}f_{1,h} & f_{2,\text{v}}f_{1,\text{v}} \\ f_{2,h}^2 & f_{2,h}f_{2,\text{v}} & f_{2,\text{v}}f_{2,h} & f_{2,h}^2 \end{pmatrix} \bullet \begin{pmatrix} \tilde{\Lambda}_{b_{h,h}} \\ \tilde{\Lambda}_{b_{h,\text{v}}} \\ \tilde{\Lambda}_{b_{\text{v},h}} \\ \tilde{\Lambda}_{b_{\text{v},\text{v}}} \end{pmatrix} \tag{7.17}
$$

which is equivalent to (7.16), but simpler looking. The matrix to be inverted now is a function of only the angles, and so need not be inverted for each frequency and can apply to a complete temporal waveform.

As previously in (7.11) this can be expressed using a 3 x 3 matrix using reciprocity. Adapting the form in (7.17) we have

$$
\begin{pmatrix} \csc^2(\theta_1)\,\tilde{X}_{1,1}^{(2)} \\ \csc(\theta_1)\,\csc(\theta_2)\,\tilde{X}_{1,2}^{(2)} \\ \csc^2(\theta_2)\tilde{X}_{2,2}^{(2)} \end{pmatrix} = \begin{pmatrix} f_{1,h}^2 & 2\,f_{1,h}f_{1,\text{v}} & f_{1,\text{v}}^2 \\ f_{1,h}f_{2,h} & f_{1,h}f_{2,\text{v}} + f_{1,\text{v}}f_{2,h} & f_{1,\text{v}}f_{2,\text{v}} \\ f_{2,h}^2 & 2\,f_{2,h}f_{2,h} & f_{2,h}^2 \end{pmatrix} \bullet \begin{pmatrix} \tilde{\Lambda}_{b_{h,h}} \\ \tilde{\Lambda}_{b_{h,\text{v}}} \\ \tilde{\Lambda}_{b_{\text{v},\text{v}}} \end{pmatrix} \tag{7.18}
$$

If desired, a similarly reduced form of (7.16) is just as easily constructed.

An important point to note about the electrically-small case is that the electric-dipole transfer function $\tilde{T}_p(s)$ is removed from the matrix as a common factor. Errors in $\tilde{T}_p(s)$ do not appear in the reconstruction of the h,v polarizations via the matrix inversion. The matrix elements are only functions of geometrical parameters (angles and range r) which one can determine accurately.

8. Special Case of $\psi_0 = 0$

If we choose the special case of $\psi_0 = 0$, this makes antenna 2 be oriented vertically, i.e.,

$$\overrightarrow{1}_2 = \overrightarrow{1}_{z_2} = \overrightarrow{1}_z \tag{8.1}$$

Then from (6.5) we have the important result

$$f_{2,h} = 0 \quad , \quad f_{2,\mathrm{v}} = 0 \tag{8.2}$$

What this says is that antenna 2 gives pure vertical polarization and that $\tilde{\Lambda}_{b_{\mathrm{v},\mathrm{v}}}$ can be directly found by transmission and reception from antenna 2 alone. As can be seen in (5.2) and (5.3), such a simple result does not similarly apply to antenna 1.

With unipolarized currents so that the antenna 2 polarization is in the $\overrightarrow{1}_{\theta_2} = \overrightarrow{1}_\theta$ direction we have

$$F_{2,h} = 0 \tag{8.3}$$

This can be substituted in (7.11) to give

$$\begin{pmatrix} \tilde{X}_{1,1}^{(1)} \\ \tilde{X}_{1,2}^{(1)} \\ \tilde{X}_{2,2}^{(1)} \end{pmatrix} = \begin{pmatrix} F_{1,h}^2 & 2\, F_{1,h} F_{1,\mathrm{v}} & F_{1,\mathrm{v}}^2 \\ 0 & F_{1,h} F_{2,\mathrm{v}} & F_{1,\mathrm{v}} F_{2,\mathrm{v}} \\ 0 & 0 & F_{2,\mathrm{v}}^2 \end{pmatrix} \cdot \begin{pmatrix} \tilde{\Lambda}_{b_{h,h}} \\ \tilde{\Lambda}_{b_{h,\mathrm{v}}} \\ \tilde{\Lambda}_{b_{\mathrm{v},\mathrm{v}}} \end{pmatrix} \tag{8.4}$$

This is readily solved as

$$\begin{aligned} \tilde{\Lambda}_{b_{\mathrm{v},\mathrm{v}}} &= F_{2,\mathrm{v}}^{-2} \tilde{X}_{2,2}^{(1)} \\ \tilde{\Lambda}_{b_{h,\mathrm{v}}} &= F_{1,h}^{-1} F_{2,\mathrm{v}}^{-1} \left[\tilde{X}_{1,2}^{(1)} - F_{1,\mathrm{v}} F_{2,\mathrm{v}} \tilde{\Lambda}_{b_{\mathrm{v},\mathrm{v}}} \right] \\ &= F_{1,h}^{-1} F_{2,\mathrm{v}}^{-1} \tilde{X}_{1,2}^{(1)} - F_{1,h}^{-1} F_{2,\mathrm{v}}^{-2} F_{1,\mathrm{v}} \tilde{X}_{2,2}^{(1)} \end{aligned} \tag{8.5}$$

$$
\begin{aligned}
\tilde{\Lambda}_{b_{h,h}} &= F_{1,h}^{-2}\left[\tilde{X}_{1,1}^{(1)} - 2F_{1,h}F_{1,v}\tilde{\Lambda}_{b_{h,v}} - F_{1,v}^{2}\tilde{\Lambda}_{b_{v,v}}\right] \\
&= F_{1,h}^{-2}\tilde{X}_{1,1}^{(1)} - 2F_{1,h}^{-2}F_{2,v}^{-2}F_{1,v}\tilde{X}_{1,2}^{(1)} + F_{1,h}^{-2}F_{2,v}^{-2}F_{1,v}^{2}\tilde{X}_{2,2}^{(1)}
\end{aligned}
$$

which can be expressed in matrix form as

$$
\begin{pmatrix} \tilde{\Lambda}_{b_{h,h}} \\ \tilde{\Lambda}_{b_{h,v}} \\ \tilde{\Lambda}_{b_{h,h}} \end{pmatrix} = \begin{pmatrix} F_{1,h}^{-2} & -2\,F_{1,h}^{-2}F_{2,v}^{-2}F_{1,v} & F_{1,h}^{-2}F_{2,v}^{-2}F_{1,v}^{2} \\ 0 & F_{1,h}^{-1}F_{2,v}^{-1} & -F_{1,h}^{-1}F_{2,v}^{-2}F_{1,v} \\ 0 & 0 & F_{2,v}^{-2} \end{pmatrix} \cdot \begin{pmatrix} \tilde{X}_{1,1}^{(1)} \\ \tilde{X}_{1,2}^{(1)} \\ \tilde{X}_{2,2}^{(1)} \end{pmatrix} \tag{8.6}
$$

thereby giving the explicit matrix inverse.

For electrically small antennas (7.18) becomes

$$
\begin{pmatrix} \csc^2(\theta_1)\tilde{X}_{1,1}^{(2)} \\ \csc(\theta_1)\csc(\theta_2)\tilde{X}_{1,2}^{(2)} \\ \csc^2(\theta_2)\tilde{X}_{2,2}^{(2)} \end{pmatrix} = \begin{pmatrix} f_{1,h}^{2} & 2f_{1,h}f_{1,v} & f_{1,v}^{2} \\ 0 & f_{1,h} & f_{1,v} \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \tilde{\Lambda}_{b_{h,h}} \\ \tilde{\Lambda}_{b_{h,v}} \\ \tilde{\Lambda}_{b_{v,v}} \end{pmatrix} \tag{8.7}
$$

This is also solved as

$$
\begin{aligned}
\tilde{\Lambda}_{b_{v,v}} &= \csc^2(\theta_2)\tilde{X}_{2,2}^{(2)} \\
\tilde{\Lambda}_{b_{h,h}} &= f_{1,h}^{-1}\left[\csc(\theta_1)\csc(\theta_2)\tilde{X}_{1,2}^{(2)} - f_{1,v}\tilde{\Lambda}_{b_{v,v}}\right] \\
&= f_{1,h}^{-1}\csc(\theta_1)\csc(\theta_2)\tilde{X}_{1,2}^{(2)} - f_{1,h}^{-1}f_{1,v}\csc(\theta_2)\tilde{X}_{2,2}^{(2)} \\
\tilde{\Lambda}_{b_{h,h}} &= f_{1,h}^{-2}\left[\csc^2(\theta_1)\tilde{X}_{1,1}^{(2)} - 2f_{1,h}f_{1,v}\tilde{\Lambda}_{b_{h,v}} - f_{1,v}^{2}\tilde{\Lambda}_{b_{v,v}}\right] \\
&= f_{1,h}^{-2}\csc^2(\theta_1)\tilde{X}_{1,1}^{(2)} - 2f_{1,h}^{-2}f_{1,v}\csc(\theta_1)\csc(\theta_2)\tilde{X}_{1,2}^{(2)} + f_{1,h}^{-2}f_{1,v}^{2}\csc^2(\theta_2)\tilde{X}_{2,2}
\end{aligned} \tag{8.8}
$$

which can be expressed in matrix form as

$$\begin{pmatrix} \tilde{\Lambda}_{b_{h,h}} \\ \tilde{\Lambda}_{b_{h,v}} \\ \tilde{\Lambda}_{b_{v,v}} \end{pmatrix} = \begin{pmatrix} f_{1,h}^{-2} & -2f_{1,h}^{-2}f_{1,v} & f_{1,h}^{-2}f_{1,v}^{2} \\ 0 & f_{1,h}^{-1} & f_{1,h}^{-1}f_{1,v} \\ 0 & 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} \csc^2(\theta_1)\tilde{X}_{1,1}^{(2)} \\ \csc(\theta_1)\csc(\theta_2)\tilde{X}_{1,2}^{(2)} \\ \csc^2(\theta_2)\tilde{X}_{2,2}^{(2)} \end{pmatrix} \tag{8.9}$$

again giving the explicit matrix inverse.

So this special case of $\psi_0 = 0$ simplifies the algebra. Antenna 2 has the properties of an O_2 antenna (including the ground) giving a pure vertical polarization over the target site. Antenna 1 does not have this special property. So a partial separation of the h,v polarizations is achieved.

9. Electrically Small Antennas With Unipolarized Currents

A common technique used to make wire antennas resonate at frequencies for which the antenna is electrically small is the addition of inductive loading. However, inductive coils do not have currents all running in one direction. A typical coil is a helix and has a magnetic-dipole moment given by

$$\begin{aligned} \vec{m} &= \pm \mathrm{INA}\, \vec{1}_a \\ \mathrm{A} &= \text{area per turn} \\ \mathrm{N} &= \text{number of turns} \end{aligned} \tag{9.1}$$

As illustrated in Fig. 9.1, such a coil can have $\vec{m}$ parallel or antiparallel to $\vec{1}_a$ depending on the sense of winding pitch. (Other orientations are also possible.) The presence of such a moment is undesirable because of how it distorts the polarization of the far field, a polarization which we would like to be governed by $\vec{p}$, the electric-dipole moment.

One technique for canceling the magnetic-dipole moment is to make the two oppose each other as illustrated in Fig. 9.1 by making their winding pitches have opposite sense. This restores the $z_a = 0$ plane as a symmetry plane, about which the currents and fields are antisymmetric. However, these coils can present other problems due to their mutual interaction. In arrays of such thin-wire elements as in Fig. 2.2B and Fig. 3.1 there may be many such coils. Consider, for example, a log-periodic antenna in which now, potentially, one element can couple to another via these coils, a transformer effect. For various possible reasons, one may wish to avoid this additional coupling. (Conceivably, one may wish in some instances to utilize this coupling.)

Another approach to removing the effects of such magnetic dipole moments is to design the coil(s) to have negligible such moments. This is accomplished by designing a coil such that the magnetic field produced remains internal to the coil structure. The general theory is developed in [2]. A simpler approximate form is given in [4]. In this simpler bisolenoidal (or multisolenoidal) form, for each loop turn its magnetic-dipole moment is cancelled by another loop turn with opposite sense, but displaced as part of a second parallel solenoid so that the magnetic fluxes do not cancel. This is an approximate solution in the sense that higher order magnetic moments (quadrupole, etc.) remain.

As illustrated in Fig. 9.2, one can think of this as a figure –8 winding on two parallel cylindrical (circular or otherwise) dielectric forms for supporting the windings. (The forms may be removed for an air-core coil if desired.) With N turns in each solenoid of length ℓ with cross-section area A the inductance is

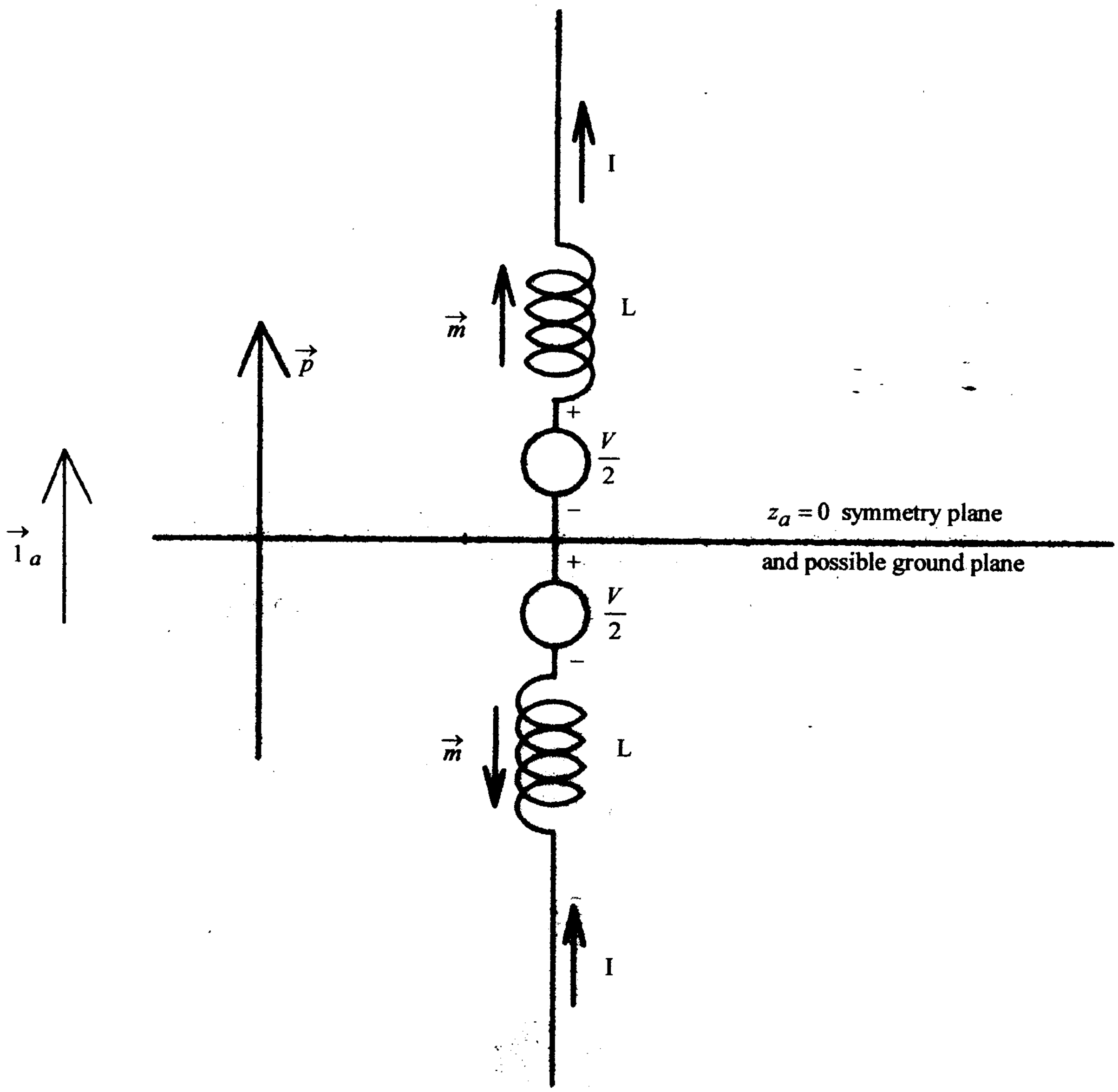


Fig. 9.1 Thin-Wire Antenna Symmetrically Loaded by Inductors.

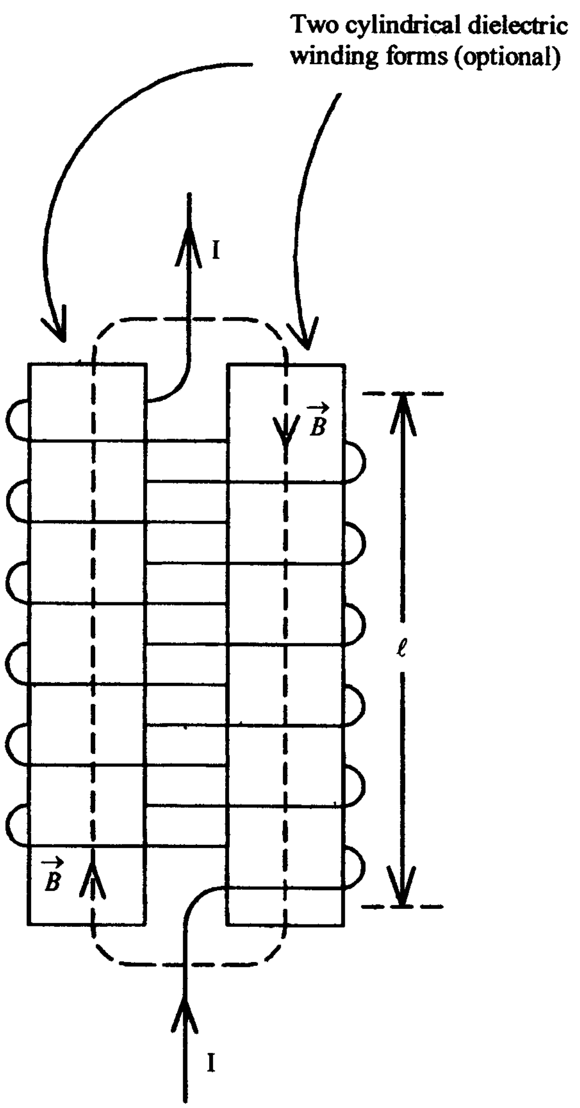


Fig. 9.2 Bisolenoidal Inductor.

$$L = 2\mu_0 N^2 \frac{A}{\ell} \qquad (9.2)$$

(valid for sufficiently large N / ℓ.) Note that the magnetic flux density $\vec{B}$ reverses direction between the parallel solenoids, and the magnetic flux leaving the end of one solenoid enters the end of the adjacent solenoid (approximately).

Besides viewing such inductors as lumped elements, one can make a distributed bisolenoidal inductor by letting ℓ be large (even the entire length of the antenna element). In this case we have an inductance per unit length

$$L' = 2\mu_0 N'^2 A$$

$$N' \equiv \text{number of turns (each solenoid) per unit length} \qquad (9.3)$$

In this case the antenna element becomes a slow-wave structure. The inductance per unit length can even be variable as $L'(z_a)$ based on a variable turns density $N'(z_a)$ and/or a variable cross-section area $A(z_a)$.

One can add $sL'(z_a)$ as an additional series impedance per unit length in a transmission-line model of a wire antenna [1], and look for useful forms of such loading. This can also be combined with resistance-per-unit length $R'(z_a)$ for damping the antenna response for desirable transient response [1]. This opens various possibilities.

One need not be limited to a single loaded-wire element, but can have arrays of same, say in log-periodic form. By this technique one can in principle have electrically small arrays with multiple-wavelength waves on the slow-wave structures, if desired. The present considerations are but an introduction to the various design possibilities.

10. Concluding Remarks

This basic idea of unipolarized antenna currents has led to various implications for antenna design for frequency-independent far-field polarization. Combination of two such antennas with orthogonal antenna currents has led to the requirement of two orthogonal symmetry planes applying to both of the antennas. Within these constraints there is still much flexibility in designing arrays of parallel thin wires. Furthermore, such wires can be symmetrically impedance loaded for various reasons, including a desire to make them electrically small so as to simplify the analysis in terms of a unipolarized electric-dipole moment.

It will be interesting to see where these ideas may lead. While the present analysis is in terms of unipolarized electric currents, what about the electromagnetic dual, i.e., unipolarized magnetic currents? The present analysis still applies with the interchange of electric and magnetic fields. The question is then how to synthesize such magnetic currents from loops to give magnetic moments without significant electric moments. This should be achievable using pairs of coils whose magnetic dipole moments add, but electric-dipole moments subtract.

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