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Transmission-Line Antennas for Sparse Dielectric Airfoils

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Abstract

This paper considers the design of TEM-transmission-line antennas as pulse receptors/transmitters. Such are suitable for mounting on sparse dielectric airfoils. Special results are found for boresight antenna response. Certain symmetrical cross sections are considered for their characteristic impedances.

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1. Introduction

The placing of antennas on aircraft is a complicated matter, depending on frequencies and pulses of interest [4, 8, 10, 11]. This paper considers another type of antenna for reception/transmission of fast pulses. This utilizes dielectric airfoils as a suitable place to locate such antennas.

The type of antenna considered is a length ℓ of TEM transmission line. The conductors are mounted on or inside a sparse dielectric airfoil (sparse so as not to interfere significantly with the antenna performance). With the response in reception well understood, the reciprocity theorem is used to characterize its response in transmission, leading to an interesting general result. Then specific transmission-line geometries are considered.

2. Combining Two-Dimensional Conducting Geometries with Sparse Dielectric Structures

Consider adding conductors (wires, strips, etc.) to a mechanical dielectric structure of approximately two-dimensional shape such as an airfoil. Let this be sparse (for low weight) and let the average dielectric constant be near to (slightly greater than) 1.0 (free space, $\varepsilon \simeq \varepsilon_0$) with permeability μ_0. The added conductors can be quite light, such as foil, while giving two-dimensional cross sections of transmission line with characteristic impedance of nearly whatever one may wish.

Consider as in Fig. 2.1 that we have the cross section of an airfoil-like dielectric structure. Somewhere near the center, or better position of maximum thickness h, place four strip conductors (foil, paint, etc.), two on the upper surface (inside or outside), and similarly two on the lower surface. To the degree practical try to locate them symmetrically with respect to an approximate symmetry plane as indicated. By appropriate connection of these strips in pairs these can be made to transmit/receive in both vertical and horizontal polarizations. Here the degree to which symmetry can be achieved will determine the degree to which polarization purity (orthogonality of the two polarizations) can be achieved [12].

Figure 2.2 shows the same configuration from a top view. The airfoil connects to some structure such as a fuselage at which location the antenna terminals for the four conductors appear for connection to transmitters/receivers. The two polarizations are ideally operated each in differential mode. The reference conductor(s) (ground) are on the equipment in the fuselage and include the fuselage itself if conducting. If desired, this reference can also be extended out in the airfoil, appropriately positioned (e.g., by symmetry) between the four antenna conductors. This may help reduce common-mode signals.

Since the fuselage and internal equipment will in general scatter fields back toward the antenna(s), one needs to account for this. In one approach a dielectric fuselage skin may be made to have small scattering, but the internal conductors will need to be made sparse and/or far enough back from the antenna port(s) (connections to the port(s) using symmetry like the possible reference conductor in Fig. 2.1). Alternately one may move the antenna port(s) (connections to the transmission line) farther out on the wing to obtain some transit-time isolation from the fuselage.

With a length ℓ of the antenna conductors one needs to be concerned with the potentially dispersive character of the airfoil. Ideally the propagation speed on the transmission lines is nearly

$$c = [\mu_0 \varepsilon_0]^{-1/2} \equiv \text{speed of light in vacuo} \tag{2.1}$$

This implies that the dielectrics have low dielectric constants and/or small cross sections compared to h. If one estimates some propagation speed

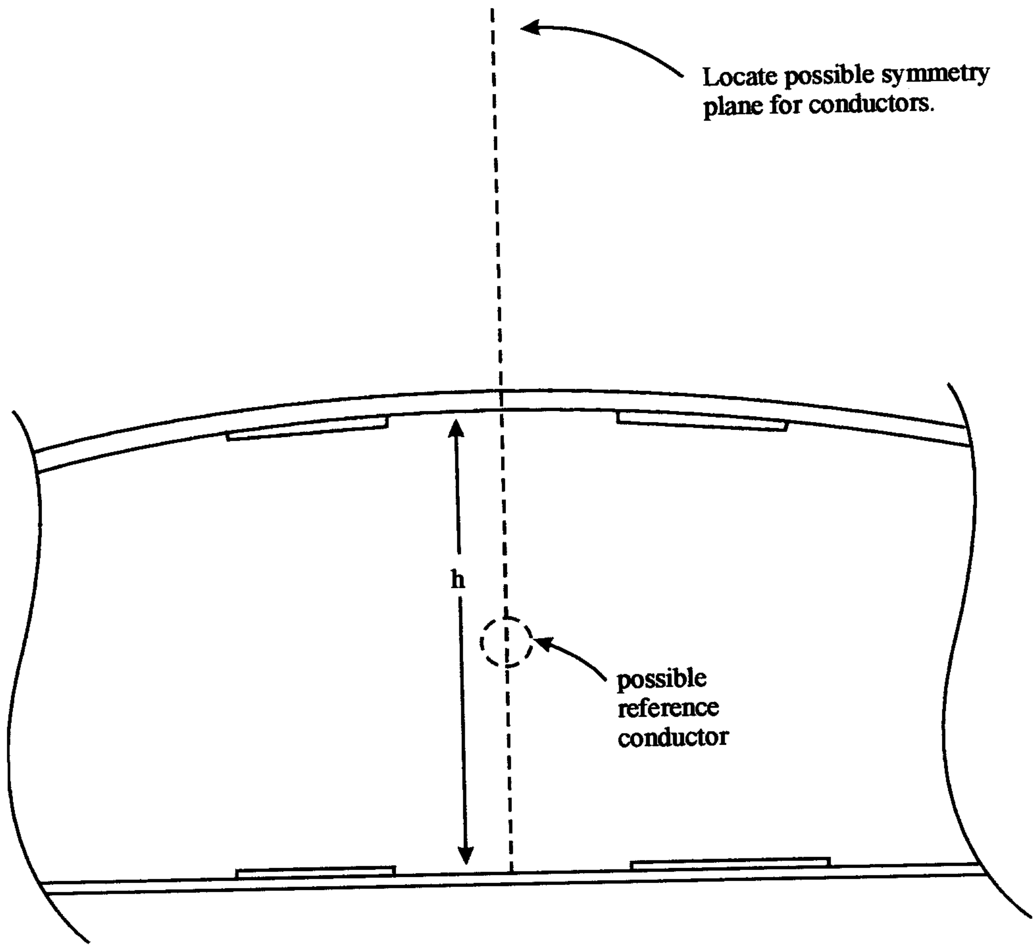


Fig. 2.1 Four Strip Conductors for Dual-Polarized Antennas on Dielectric Airfoil.

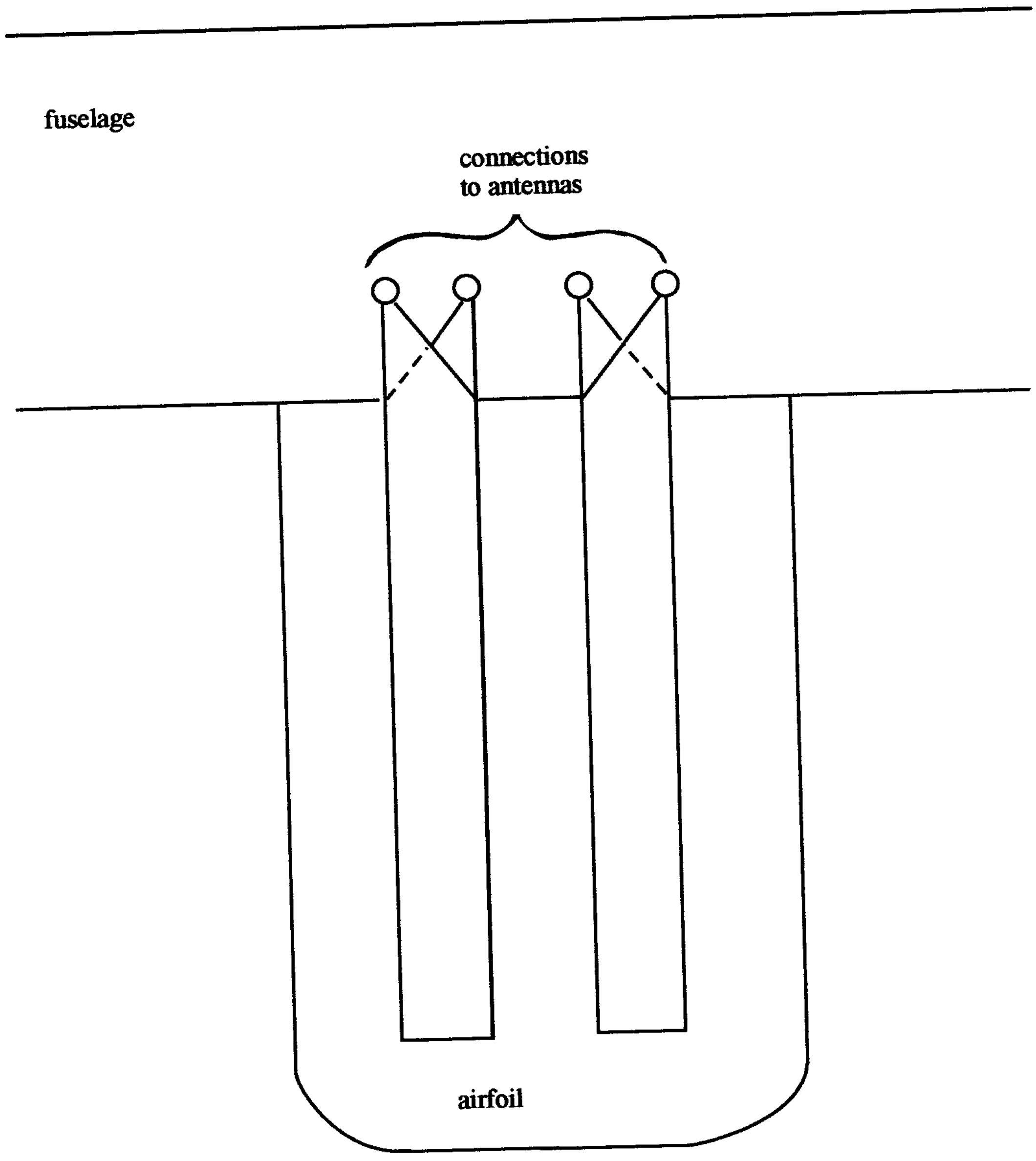


Fig. 2.2 Conductors on Airfoil: Top View

$$v < c \tag{2.2}$$

then one can compute

$$t_d = \frac{\ell}{v} - \frac{\ell}{c} \tag{2.3}$$

as some dispersion time which should be less than characteristic times in pulses of interest. There may also be conductors in the airfoil (e.g., to operate control surfaces). In this case currents on such conductors may need to be suppressed (e.g., by chokes such as ferrites).

Here we consider the case that the antenna conductors form uniform transmission lines, for simplicity of response and analysis. In some cases, however, it may be necessary (or even desirable) to taper these transmission lines.

3. Limited-Angle-of-Incidence, Limited-Time Electric Sensors in Transmission

A previous paper [7] has considered parallel-plane transmission lines in receptions. This begins with the recognition that, as in Fig. 3.1, an incident electric field parallel to the y axis has no scattering from perfectly conducting sheets on planes of constant y. Defining an open circuit voltage by

$$V_{o.c.} = -\int_{\frac{h}{2}}^{\frac{h}{2}} E_y^{(inc)}\Big|_{x,z\ =\ \text{const.}} dy \quad = -h\, E_y^{(inc)} \tag{3.1}$$

we have a fundamental property of the antenna response. For convenience we can take the integration path on $(x,z) = (0,0)$.

At this point we can note that the shapes of the top and bottom plates are arbitrary. They need not even have the same shapes.

Letting the direction of incidence be denoted by $\vec{1}_i$, we have the constraint

$$\vec{1}_i \bullet \vec{1}_y = 0 \tag{3.2}$$

which still allows $\vec{1}_i$ to vary over an angle of 2π (360°), with the same (3.1) result. This defines an effective-height vector of

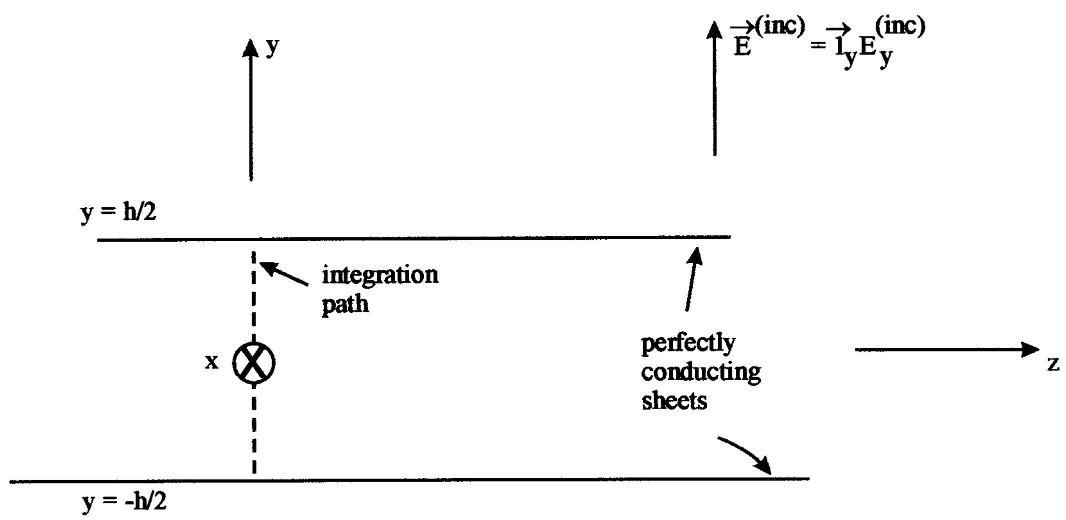
$$\vec{h}_V(\vec{1}_i) = -h\ \vec{1}_y \tag{3.3}$$

independent of frequency, with

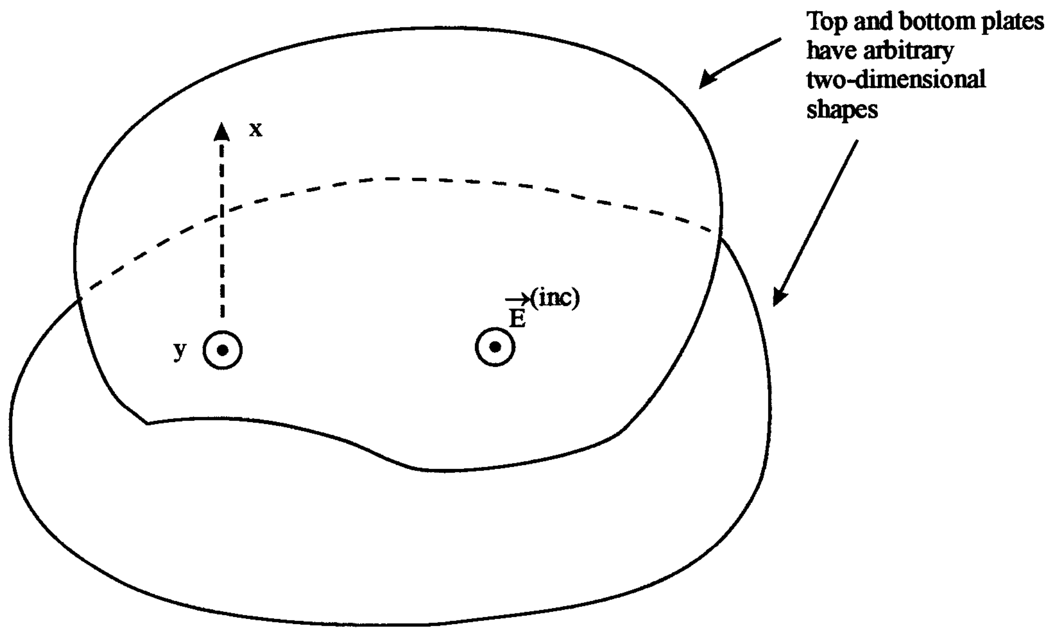
$$V_{o.c.} = \vec{h}_V(\vec{1}_i) \bullet \vec{E}^{(inc)} \tag{3.4}$$

applying in both frequency and time domains.

Appealing to the antenna reciprocity theorem [6] we have the far field radiated by a current source I_s on the same path as



A. Side View



B. Top View

Fig. 3.1 Incident Electric Field Perpendicular to Perfectly Conducting Sheets

$$\tilde{\vec{E}}_f(\vec{r},s) = \frac{e^{-\gamma r}}{r} \tilde{\vec{F}}_I(\vec{1}_0,s)$$

$$\vec{1}_0 = -\vec{1}_i$$

$$\tilde{\vec{F}}_I(\vec{1}_0,s) \cdot \vec{1}_0 = 0$$

$$\gamma \equiv \frac{s}{c}$$

$$\sim \; \equiv \text{two-sided Laplace transform over time } t \tag{3.5}$$

$$s \;\; \Omega + j\omega \; \equiv \text{Laplace-transform variable or complex frequency}$$

$$\tilde{\vec{F}}_I(\vec{1}_0,s) = -s\frac{\mu_0}{4\pi} \vec{h}_V(\vec{1}_i,s)$$

$$= s\frac{\mu_0}{4\pi} h \; \vec{1}_y$$

which is a very simple result. Interpreting this, let the incident field be of the form of a step function, producing a step-function open-circuit voltage. A step-function source current then produces a far field that approximates a delta function (similar to the discussion in [5]).

Note that the shape of the two plates does not affect the result. This is limited to $\vec{1}_i$ and $\vec{1}_0$ transverse to the $\vec{1}_y$ direction. Ideally the far-field observer is on or near the $y = 0$ plane. In any event for a fixed y, as the observer goes to infinity the direction of propagation asymptotically approaches a perpendicular to $\vec{1}_y$.

An alternate approach to this results notes that in transmission, no matter what are the currents on the two sheets, they are perpendicular everywhere to $\vec{1}_y$. The currents parallel to $\vec{1}_0$ do not contribute to the far-field integrals. The currents perpendicular to $\vec{1}_0$ must also give zero far-field integrals to satisfy reciprocity.

Now whence comes the more complicated properties of such a radiating antenna? These appear through the antenna impedance

$$\tilde{Z}_a(s) = \frac{\tilde{V}_a(s)}{\tilde{I}_a(s)} = \frac{\text{voltage at antenna port in transmission}}{\text{current into antenna port in transmission}} \tag{3.6}$$

when coupled to a source impedance and some associated voltage or current source. Here is where shape of the conducting sheets is important.

If we now choose the top and bottom plates in the form of a transmission line of characteristic impedance Z_c, we then have

$$\tilde{Z}_a(s) = Z_c \frac{1 + e^{-2st_r}}{1 - e^{-2st_r}}$$

$$t_r \equiv \frac{\ell}{v} \equiv \text{transit time of transmission line} \qquad (3.7)$$

This assumes that the cross-section of the line is sufficiently small compared to both ℓ and wavelengths of interest. There are also special things one can do near the antenna port to improve the high-frequency (short-time) performance [7].

In an early-time sense, before reflections from the open end reach the antenna port at $t = 2t_r$, we can think of $\tilde{Z}_a$ as a constant Z_c. This gives a particularly simple antenna response provided that the source impedance is also a constant R (say the characteristic impedance of a transmission line such as a coaxial cable). There is a simple voltage-divider relationship for the antenna response in both transmission and reception.

In Section 2 we assumed that the transmission-line conductors were flat plates perpendicular to the incident electric field. Here we may observe that more general transmission-line cross sections (e.g., two wires, tubes, coplanar sheets, etc.) can also be used. While the analysis is not as simple one can compute an effective plate separation $\vec{h}_{eq}$ related to the electric dipole moment per unit length in transmission. There is also the question of response characteristics for wavelengths of the order of h or less. Furthermore, one need not be limited to two conductors, but multiple ones, such as in Fig. 2.1 are also possible.

4. Two-Dimensional Rotation Symmetry

There are various cross-section shapes that such a transmission-line antenna can take. Noting the desire of symmetry for polarization decomposition one can look at various symmetrical configurations. In Section 2 axial symmetry planes have been considered. For completeness one can consider two-dimensional rotational symmetry C_N. This consists of structures which on rotation by $2\pi/N$ replicate themselves. For a discussion of the group structure see [15, 16].

Figure 4.1 shows a particularly simple form such symmetry might take. Simply space N wires, each of radius a, around a circle of radius h. These wires may be "fat", i.e., approximately perfectly conducting tubes (circular cylinders). As such there are also N axial symmetry planes giving C_{Na} symmetry ($2N$ group elements). Such a system of conductors has a characteristic impedance matrix which is bicirculant, giving a simple analytic form to the eigenvectors due to reciprocity [16].

In Fig. 4.1A (C_{2a} symmetry) we have the simple example of a single-polarization antenna (polarization in the $\vec{1}_x$ direction) with, say, V on conductor 1 and $-V$ on conductor 2. Here we consider only differential modes for which the sums of the voltages, currents and charges per unit length are all zero. One might have a reference conductor (zero volts) on the z axis preserving the rotation symmetry.

A simple dual-polarization configuration is given in Fig. 4.1C (C_{4a} symmetry). In this case one might have V on conductor 2, $-V$ on conductor 4, and 0 on conductors 1 and 3, giving polarization in the $\vec{1}_x$ direction. Rotating the potentials by $\pi/2$ with V on conductor 3, $-V$ on conductor 1, and zero on the others, gives a polarization in the $\vec{1}_y$ direction. This is not the only way to achieve two orthogonal polarizations. For example, one can have V on conductors 2 and 3 with $-V$ on conductors 1 and 4, giving polarization in the $[\vec{1}_x + \vec{1}_y]/\sqrt{2}$ direction. The orthogonal polarization has V on conductors 1 and 2 with $-V$ on conductors 3 and 4 giving polarization in the $[\vec{1}_x - \vec{1}_y]/\sqrt{2}$ direction.

For $N \geq 4$ and an integer power of 2 we have various similar ways to construct two orthogonal polarizations by a rotation of the potentials by $\pi/2$. There is another interesting way of constructing two *independent* polarizations when $N \geq 3$ and an integer multiple of 3. In this case we can construct three polarizations by rotation of the potentials through successive angles of $2\pi/3$. by taking a linear combination of two of these polarizations we can construct a polarization which is orthogonal to the remaining (third) polarization.

In Fig. 4.1B (C_{3a} symmetry) we can place charge per unit length $q/2$ on conductors 1 and 2 with $-q$ on conductor 3, giving polarization in the $\vec{1}_x$ direction. By placing q on conductor 2, $-q$ on conductor 1, and zero on conductor 3 we have polarization in the $\vec{1}_y$ direction.

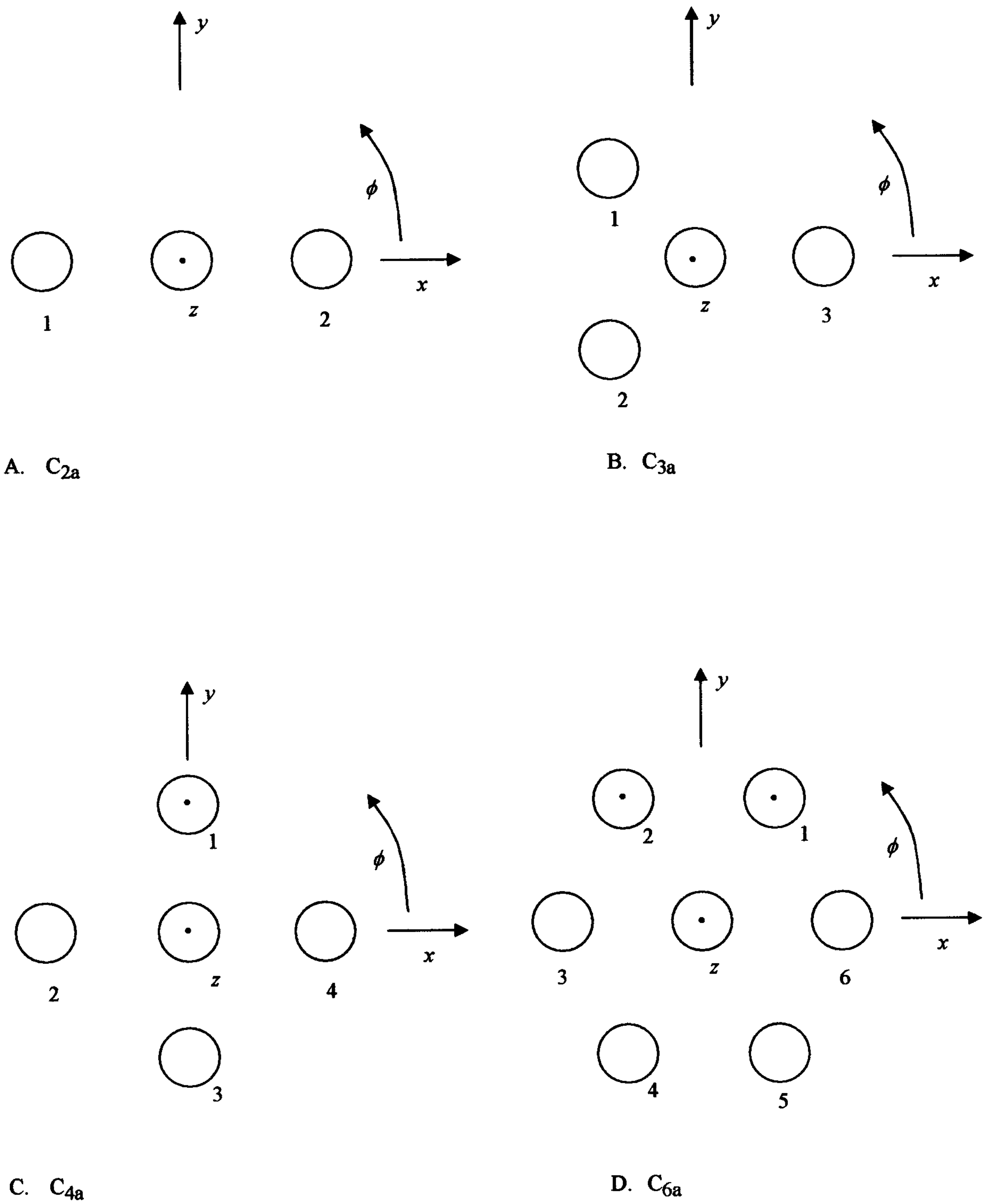


Fig. 4.1 Examples of Two-Dimentional Rotation Symmetry with Axial Symmetry Planes (C_{Na}) Using Perfectly Conducting Circular Cylinders.

A more sophisticated version appears in Fig. 4.1D. In this case begin with q on each of conductors 4 and 5, -q on each of conductors 1 and 2, and zero on each of conductors 3 and 6, giving polarization in the $\vec{1}_y$ direction. To construct an orthogonal polarization rotate twice by $2\pi/3$, adding the charges per unit length. This has q on conductors 2 and 4, 2q on 3, -q on 1 and 5, and -2 on 6, giving a polarization in the $\vec{1}_x$ direction. This geometry has other interesting aspects. The case above for $\vec{1}_y$ polarization also gives a highly uniform field on the z axis (the two-dimensional analog of a Helmholtz coil) [1, 2, 9].

5. Self-Complementary and Reciprocation Symmetry

Concepts of symmetry in two dimensions can be extended yet further. As discussed in [13, 16] our structures can be self complementary. In this context, self complementarity is based on the interchange of electric and magnetic potentials in the complex potential

$$\begin{aligned} w(\zeta) &= u(\zeta) + jv(\zeta) \equiv \text{complex potential} \\ \zeta &= x + jy \equiv \text{complex coordinate} \end{aligned} \tag{5.1}$$

This requires an interchange of electric and magnetic boundaries, which in the present context leaves the geometry unchanged except for a rotation by

$$\phi_c = \frac{\phi_1}{2} = \frac{\pi}{N} \tag{5.2}$$

This fits into C_N symmetry defined by

$$\begin{aligned} C_N &= \left\{ (C_N)_\ell \middle| \ell = 1, 2, \dots, N \right\} \\ (C_N)_\ell &= (C_N)_1^\ell = \text{rotation by } \phi_\ell \\ \phi_\ell &= \ell \phi_\ell \quad , \quad \phi_\ell = \frac{2\pi}{N} \end{aligned} \tag{5.3}$$

This in turn defines the self-complementary rotation group C_{Nc} with $2N$ elements.

Another symmetry of interest is reciprocation symmetry with respect to a circle on $\Psi = b$ in complex cylindrical coordinates (Ψ, ϕ) with

$$\begin{aligned} \zeta &= \Psi e^{j\phi} \\ x &= \Psi \cos(\phi) \quad , \quad y = \Psi \sin(\phi) \end{aligned} \tag{5.4}$$

First introduce the analytic transform

$$\begin{aligned} \zeta_1 &= x_1 + jy_1 = \Psi_1 e^{j\phi_1} = \frac{b^2}{\zeta} \\ \Psi_1 &= \frac{b^2}{\zeta} \quad , \quad \phi_1 = -\phi \end{aligned} \tag{5.5}$$

which is conformal. Then reciprocation (non-analytic) is given by

$$\zeta_2 = x_2 + jy_2 = \Psi_2 e^{j\phi_2} = \frac{b^2}{\zeta^*}$$
$$\Psi_2 = \frac{b^2}{\Psi} \quad , \quad \phi_2 = \phi \tag{5.6}$$

One consequence of this symmetry is that the characteristic impedance of such a transmission line has an equal contribution from $\Psi > b$ and $\Psi < b$. So if we know the impedance from some $\Delta u / \Delta v$ for the inside domain, one merely multiplies by 1/2 to account for the external domain.

Examples of this compound symmetry are given in Fig. 5.1. If the wave impedance is

$$Z_0 = Y_0^{-1} = \left[\frac{\mu_0}{\varepsilon_0}\right]^{1/2} \text{ (e.g., for free space)} \qquad 5.7)$$

one can calculate the impedance parameters for various self complementary reciprocal configurations. The configuration in Fig. 5.1A with only two conductors has the well-known differential-mode characteristic impedance [3]

$$Z_c = f_g Z_0 \quad , \quad f_g = \frac{1}{2} \tag{5.8}$$

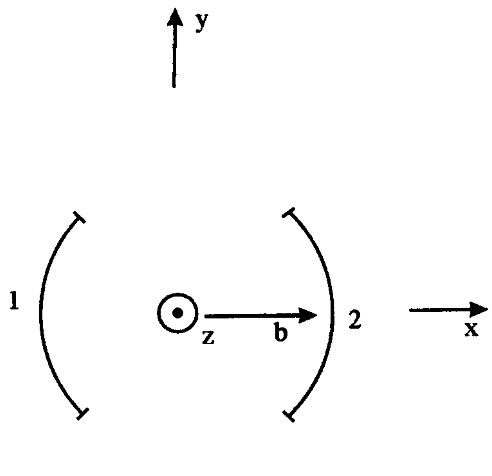
(it is also found that this produces a highly uniform field near the x axis.)

Considering only differential modes (sum of all charges and all currents on the N conductors equal to zero) one can construct a characteristic admittance matrix (bicirculant)

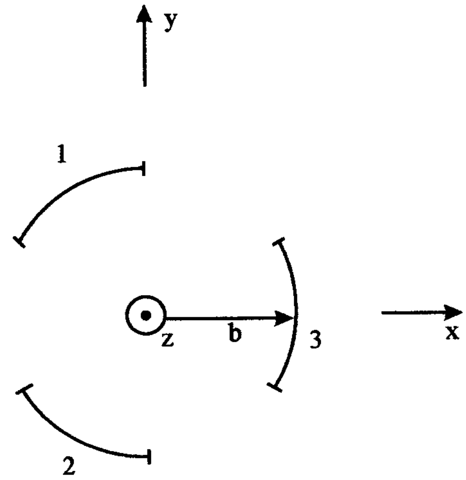
$$\left(Y_{c_{n,m}}\right) = \begin{pmatrix} Y_1 & Y_2 & Y_3 \cdots Y_N \\ Y_2 & Y_1 & Y_2 \cdots Y_N \\ Y_3 & Y_2 & Y_1 \cdots Y_N \\ \vdots & \vdots & \vdots \cdots Y_N \\ Y_N & \cdots\cdots & \cdots Y_N \end{pmatrix}$$

$$\sum_{p=1}^{N} Y_\ell = 0$$

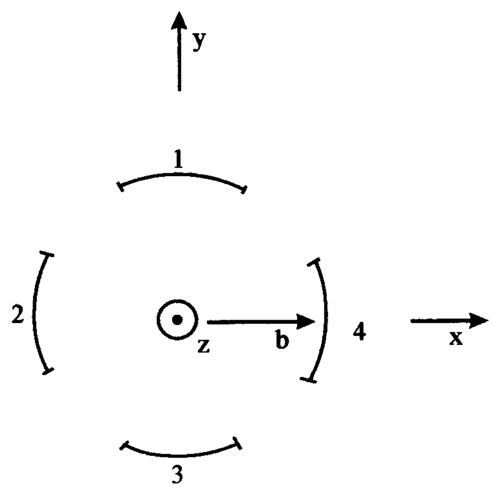
$$Y_\ell = \frac{4Y_0}{N} \frac{\sin\left(\frac{\phi_1}{2}\right)}{\cos([\ell-1]\phi_1) - \cos\left(\frac{\phi_1}{2}\right)} \tag{5.9}$$



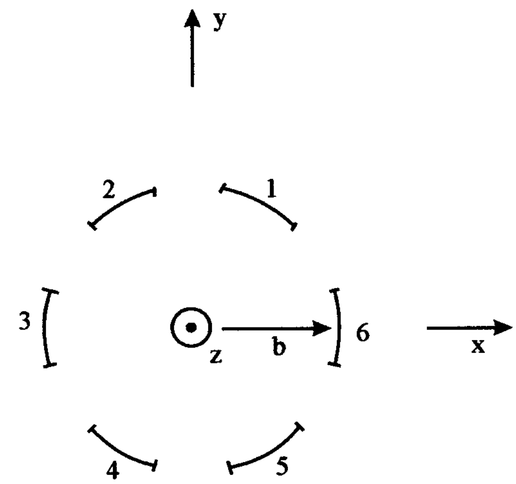
A. $C_{2a} \otimes$ reciprocation



B. $C_{3a} \otimes$ reciprocation



C. $C_{4a} \otimes$ reciprocation



D. $C_{6a} \otimes$ reciprocation

Fig. 5.1 Examples of Self-Complementary Reciprocation Symmetry with Axial Symmetry Planes

From this the characteristic impedance of various connections of conductors to give a single differential mode may be computed.

Considering $N \geq 3$ there are various possible conductor connections and resulting characteristic impedances. One can consult [14] for a table of these for N from 2 through 6. We can list some of the interesting cases. For N = 3 we have

$$\left.\begin{array}{ll}\text{conductor,} & 1, + \\ \text{conductor} & 2, - \\ \text{conductor} & 3, 0\end{array}\right\} f_g = \frac{1}{\sqrt{3}} \simeq 0.5774$$

$$\left.\begin{array}{ll}\text{conductor,} & 1 \text{ and } 2, + \\ \text{conductor} & 3 \quad\quad , -\end{array}\right\} f_g = \frac{\sqrt{3}}{4} \simeq 0.4330 \tag{5.10}$$

For N = 4 we have

$$\left.\begin{array}{ll}\text{conductor,} & 1, + \\ \text{conductor} & 3, - \\ \text{conductor} & 2 \text{ and } 4, 0\end{array}\right\} f_g = \frac{1}{\sqrt{2}} \simeq 0.7071$$

$$\left.\begin{array}{ll}\text{conductor,} & 2 \text{ and } 2, + \\ \text{conductor} & 3 \text{ and } 4, -\end{array}\right\} f_g = \frac{1}{2\sqrt{2}} \simeq 0.3536 \tag{5.11}$$

For N = 6 we have

$$\left.\begin{array}{ll}\text{conductor,} & 1 \text{ and } 2, + \\ \text{conductor} & 4 \text{ and } 5, - \\ \text{conductor} & 3 \text{ and } 6, 0\end{array}\right\} f_g = \frac{1}{2} = 0.5 \tag{5.12}$$

6. Concluding Remarks

This paper now generalizes some previous results concerning TEM-transmission-line antennas. Using the reciprocity relationship a particularly simple form of the far field is found for the response to a current source driving two arbitrarily shaped parallel plates (perfectly conducting) on a plane (mathematical, infinite) parallel to and between the two conducting plates. Specializing to TEM transmission lines (two-dimensional structures) the characteristic impedances are treated for certain symmetrical cases. Together with the length of the line and the impedance(s) at the antenna port(s) this characterizes the antenna(s) on boresight in both transmission and reception.

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