Sensor and Simulation Notes

Note 512

April 2006

Addition of a Lens Before the Second Focus of a Prolate-Spheroidal IRA

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Abstract

This paper gives basic physical considerations for the design of a dielectric lens for concentrating the fields on a target, thereby increasing the fields and lowering the spot size. This is used in conjunction with an incoming spherical wave centered on the target, such as at the second focus of a prolate-spheroidal reflector.

This work was sponsored in part by the Air Force Office of Scientific Research.

1. Introduction

Recent papers [1-4] have considered the design of a prolate-spheroidal reflector for concentrating a pulse from one of the foci onto a target at the second focus. Such a pulse is similar to that generated by a reflector impulse radiating antenna (IRA) [9], except that now the pulse is focused in the near field, instead of at infinity. The idea is to get a very fast, very intense electromagnetic pulse to illuminate the target (e.g., a tumor). By very fast we mean fast enough to get the spot size at the focus down to the target size, or as close to this as technology allows. At the same time a small spot size also implies large fields.

One problem with placing fields on the target concerns the dielectric properties of the target medium and its surroundings. If the wave incident on the target is in air, but the target medium has a large relative permittivity (say about 81 for water), then there will be a significant reflection of the pulse, leading to a smaller field in the target medium. This paper discusses the addition of a lens to better match the wave to the target, and to reduce spot size, thereby increasing the field on the target.

Figure 1.1 illustrates the concept. We have the prolate sphere with major radius *a* and minor radius *b*. The focal points are at

$$\begin{aligned} \pm \vec{r}_0 &= \pm z_0 \vec{1}_z \\ z_0 &= \left[a^2 - b^2 \right]^{1/2} \end{aligned} \tag{1.1}$$

The high-voltage pulse is launched at $-\vec{r}_0$, guided by conical conductors to the prolate sphere, intersecting it at

$$\left(\Psi, z\right) = \left(\Psi_p, z_p\right) \tag{1.2}$$

Where the prolate sphere is truncated to surface S'_p. As in [4], the TEM wave reflected from S'_p is evaluated on the aperture S_a at $z = z_p$. Aperture integrals are then performed to find the fields at the target at $+\vec{r}_0$. For a fast rising pulse at the launch, there is an important delta-function like pulse at the target [4 (5.1)].

Let us also consider a lens of radius r_1 centered on $\vec{r}_0$. It is this lens we wish to optimize by appropriate choice of its permittivity profile $\varepsilon_\ell(\vec{r}_\ell)$ to maximize this delta-function-like pulse at $\vec{r}_0$.

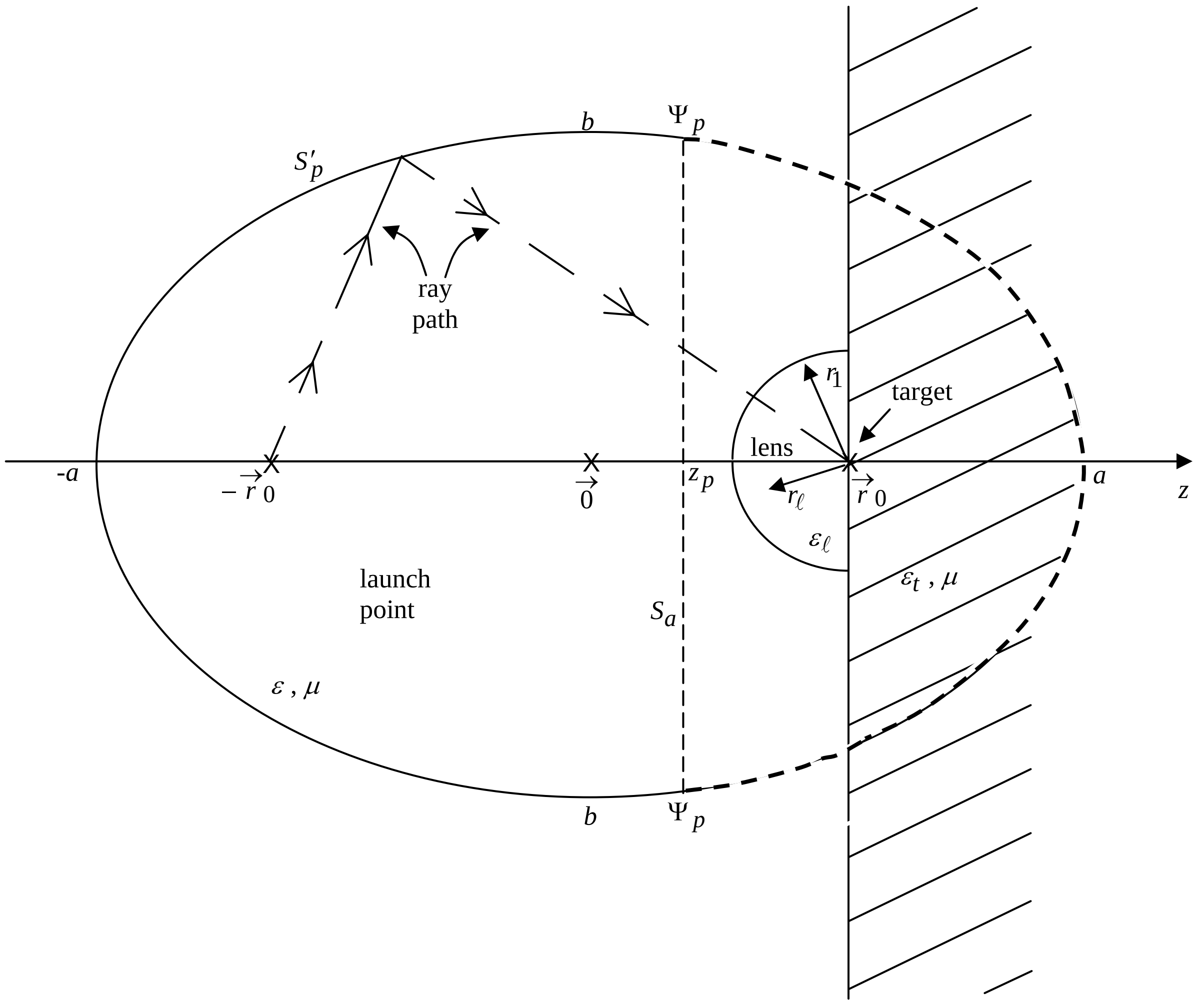

Fig. 1.1 Addition of Lens With Prolate-Spheroidal Reflector

2. Transmission Into Target Medium Without Lens

Approximating the fields as a plane wave the transmission from one medium to another is

$$T \equiv \frac{E_{transmitted}}{E_{incident}} = \frac{2Z_t}{Z_t + Z_0} = 2\left[1 + \frac{Z_0}{Z_t}\right]^{-1}$$

$$Z_0 = \left[\frac{\mu}{\varepsilon}\right]^{1/2} = \text{ wave impedance of reference medium (e.g., free space)}$$

$$Z_t = \left[\frac{\mu}{\varepsilon_t}\right]^{1/2} = \varepsilon_{rt}^{-1/2} Z_0 = \text{ wave impedance of target medium} \tag{2.1}$$

$$\varepsilon_{rt} \equiv \frac{\varepsilon_t}{\varepsilon} \equiv \text{ relative permittivity of target medium}$$

$$T = 2\left[1 + \varepsilon_{rt}^{1/2}\right]^{-1}$$

As an example we may have

$$\varepsilon_{rt} = 81 \text{ (water)}$$

$$T = 0.2 \tag{2.2}$$

This is a significant and undesirable reduction of the field.

3. Lens With $\varepsilon_\ell = \varepsilon_t$

Suppose now that we choose

$$\varepsilon_\ell = \varepsilon_t \tag{3.1}$$

So that the lens has the same relative permittivity as the target medium. Then the transmission into the lens at its boundary $r_\ell = r_1$ is just given by the same previous equations, and nothing is gained.

However, at the focal point the fields are not a plane wave. In the spirit of a gedankenexperiment let us imagine that we choose

$$r_1 \equiv r_p = \left[\Psi_p^2 + \left[z_0 - z_p \right]^2 \right]^{1/2} \tag{3.2}$$

So that the lens occupies the full aperture area S_a. Then the fields from the refleetor are transmitted with coefficient

$$T_0 = \left[1 + \varepsilon_{rt}^{1/2} \right]^{-1} \tag{3.3}$$

which is 0.2 in the example.

Now the fields on S_a are T_0 times as much as before. Turning to our formula for the delta-function part of the field at the target [4 (5.1)] we find that this is inversely proportional to

$$\begin{aligned} \mathrm{v} &= \left[\mu\, \varepsilon_\ell \right]^{-1/2} = c\, \varepsilon_{r\ell}^{-1/2} \\ &= \text{speed of wave in lens medium} \\ c &= \left[\mu\, \varepsilon \right]^{-1/2} = \text{wave speed in reference medium (speed of light in vacuo)} \\ \varepsilon_{r\ell} &\equiv \text{relative permittivity of lens medium} \end{aligned} \tag{3.4}$$

This gives an enhancement factor

$$F_0 = \varepsilon_{r\ell}^{1/2} = \varepsilon_{rt}^{1/2} \tag{3.5}$$

for a net factor for the impulsive part of the field at $\vec{r}_0$ of

$$F_0 T_0 \ = \ 2\,\varepsilon_{r\ell}^{1/2}\left[1+\varepsilon_{r\ell}^{1/2}\right]^{-1} = \ 2\left[1+\varepsilon_{r\ell}^{-1/2}\right] \tag{3.6}$$

For our example we have

$$\begin{aligned} &\varepsilon_{r\ell} \ = 81 \quad \text{(water)} \quad , \quad F_0 \ = 9 \\ &F_0 T_0 \ = \ 1.8 \end{aligned} \tag{3.7}$$

which is a significant increase in the field.

Whence does this increase come? One way to look at this is spot size around the target. As discussed in [(Section 6)], the spot size containing the narrow beam is characterized for positions near $\vec{r}_0$ by

$$\begin{aligned} &\Delta\Psi \ \simeq \ \frac{a}{b}\,\mathrm{v}t_\delta \ = \ \varepsilon_{r\ell}^{-1/2}\,\frac{a}{b}\,c\,t_\delta \\ &t_\delta \ \text{ width of } \delta\text{-like pulse} \end{aligned} \tag{3.8}$$

So we can define a spot-size factor as

$$S_0 \ = \ \varepsilon_{r\ell}^{-1/2} \tag{3.9}$$

For our example

$$\begin{aligned} &\varepsilon_{r\ell} \ = 81 \quad \text{(water)} \\ &S_0 \ = \ 1/9 \end{aligned} \tag{3.10}$$

The power per unit area going as one over the area, the fields go as one over the radius, giving an increase of S_0^{-1}, consistent with (3.7).

4. Lens With Intermediate ε_ℓ

Next, let our lens of radius r_1 (as in the previous section) be characterized by a constant intermediate value of

$$1 < \varepsilon_{r\ell} < \varepsilon_{rt} \tag{4.1}$$

Then we have two plane-wave transmission coefficients

$$\begin{aligned} T_1 &= 2\left[1+\varepsilon_{r\ell}^{1/2}\right]^{-1} \\ T_2 &= 2\left[1+\left[\frac{\varepsilon_{rt}}{\varepsilon_{r\ell}}\right]^{1/2}\right]^{-1} \end{aligned} \tag{4.2}$$

With a net transmission coefficient

$$T = T_1 \; T_2 = \frac{4}{1+\varepsilon_{r\ell}^{1/2}+\left[\dfrac{\varepsilon_{rt}}{\varepsilon_{r\ell}}\right]^{1/2} + \varepsilon_{r\ell}^{1/2}} \tag{4.3}$$

This is maximized at

$$\begin{aligned} \varepsilon_{r\ell} &= \varepsilon_{rt}^{1/2} \\ T &= \left[\frac{2}{1+\varepsilon_{rt}^{1/4}}\right]^2 \end{aligned} \tag{4.4}$$

For our example this is

$$\begin{aligned} &\varepsilon_{rt} = 81 \ \text{(water)} \\ &\varepsilon_{r\ell} = 9 \\ &\mathrm{T} = 0.25 \end{aligned} \tag{4.5}$$

which is some improvement over (2.2).

However, the spot size, being proportional to $\varepsilon_{r\ell}^{-1/2}$ gives an enhancement factor of

$$F_1 = \varepsilon_{r\ell}^{1/2} = \varepsilon_{rt}^{1/4} \tag{4.6}$$

For our example this is

$$\begin{aligned} &\varepsilon_{rt} = 81 \quad (\text{water}) \ , \quad F_1 = 3 \\ &F_1 T = 0.75 \end{aligned} \tag{4.7}$$

which is not as good as in (3.7). Lowering the ε in front of the target gives this reduction.

So, let us keep ε_t in front of the target and make our lens in two stages. For the present discussion let the two boundaries be near r_1, separated by a thickness larger than $\mathrm{v}t_\delta$ in the first ε_ℓ medium. Then the spot size reduction is $\varepsilon_{rt}^{-1/2}$, giving

$$F_0 T = \varepsilon_{rt}^{1/2} \left[\frac{2}{1+\varepsilon_{rt}^{1/4}} \right]^2 \tag{4.8}$$

For our example, this is

$$\begin{aligned} &\varepsilon_{rt} = 81 \quad (\text{water}) \\ &F_0 T = 2.25 \end{aligned} \tag{4.9}$$

an increase over (3.7).

The essential factor is the spot-size reduction suggesting that it is important to surround the target with at least a portion of the lens with permittivity ε_t. How far out this should extend is an important question.

5. Lens With Graded ε_ℓ

Extending the results of the previous section we can envision a set of lens media with progressively increasing ε_ℓ as one goes from $r_\ell = r_1$ back toward $r_\ell = 0$ (the target). Taking the limit we can have a continuous variation of ε_ℓ with

$$
\begin{aligned}
&r_1 \geq r_\ell \geq r_2 \\
&\varepsilon_{r\ell}(r_1) = 1 \\
&\varepsilon_{r\ell}(r_2) = \varepsilon_{rt}
\end{aligned}
\tag{5.1}
$$

The wave propagating through this takes the same form as that in a transmission-line transformer. The high-frequency/early-time transfer function is

$$
T = \left[\frac{Z_0}{Z_t}\right]^{1/2} = \varepsilon_{rt}^{-1/4} \tag{5.2}
$$

For our example this is

$$
\begin{aligned}
&\varepsilon_{rt} = 81 \ \text{(water)} \\
&T = 1/3 \simeq 0.33
\end{aligned}
\tag{5.3}
$$

which is a further improvement over (4.5).

As discussed in [8] an exponential variation of the characteristic impedance of the transmission line (for constant wave speed) along the line is somewhat optimal. It minimizes the droop of the step response after the initial rise characterized by (5.2). This says that $r_1 - r_2$ should be large enough that this droop is insignificant. The profile of $\varepsilon_{r\ell}$ can be modified to take account of the variable wave speed $\mathrm{v}(r_\ell)$ in terms of a radial coordinate based on local transit time in the lens.

Keeping ε_t in the lens after the graded portion, then the spot size is still reduced proportional to $\varepsilon_{rt}^{1/2}$. The net transmission improvement is then

$$
F_0 T = \varepsilon_{rt}^{1/2} \varepsilon_{rt}^{-1/4} = \varepsilon_{rt}^{1/4} \tag{5.4}
$$

For our example

$$
\begin{aligned}
&\varepsilon_{rt} = 81 \ \text{(water)} \\
&F_0 T = 3
\end{aligned}
\tag{5.5}
$$

which is a yet further increase over (4.9).

6. Reduction of Lens Size

Thus far we have been considering a rather large lens characterized by r_1 extending into the aperture plane as in (3.2). This could be overly massive and unwieldy. How much smaller can we make r_1 and still obtain the improvements discussed in the previous sections?

Letting r_1 (lens radius) be less than $z_0 - z_p$, we have the case of a wave which is initially a spherical TEM wave passing through S_a. However, since this wave is not being guided toward $\vec{r}_0$ by appropriate conical conductors, this complicates the situation. Initially, the wavefront heading toward $\vec{r}_0$ inside a cone of half angle $\theta_{\ell 0}$ is TEM (Fig. 6.1), since the truncation of the wave at the reflector rim propagates a wave behind the wavefront. The wave incident on the lens for $\theta_\ell < \theta_{\ell 0}$ is initially TEM, implying that the previous results hold *initially*. However, for

$$\psi \equiv \theta_{\ell 0} - \theta_\ell > 0 \tag{6.1}$$

There is a short time, depending on ψ, for which the TEM results hold. Depending on how small is the width of t_δ (based on the source risetime (as in 3.8)) there is a portion of the lens for $\psi > 0$ for which the TEM incident fields do not strictly apply. For the previous results to approximately hold this region of non-TEM wave for a length of time t_δ needs to be a small fraction of the lens surface for $\theta_\ell < \theta_{\ell 0}$.

What we need then is *d* in Fig. 6.1 for which we require

$$\begin{aligned} & d - r_p + r_1 \le c\, t_\delta \\ & r_p = \left[\Psi_p^2 + \left[z_0 - z_p \right]^2 \right]^{1/2} \end{aligned} \tag{6.2}$$

For this we have

$$d = \left[\left[\Psi_p - r_1 \sin\left(\theta_{\ell 0} - \psi\right) \right]^2 + \left[z_0 - z_p - r_1 \cos\left(\theta_{\ell 0} - \psi\right) \right]^2 \right]^{1/2} \tag{6.3}$$

Expanding this for small ψ to second order we find (there also being a quicker way from Fig. 6.1)

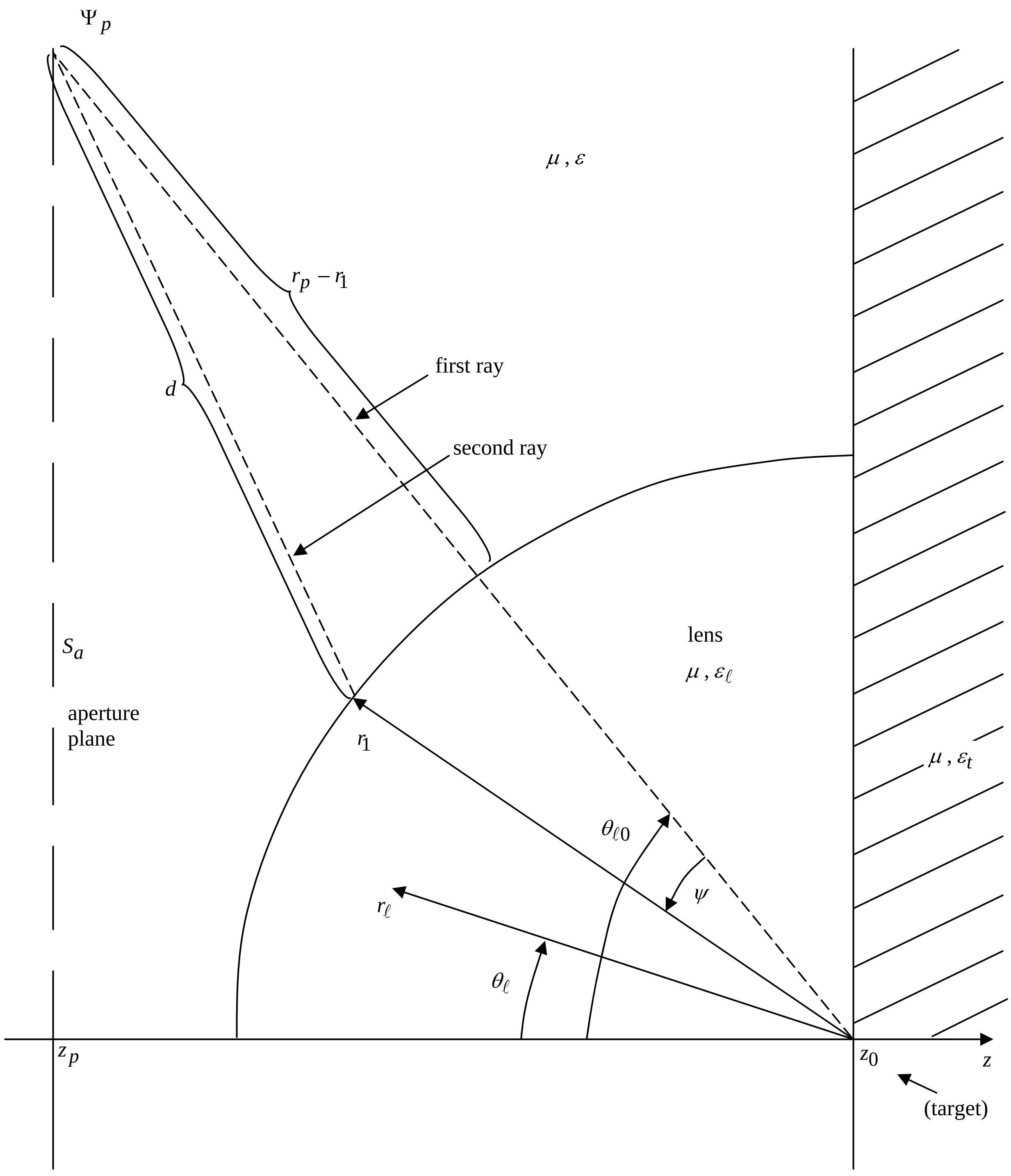

Fig. 6.1 Ray Paths for Time of Validity of TEM Wave

$$d - r_p + r_1 \simeq \frac{\psi^2}{2} \frac{r_p - r_1}{r_p r_1} < c\, t_\delta \tag{6.4}$$

We need

$$\frac{\psi}{\theta_{\ell 0}} << 1 \tag{6.5}$$

for the TEM result to approximately hold. Thus we need

$$\psi \simeq \left[2 \frac{r_p - r_1}{r_p r_1} c\, t_\delta \right]^{1/2} << \theta_{\ell 0} = \arctan\left(\frac{\Psi_p}{z_0 - z_p} \right) \tag{6.6}$$

The smallness of $\psi / \theta_{\ell 0}$ gives some measure of how well the TEM approximation works.

How small might the lens be? For this we have

$$\begin{aligned} & r_1 << r_p \\ & \psi \simeq \left[\frac{2}{r_1} \left[1 - \frac{r_1}{r_p} \right] c\, t_\delta \right]^{1/2} \simeq \left[\frac{2\, c\, t_\delta}{r_1} \right]^{1/2} \end{aligned} \tag{6.7}$$

So, we need $r_1 >> c\, t_\delta$ for good performance. Since we may wish to have a small lens then we may wish a progression of dimensions as

$$r_p >> r_1 >> c\, t_\delta \tag{6.8}$$

in the overall design.

7. Embeded Target

Suppose, now, that the target is embeded in a medium of the permittivity ε_t at some depth d_t as indicated in Fig. 7.1. In this case, one can apply the previous results by adding a segment of a sphere of radius r_t and permittivity ε_t centered on the *z* axis. Its largest thickness is $r_t - d_t$. Its angular extent is $\geq \theta_{\ell 0}$ (Fig. 6.1) so that the incoming spherical wave propagates through a region with a permittivity independent of θ_ℓ. The lens then continues out to a radius r_1 centered on the target at $\vec{r}_0$ with permittivity profile as discussed in previous sections.

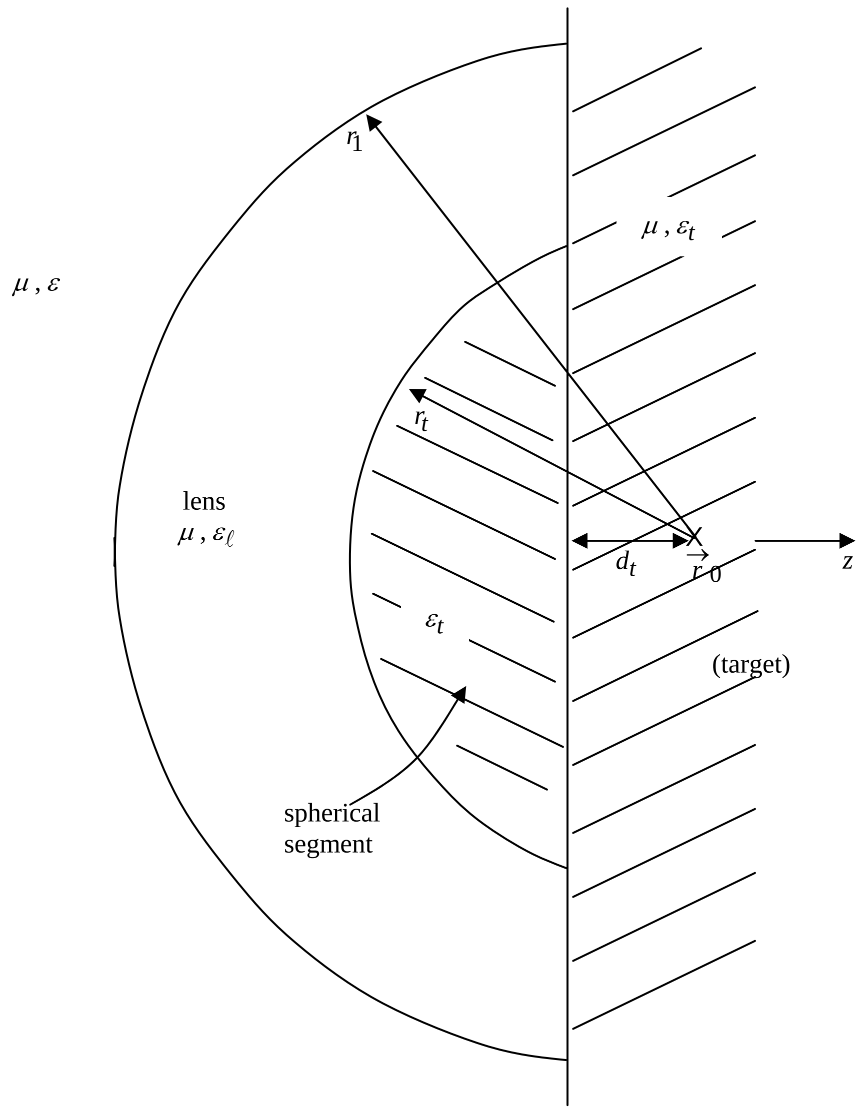

Fig. 7.1 Embeded Target

8. Concluding Remarks

As we can see, there are many considerations for the design of a lens focused on a target, in conjunction with the focusing prolate-spheroidal reflector. How large should the lens be, and what permittivity profile should be used? This is strongly influenced by the risetime of the pulser since this determines the width t_δ of the impulsive part of the waveform incident on the target.

In synthesizing an appropriate lens there is the problem of selecting appropriate materials with desirable permittivities to approximate the desired $\varepsilon_{r\ell}(r_\ell)$. Not only must these materials be able to withstand the large electric fields, but also they must maintain a nearly constant real permittivity over the frequency range of interest for the impulsive part of the waveform. Some measurements of the frequency characteristics have been made [5-7]. More such measurements need to be made, including for artificial dielectrics that one might synthesize.

One might also contemplate a more exact analysis of the wave propagation to the target. This might take the form of a direct numerical simulation of the Maxwell equations. Alternately one might expand the incoming wave in spherical vector wave functions, propagate each of these through the spherically stratified media, and sum the terms at the target.

As one can see, the present paper only begins the design process for such a lens.

References

1. C. E. Baum, “Producing Large Transient Electromagnetic Fields in a Small Region: An Electromagnetic Implosion”, Sensor and Simulation Note 501, August 2005.

2. C. E. Baum, “Combining Multiple Prolate-Spheroidal Reflectors as a Timed Array With a Common Near-Field Focus”, Sensor and Simulation Note 504, November 2005.

3. C. E. Baum, “Prolate Spheroidal Scatterer for Spherical TEM Waves “, Sensor and Simulation Note 508, January 2006.

4. C. E. Baum, “Focal Waveform of a Prolate-Spheroidal IRA”, Sensor and Simulation Note 509, February 2006.

5. C. Courtney, W. Motil, T. Bowen, and S. Blocher, “Measurement Methods and the Characterization of the Electromagnetic Properties of Materials”, Measurement Note 48, December 1996.

6. E. G. Farr and C. A> Frost, “Time Domain Measurement of the Dielectric Properties of Water in a Coaxial Test Fixture”, Measurement Note 49, December 1996.

7. E. G. Farr and C. A. Frost, “Impulse Propagation Measurements of the Dielectric Properties of Water, Dry Sand, Moist Sand, and Concrete”, Measurement Note 52, November 1997.

8. C. E. Baum and J. M. Lehr, “Nonuniform-Transmission-Line Transformers for Fast High-Voltage Transients”, Circuit and Electromagnetic System Design Note 44, February 2000; IEEE Trans. Plasma Science, 2002, pp. 1712-1721.

9. C. E. Baum, E. G. Farr, and D. V. Giri, “Review of Impulse-Radiating Antennas”, ch. 16, pp. 403-439, in W. R. Stone (ed.), *Review of Radio Science 1996-1999*, Oxford U. Press, 1999.