Sensor and Simulation Notes

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A Sensor for Voltage, Current, and Waves in Coaxial Cables

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Abstract

This paper considers a sensor for measuring the time derivatives of waves propagating in the two directions on a coaxial transmission line. The signals from the two ports can also be added and subtracted to give the time derivatives of voltage and current, respectively.

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1. Introduction

Many sensors have been developed for the measurement of I-dot (time derivative of current) in various geometries, including in coaxial-cable geometry as an insertion unit. (See [11] and references therein.). The present paper generalizes the cable case to include V-dot (time derivative of voltage) and combinations of the two. These results have been languishing in my notebooks for many years, so I thought that it was time that I documented them.

A result of some interest and utility concerns the balancing of these two measurements so that one can separate waves propagating in two directions (which we term right and left) on the coaxial cable. This makes a directional coupler for use in transmission lines. It is analogous to the BTW (balanced transmission-line wave) sensor for plane waves in space, such a sensor also being a directional coupler for such waves [5]. Such a field sensor is but a receiving version of a $\vec{p} \times \vec{m}$ antenna (balanced electric- and magnetic-dipole moments at right angles to each other) in transmission [3, 4, 6, 8-10].

2. Sensor Geometry

The sensor geometry is given in Fig. 2.1. Notice the two gaps (circumferential slots) of width w in the outer conductor of the coax which we idealize as the surface S_0. It is here that the signals are introduced into the sensing cables. Each gap is loaded by the characteristic impedance (constant resistance) ζ_c (typically 50 Ω) of a coaxial cable transmitting a signal to the recording instrument(s) where each is terminated in a resistance ζ_c. Note that each of these cables is bonded to and perhaps recessed in S_0 so as to be topologically part of it. In one design the center conductors of the two sensing cables cross the signal gaps, connecting to the other side. A more elaborate design has each such cable connecting to two cables (in parallel) of characteristic impedance $2\zeta_c$ (say 100 Ω) and of equal length. Each of these in turn connects across the appropriate gap at opposite circumferential positions. This is a feature of various D-dot, B-dot, and I-dot sensor designs [11] for improved signal averaging around gaps and improved high-frequency response.

The cable of characteristic impedance Z_c (from which signals are to be sensed) has voltage V and current I (positive to the right). In terms of the various parameters we have

$$Z_c = \left[\frac{\mu_0}{\varepsilon_1}\right]^{\frac{1}{2}} \frac{1}{2\pi} \ell n\left(\frac{\Psi_{1-}}{\Psi_0}\right) \tag{2.1}$$

which may also (but not necessarily) be 50 Ω. Note that the inner radius Ψ_{1-} of S_0 is used with the inner-conductor radius Ψ_0 due to the small skin depth in the conductors at the high frequencies of interest.

Note that Ψ_{1+} is the outer radius of S_0, which together with Ψ_2 (the inner radius of the surrounding conducting cylindrical shell) and ℓ_I (the inner spacing of the conducting disks closing this surrounding cylinder to S_0) are the appropriate dimensions of the outer sensing volume. Note that the constitutive parameters of this sensing volume need not be the same as those of the original cable. Typically the permeability is μ_0 in both cases, but different dielectrics with permittivities ε_1 (coax) and ε_2 (sensing volume) may be employed. Note that the width w of each sensing gap needs to be small compared to other dimensions, including $\Psi_{1-} - \Psi_0$, $\Psi_2 - \Psi_{1+}$, $\ell_V / 2$, and $[\ell_I - \ell_V]/2$ so that we may accurately choose ℓ_V as the distance between the centers of the two gaps (for electric-flux interception). This also makes the length of the coaxial center conductors crossing the gaps small for low inductance and small correction to the magnetic flux interception based on Ψ_{1+}.

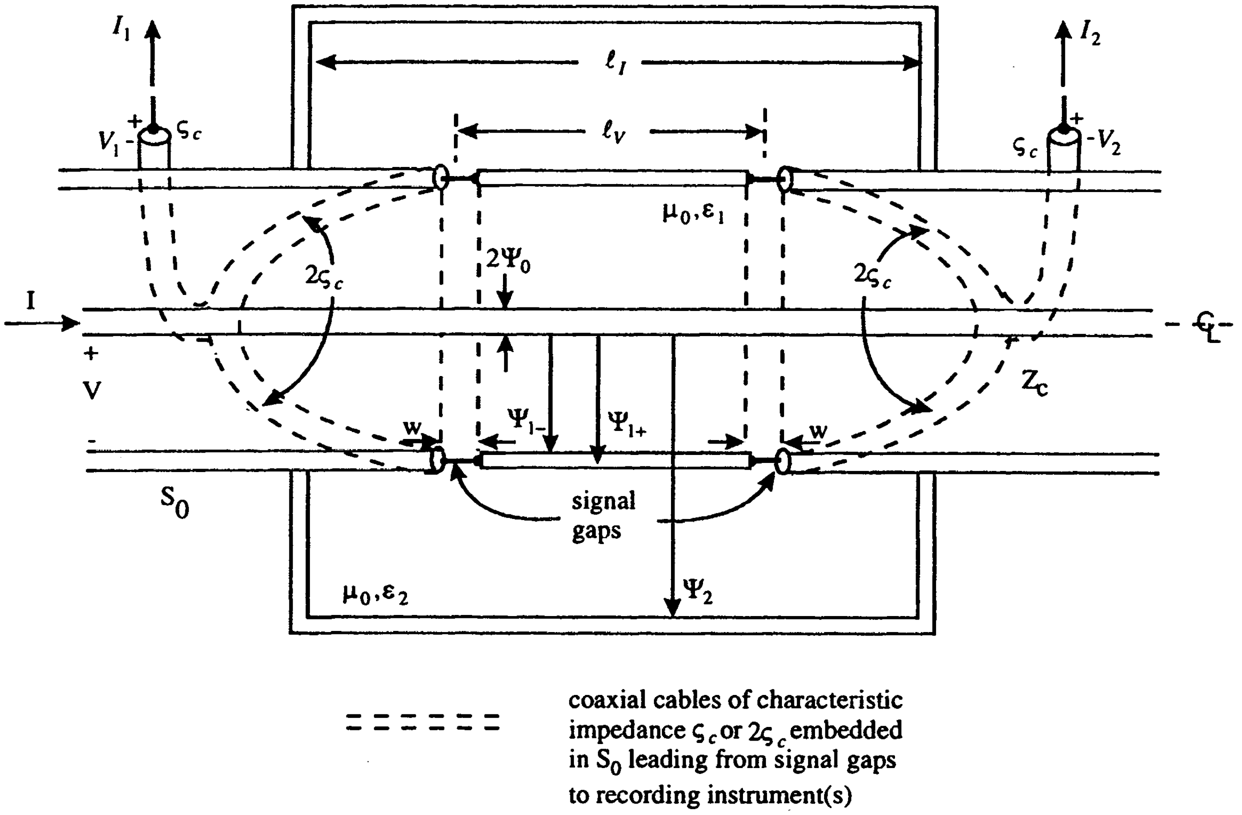

Fig. 2.1 Combined V-Dot and I-Dot Sensor For Coaxial Signals.

3. Mutual Inductance

The mutual inductance is just [1, 2]

$$M = \frac{\mu_0}{2\pi} \ell_I \ell n\left(\frac{\Psi_2}{\Psi_{1+}}\right) \tag{3.1}$$

based on the magnetic flux in the sensing volume per unit current (I). This gives a sensor output (differential) as

$$V_1 - V_2 = M \frac{dI}{dt} \tag{3.2}$$

For this result to hold the frequency should be low enough that

$$\begin{aligned} |s| L_s &<< 2\zeta_c \\ s &= \Omega + j\omega \equiv \text{complex frequency or two-sided Laplace-transform variable} \\ L_s &\equiv \text{self inductance} \end{aligned} \tag{3.3}$$

where L_s is a little more than M due to the extra inductance of the wires crossing the gaps.

There is also a series insertion impedance as

$$Z_s = [sL_s]//[2\zeta_c] = \left[\frac{1}{sL_s} + \frac{1}{2\zeta_c}\right]^{-1} \simeq sL_s \text{ for low frequencies} \tag{3.4}$$

This needs to be small compared to $2Z_c$ for negligible reflection on the coax being measured. This is consistent with (3.3) if Z_c and ζ_c are comparable.

Concerning accuracy, the considerations in [7] are applicable here.

4. Mutual Capacitance

The capacitance per unit length of the coax being sensed is

$$C' = 2\pi\varepsilon_1 \, \ell n^{-1}\left(\frac{\Psi_{1-}}{\Psi_0}\right) \tag{4.1}$$

A positive voltage V induces a negative charge on the section of length ℓ_V, thereby delivering a positive charge to the recording instrument(s) as

$$\begin{aligned} Q &= C \quad V = \int_{-\infty}^{t} \left[I_1(t') + I_2(t') \right] dt' \\ C &= \ell_V \quad C' \\ I_1 + I_2 &= C \, \frac{dV}{dt} \\ V_1 + V_2 &= \zeta_c \, C \, \frac{dV}{dt} \end{aligned} \tag{4.2}$$

As we can see that this appears in the common mode of the two sensing cables.

For this result to hold the frequency should be low enough that

$$|s| \, C << \frac{2}{\zeta_c} \tag{4.3}$$

noting that the two sensing coaxes of characteristic impedance appear in parallel in this case.

Associated with this voltage measurement there is a parallel insertion admittance. One can estimate this by subtracting the admittance of the unperturbed coax section (admittance $s\,C$) from the admittance of the section as perturbed, giving

$$Y_s + s\,C = \left[\frac{1}{s\,C} + \left[s\,C_{ext} + \frac{2}{\zeta_c} \right]^{-1} \right]^{-1} \tag{4.4}$$

where C_{ext} represents the capacitance of the section of S_0 of length $\ell_V - 2w$ to the other boundaries of the sensing volume. Note that as C_{ext} becomes large the right side of (4.3) $\rightarrow s\,C$ (i.e., no insertion admittance). While the right side of (4.3) is passive (a positive real function), if $s\,C$ is subtracted this becomes active. So it is convenient to include $s\,C$ with Y_s to give the transverse admittance of the section of the main coax with length ℓ_V.

5. Sensor Operation

Section 3 shows that the differential signal $V_1 - V_2$ on the sensing coaxes (with equal delays) is proportional to the time derivative of I. Section 4 shows that the common-mode signal $V_1 + V_2$ on the sensing coaxes is proportional to the time derivative of V. Let us now consider the properties of V_1 and V_2 separately.

Consider waves propagating on the coax. These have the form

$$\frac{V}{I} = \pm Z_c$$

$$+ \Rightarrow \text{ right-propagating wave} \tag{5.1}$$

$$- \Rightarrow \text{ left-propagating wave}$$

For purposes of discussion consider a right-going wave.

Suppose for a right-going wave we could set $V_2 = 0$. (V_1 monitors right-going waves. By symmetry, V_2 monitors left-going waves.) From (3.2) we have

$$V_1 - V_2 = M\frac{dI}{dt} = V_1 = \frac{M}{Z_c}\frac{dV}{dt} \tag{5.2}$$

From (4.2) we have

$$V_1 + V_2 = \zeta_c\, C\frac{dI}{dt} = V_1 \tag{5.3}$$

Combining, we have

$$= \frac{M}{Z_c} = \zeta_c\, C \tag{5.4}$$

Rearranging we have (using (3.1) and (4.1))

$$\frac{M}{C} = Z_c\zeta_c = \frac{\mu_0}{\varepsilon_1}\frac{1}{4\pi^2}\frac{\ell_I}{\ell_V}\,\ell\text{n}\left(\frac{\Psi_2}{\Psi_{1+}}\right)\ell\text{n}\left(\frac{\Psi_{1-}}{\Psi_0}\right) \tag{5.5}$$

as the basic condition to be met.

As a simple (yet practical case) if we have equal characteristic impedances (say 50 Ω) we have

$$Z_c = \zeta_c = \left[\frac{\mu_0}{\varepsilon_1}\right]^{\frac{1}{2}} \frac{1}{2\pi} \ell n\left(\frac{\Psi_{1-}}{\Psi_0}\right) \tag{5.6}$$

In (5.5) this gives

$$\frac{\ell_I}{\ell_V} = \frac{\ell n\left(\frac{\Psi_{1-}}{\Psi_0}\right)}{\ell n\left(\frac{\Psi_2}{\Psi_{1+}}\right)} \tag{5.7}$$

with $\ell_I > \ell_V$ this is readily achievable.

6. Concluding Remarks

So by appropriate attention to the various sensor dimensions one can measure on separate channels (V_1 and V_2) the time derivatives of right- and left-going waves in coaxial transmission lines. Sum and difference signals are proportional to the time derivatives of voltage and current respectively. Of course, one can integrate these signals with respect to time, if desired.

As a wave sensor, accuracy in design of M and C is important. One can measure how well the proper balance has been achieved by sending a wave in one direction on the coax. Then by measuring the signal from the port (1 or 2) which should be zero, one can see how small this is compared to the signal from the other port. Since the formulae are developed under the assumption that the sensor is electrically small for frequencies of interest, the high frequencies may need to be limited for this comparison.

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