Sensor and Simulation Notes

Note 470

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Compact, Low-Impedance Magnetic Antennas

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Abstract

In designing a damped-sinusoid electromagnetic radiator one is sometimes confronted with significant space limitations. This paper explores the design of loop antennas and associated feeds for such applications. We find that subdividing the loop can be sometimes advantageous.

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1. Introduction

If one wishes to radiate large fields from some small hypoband (pulse of approximately single frequency) source/antenna, one has a challenging engineering problem. If the source involves a resonant transformer or Marx generator charging some capacitor, one is concerned about how fast this can be switched into the load (antenna). This switching time limits how high in frequency one can make the oscillation involving the source and antenna [1, 2]. For a small antenna (compared to a radian wavelength) a loop type of antenna is attractive in that in the near field the ratio of electric to magnetic field can be made smaller than the wave impedance, say Z_0, the impedance of free space (or some other dielectric medium). This in turn allows one to radiate more power for a given voltage limitation on the antenna.

Here we look at loops in low-impedance configurations in confined volumes. The object is to maximize the magnetic moment (and its time derivative) of the form m = IA by maximizing the current and loop area. The reader can note that the source occupies some of the volume and thereby can reduce the loop area. Appropriate design of the source (making it some what "transparent" to the fields) can help in this regard.

2. Fitting the Antenna in a Rectangular Parallelepiped Volume

As a canonical example let us consider a rectangular parallelepiped as a volume into which to fit a loop-type antenna. (Various other shapes such as a sphere or truncated circular cylinder might also be considered.) Basically we need to maximize the loop area and minimize the loop inductance L_0 to minimize the stored inductive energy $L_0 I_0^2 / 2$ for a given current I_0

Consider the rectangular parallelpiped with sides a, b, w, and w as the smallest of these as indicated in Fig. 2.1. Then an appropriate choice for the path of the loop currents is along the boundaries of width w giving an equivalent area

$$\vec{A}_{eq}^{(0)} = A_{eq}^{(0)} \vec{1}_z , \quad A_{eq}^{(0)} = ab$$
$$\vec{m}^{(0)} = I_0 \vec{A}_{eq}^{(0)} \tag{2.1}$$

As discussed in various papers [4 and references therein], for a given loop area one can minimize the inductance by maximizing the conductor widths, i.e., equal to w in this case. The inductance of such a single-turn loop can be estimated via [3]

$$L_0 \simeq \frac{\mu_0}{\pi}\left[a\, \ell n\left(\frac{8a}{w}\right) + b\, \ell n\left(\frac{8b}{w}\right) + 2\left[a^2 + b^2\right]^{1/2} - a \operatorname{arcsin} h\left(\frac{a}{b}\right) - b \operatorname{arcsin} \mathrm{h}\left(\frac{a}{b}\right) - 2[a+b]\right] \tag{2.2}$$

$$\mu_o = 4\pi \times 10^{-7}\ H/m = \text{permeability of free space}$$

where the conductor has been assumed perfectly conducting and the circular wire has been replaced by the strip of width w, equivalent to a wire of radius $w/4$. For a square loop $a = b$ and the formula simplifies to

$$L_0 \simeq \frac{2}{\pi}\mu_0\, a\left[\ell n\left(\frac{4a}{\mathrm{w}}\right) - 0.77401\right] \tag{2.3}$$

For comparison the inductance of a circular loop of radius d is [5]

$$L_0 \simeq \mu_0 d\left[\ell n\left(\frac{32d}{w}\right) - 2\right] \tag{2.4}$$

These formulae are valid for $w \ll a,b$.

Note that except for a logarithmic correction the inductance is proportional to the linear dimensions (a or b) while the area is proportional to ab. Thus it is important to make the loop as large as possible (within the limitation of being electrically small). It is in this sense that we wish to fill the available volume with the antenna.

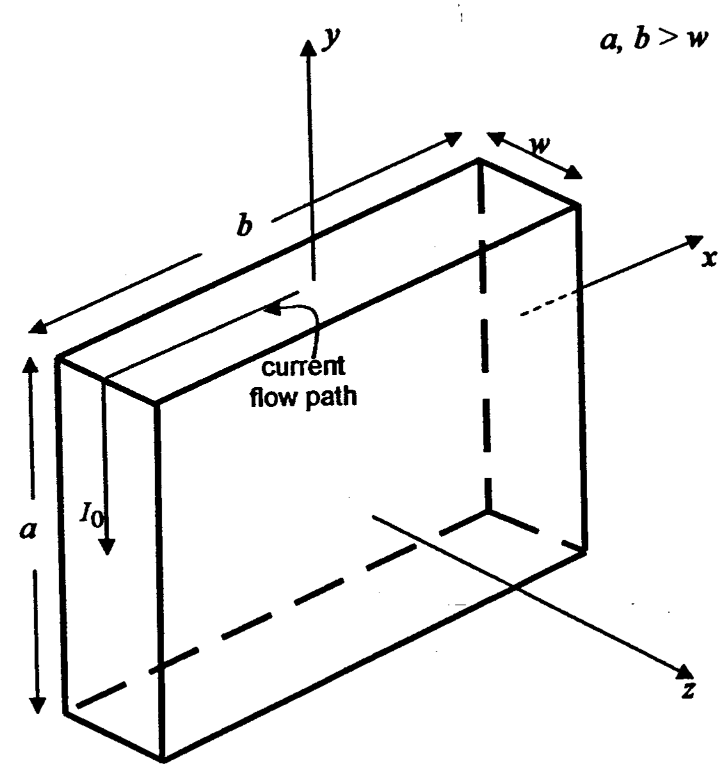


Fig. 2.1 Rectangular Parallelepiped Antenna Volume

3. Subdividing Loop to Lower Impedance

There are various ways to match the loop antenna to a source, e.g., by use of a transformer. For present purposes one may more effectively incorporate this into the loop itself.

As illustrated in Fig. 3.1 one can subdivide the loop into N loops. If we retain I_0 and L_0 as parameters for the overall or collective loop, then we have

$$I = N I_0 \equiv \text{current provided by source}$$

$$L = \frac{L_0}{N^2} \equiv \text{inductance presented to source} \tag{3.1}$$

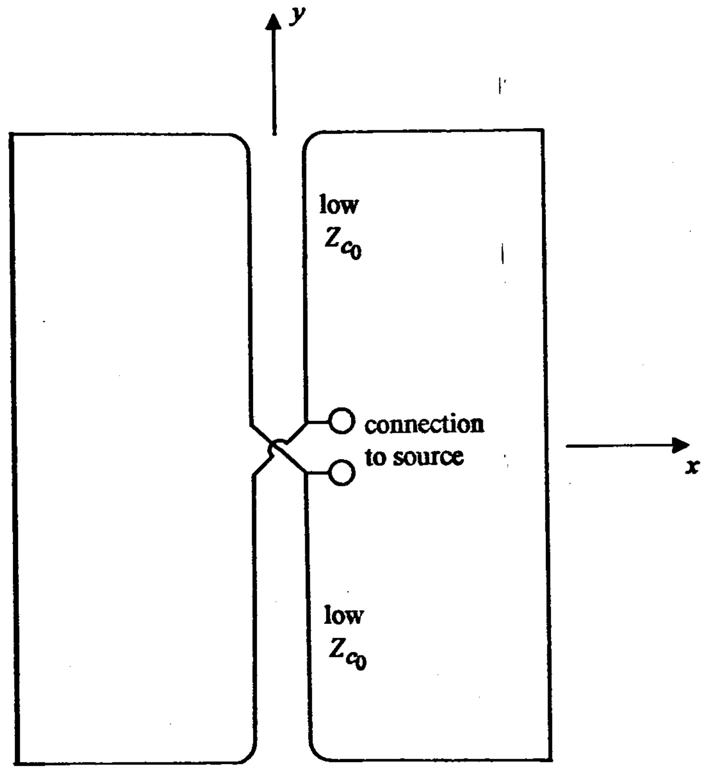
These are very similar to transformer equations, except that now the subdivided loop is the transformer. There is also a correction to the above associated with the leads connecting from the source to the loop perimeter. Such connections can be approximated by transmission lines of characteristic impedance Z_{c_0} and transit time t_0, giving for low frequencies an inductance $t_0 Z_{c_0}$ (ideally small). N of these in parallel give an inductance $t_0 Z_{c_0} / N$ which can be added to L in (3.1) in the form

$$L_1 = L + \frac{t_0 Z_{c_0}}{N} = \frac{L_0}{N^2} + \frac{t_0 Z_{c_0}}{N} \tag{3.2}$$

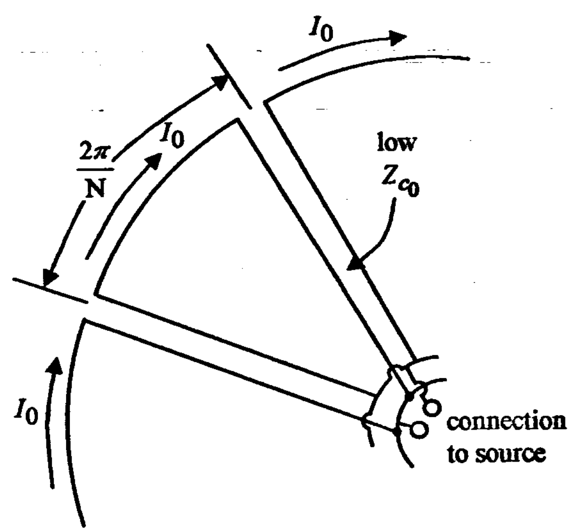
For the simple division of currents and inductances to hold one can enforce various symmetries on the antenna [6]. The point symmetries are of interest here, specifically reflection and two-dimensional rotation. Considering the antenna volume in Fig. 2.1 one can have a transverse (to z) symmetry plane (the xy plane) dividing the small dimension w. This assures that the magnetic moments is of the form $m \vec{1}_z$.

Axial symmetry planes are also possible. For $a \neq b$ there are two possibilities, the xz and yz planes. As in Fig. 3.1A, one might use one such symmetry plane to divide the loop ($N = 2$). This reflection symmetry assures that the currents in the two subloops are the same. Using both symmetry planes can give four subloops with the two reflection planes giving four equal currents. (For high-frequencies such that a and b and the connections are not electrically small, the situation is more complicated.

One can go to larger N using discrete rotation symmetry C_N in which the structure is invariant to rotation about the z axis by an angle



A. N = 2: reflection symmetry R_x



B. General N: C_N symmetry

Fig. 3.1 Subdividing Loop

$$\phi_0 = \frac{2\pi}{N} \tag{3.3}$$

and integer multiples of this. Figure 3.1B illustrates a section of this. Note that the basic loop need not be circular, but could be star-shaped or otherwise as long as the C_N symmetry is preserved. This should be applied to the feed connections as well. If $a = b$ then a C_4 antenna can fill the rectangular parallelpiped in Fig. 2.1, but $N > 4$ will not fill the volume. For $a \neq b$ we can have C_2 symmetry as in Fig. 3.1A, larger N does not fill the volume.

4. Resonant Circuit

The loop antenna is connected to some kind of source. For the present discussion let this be a capacitor of capacitance C, charged to an initial voltage V_0, and switched to the load at time $t = 0$ through an ideal closing switch as indicated in Fig. 4.1. From Section 2 we can construct an equivalent circuit involving both the loop and the transmission-line feeds as in Fig. 4.1A.

For present purposes let us consider the simpler equivalent circuit in Fig. 4.1B. In this case the feed inductance is combined with the antenna inductance as in (3.2). This is a low-frequency approximation in which the feed capacitance is neglected. In this form we can write

$$\tilde{Z}(s) = s L_1 + \frac{1}{sC} = \text{circuit impedance}$$

$$\tilde{V}_s(s) = \frac{V_0}{s} = \text{source voltage}$$

$$\tilde{I}(s) = \frac{V_0}{s\tilde{Z}(s)} = \text{current delivered to load} \qquad (4.1)$$

$$\sim \;\equiv\; \text{two-sided Laplace transform over time } t$$

$$s \equiv \Omega + j\omega \equiv \text{Laplace-transform variable or complex frequency}$$

Defining for convenience

$$\omega_0 = [L_1 C]^{-1/2}$$

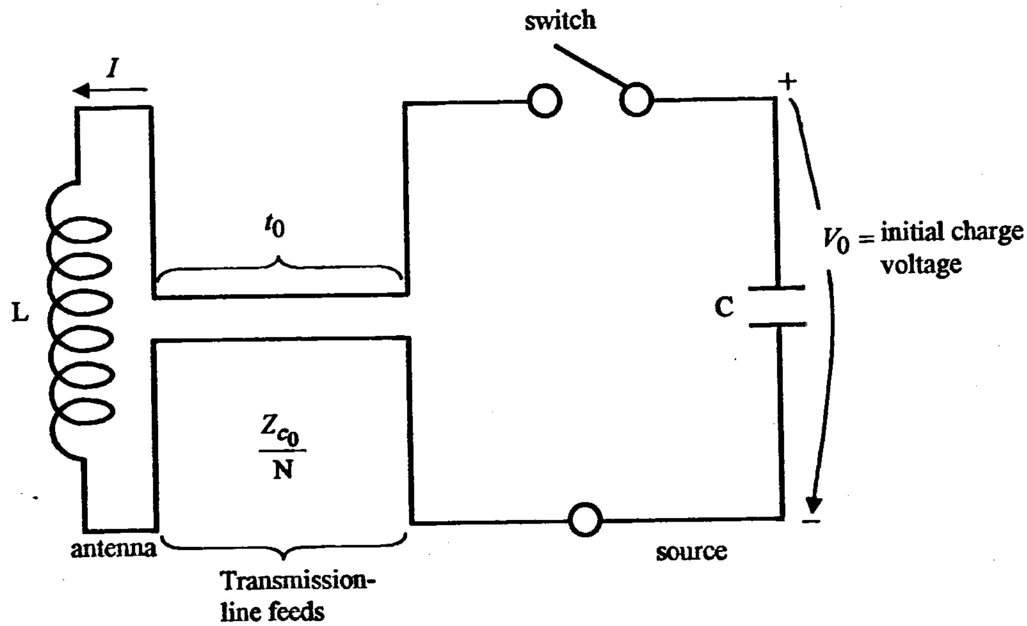
$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi[L_1 C]^{1/2}} \equiv \text{resonance frequency} \qquad (4.2)$$

we have

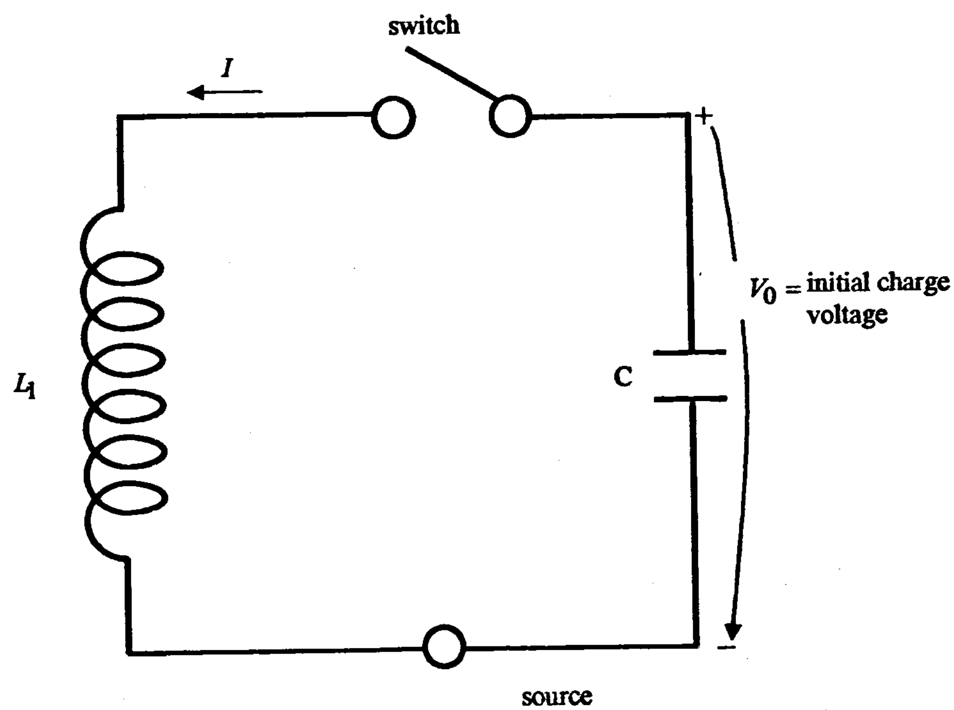
$$\tilde{I}(s) = \frac{C V_0}{1 + s^2\omega_o^{-2}} = \frac{1}{s^2 + \omega_0^2}\frac{V_0}{L_1}$$

$$I(t) = \frac{V_0}{\omega_0 L_1}\sin(\omega_0 t)\,u(t) = \left[\frac{C}{L_1}\right]^{1/2} V_0 \sin(\omega_0 t)\,u(t)$$

$$I_0(t) = \frac{1}{N}\left[\frac{C}{L_1}\right]^{1/2} V_0 \sin(\omega_0 t)\,u(t)$$



A. More complete circuit



B. Simplified (low-frequency) circuit

Fig. 4.1 Equivalent Circuits

$$
\begin{aligned}
I_{0_{max}} &= \frac{1}{N}\left[\frac{C}{L_1}\right]^{1/2} V_0 = \left[\frac{C}{L_0 + N\, t_0\, Z_{c_0}}\right]^{1/2} V_0 \\
\frac{d}{dt} I_0(t) &= \frac{\omega_0}{N}\left[\frac{C}{L_1}\right]^{1/2} V_0 \cos(\omega_0 t)\, u(t) = \frac{V_0}{N\, L_1} \cos(\omega_0 t)\, u(t) \\
\dot{I}_{0_{max}} \equiv \frac{d}{dt} I_0(t)\bigg|_{max} &= \frac{V_0}{N\, L_1} = \frac{V_0}{\frac{L_1}{N} + t_0 Z_{c_0}}
\end{aligned}
\tag{4.3}
$$

These give us some expressions we can explore for optimizing performance.

Consider first the frequency. Here we can see that for a given source capacitance C (with V_0 giving the source energy) we can raise the frequency by lowering L_1. From (3.2) we can see that this is accomplished for a given overall loopsize (hence L_0) by increasing N. So here is one potential advantage in subdividing the loop. Note, however, that one cannot extend these results too high in frequency since the formulae are based on an assumption that the loop and feed are electrically small.

The maximum current $I_{0_{max}}$ in the loop and hence the maximum magnetic moment

$$
m_{max}^{(0)} = I_{0_{max}} A_{e_q}^{(0)} \tag{4.4}
$$

is basically proportional to $[C / L_0]^{1/2} V_0$ for small $N\ t_0\ Z_{c_0}$. For a constrained L_0 then one can increase C but this also lowers frequency unless one compensates by increasing N (lowering L_1). So appropriate choice of N may help here.

The maximum time rate of change of the current, and hence of the magnetic moment

$$
\dot{m}_{max}^{(0)} = \dot{I}_{0_{max}} A_{eq}^{(0)} = \frac{V_0 A_{eq}^{(0)}}{\frac{L_1}{N} + t_0 Z_{c_0}} \tag{4.5}
$$

is significantly increased by increasing N. This is limited in turn by how small one makes $t_0\ Z_{c_0}$.

5. Concluding Remarks

This paper has explored the possibility of loop antennas in confined volumes for radiating pulsed oscillatory waveforms. In particular, there is an advantage in some cases in subdividing the loop. The results need to be tempered due to the approximations used. Besides assuming an electrically small antenna and feed, the radiation resistance and other losses have been neglected. This will damp the oscillation.

The source also needs to be considered. In general it is not just a capacitor, but includes various other components. This also needs to fit in the volume such as in Fig. 2.1. For symmetry, one might prefer it to be located near the center of the volume where the loop feeds come together. The source conductors can also interfere with the magnetic field produced by the loop. To the extent practical such conductors should not form closed current paths allowing currents to circulate around the z axis, thereby reducing the equivalent area of the loop.

Here we have general considerations for optimizing loop design. One could make more detailed models for the various components such as the antenna and feeds, as well as the source.

References

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