Sensor and Simulation Notes

Note 477

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Limited-Angle-of-Incidence and Limited-Time Magnetic Sensors

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Abstract

This paper considers sensors for measurement of fast electromagnetic pulses. Instead of the time derivative of the electromagnetic wave characteristic of electrically small dipoles (electric, magnetic), these have a response proportional to the incident waveform or its time integral. However, these electrically large sensors have limitations concerning constraints on direction of incidence and polarization. Whereas previous results have concerned sensors for the incident electric field, the present paper concerns the dual case of the incident magnetic field.

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1. Introduction

A class of electric sensors, for which knowledge and control of parameters of the incident electric field, was discussed in a previous paper [7]. There control of the direction of incidence of a uniform plane wave was used to define two types of such sensors. One had the output tproportional to the time waveform of the incident field, while the second had it proportional to the time integral of the incident field. The first utilized TEM transmission lines with perfect conductors immersed in the external dielectric medium. The second had two perfectly conducting cones (of possibly general cross sections) with common apex, again immersed in the external dielectric medium.

By comparison with electrically small dipoles (electric and magnetic) which measure a particular component of the incident field (which need not be a single plane wave), this other class of sensors needs constraints on the form of the incident wave in order to have a quantitative relation to a particular component of the incident field. On the other hand, this type gives considerably larger signals to a resistive load (such as the characteristic impedance of a coaxial cable). The price it pays, however, is that the necessary truncation of the TEM transmission line and cones (to give finite, practical size) makes the simple response functions (giving waveform or its integral0 apply for only a short time related to transit times (at light speed in the external dielectric medium) involving the signal access port, the truncation positions, and, in some cases, the direction of incidence of the wave to be measured. So these are appropriate for early-time measurements, but have problems (complex transfer functions) in frequency domain.

A natural question to investigate concerns the possible design of magnetic sensors with similar advantages and limitations. This can be thought of as the dual problem to the electric-sensor design. As one might expect, the Babinet principle leading to the idea of complementary antennas plays a role here.

2. Current Proportional to Incident Magnetic Field

In the spirit of the Babinet principle, now let us construct a TEM transmission line in a ground plane. Figure 2.1 shows a way to do this. Consider an infinite perfectly conducting ground plane, the $y = 0$ plane. Let there be an incident plane wave of the form

$$
\begin{aligned}
\vec{E}^{(inc)}(\vec{r},s) &= E_0 \tilde{f}(s) \vec{1}_p e^{-\gamma \vec{1}_1 \cdot \vec{r}} \\
\vec{H}^{(inc)}(\vec{r},s) &= H_0 \tilde{f}(s) \vec{1}_z e^{-\gamma \vec{1}_1 \cdot \vec{r}} \\
\vec{E}^{(inc)}(\vec{r},t) &= E_0 \vec{1}_p f\left(t - \frac{\vec{1}_1 \cdot \vec{r}}{c}\right) \\
\vec{H}^{(inc)}(\vec{r},t) &= H_0 \vec{1}_z f\left(t - \frac{\vec{1}_1 \cdot \vec{r}}{c}\right) \\
E_0 &= Z_0 H_0 \\
Z_0 &= \left[\frac{\mu_0}{\varepsilon_0}\right]^{1/2} \equiv \text{wave impedance of free space} \\
c &= [\mu_0 \varepsilon_0]^{-1/2} \equiv \text{speed of light} \\
\sim &\equiv \text{two-sided Laplace transform over time } t \\
s &\equiv \Omega + j\omega \equiv \text{Laplace-transform variable or complex frequency} \\
\gamma &\equiv \frac{s}{c} \equiv \text{propagation constant}
\end{aligned}
\tag{2.1}
$$

We have the constraints

$$
\begin{aligned}
&\vec{1}_p \cdot \vec{1}_z = 0 \ , \ \vec{1}_1 \cdot \vec{1}_z = 0 \\
&\vec{1}_1 \equiv \text{direction of incidence} \\
&\vec{1}_p \equiv \text{polarization (of electric field)} \\
&\vec{1}_z \equiv \text{orientation (polarization) of magnetic field (parallel to ground plane)}
\end{aligned}
\tag{2.2}
$$

With the magnetic field parallel to $\vec{1}_z$, then the surface current density on the ground plane is given by

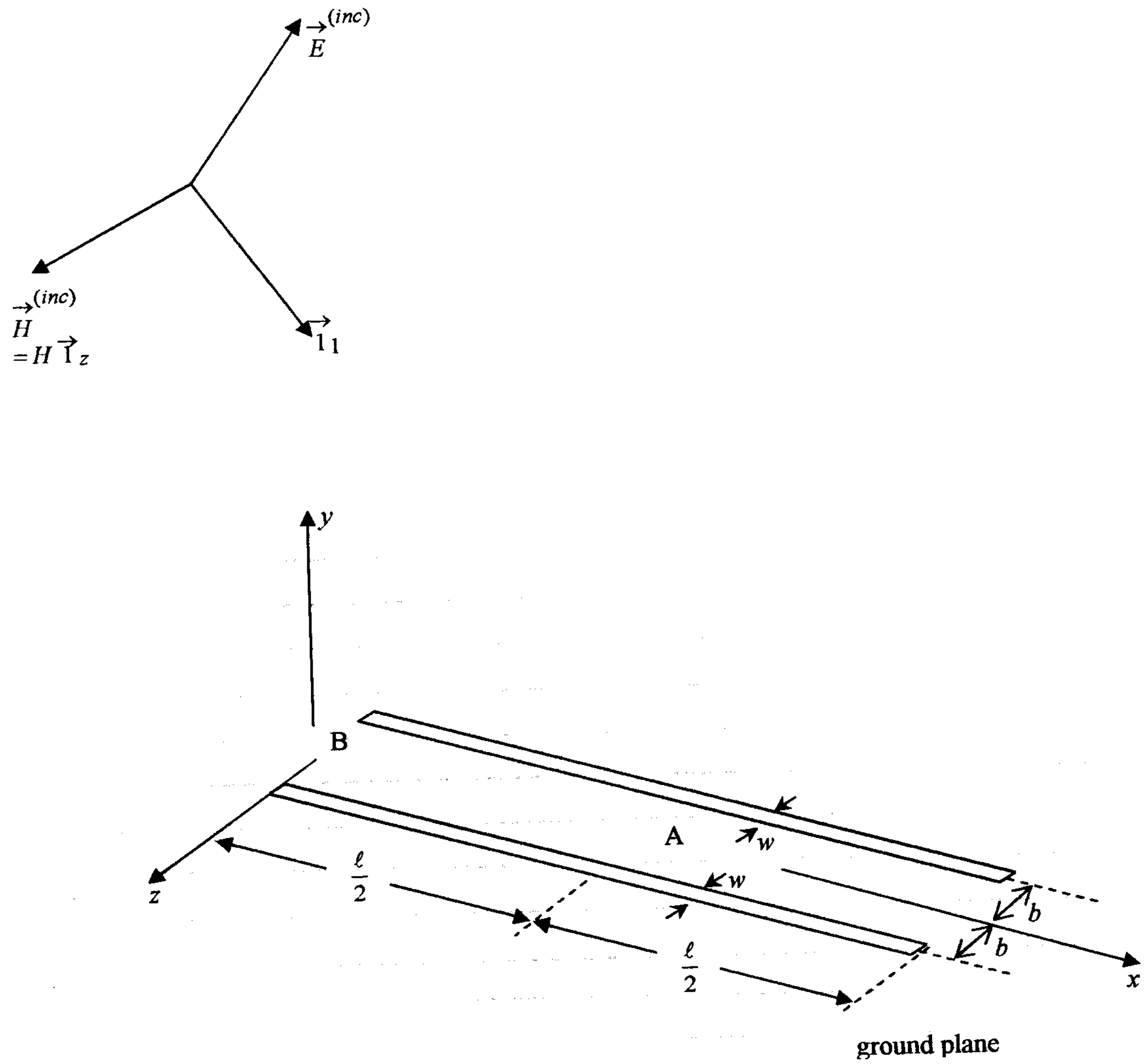


Fig. 2.1. Sensor Structure with Narrow Slots Perpendicular to $\vec{1}_y$

$$\tilde{\vec{J}}_s(\vec{r}_s,s) = 2H_0\,\vec{1}_x\;\tilde{f}(s)e^{-\gamma\vec{1}_1\cdot\vec{1}_x\,x}$$

$$\vec{J}_s(\vec{r}_s,t) = 2H_0\,\vec{1}_x\;f\left(t-\frac{\vec{1}_1\cdot\vec{1}_x\;x}{c}\right)$$

$$\vec{1}_1\cdot\vec{1}_x = \cos(\psi)$$

$$\psi = \text{angle between } \vec{1}_1 \text{ and ground plane} \tag{2.3}$$

$$\vec{r}_s = x\,\vec{1}_x + z\,\vec{1}_z = \text{coordinate on ground plane}$$

The surface current density being parallel to the x axis, then we can cut *thin* slots oriented in this same direction in the ground place without significantly disturbing the flow of the surface current density.

Figure 2.1 shows the first stage in the design of this sensor. There are two slots, each of width w, separated a distance 2b between slot centers. This gives what we can call a short-circuit current density on the strip of

$$\begin{aligned}
\tilde{I}_{s.c.}(x,5) &= 2b\;\vec{1}_x\cdot\tilde{\vec{J}}_s(\vec{r}_s,s)\\
&= 4bH_0\tilde{f}(s)e^{-\cos(\psi)x}\\
I_{s.c.}(x,t) &= 4bH_0 f\left(t-\frac{\cos(\psi)x}{c}\right)\\
&\quad w \ll b
\end{aligned} \tag{2.4}$$

Note that the waveform of the incident field is preserved, with a simple time delay depending on location x along the strip where one may wish to sample this current. In Fig. 2.1 two locations, B at the beginning, and A halfway ($x=\ell/2$) along the strip, are indicated. For simplicity let us remove this time delay and reference the waveform to the time of arrival of the wave at the measurement location giving

$$I_{s.c.}(t) = 4bH_0 f(t) \tag{2.5}$$

Note now that $I_{s.c.}(t)$ is not a function of ψ, but it does depend on the special choice of the orientation $\vec{1}_z$ of the magnetic field. Other orientations in general induce a surface-current-density component in the z direction, perpendicular to the slots. In such a case the slots greatly disturb the surface current density. For the special case that the projection of the incident magnetic field on the ground plane is still parallel to $\vec{1}_z$ the surface current density is still parallel to $\vec{1}_x$ but this requires a change in $\vec{1}_1$ making it not parallel to the xy plane. For present purposes we neglect this special case.

This strip forms a transmission line which can propagate signals along it using the portions outside the two slots as a reference conductor. Figure 2.2A shows a cross section of the strip transmission line. In calculating the characteristic impedance of such a transmission line, a conformal transformation involving a complex potential

$$
\begin{aligned}
w(\zeta) &= u(\zeta) + jv(\zeta) \\
\zeta &= y + jz \equiv \text{complex coordinate}
\end{aligned}
\tag{2.6}
$$

can be used. With u_0 as the potential of the strip (and 0 for the reference conductor) we have

$$Z_c = Y_c^{-1} = \frac{u_0}{4v_0} Z_0 \tag{2.7}$$

where $4v_0$ is the change in the stream function in going around the strip. The complementary problem in Fig. 2.2B has two strips, each of width w, separated by an average distance of 2b. In this case the complementary structure has a characteristic impedance of

$$Z_c^{(c)} = Y_c^{(c)^{-1}} = \frac{2v_0}{2u_0} Z_0 = \frac{v_0}{u_0} Z_0 \tag{2.8}$$

where $2v_0$ represents the potential between the strips and $2u_0$ represents the change in u in going completely around one of the strips. Thus we have

$$Z_c Z_c^{(c)} = \frac{Z_0^2}{4} = \left[Y_c Y_c^{(c)}\right]^{-1} \tag{2.9}$$

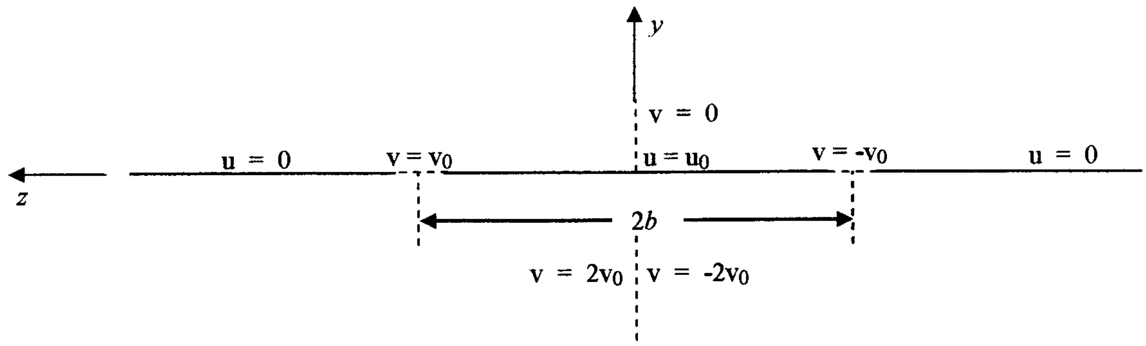
which is the relationship of the complementary parameters.

We now approximate the two-strip transmission line by a two-wire transmission line, each wire being of radius [8]

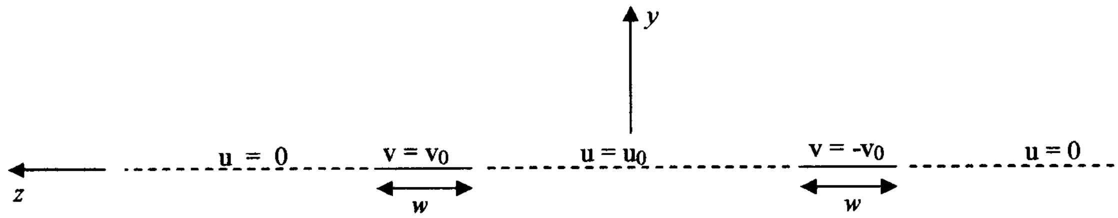
$$a = \frac{w}{4} \tag{2.10}$$

appropriate for $w << b$, as indicated in Fig. 2.2C. The characteristic impedance is then [1]

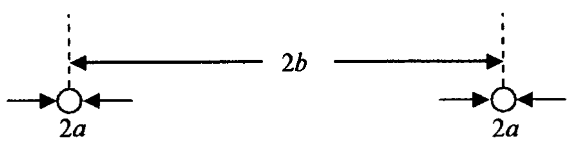
$$Z_c^{(c)} \simeq \frac{Z_0}{\pi} \ell n\left(\frac{b}{a} + \left[\left[\frac{b}{a}\right]^2 - 1\right]^{1/2}\right) \simeq \frac{Z_0}{\pi} \ell n\left(\frac{2b}{a}\right) \tag{2.11}$$



A. Strip transmission line



B. Two-strip transmission line (complement)



C. Approximate equivalent two-wire transmission line

Fig. 2.2. Strip Transmission Line

Hence, we have for the original strip transmission line

$$Y_c = Z_c^{-1} \simeq \frac{4Z_0}{\pi} \ell n\left(\frac{2b}{a}\right) \tag{2.12}$$

Note that the metal sheet is assumed to be thin compared to w; otherwise the characteristic admittance will be increased somewhat.

Next insert a resistive load R at a break in the strip as indicated in Fig. 2.3. This resistance is taken as the characteristic impedance of a coaxial cable which loads the port opened in the strip. This cable shield is bounded to the strip (preferably on the side away from the incident field) for the case of the port in the middle of the strip (Fig. 2.3A); it continues at the strip end bonded to the ground plane. In this case we need

$$R = \frac{Z_c}{2} \tag{2.13}$$

if we wish to match the coaxial cable to the two halves of the strip (in parallel). This matched case has a current into the cable of

$$I(t) = \frac{1}{2} I_{s.c.}(t) \quad \text{for} \quad t < \frac{\ell}{c} \tag{2.14}$$

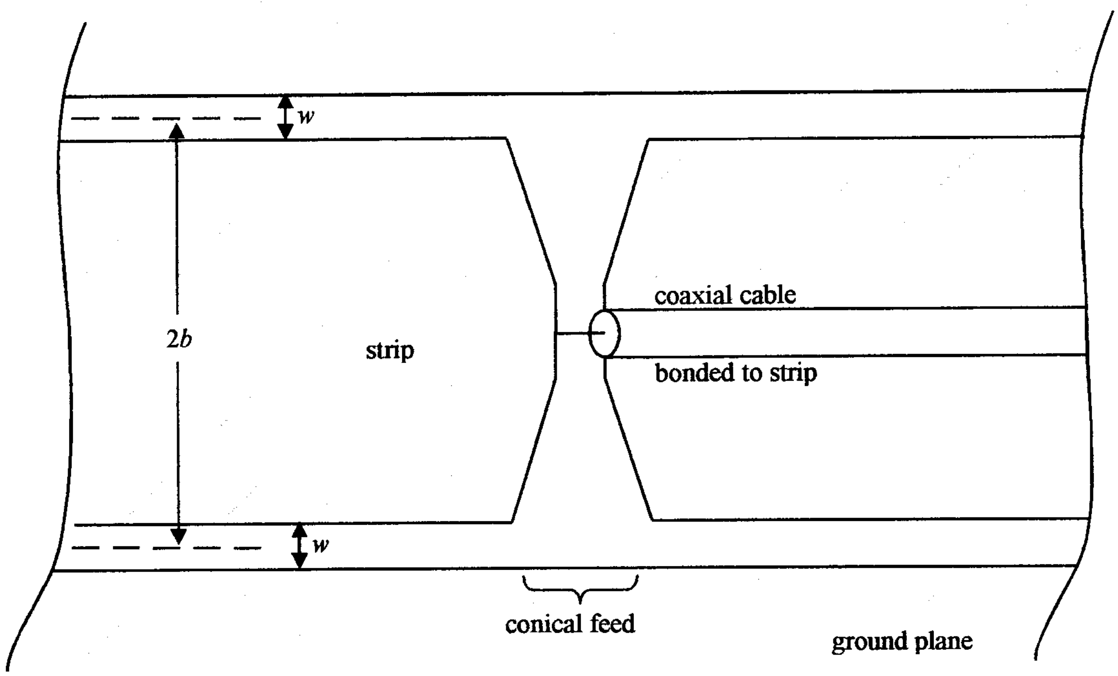
The time ℓ / c is when the reflections arrive at the port simultaneously from both ends. The conical feed into the cable also has ideally a characteristic impedance of R which can be readily calculated from the formulas and tables in [3]. Other mismatched choices can also used for which we have

$$\begin{aligned} I(t) &= \left[1+\frac{2R}{Z_c}\right]^{-1} I_{s.c.}(t) \\ V(t) &= R\,I(t) = \text{voltage into coaxial cable} \end{aligned} \tag{2.15}$$

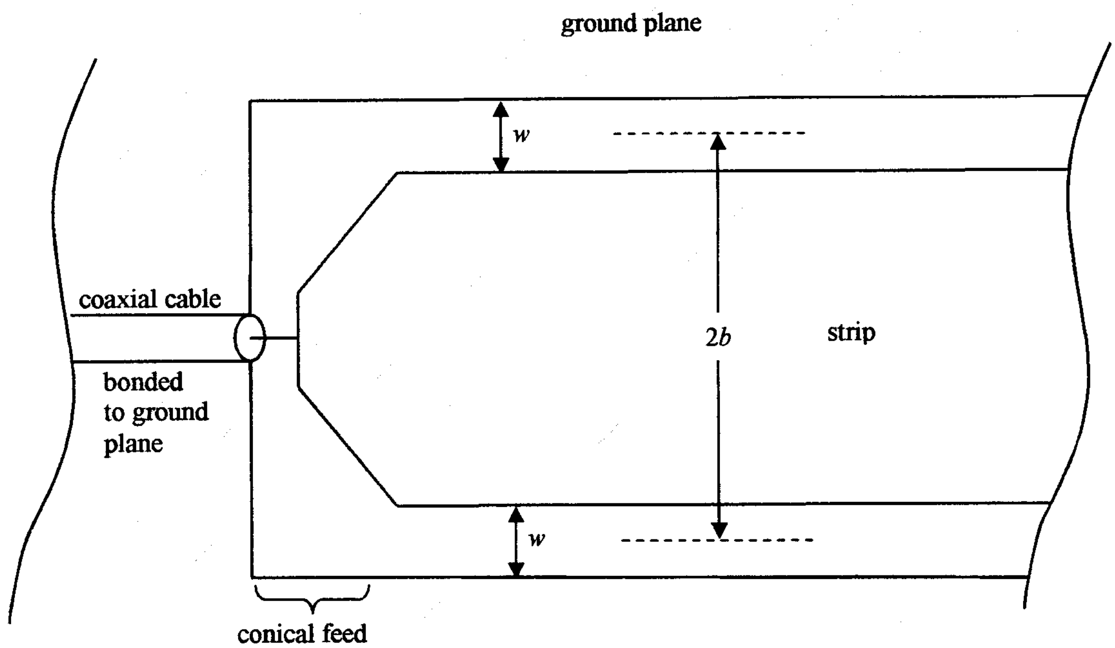
In this case the reflections from the strip ends do not terminate in the coax, but begin a series of multiple reflections.

For the case of the port at the end of the strip (Fig. 2.3B), the cable shield is bonded to the ground plane for removing the signal to the recorder. In this case we need

$$R = Z_c \tag{2.16}$$



A. Port in middle of strip (position A)



B. Port at beginning (or end) of strip (position B).

Fig. 2.3. Construction of Signal Port in Strip Transmission Line

to match the cable to the strip. This matched case has a current into the cable of

$$I(t) = \frac{1}{2} I_{s.c.}(t) \text{ for } t < \frac{2\ell}{c} \tag{2.17}$$

twice as long as the previous case. The conical feed can also be made to have characteristic impedance R. Using the complementary relationship (as in (2.9)), one can solve for the characteristic impedance using stereographic and conformal transformations given in [6]. Other choices of R give

$$I(t) = \left[1 + \frac{R}{Z_c}\right]^{-1} I_{s.c.}(t) \tag{2.18}$$

again with multiple reflections from the far end of the strip.

An interesting special point concerns which end of the strip one should use for the antenna port. Both ends in principle work equally well. (At the highest frequencies the design of the port may favor a particular end, based on the direction of incidence.) This raises the possibility that both ends can be used for ports, giving two signals (opposite polarities). This extra information tells something about the direction of incidence.

3. Current Proportional to Time Integral of Incident Magnetic Field

The second kind of sensor discussed here is based on a biconical slot in the ground plane with signal port at the apex as in Fig. 3.1. One might call this a symmetrical bow-tie slot antenna. A previous paper [4] considered an electrically small (short) slot as a means of measuring the time derivative of the surface current density. The reader can go there to find detailed discussion of this case, including the Babinet equivalence which need not be repeated here.

For present purposes we let the complementary bow-tie antenna be electrically large. In time domain this can be analyzed as an infinitely large biconical slot, valid for a time before scattering from the truncations can reach the signal port A at the cone apex. As one can see in Fig. 3.1 this comes from the incident plane wave scattering from an end of the antenna, the incident wave reaching there generally before any wave scattering from the antenna port.

This antenna need not have three orthogonal symmetry planes (*xy, yz, zx*). Various asymmetrical (two-dimensional asymmetry) geometries are possible. An example with two symmetry planes (*xy, zx*) is shown in Fig. 3.2. For such shapes, let us constrain $\vec{1}_1$ to have

$$\vec{1}_1 \cdot \vec{1}_z = 0 \quad (\vec{1}_1 \text{ parallel to } xy \text{ plane}) \tag{3.1}$$

making the scattering from both slot ends arrive at the port at the same time. For small ψ (in (2.3)) the design in Fig. 3.2 gives more clear time (approaching $2\ell / c$), but as ψ increases toward π this clear time for the measurement becomes quite small. On the other hand the more symmetric design in Fig. 3.1 makes the clear time the same for $\psi = \psi_0$ and $\psi = \pi - \psi_0$. For small conical slot half angle θ_h this clear time is approximately independent of ψ, a favorable feature.

The sensitivity of the antenna is proportional to the magnetic field parallel to the *z* axis. So let us choose the polarization such that

$$\vec{1}_h = \vec{1}_z \tag{3.2}$$

as in (2.1), making the incident electric field parallel to the xy plane.

The characteristic impedance of the biconical slot can be found in [3]. For small θ_h this is

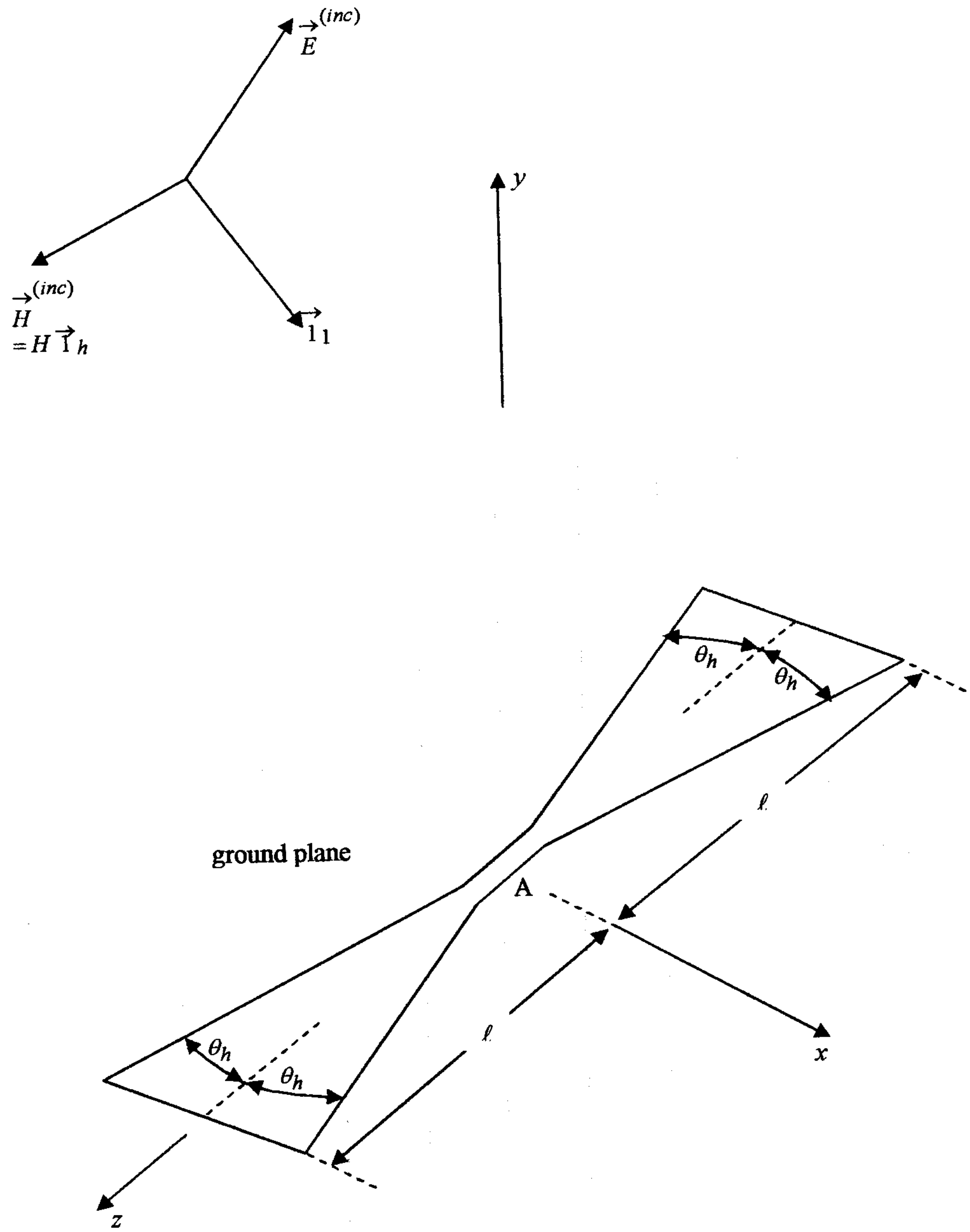


Fig. 3.1. Symmetrical Bow-Tie Slot Antenna: Three Symmetry Planes

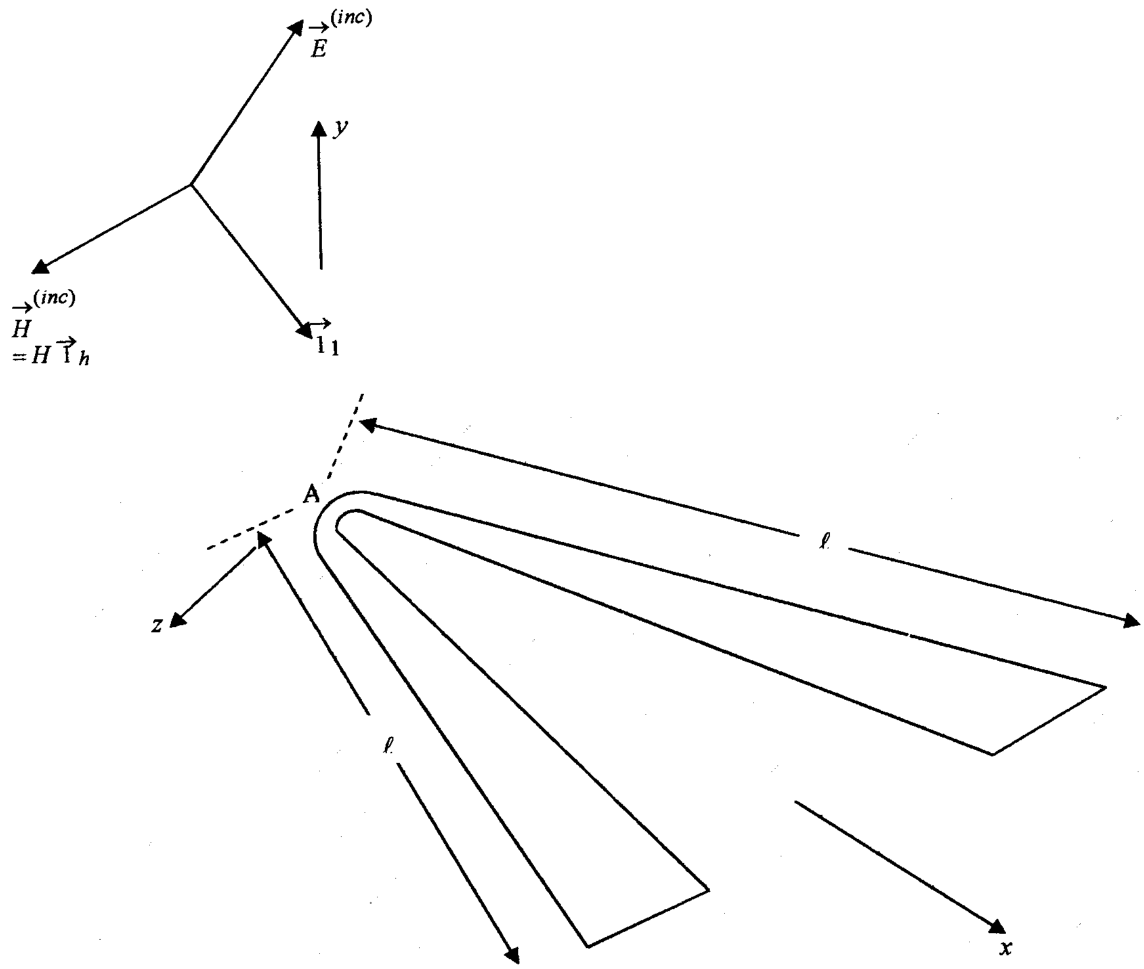


Fig. 3.2. Conical Slot Antenna with Two Symmetry Planes

$$Z_c \simeq \frac{\pi Z_0}{4}\left[\ell n\left(2\cot\left(\frac{\theta_h}{2}\right)\right)\right]^{-1} \simeq \frac{\pi Z_0}{4}\ell n^{-1}\left(\frac{4}{\theta_h}\right) \tag{3.3}$$

Note also that this is approximately the same as a circular bicone with half-cone angle $\theta_h/2$ as in the equivalence in (2.10). From (2.9) we have the dual or complementary characteristic impedance for later use as

$$Z_c^{(d)} = \frac{Z_0^2}{4Z_c} \simeq \frac{Z_0}{\pi}\ell n\left(2\cot\left(\frac{\theta_h}{2}\right)\right) \simeq \frac{Z_0}{\pi}\ell n^{-1}\left(\frac{4}{\theta_h}\right) \tag{3.4}$$

valid for small θ_h.

At this point we can note the desirability of small θ_h. Not only does this occur for matching (not essential) to typical coax impedances (e.g., 50 Ω), but small θ_h makes the response approximately independent of $\psi(\vec{1}_1$ within the constraints). We can analyze the reception properties in this case as that of a symmetrical bicone antenna of cone half-angle $\theta_h/2$, in its response to an incident electric field in the complementary problem [11].

Following [11] we define dual fields via

$$\vec{E}^{(d)}(\vec{r},t) = Z_0\vec{H}_s(\vec{r},t) \quad , \quad \vec{H}_s^{(d)}(\vec{r},t) = -\frac{1}{Z_0}\vec{H}_s(\vec{r},t) \tag{3.5}$$

Taking symmetric and antisymmetric parts with respect to the ground plane ($z = 0$), and evaluating the incident fields on this plane leads to a solution for the slot response in terms of the complementary bicone responding to the dual electric field. From [9, 10] we have

$$\begin{aligned}
\tilde{I}_{s.c.}(s) &= 2\vec{H}^{(inc)}(\vec{r},s)\cdot\vec{h}_e^{(d)}(\vec{1}_1,s)\\
&= 2H_0\tilde{f}(s)\ \tilde{h}_e^{(d)}(\vec{1}_1,s)\\
\tilde{h}_e^{(d)}(\vec{1}_1,s) &= \text{effective height of complementary bicone antenna}\\
\tilde{V}_{o.c.}(s) &= Z_c\,\tilde{I}_{s.c.}(s)
\end{aligned} \tag{3.6}$$

Antennas in transmission are characterized by

$$\vec{E}_f(\vec{r},s) = \frac{e^{-\gamma r}}{r}\vec{F}_V(\vec{1}_r,s)\tilde{V}(s) \tag{3.7}$$

For a bicone of half angle $\theta_h / 2$ we have (in the direction - $\vec{1}_1$) [2]

$$F_V^{(d)}(-\vec{1}_1) \simeq \left[2\, \ell n\left(\cot\left(\frac{\theta_h}{4}\right)\right)\right]^{-1} \simeq \frac{1}{2}\, \ell n^{-1}\left(\frac{4}{\theta_h}\right) \tag{3.8}$$

From reciprocity [5] we have

$$\begin{aligned}
F_V^{(d)}(-\vec{1}_1) &= s\frac{u_0}{4\pi}\vec{h}_1^{(d)}(\vec{1}_1, s) = \frac{s\mu_0}{4\pi Z_c^{(d)}}\vec{h}_e^{(d)}(\vec{1}_1, s) \\
\vec{h}_e^{(d)}(\vec{1}_1, s) &= \frac{4\pi Z_c^{(d)}}{s\mu_0} F_V^{(d)}(-\vec{1}_1) = \frac{\pi Z_0^2}{s\mu_0\, Z_c} F_V^{(d)}(-\vec{1}_1) \\
&\simeq 2\frac{c}{s}
\end{aligned} \tag{3.9}$$

which is *quite simple*, showing the property of a temporal integrator, independent of $\vec{1}_1$. Continuing we have

$$\begin{aligned}
\tilde{I}_{s.c.}(s) &= 2H_0\tilde{f}(s)\tilde{h}_e^{(d)}(s) \simeq 4\, H_0\frac{c}{s}\tilde{f}(s) \\
\tilde{V}_{s.c.}(s) &= 2H_0\tilde{f}(s)Z_c\,\tilde{h}_e^{(d)}(s) \simeq 4\, H_0\frac{c}{s}\tilde{f}(s) \\
&\simeq \pi E_0 \ell n^{-1}\left(\frac{4}{\theta_h}\right)\frac{c}{s}\tilde{f}(s) \\
\tilde{V}(s) &= \frac{R}{R+Z_c}\tilde{V}_{o.c.}(s) \\
&= \frac{1}{2}\tilde{V}_{o.c.}(s) \text{ for matched case}
\end{aligned} \tag{3.10}$$

R $\equiv$ resistive load at antenna port provided by terminated coaxial cable

In time domain then we have

$$\begin{aligned}
V(t) &\simeq \frac{R}{R+Z_c} 4\, E_0 Z_c\, \varepsilon_o \int_0^t f(t')dt' \\
&\simeq \frac{R}{R+Z_c}\pi\, E_0 c\, \ell n^{-1}\left(\frac{4}{\theta_h}\right)\int_0^t f(t')dt'
\end{aligned} \tag{3.11}$$

While one can write the result in terms of E_0, it is still the z component of the incident magnetic field (independent of $\overrightarrow{1}_1$) to which the sensor responds.

Again this assumes that the port signal begins at $t = 0$, and we limit times to $t < \ell / c$ (for small θ_h). It may be convenient to leave formulas in terms of Z_c. This is tabulated in [3] for various choices of Z_c (e.g., 50 Ω has $\theta_h \simeq 0.617°$, 100 Ω has $\theta_h \simeq 11.85°$). For typical impedances θ_h is rather small, consistent with our small-angle approximation.

4. Concluding Remarks

So now we have a class of sensors, dual to those discussed in [7], for limited angle of incidence and limited time, suitable for early-time pulse measurements. The reader can then consult [7] for more details.

Note that various approximations are involved in some of the formulae. One can use more exact expressions from which these are derived, or restrict the parameter range appropriately. Thereby, one can make the sensor response accurately calculable. There are also physical assumptions concerning the construction of such sensors. For example, the ground plane has been assumed to have negligible thickness; a thick ground plane will lower the characteristic impedances associated with the slot antennas.

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